The principles of electric wave telegraphy and telephony


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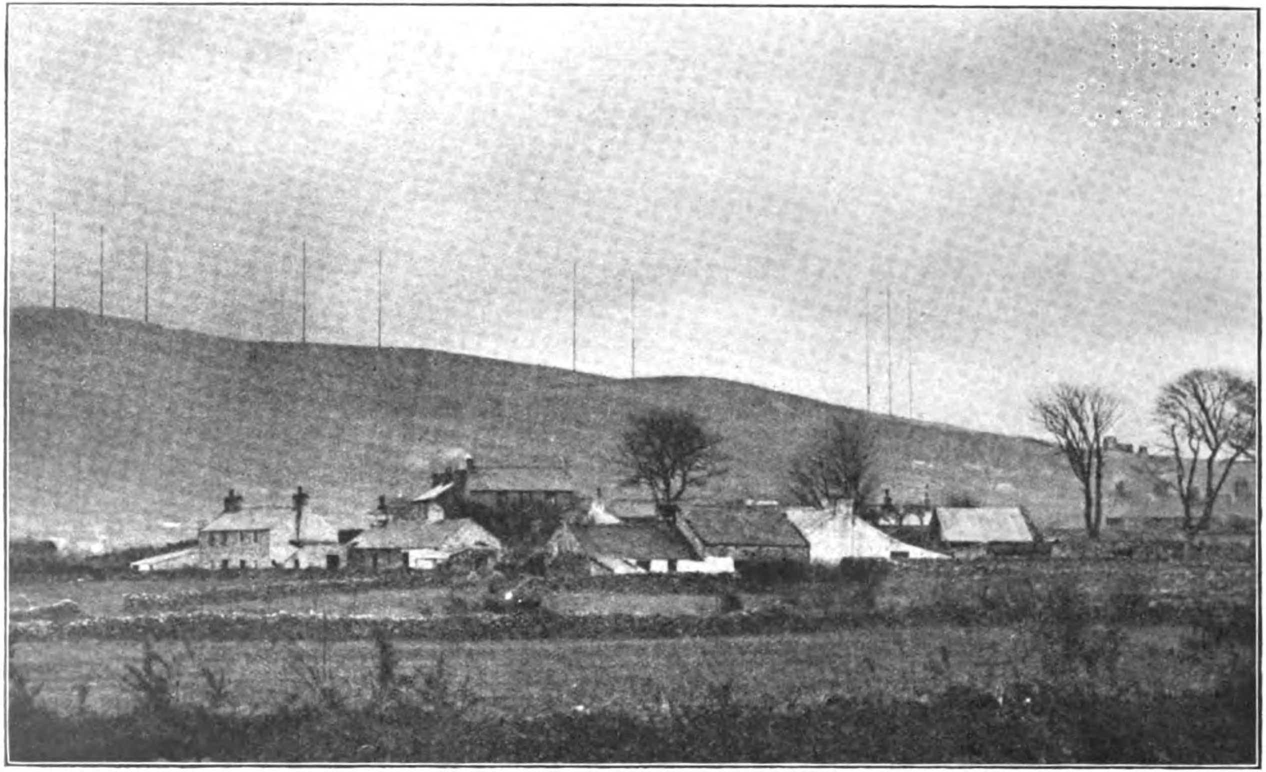

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The steel masts are shown against the sky-line, and the station itself is on the extreme right of the picture between the two trees on the crest of the hill.

[*See p.* 579.

THE PRINCIPLES OF
ELECTRIC WAVE TELEGRAPHY AND TELEPHONY

BY
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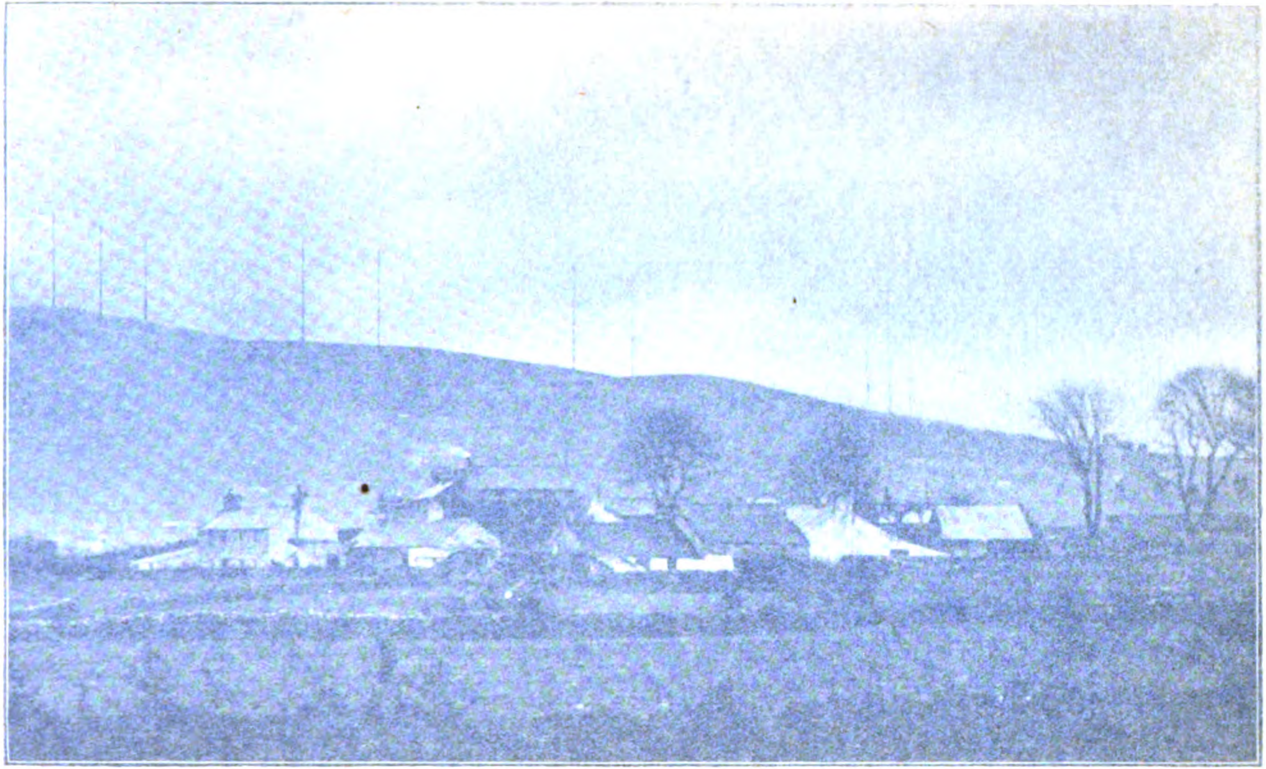

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Page 30

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PREFACE TO THE FOURTH EDITION

In the eight years that have elapsed since the publication of the second edition of this book there have been large additions both to theoretical knowledge and to the practical arts of Radiotelegraphy and Radiotelephony. Hence, in preparing for the press a third edition, the Author found it necessary to make very considerable alterations and additions.

The third edition was therefore by no means merely a reproduction of the second, but the book was in great part, by kind permission of the publishers, Messrs. Longmans, Green, & Co., re-written and re-arranged.

A new chapter was added on the Transmission of Radiotelegraphic Waves over the earth, and to make room for the new matter without unduly enlarging the book, much of the merely historical portions and of the descriptions of apparatus which has become antiquated, have been removed. In this fourth edition it has been necessary to economize space and paper as much as possible by certain additional deletions of antiquated matter. Nevertheless, additions have been made to bring the book up to date, and yet by the use of rather smaller type the total bulk of the volume reduced.

The aim of the Author has been to deal with principles rather than give the fullest possible account of actual apparatus. The practical Radiotelegraphist has now at his disposal a small library of books upon the subject, which in many cases are devoted to particular types of apparatus. Nevertheless it is hoped that the present treatise will in its revised form serve to give a comprehensive view of the subject particularly on its scientific side, and that part of it which is concerned with quantitative measurements and the underlying theory. The immense attention which has been given to this attractive subject is a consequence of the interesting physical and important practical aspects of it, and no one at present can hope to make additions to it who has not a very firm grasp of the electrical principles and facts which lie at its root, as well as broad acquaintance with what has been invented or discovered.

The Author desires to place on record his thanks to the following firms, publishers, proprietors of journals, societies, authors, and editors who have kindly permitted the use or reproduction of illustrations and diagrams belonging to them :—

To Messrs. Marconi's Wireless Telegraph Company, Limited, and The Wireless Press, Limited, for the many illustrations connected with the Marconi apparatus and system. To the Royal Society and Professor Karl Pearson, F.R.S., and Dr. Alice Lee, for permission to use the diagrams in Plates II., III., IV., and V., and to Professor A. E. H. Love, F.R.S., for

the diagrams in Plate V. To Admiral Sir Henry Jackson, F.R.S., R.N., for permission to use the diagrams and copious extracts from his paper in the *Proceedings of the Royal Society*, abstracted in Chapter IX. To the Physical Society and to the Royal Institution, and to Senatore Marconi and to Mr. W. Duddell, for permission to use diagrams illustrating lectures by them. To the Royal Society of Arts for the same favour in connection with illustrations of Cantor Lectures given there by the Author.

Also to the Editor, Dr. A. N. Goldsmith, and to the Institute of Radio-Engineers, New York, for diagrams which have appeared in the *Proceedings* of that Institute. To the Proprietors of *The Electrician*, *The Electrical Review*, *The Philosophical Magazine*, *Electrical Engineering*, *The Electrical World of New York*, and *The Physical Review*, for the use of diagrams which have illustrated articles in these journals, as well as to the Authors themselves.

The Institution of Electrical Engineers has also kindly permitted use to be made of diagrams which have appeared in papers read before it.

Many private firms, such as *The Cambridge Scientific Instrument Company*, Messrs. Robert Paul & Company, Mr. Leslie Miller, Messrs. Isenthal, and Mr. K. Schall, have also given permission to make use of illustrations in their trade circulars. Other private workers, such as Dr. Hemsalech, Mr. A. Campbell, Professor E. Wilson, Professor G. W. Pierce, and others, have permitted free use to be made of diagrams in papers published by them. To all these the Author begs to return thanks for the kindness. In many cases where it has been desired the acknowledgment of the ownership has been made under the diagram in place.

Finally the Author desires to mention his obligations to various readers who have drawn his attention to misprints in previous editions, and to say that these errata have been corrected. It is hoped that they have been completely deleted, and that the book in its amended and extended form will continue to be of service to students of this fascinating subject.

J. A. F.

THE PENDER ELECTRICAL LABORATORY,
UNIVERSITY COLLEGE, LONDON,
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TABLE OF CONTENTS

PART I

ELECTRIC OSCILLATIONS

CHAPTER I

PART II

ELECTRIC WAVES

CHAPTER IV

CHAPTER V

CHAPTER VI

PART III

ELECTRIC WAVE OR RADIOTELEGRAPHY

CHAPTER VII

CHAPTER X

LIST OF PLATES

THE PRINCIPLES OF

ELECTRIC WAVE TELEGRAPHY AND TELEPHONY

PART I.—ELECTRICAL OSCILLATIONS

CHAPTER I

THE PRODUCTION OF HIGH FREQUENCY CURRENTS AND ELECTRIC OSCILLATIONS

1. High Frequency Electric Currents—Damped and Undamped Electric Oscillations.—In that form of telegraphy called Radio-telegraphy, Hertzian, or Electric Wave Telegraphy, or more commonly, Wireless Telegraphy, the principles and practice of which are the subject of the present treatise; we are closely concerned with the properties of alternating electric currents of very high frequency. Hence it is necessary briefly to refer to them.

An alternating electric current is defined to be one which periodically changes direction in its circuit. For a certain time it flows in one direction in some conductor with varying strength, and then reverses and flows for an equal time in the opposite direction. The time in fractions of a second which elapses between the commencement of the current in one direction and beginning again in the same direction is called *a complete period* or cycle, and will be denoted in this treatise by the letter T. The number of complete periods per second is called *the frequency* of the current, and is denoted by n. The quantity $2\pi n$ or $\frac{2\pi}{T}$ is of the nature of an angular velocity, and will be represented by the letter p. It is also the number of periods in 2π seconds.

We have furthermore to distinguish between the *instantaneous value* of the current, or its value at any instant, and the *maximum value*. The former will be denoted by a small letter such as i, whilst the maximum value of the same quantity during the period will be represented by a large letter I of the same type.

A *high frequency alternating electric current* may be defined to be an alternating current of which the frequency is reckoned in thousands. There is no absolute demarcation between high and low frequency. The terms are of course relative. If, however, the frequency is such that the number of periods per second is, say, 1000 or upwards, then it would generally be called a high frequency current, whereas if the frequency was such as to be reckoned in hundreds, or less than a hundred, it would in general be called a low frequency current.

An *electric oscillation* may be defined to be an alternating electric current of extremely high frequency reckoned, say, in hundreds of thousands or millions per second, but here again the distinction between so-called high frequency electric currents and electric oscillations is more a matter of terminology than any precise difference in frequency. We are, however, concerned with two classes of electric oscillations, the difference between which is important. In one case the oscilla-

tions or high frequency currents continue with undiminished amplitude or maximum value. They are then called *undamped* or *persistent oscillations*. If, on the other hand, the oscillations after beginning with a certain amplitude die away, then cease, and after a time begin again with the original amplitude, they are called *damped oscillations*, and each group is called *a train of oscillations*. If the decay of amplitude in each train is very rapid, it is called a *strongly damped* oscillation train, and if the rate of decay is small, it is said to be *feebly damped*.

We may graphically represent a high frequency electric current or undamped electric oscillation in the usual manner by a repeated sinoidal curve, since in nearly all the cases likely to occur in practice the variation of current from moment to moment during the complete period is a simple sine function of the time. Hence we may proceed as follows: Let a horizontal line AX (see Fig. 1) be

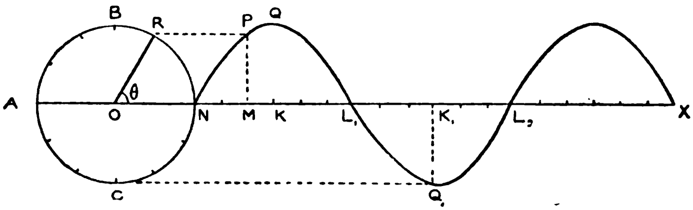


FIG. 1.—Delineation of a Simple Periodic or Sine Curve.

taken as a time axis, and equidistant points, N, L_1, L_2, X, etc., set off so that distances such as NL_2 or L_1X represent one complete period denoted by T.

Then with some point O in this line AX as centre describe a circle ABN. Let the radius OR of this circle be taken to represent, to some suitable scale, the maximum value of the current during the period. Imagine the radius OR to revolve in a counter-clockwise direction with a uniform angular velocity. Let a horizontal (dotted) line, RP, be drawn at every instant through the extremity of the radius OR. Let another point, M, be supposed to move uniformly along OX, and through it a vertical (dotted) ordinate, MP, be drawn. Let the point M move uniformly through a distance NL_2 in the time taken by the radius OR to revolve once with uniform angular velocity. Assume that OR starts from the position ON, and that the point M also starts from N. Then the locus of the point of intersection P of the vertical ordinate MP with the horizontal line RP will trace out a sinoidal curve, NQL_1Q_1. The length of the ordinate MP will always be equal to the radius of the circle OR multiplied by the sine of the *phase angle* RON $=\theta$. Let the radius of the circle be denoted by I taken to represent the maximum value of the current during its period. Let the radius OR revolve through the angle RON $=\theta$ in the time t with angular velocity p. Hence $\theta=pt$, and if we denote MP by i, then i is the instantaneous value of the current, and we have—

$$i=I \sin pt \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The value of the maximum current I is called the *amplitude* of the oscillation and the angle pt is called its *phase*. The above expression (1) is therefore the equation of the wavy curve, called a *sine curve*, and is also the analytical expression for a high frequency alternating current, or persistent, or undamped electric oscillation.

We can in the same manner describe a line representing graphically the nature of a damped electric oscillation if we employ a *logarithmic spiral* instead of a circle in a construction similar to that in Fig. 1.

A logarithmic spiral is the curve described by the extremity of a radius vector, the length of which varies so that the logarithm of its length bears a constant ratio to the phase angle the radius vector makes with some fixed straight line.

Thus in Fig. 2 the spiral curve is described by the extremity R of a radius OR ($=r$) which revolves uniformly round O, the length OR varying so that the ratio of log r to the angle RON ($=\theta$) is constant. Hence the polar equation of the left-handed logarithmic spiral as drawn is $r=a^{-\theta}$, where a is some constant.

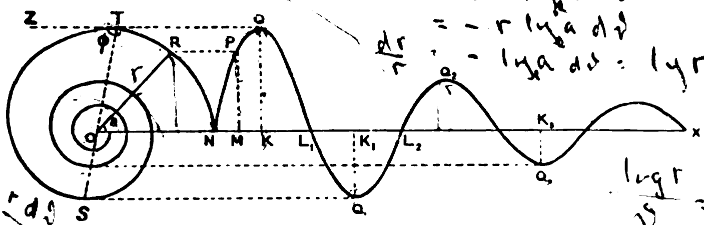
The exponent has a negative sign, because r diminishes as θ increases in the case of the spiral as delineated. Suppose, then, that we draw a time axis, OX, and assume a point, M, to move uniformly along it. Also let the radius vector OR move counter-clockwise with a uniform angular velocity. Let a perpendicular MP drawn through M move with it, and through R draw a horizontal line, RP. The locus of the point of intersection P, of the lines RP and MP as the points R and M move in their respective modes, will describe a decrescent wavy line. The

$dr = -a^{-\theta} \log a \, d\theta$

$= -r \log a \, d\theta$

$\frac{dr}{r} = -\log a \, d\theta : \log r = -\text{const}\,\theta$

$r = e^{-a\theta}$



$\frac{\log r}{\theta} = \log_e a = \text{const}$

FIG. 2.—Delineation of a Damped Periodic Curve.

or $\cot \phi = \log_e a = \text{const}$

equation of this line is found as follows :—Since the angle RON$=\theta$, the ordinate MP$=r\sin\theta$. Also $r=a^{-\theta}$. Hence if we write i for MP, we have—

$$i=a^{-\theta}\sin\theta.$$

Let p be the angular velocity of OR. Accordingly, if OR moves through the angle RON in a time t, we can write pt for θ. Also it is convenient to substitute $I\epsilon^{-at}$ or $I\epsilon^{-kpt}$ for $a^{-\theta}$ where ϵ is the base of the Napierian logarithms, and I, a, or k are certain constants.

We then obtain the equation of the wavy line NQQ_1Q_2 in the form—

$$i=I\epsilon^{-at}\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and this therefore is the mathematical expression for a damped electric oscillation.

If I_1 denotes the maximum value KQ of the first oscillation, I_2 that of the second K_1Q_1 in the opposite direction, and so on ; then it is easy to see that i has the value I_1 when $t=\frac{\phi}{\pi}\frac{T}{2}$, and the value I_2 when $t=\frac{\phi}{\pi}\frac{T}{2}+\frac{T}{2}$, where ϕ is the angle TON$=$OTZ (see Fig. 2). Hence it follows by substitution in (2) that,

$$\frac{I_1}{I_2}=\epsilon^{aT/2} \text{ or } \log_\epsilon\frac{I_1}{I_2}=\frac{aT}{2}=\frac{\delta}{2} .$$

The quantity aT or δ is called the *logarithmic decrement of the oscillations.* The quantity $\epsilon^{-\delta}$, where ϵ is the base of Nap. logs, is called *the damping.* Hence we have—

$$a=n\delta=n\log_\epsilon\frac{I_1}{I_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We can therefore write the equation of a damped electrical oscillation in one of two equivalent forms, thus—

$$i=I_1\frac{\epsilon^{a\frac{\phi}{\pi}\frac{T}{2}}}{\sin\phi}\epsilon^{-at}\sin pt=I_1\frac{\epsilon^{\delta\phi/2\pi}}{\sin\phi}\epsilon^{-at}\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The ratio of one maximum oscillation to the next in the same direction is that of ϵ^{δ} to unity.

2. The Practical Generation of Undamped and Damped Electric Oscillations.—A number of arrangements have been devised for generating high frequency currents and electric oscillations. Some of these appliances create damped and some undamped oscillations. The oldest of these methods is that in which the oscillatory discharge of a condenser is employed to create intermittent trains of damped oscillations. Other well-known agencies are available for the production of undamped oscillations, viz. the high frequency alternator, the direct current electric arc shunted by an inductive resistance in series with a capacity, and the thermionic valve generator. The high frequency alternator is more especially applicable for the production of high frequency alternating currents, and the arc methods for the generation of undamped electric oscillations. It is convenient to discuss these in the following order: (1) high frequency alternators; (2) the production of damped oscillation by condenser discharges; (3) the generation of undamped oscillations by the electric arc, and other means.

3. Production of High Frequency Currents by High Frequency Alternators.—Machines for the direct production of undamped high frequency alternating currents are called *high frequency alternators.* Previously to the year 1900, no one had attempted to build an alternator yielding a current having a frequency of more than about 20,000, and even for some years afterwards, although efforts had been made to obtain higher frequencies, only machines of extremely small power had been actually built. Nevertheless inventors persevered, recognizing that high frequency alternators would be of great utility in radiotelegraphy and radiotelephony. It was evident at an early stage that, for this purpose, machines giving a frequency of less than 40,000 or 50,000 and some power reckoned in kilowatts would not be of much use. Lately high power machines have been developed either by increasing the frequency and power of ordinary alternators starting from the early work of Elihu Thomson and Nikola Tesla in 1889 and 1890 on this subject, or else by employing an alternating current field excitation which acts on a suitable rotor, frequency-raising alternators such as those of Goldschmidt, Latour, and Béthenod have been invented.

The difficulties met with in the former type are shown by the limitations of frequency and power exhibited by the early Tesla high frequency alternators made about 1890, which were capable of producing currents of 10 amperes or so having a frequency of not more than about 10,000 or 12,000.

One form of Tesla high frequency alternator was constructed as follows (see Fig. 3): It consists of a fixed ring-shaped field magnet with magnetic poles projecting inwards and a rotating armature in the form of a flywheel.[1] This wheel, J (see Fig. 4), was turned down on the edge, forming a kind of flanged pulley, and this groove is wound full of annealed iron wire insulated with shellac. Pins, L, were set in the sides of the ring J, and flat coils, M, of insulated wire wound over the periphery of the armature wheel and around the pins. These coils were connected together in series, and the ends of the series carried through a hollow shaft, H, to slip rings, P, P, from which the currents were taken off by brushes, O, O. The field magnet consisted of a kind of toothed wheel, with the teeth turned inwards (see Figs. 3 and 4), and an insulated wire or strip was wound zigzag fashion between these teeth, so that when a continuous current was passed along this conductor, the teeth were made alternately North and South magnetic poles. It is quite possible thus to produce a magnet having 400 radial poles in the circumference, and also easy to put 400 coils on the armature. Hence if such a machine is driven at a speed of 3000 revolutions per minute, or 50 per second, it produces an alternating current having a periodicity of 10,000 ~. A machine of this kind can be constructed to give a current of, say, 10 amperes. In the machine above described, which was capable of giving an alternating electromotive force of about 100 volts, the field magnet consisted of a ring of wrought iron 32 inches outside diameter, about 1 inch thick; the inside diameter was about 30 inches. The distance between the teeth was about $\frac{3}{16}$ inch, and each field magnet tooth

[1] See *The Electrical Engineer* of New York, March 18, 1891, vol. xi. p. 338.

was about $\frac{3}{16}$ inch thick. On the armature 384 coils were connected in two series. The width of the armature was $1\frac{1}{4}$ inch. With magnetic teeth placed so close it was necessary to have an extremely small clearance between the armature coils and the magnet, to avoid excessive leakage or loss of useful magnetic flux; hence it was impossible to use wire for the armature thicker than No. 26, Brown and

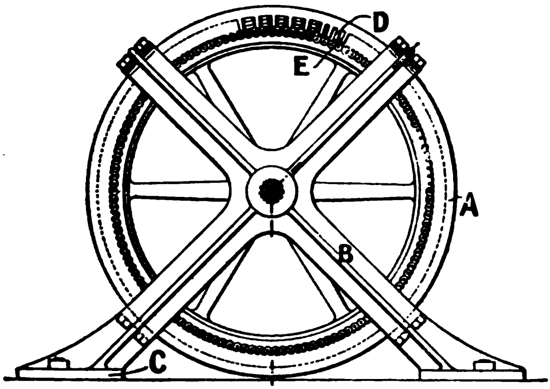


FIG. 3.—Tesla High Frequency Alternator. (*Side view*).

Sharp gauge. This size is equivalent to No. $28\frac{1}{2}$ British S.W.G. The armature wires must be wound with great care, otherwise they are apt to fly off in consequence of the great peripheral speed. It is practicable to run such an armature

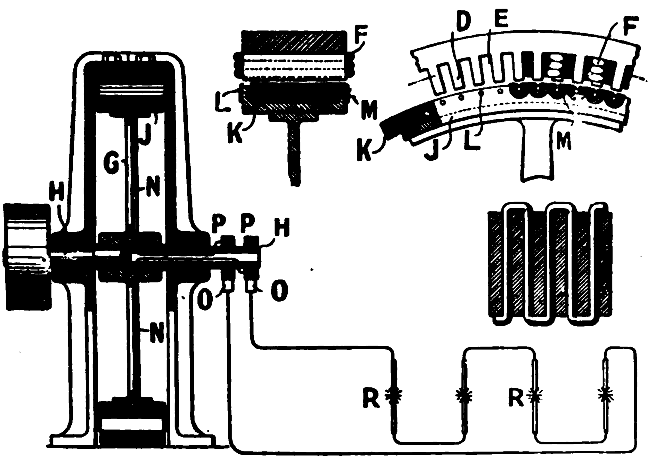


FIG. 4.—Tesla High Frequency Alternator. (*End view.*)

at a speed of 3000 revolutions per minute, equivalent to a peripheral speed of 375 feet per second.

In another type of machine constructed by Tesla, magnetic leakage was avoided by making adjacent poles on the same side of the armature of the same polarity. In this second form the armature consisted of a copper plate in the form of a disc with a large hole in it (see Figs. 5 and 6). The plate was cut through by radial slits alternately at the inside and outside edge, so as to divide the plate up into a zigzag

strip. This plate was clamped on a central boss fixed on a shaft (see Fig. 5), and caused to revolve between the two parts of a field magnet having a large number of inward projecting poles, all those on one side being of the same polarity and facing

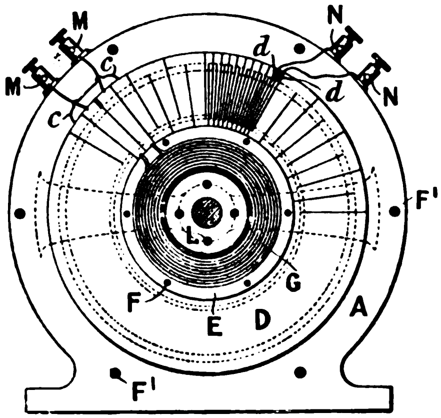


FIG. 5.—Tesla High Frequency Alternator: Disc type. (*Side view.*)

an equal number of like poles on the opposite side, of the opposite polarity (see Fig. 6). In this manner, the disc was perforated by the magnetic flux passing

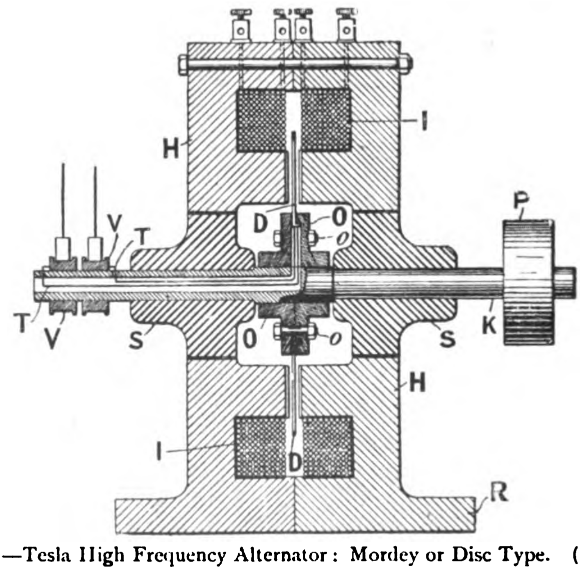


FIG. 6.—Tesla High Frequency Alternator: Mordey or Disc Type. (*Section.*)

across from one set of poles to another, and the passage of the strips into which the disc is cut up, into and out of these streams of the magnetic flux, gives rise to the electromotive force in the armature. The armature winding therefore consisted of a single disc-shaped conductor equivalent to a zigzag winding, and this was driven at a high speed, so that the radial elements of the armature cut across

streams of magnetic flux. A very strong excitation could therefore be employed without producing any wasteful leakage flux. The essential drawback of this construction is that unless the slits in the armature are very close together, so that the width of the radial bar or slice is not more than $\frac{1}{32}$ inch, there is considerable heating of the armature, due to eddy currents set up in it. In one machine of the last type, constructed by Tesla, the field had 480 polar projections on each side, and from this machine it was possible to obtain a current having a frequency of 15,000 complete periods per second. When a machine of this description, having a disc of considerable diameter, is driven at a speed of 3000 R.P.M., very accurate balancing is necessary, or otherwise dangerous vibrations will be set up in the machine. Great rigidity and accuracy of work are therefore necessary in all parts of the machine, because the clearance between armature and field magnets must necessarily be very small.

Generally speaking, it is not easy to obtain by the devices above described a frequency higher than 10,000 periods per second. Very excellent mechanical workmanship and perfect balance are necessary to be able to run any form of disc armature, having a diameter of 30 cms. or so, at a speed of 50 revolutions per second. Such an armature must carry 400 coils to be enabled to give even this frequency.

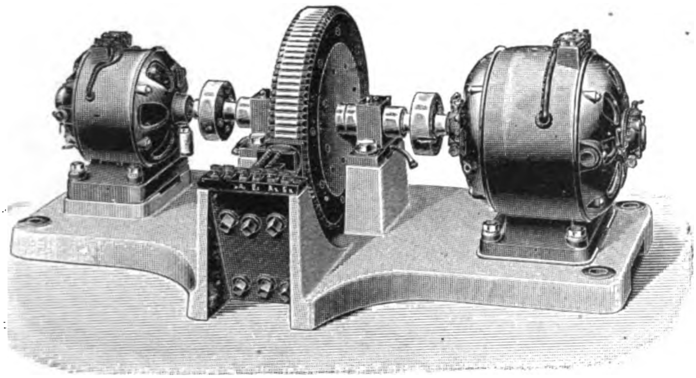


FIG. 7.—Siemens' High Frequency Inductor Alternator.

In consequence of the difficulty of balancing a wound armature, the inductor form of alternator is now adopted for high frequency machines. In this case the revolving part is merely a steel disc having teeth or notches cut on its edge. If two chisel-shaped magnetic poles are placed on either side of such a disc, and if these poles carry armature coils wound on them, then as the notched steel disc rotates it varies the magnetic reluctance of the magnetic circuit, and hence the flux passing through the armature coils. In Fig. 7 is shown a view of such a high frequency inductor alternator made by Messrs. Siemens Bros., the toothed inductor disc being driven by a motor which also drives on the same shaft a small dynamo which provides the exciting current for the field magnets of the alternator. The frequency and electromotive force are, of course, determined by the speed of this disc. It is easy by it to produce an alternating current of a frequency of 5000.

About the year 1907, R. A. Fessenden in the United States directed his attention to the design of alternators capable of producing larger output and with a frequency of the order of 50,000. Recognizing the difficulties which arise from magnetic leakage when poles of opposite sign are interspaced, he, like Tesla, adopted the Mordey type of alternator, in which the field magnets have poles of the same sign on one side, and the magnet consists of a pair of revolving discs with opposed teeth facing inwards, those on one side being all N. poles and those on the opposite side S. poles. The fixed armature in the form of a disc is placed between these rows of poles, and in some cases it appears a double

armature is employed. The two magnet discs resemble crown wheels with small teeth, between which the wire for carrying the exciting current is wound. One machine thus made by him was said to be capable of giving an alternating current with a frequency of 80,000. In practice it seems to have been limited to 60,000, with an output of 250 watts and an electromotive force of 60 volts when running

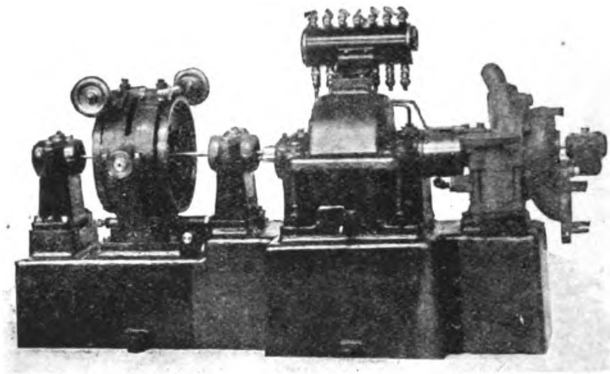


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FIG. 8.—Fessenden's High Frequency Turbo-Alternator.

at a speed of 10,000 R.P.M. At a speed of 8400 R.P.M. it gave a frequency of 50,000 and a voltage of 65. The field magnet of this machine is described as having 360 poles. Another type of alternator coupled direct to a De Laval steam

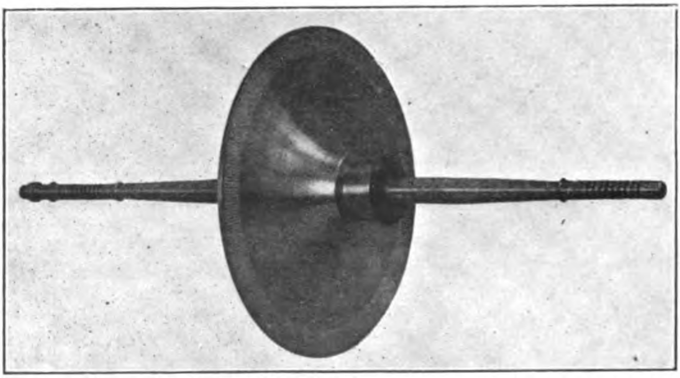

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FIG. 9.—Inductor Disc of the Alexanderson Inductor High Frequency Alternator.

turbine has been constructed and described by Fessenden (see *The Electrician*, vol. 61, p. 441, 1908). De Laval steam turbines are now made in small sizes to run at 30,000 R.P.M. Hence when coupled direct to a shaft running at this speed an armature of comparatively small diameter will give the required high frequency. In the case of the machine shown in Fig. 8 the alternator gives a current at 225 volts and a frequency of 75,000, with about 2·5 kilowatts output.

The machine is of the double armature type, with 300 coils on each armature, and a field magnet with 150 teeth. The two air gaps are only $\frac{1}{16}$ inch in length. The steam pressure used with the turbine is 100 lbs. per sq. inch.

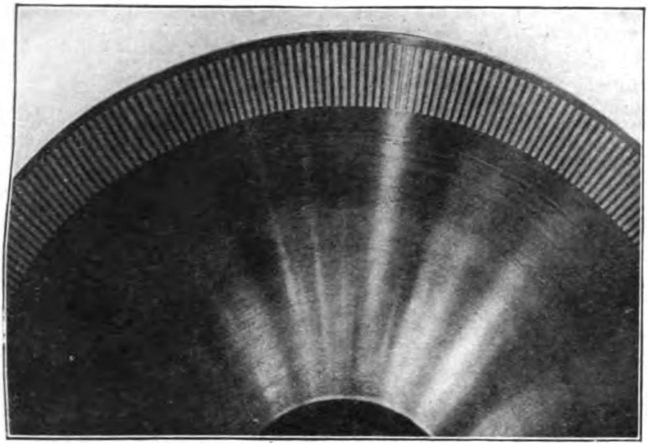

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FIG. 10.—View of part of the Inductor Disc of Alexanderson Alternator showing slots or holes in edge.

Appreciating the special difficulties involved in the construction of a high-power high frequency alternator with moving coils, E. F. W. Alexanderson, of the General Electric Company in America, directed his attention in 1908 to the design of an inductor alternator.[2] In this type of machine both the field magnet coils and the armature coils are fixed. The only moving part of the machine consists of a steel disc having slots or holes cut out in it near the edge. This disc is carried on a flexible steel spindle and can be revolved at a very high speed. To prevent churning of the air and consequent loss of power the slots are filled up with a non-magnetic material (see Figs. 9 and 10). This disc revolves with its edge between the fixed field and armature coils which are carried on a fixed frame, and the function of the slots in the steel disc is to change the total magnetic flux passing through the armature coils and so create in them an alternating electromotive force. This action will be understood from Fig. 11, which represents a cross section of half the machine. The parts D and B are portions of the ring frame, and A are the field coils. The magnetic flux created by these coils passes round through certain laminated iron teeth E, and the armature winding is a simple zigzag circuit wound over these teeth. The steel disc

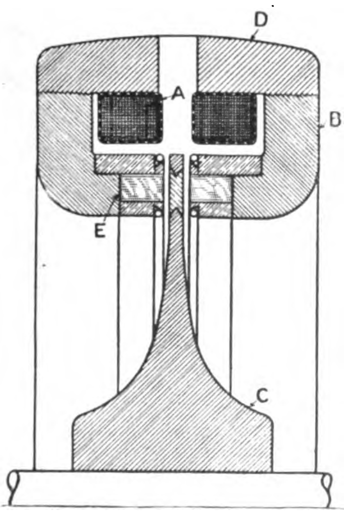

FIG. 11.—Half Cross section of Alexanderson High Frequency Alternator.

[2] See *The Electrician*, vol. 63, p. 541, 1909. On an alternator for one hundred thousand cycles, by E. F. W. Alexanderson.

passes between these teeth with very little clearance, and according as a space or a slot is passing so will the flux linked with the armature winding be varied. If the steel disc has 300 slots in its edge, and is revolved at a speed of 20,000 R.P.M., this will create an alternating current having a frequency of 100,000. To obtain this high speed and yet retain the advantages of driving by an electric motor a special form of 1 to 10 speed-raising gearing is employed. A general view of this motor-driven high frequency alternator is shown in Fig. 12, in which the alternator is seen on the left hand and the driving motor on the right.

The revolving disc is made of chrome-nickel steel, and is carried on a flexible shaft which allows the disc to rotate about its mass centre at a high speed, thus avoiding any strain on the bearings. The shaft is carried on two end bearings, but there are intermediate loose bearings which limit the play of the shaft at those critical speeds which induce vibration. These bearings have forced lubrication. The machine shown in the figure has an output of 2 kilowatts at a voltage of 110 volts, and short circuit current of 20 amperes with frequency of 100,000. Mr Alexanderson stated in 1915 that a similar machine was being built for an output of 50 kilowatts at a frequency of 50,000. The above described high

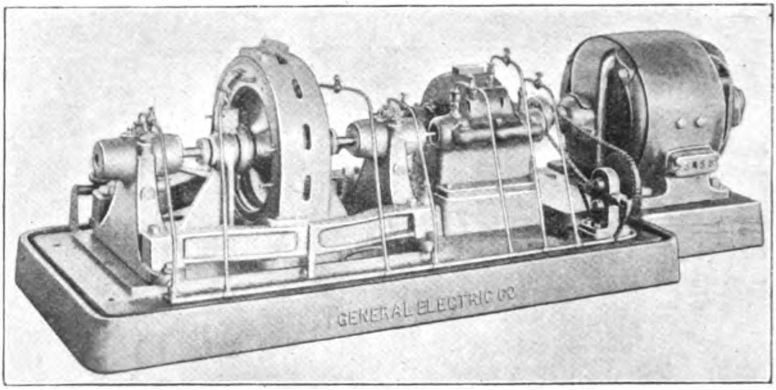

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FIG. 12.—Alexanderson Motor-Driven 2 k.w. High Frequency Alternator.

frequency alternators act on just the same principles as the ordinary low frequency alternators used for power and lighting, but a high frequency generator on quite a different principle has been invented by Dr Rudolph Goldschmidt,[3] which enables frequency to be raised, just as a transformer raises voltage or current. To understand the scientific principle of this machine, it is first necessary to recall to mind that any alternating magnetic field, which is constant in direction but varies in strength according to a sine law, can be considered to be the resultant of two magnetic fields of constant strength equal to half the maximum value of the pulsating field; these two constant fields changing their direction continually by rotating with equal angular velocity in opposite directions. Let OA, OB (see Fig. 13) be the two fields of constant strength rotating uniformly in opposite directions. Then their resultant OC is constant in direction, but has a sinoidally varying strength with a maximum value equal to twice OA or OB. Hence we can

[3] See R. Goldschmidt. British Patent Specifications, No. 27260 of 1907; No. 17835 of 1908; No. 11294 of 1910. Also *The Electrician*, vol. 66, p. 744, 1911, and *The Electrician*, vol. 69, p. 615, 1912. See also *Pall Mall Gazette*, 5th December 1912, in which it is mentioned that the author had some share in directing attention to the problem of the construction of a high frequency alternator for radiotelegraphy. For a good description of the Goldschmidt alternator the reader is referred to an article by Mr E. E. Mayer in the *Proceedings of the Institute of Radio Engineers of New York, U.S.A.*, for March 1914; vol. 2, p. 69.

consider any alternating magnetic field as resolvable into a pair of oppositely rotating constant fields.

Goldschmidt's high frequency alternator has a fixed coil or set of coils called the stator, and in the interior revolves a movable coil or coils called the rotor. In the conventional diagram in Fig. 14 only one stator coil, S, is shown, and one rotor coil, R. Imagine that the stator has a continuous electric current sent through it from a battery B, and that the rotor is made to revolve with an angular velocity ω in the field of the stator. The rotor coil will then have an alternating current induced in it. Relatively to the rotor the magnetic field due to this last-named current is fixed but pulsating in strength. It may therefore be resolved into two oppositely rotating constant fields. One of these revolves through space with twice the angular velocity of the rotor, and the other revolves in the opposite direction to the rotor and is stationary with respect to the stator coil. The former field cuts through the stator coil and induces in it an alternating current of twice the frequency of that in the rotor. Next, suppose that both the rotor and the stator coils have their circuits completed by condensers in series with inductance

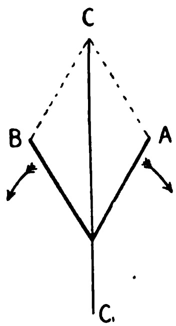


FIG. 13.—Resultant of Two Oppositely Rotating Vectors.

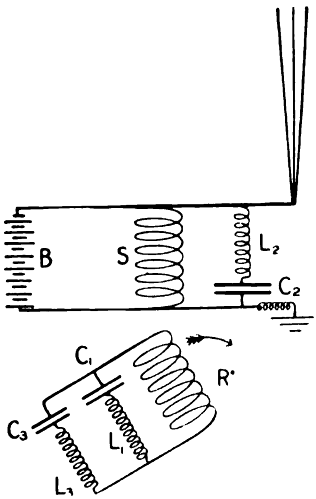


FIG. 14.—Diagram of Connections of Goldschmidt Alternator.

coils, the capacity and inductance being so adjusted in accordance with the rules explained in the later sections of this chapter that they are in tune for certain frequencies. It is shown below that if a capacity C is in series with an inductance L the circuit responds most readily to electromotive forces having a frequency $1/2\pi\sqrt{CL}$, but permits only the flow of a very small current through it if the frequency of the impressed E.M.F. differs very little from the above value. If then the rotor coil is closed by a circuit composed of a condenser C_1 and an inductance L_1 adjusted to respond to the currents of the frequency $\omega/2\pi$ or the fundamental frequency, the currents set up by the revolution of the rotor will circulate in this circuit, and will, as above mentioned, create a field which cuts through the stator coil with twice the angular velocity of the rotor.

If then the stator coil S is closed by an inductive circuit L_2 with capacity C_2 and if these are adjusted to the right value, these stator currents of frequency $2\omega/2\pi$ will circulate in it. The corresponding stator field can be resolved into two components rotating in opposite directions with twice the angular velocity of the rotor. These will cut through the rotor coil and induce in it currents of frequencies $\omega/2\pi$ and $3\omega/2\pi$, because relatively to the rotor they cut it with angular velocities either

equal to, or else three times, that of the rotor. But the rotor is already short-circuited by a circuit which can pass a current of the fundamental frequency. If then we add another condenser C_3 of suitable capacity we can also provide a path for the currents of threefold frequency. This last current in turn produces a field which can be resolved into two oppositely rotating fields which cut the stator coil. One of these cuts with an angular velocity twice that of the rotor, and the other with four times that of the rotor.

The stator coil is short-circuited for currents of twice the rotor frequency. If, however, we attach an antenna to the stator circuit or any other equivalent capacity adjusted to be tuned for a fourfold frequency we can create in that circuit alternating currents having a frequency fourfold that of the fundamental.

Hence we can by this means multiply up frequency so that starting with an excitation obtained by a single-phase alternating current having a frequency of 10,000 we can generate one of 40,000, or to any required higher frequency.

Such alternators can now be built for large power, and without any dangerous speeds of revolution can produce alternating currents of a frequency high enough for radiotelegraphy. In Fig. 15 is shown the external appearance of a motor-driven Goldschmidt high frequency alternator. The inventor has already constructed machines of 200 kilowatts output at a frequency of 50,000, and for higher frequencies at lesser outputs.

The theory of the Goldschmidt alternator has been considered very fully by Dr T. R. Lyle, in a paper read before the British Association at Birmingham.[4] It has also been discussed by Sir Oliver Lodge.[5] At first sight it might appear that such a machine could not have a high efficiency, since the various currents of intermediate frequency constituting the steps by which the ultimate high frequency is reached, would dissipate energy in this closed inductive-capacity circuit to a considerable amount.

The reason this does not happen is that these intermediate currents occur in pairs of equal frequency but of opposite phase, and so tend to nullify each other.

Thus suppose we start with an alternating current in the stator of frequency n; we have produced in the stator and rotor circuits currents of the following frequencies by the reaction of the revolving fields and rotating rotor, viz.:—

In the Stator Circuit.	In the Rotor Circuit.
n	$2n$ and O
$3n$ and n	$4n$ and $2n$
$5n$ and $3n$	$6n$ and $4n$

Now a little consideration will show that the second current of frequency n induced in the stator by the first current of frequency $2n$ in the rotor, is opposite in phase to the original inducing current of frequency n in the stator. Just as the field due to the armature current of a D.C. dynamo is opposed in direction to the permanent or exciting field, and just as the secondary current in an ordinary iron core A.C. transformer is nearly opposite in phase to the primary current, so the secondary induced current of frequency n in the stator is about 180° different in phase from the original stator current of frequency n. In the same way all the pairs of currents both in stator and rotor of identical frequency cut each other out or more or less neutralize each other. Thus the pairs of currents of $3n$ frequency in the stator are opposite in phase, and also the pair of currents of $2n$ and $4n$ frequency in the rotor. Hence we are left only with the unneutralized current of highest frequency. If there were no magnetic leakage and no dissipation of energy in the various closed inductive-capacity circuits in which the lower harmonics circulate there would be a complete extinction of all the intermediate harmonics, and all the applied energy of low frequency would be transformed into oscillations of high frequency.

As a matter of fact there is a loss at each "reflection" due to iron, copper, and

[4] See *The Electrician*, vol. 71, p. 1001, 1913, on the "Goldschmidt Alternator," by T. R. Lyle.

[5] See Sir Oliver Lodge, on a "Dynamo for Maintaining Electrical Vibrations of High Frequency," *Phil. Mag.*, June 1913.

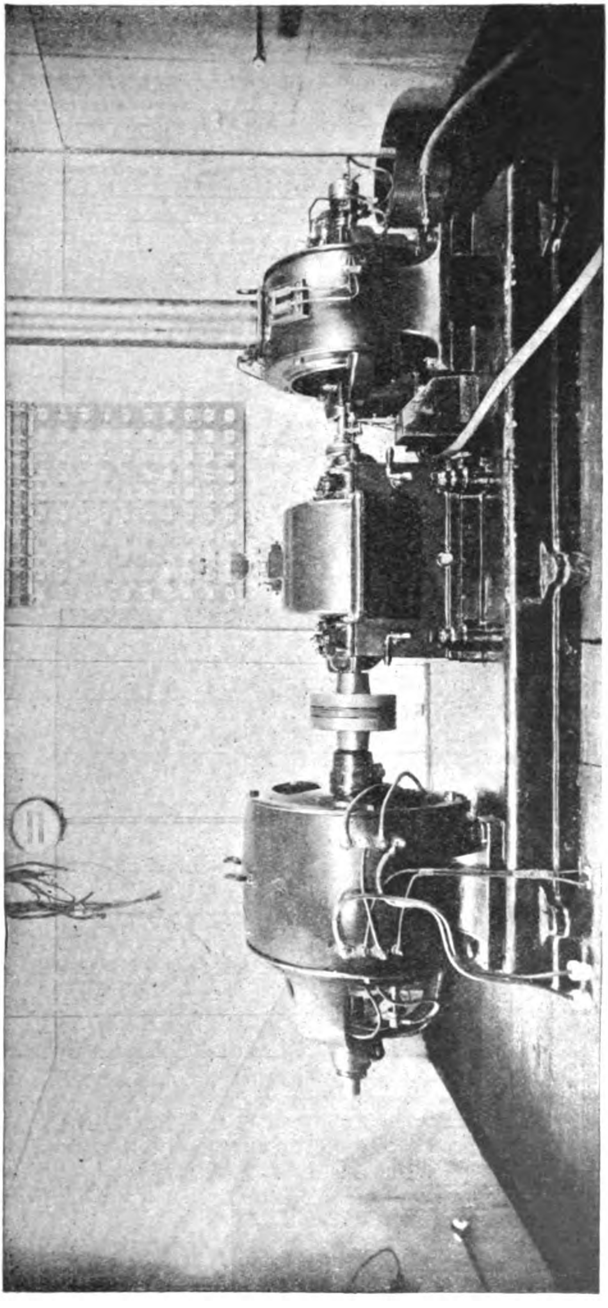


Fig. 15.—View of Goldschmidt's High Frequency Alternator.

dielectric losses, and also the magnetic leakage will decrease the output of high frequency current per unit of energy of applied low frequency current.

Hence it is important to start with a fairly high frequency and limit as much as possible the number of frequency transformations.

This introduces certain mechanical difficulties. There is first a limit to the safe peripheral speed, say of 200 metres per second at the circumference of the rotor. Also there is a practical limit to the revolutions per minute of the driving motor, say 3000 R.P.M. Then there is a limit below which the final frequency must not fall if the machine is to be of use for radiotelegraphy, say 50,000. This gives us a limit to the width of the rotor teeth. For if we desire to have not more than five frequency transformations we must start with a frequency of 10,000. If the rotor is, say, 1·25 metres (=4 feet) in diameter and has 400 cms. perimeter and has 400 poles of 1 cm. pitch, then at a speed of 3000 R.P.M. we have a frequency of $(400 \times 3000) \div 120 = 10{,}000 = n$. These values are, however, practicable. To save room and reduce capacity and inductance the rotor and stator windings are each a simple zigzag winding of one wire in each slot.

The wire itself is composed of extremely fine high conductivity copper, insulated and stranded so as to nullify skin effect. The whole strand has to be well insulated. The iron cores of the stator and rotor are built of sheet steel 0·002 inch (=0·05 mm.) thick, insulated with paper 0·0012 inch thick. Thus the cores of the machine are more than 30 per cent. paper. The clearance of the rotor is only $\frac{1}{32}$ inch (=0·8 mm.), hence the form must be truly cylindrical with respect to the axis of rotation.

As the peripheral speed is 200 metres per second and the rotor of the 150 kilowatts machine weighs 5 tons, the design of the bearings and lubrication involved great consideration. The slots in the rotor and stator have to be absolutely parallel, and a deviation of 1 part in a 1000 from true parallelism involves a loss of 20 per cent. in the output of the machine.

The difficulties of collecting the currents at the brushes and preventing loss of current by stray capacity paths, required great thought to overcome them. Nevertheless these difficulties of construction have been overcome in machines built up to sizes of 150 kilowatts or 200 kilowatts.

The theory of the Goldschmidt alternator has also been very fully discussed by Professor Pupin as a particular case of the general theory of asymmetrical rotors in unidirectional magnetic fields.

If a fixed coil and a rotating coil are in presence of each other and a steady E.M.F. is applied to the two in series, then the whole system will have a pulsating reactance and periodic currents will be set up in it, the frequency of which may be enhanced by suitable resonance. The full details of Pupin's theory are given in *The Proceedings of the Institute of Radio Engineers*, vol. ii., December 1915, p. 385, in an article by B. Liebowitz, to which we must refer the reader.

In connection with high frequency alternators it is necessary to mention the alternators of MM. Marius Latour and Béthenod. If an alternator is excited with a single phase alternating current, we may consider this fixed alternating field to be resolved into two oppositely rotating constant fields, as above explained. If there are p pairs of poles, and if the alternator armature makes N turns per second, and if the frequency of the exciting current is f, then the rotation of the armature produces two induced currents of frequency $f - Np$ and $f + Np$. We may by suitable resonance exalt the amplitude of the latter. If then we pass this current of augmented frequency into the field-circuit of a second alternator and join up in cascade a series of such alternators, the current from each armature going into the field of the next machine, we have a final equation for the frequency f_m in the m^{th} alternator

$$f_m = f_1 + (m-1)Np = mNp,$$

since $f_1 = Np$.

Hence by the use of m alternators in cascade we can augment frequency m times. By the use of four rotating armatures or rotors on one shaft, we can in this manner quadruple the frequency of the first armature.

We shall discuss in the last two sections of this chapter other mechanical means for producing continuous oscillations.

4. Production of Damped Electric Oscillations by the Discharge of a Condenser.—We have in the next place to consider the methods employed for the production of damped oscillations. These are always created by the discharge of a condenser of some kind. If two conductors receive electrical charges of opposite sign, in other words, are brought to different potentials, and if they are suddenly connected through a conductor having inductance but small resistance, the equalization of their potentials takes place by means of a discharge, consisting of a series of decadent electrical oscillations, or movements of electricity, to and fro along the conductor.

The nature of this phenomenon is best explained by considering a hydrodynamic analogue. Suppose two air-tight reservoirs to be connected by a wide pipe having in it a valve which can suddenly be opened. Let one vessel contain air under great pressure, and let the other vessel be exhausted. Then the difference of air pressure between the vessels is analogous to the difference of electric potential of the electric conductors. If then the valve in the pipe is opened, air rushes from the full to the empty vessel, but owing to its inertia it overshoots the mark, and after equalizing the pressure, for an instant reverses the relative pressure conditions of the vessels, and the pressure is finally equalized only after a series of gradually subsiding to-and-fro movements of air in the pipe have taken place. Each vessel has successively the state of higher and lower pressure, but in decrescent degree.

The conditions for the establishment of such air oscillations between the two vessels are, however, that the pipe be very suddenly opened, and it must offer but little resistance to the movement of the air. If the pipe throttles the air motion, then the pressure would sink gradually in one vessel and rise in the other, but there would be no aërial oscillations. In the same manner, if the equalization of the electric potentials of the charged conductor takes place through a wire of high resistance, electric oscillations are not produced.

We may employ another mechanical illustration of the same effect, as follows:—Suppose a glass **U**-tube to be partly filled with mercury, and the mercury to be displaced so as to be higher in one limb than the other. There is then a force due to the difference of level urging the fluid to return to an equal height in the two limbs. Let the mercury be allowed to return, but be constrained so that it is released slowly; it goes back to its original position without oscillations. If, however, the constraint is suddenly removed, then, owing to inertia of the mercury, it overshoots the position of equilibrium and oscillations are created. If the tube is rough in the interior or the liquid viscous, these oscillations will quickly subside, being damped out by friction, but, other things being equal, the denser the liquid the more prolonged will be the time of the oscillations.

The quality we call inertia in material substances corresponds in effect with the inductance of an electric circuit, and the frictional resistance experienced by a liquid in moving in the tube, with the electric resistance of a circuit. If we suppose the **U**-tube to include air above the mercury, and to be closed up at its ends, the compressibility of the enclosed air would correspond to the electrical capacity in a circuit.

The necessary conditions for the creation of mechanical oscillations in a material system or substance are that there must be a self-recovering displaceability of some kind, and the matter displaced must possess density or inertia. In other words, the thing moved must tend to go back to its original position when the disturbing or restraining force is withdrawn, and must overshoot the position of equilibrium in so doing. Frictional resistance causes decay in the amplitude of the oscillations by dissipating their energy as heat.

In the same way the essential condition for establishing electrical oscillations in a circuit is that it must connect two bodies having electrical capacity with respect to one another, such as the plates of a condenser, and the circuit must itself possess inductance and low resistance. Under these conditions the sudden release of the electrical strain results in the production of an oscillatory electric current in the circuit, provided the resistance of the circuit is less than a certain critical value. We have these conditions present when the two coatings of a charged Leyden jar are connected by a thick copper wire.

Since every charged conductor is merely one coating or surface of a particular type of condenser, it follows that most cases of electric discharge in the form of a spark are oscillatory in character. It is probable that many lightning flashes are oscillatory discharges on a gigantic scale. In a later chapter we shall consider the methods by which the existence of oscillations set up, even when a charged metal ball is discharged to earth by a spark taken by the knuckle, can be demonstrated.

5. General Theory of the Discharge of a Condenser.—It was long ago suggested that the discharge of a Leyden jar does not always consist in the flow of a transient unidirectional current through the discharging circuit, but is in some cases an alternating current diminishing gradually in strength. Joseph Henry, in 1842, came to this conclusion, guided to it no doubt by his observations on the irregular effects attending the magnetization of steel needles by Leyden jar discharges. He remarks[6]—

"The discharge, whatever may be its nature (that is, of a Leyden jar), is not correctly represented by the single transfer of imponderable fluid from one side of the jar to the other. The phenomena require us to admit *the existence of a principal discharge in one direction, and then several reflex actions backwards and forwards, each more feeble than the preceding, until equilibrium is obtained.* All the facts are shown to be in accordance with this hypothesis, and a ready explanation is afforded by it of a number of phenomena which are found to be described in the older works on Electricity, but which have until this time remained unexplained."

Von Helmholtz, whose penetrating genius opened up so many new ideas, in his celebrated essay "Die Erhaltung der Kraft" ("The Conservation of Force") read before the Physical Society of Berlin, 23rd July 1847, said—

"We assume that the discharge of a jar is not a simple motion of the electricity in one direction, but a backward-and-forward motion between the coatings, in oscillations which become continually smaller until the entire *vis viva* is destroyed by the sum of the resistances."

Lord Kelvin published in 1853 a classical paper, "On Transient Electric Currents,"[7] in which the discharge of the Leyden jar was mathematically treated in a manner which elucidated important facts. He recognized the influence which the "electro-dynamic capacity," or, as we now call it, the *inductance*, of the discharge circuit had upon the effects, and he established an equation of energy which expresses the fact that the energy of the charged jar at any instant is partly being dissipated as heat in the discharging circuit, and partly conserved as current energy in that circuit.

Consider the case of a charged Leyden jar or condenser discharged through a circuit having resistance and inductance. In the act of discharge the electrostatic energy stored up in the condenser is converted into electric current energy and dissipated as heat in the connecting circuit. At any moment the rate of decrease of the energy in the jar is equal to the rate of dissipation of energy in the discharging circuit plus the rate of change of the kinetic or magnetic energy associated with the circuit.

If we confine our consideration of the problem to the limited case in which the discharge current is of such frequency that the motion of electricity in the discharge circuit is at every instant in the same direction in all parts of this circuit, and uniformly distributed over the cross section of this circuit, we can set out the elementary theory following Lord Kelvin's method as follows:—

If the capacity of the jar is represented by C, the resistance of the discharge circuit by R, and the inductance of that circuit by L, then an equation of energy may be stated mathematically, as follows:—

[6] "The Scientific Writings of Joseph Henry," vol. i. p. 201. Washington, 1886.

[7] "On Transient Electric Currents," by Prof. William Thomson, *Phil. Mag.*, 1853, ser. 4, vol. 5, p. 393.

$$-\frac{d}{dt}\left[\tfrac{1}{2}\frac{q^2}{C}\right]=\frac{d}{dt}[\tfrac{1}{2}Li^2]+Ri^2 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

$\frac{1}{2}$ ... $= \frac{qV}{2}$

$V = \frac{q}{C}$

$$\text{or} \quad L\frac{di}{dt}+Ri=-\frac{1}{C}\int i\,dt$$

$\therefore E_n = \frac{1}{2}\frac{q^2}{C}$

$$\text{or} \quad \frac{d^2q}{dt^2}+\frac{R}{L}\cdot\frac{dq}{dt}+\frac{1}{LC}q=0. \quad . \qquad (6)$$

$-LC\ddot q + RC\dot q + q = 0$

$$\text{or} \quad TT'\ddot q+T'\dot q+q=0. \qquad (7)$$

$\therefore RC = T' \quad LC = \frac{L}{R}\cdot T' = TT'$

The above equation (5) is merely the symbolical expression of the fact that at any instant the rate of loss of energy by the condenser is equal to the sum of the rate of dissipation of energy in the circuit, and the rate of storage of energy in the magnetic field round it.

In equation (7) T is written for $\frac{L}{R}$ and T' for CR, whilst $\dot q$ and $\ddot q$ stand for the first and second time differentials of q.

The above differential equation belongs to a class which occurs in numerous physical investigations, and its solution in the last form consists in finding the value of the quantity of electricity q or the charge of the jar at any instant in terms of the time and the three constants, L, R, and C. An equation of this kind has two solutions according to the relations of the constants.

It is easy to show, following Lord Kelvin, that the nature of the solution of the above quotation (6) is determined by the relative values of the quantities $\frac{L}{R}$ and LC, or by $\frac{L}{R}$ and $\frac{L}{R}$CR. If $\frac{R^2}{4L^2}$ is greater than $\frac{1}{LC}$, that is, if R is *greater than* $\sqrt{\frac{4L}{C}}$, or if $\frac{CR}{4}$ is greater than $\frac{L}{R}$, the charge in the jar dies away gradually as the time increases, in such a manner that the discharge current is always in one direction.

The ratio $\frac{L}{R}$ is called the *time-constant* (T) of the discharge circuit, and the product CR is called the time-constant (T') of the condenser circuit. Hence the above condition amounts to saying that the discharge is unidirectional when T is less than $\frac{1}{2}\sqrt{TT'}$, that is, when the time-constant of the inductive circuit is less than half the geometric mean of the time-constants of the inductive circuit and the condenser circuit.

The solution of equation (6) and the determination of these conditions offers no difficulty.

Assume $q=A\epsilon^{mt}$, where A is some constant, ϵ is the base of the Napierian logarithms, and m a quantity to be determined. Then by substitution we have—

$$\frac{d^2q}{dt^2}+\frac{R}{L}\cdot\frac{dq}{dt}+\frac{q}{LC}=q\left(m^2+\frac{R}{L}m+\frac{1}{LC}\right)=0$$

Hence
$$m^2+\frac{R}{L}m+\frac{1}{LC}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Solving the above quadratic equation, we have—

$$m=-\frac{R}{2L}\pm\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}$$

Therefore if $\frac{R^2}{4L^2}$ is greater than $\frac{1}{LC}$, the roots of the quadratic (8) are real, and the solution of (6) takes the form—

$$q=A_1\epsilon^{m_1t}+A_2\epsilon^{m_2t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

In the above equation A_1 and A_2 are constants, and m_1 and m_2 are the two real roots of (8).

If we call Q the total charge of the jar at the instant when the discharge begins, and reckon the time t from that instant, then when $t=0$ we have $q=Q$. Also the current i flowing out of the jar $=-\frac{dq}{dt}$, and i is zero when $t=0$.

Hence from (9), under these conditions, we have—

$$A_1 + A_2 = Q \quad \text{and} \quad A_1 m_1 + A_2 m_2 = 0.$$

Therefore $$A_1 = Q\frac{m_2}{m_2 - m_1} \qquad \text{and } A_2 = -Q\frac{m_1}{m_2 - m_1}$$

Hence the complete solution of equation (6) in the case of the above defined conditions is—

$$q = \frac{Q}{m_2 - m_1}(m_2\epsilon^{m_1 t} - m_1\epsilon^{m_2 t}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where $$m_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = -\alpha + \beta$$

and $$m_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = -\alpha - \beta \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The current i at any instant flowing out of the condenser is found by differentiating equation (10) with respect to t.

Therefore $$i = -\frac{m_1 m_2 Q}{m_2 - m_1}(\epsilon^{m_1 t} - \epsilon^{m_2 t}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

But when $t=0$, $i=0$, and when $t=\infty$, $i=0$. Hence at some instant the current has a maximum value, and by differentiating equation (12) it is easily found that at a time $t = \frac{\log_\epsilon m_1 - \log_\epsilon m_2}{m_2 - m_1}$, the current i has a maximum value.

Accordingly this result shows us that when the resistance, inductance, and capacity are so related that $\frac{R^2}{4L^2}$ is greater than $\frac{1}{LC}$, or which is the same thing, when $\frac{CR}{4}$ is *greater than* $\frac{L}{R}$, then the discharge from the condenser is unidirectional, but rises up to a maximum value and then decays (see Fig. 16).

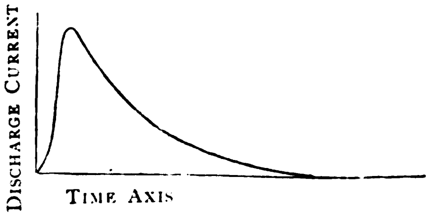


FIG. 16.—Curve representing the Dead-beat Discharge Current of a Condenser.

On the other hand, if $\frac{CR}{4}$ is *less than* $\frac{L}{R}$, the roots of the quadratic (8) are unreal, and may be written in the form:—

$$\left.\begin{aligned} m_1 &= -\alpha + j\beta \\ m_2 &= -\alpha - j\beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where $$j = \sqrt{-1}, \ \alpha = \frac{R}{2L}, \text{ and } \beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$$

In this case the solution of (6) is—

$$q = A_1\epsilon^{-(\alpha - j\beta)t} + A_2\epsilon^{-(\alpha + j\beta)t} \quad . \quad . \quad . \quad . \quad (14)$$

Bearing in mind the exponential values of the sine and cosine, viz.—

$$\sin\theta = \frac{\epsilon^{j\theta} - \epsilon^{-j\theta}}{2j}, \ \cos\theta = \frac{\epsilon^{j\theta} + \epsilon^{-j\theta}}{2},$$

we can write equation (14) in the form—

$$q = \epsilon^{-\alpha t}\{(A_1 + A_2)\cos\beta t + j(A_1 - A_2)\sin\beta t\}.$$

Hence from the values of A_1 and A_2 already obtained we arrive at the equation—

$$q = \frac{Q\epsilon^{-\alpha t}}{m_2 - m_1}\{(m_2 - m_1)\cos\beta t + j(m_2 + m_1)\sin\beta t\} \quad . \quad . \quad . \quad (15)$$

as an expression for q.

Therefore since the discharge current $i = -\frac{dq}{dt}$, we have by differentiation of (15)—

$$i = \frac{Q\epsilon^{-\alpha t}}{m_2 - m_1}\{(m_2 - m_1)(\alpha\cos\beta t + \beta\sin\beta t) + j(m_1 + m_2)(\alpha\sin\beta t - \beta\cos\beta t)\}$$

and from the values for m_1 and m_2 given above we have finally—

$$i = Q\epsilon^{-\alpha t}\left(\frac{\alpha^2 + \beta^2}{\beta}\right)\sin\beta t \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If in equation (15) we substitute the values of m_1 and m_2 given in equation (13) we have—

$$q = Q\epsilon^{-\alpha t}\left(\cos\beta t + \frac{\alpha}{\beta}\sin\beta t\right).$$

Also if v is the potential difference of the plates of the condenser at the time t, and V their initial potential difference, $Q = CV$ and $q = Cv$, where C is the capacity. Hence—

$$v = V\epsilon^{-\alpha t}\left(\cos\beta t + \frac{\alpha}{\beta}\sin\beta t\right).$$

In all practical cases of oscillatory circuits the ratio $\frac{\alpha}{\beta}$ is small compared with unity, and then $\beta = \frac{1}{\sqrt{LC}}$. Lower down (see equation (20)) this last quantity β is shown to be equal to $2\pi n = p$, when $R = 0$ or $\alpha = 0$. Hence under these conditions the above equation and also equation (16) take the form—

$$v = V\epsilon^{-\alpha t}\cos pt. \qquad i = CpV\epsilon^{-\alpha t}\sin pt \quad . \quad . \quad . \quad (17)$$

These last equations are of the same form as the expression $i = I\epsilon^{-\alpha t}\sin pt$ given on page 3 as the equation for the wavy line obtained by the projection of the point moving along a logarithmic spiral. They show, therefore, that both the current in the circuit and the potential difference of the condenser plates decay in accordance with the law of a damped oscillation train.

It is necessary, however, to call attention at this point to the fact that when circuits are traversed by high frequency currents the resistance R and the inductance L of the discharge circuit which make their appearance in the above equations have not the same numerical values as the resistance and inductance involved when steady continuous currents are passing through the circuit. Accordingly, the above statements as to the condition under which the oscillatory form of discharge is produced are subject to a certain correction, but, broadly speaking, we may say that when the resistance of the discharge circuit is very low the discharge will take the oscillatory form.[8] If we examine the equation (16) for the discharge current, we see that it shows that the current is zero at intervals of time corresponding to $\sin\beta t = 0$. It follows that these times of zero current are therefore spaced out at equal intervals, each equal to $\frac{\pi}{\beta}$. Also the maximum values of the currents in either direction decay away in geometric progression as the times

[8] See sections 1 and 2, Chap. II., of this treatise. When the frequency is so low that the discharge current is uniformly distributed over the cross section of the conductor, or when the conductor is so laminated that this is the case, the quantity R in the equations above is the ordinary or ohmic resistance and L is the ordinary inductance, but when the frequency is so high that the current is not so distributed, then the resistance R and inductance L must be replaced by the high frequency resistance and inductance of the circuit.

increase in arithmetic progression. The discharge current in the two cases, viz. the *dead-beat* case and the *oscillatory* case, corresponding to the equations (12) and (16), can therefore be represented graphically by the two curves shown in Fig. 16 and Fig. 17.

The ordinates of the curve in Fig. 16 represent the discharge current at various instants during the discharge in the dead-beat or non-oscillatory case, and the ordinates of the curve in Fig. 17, the currents in the oscillatory case. In this last, the ordinates above the datum line represent currents in one direction, and those below, currents in the opposite direction. The gradual decrease of the maximum ordinates indicates the damping.

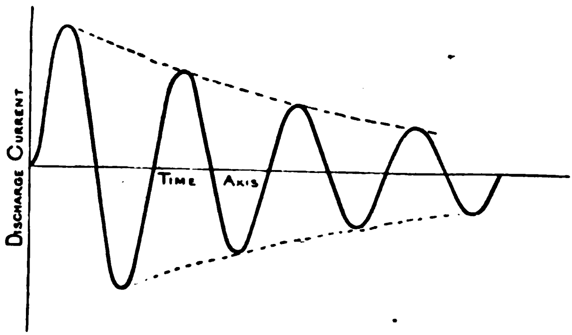


FIG. 17.—Curve representing the Damped Oscillatory Discharge Current of a Condenser.

The Napierian logarithm of the ratio of any maximum current or ordinate to the next maximum in the same direction multiplied by the frequency, gives us the value of the damping coefficient a as shown in section 2. Accordingly, we have $a=\frac{R}{2L}=n\delta$, and $\beta=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$. Taking $\frac{T}{2}$ to represent the interval of time between two successive values of zero discharge current, when it is oscillatory, we see from the above that—

$$T=\frac{2\pi}{\beta}=\frac{2\pi}{\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Hence the oscillations are isochronous, and their frequency $n=\frac{1}{T}$.

$$n=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If R is so small that $\frac{R^2}{4L^2}$ can be neglected in comparison with $\frac{1}{LC}$, then the frequency is given by the expression—

$$n=\frac{1}{2\pi\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

In this equation (20) the quantities C and L must be measured in homologous units when the expression is employed in practical calculations. That is to say, C and L must both be expressed or measured in electromagnetic units or both in electrostatic units or else in practical units, viz. in *farads* and *henrys*.

In the majority of cases with which we are concerned in radiotelegraphy the resistance of the oscillatory circuit is negligible, the capacity is small, and conveniently measured in *microfarads* or fractions of a microfarad, and the inductance is best expressed in absolute C.G.S. electromagnetic units, viz. in *centimetres*.

Bearing in mind that a microfarad is 10^{-6} of a farad, or 10^{-15} of an absolute electromagnetic unit of capacity, we can convert the above formula (20) for the frequency n into the form—

$$n = \frac{5{\cdot}033 \times 10^6}{\sqrt{\text{capacity in microfarads} \times \text{inductance in centimetres}}} \quad . \quad . \quad . \quad (21)$$

The constant 5·033 is the value of $\frac{\sqrt{1000}}{2\pi}$, which is required in the transformation of the units, and in practice may be taken as equal to 5. We shall frequently have occasion to make use of the above formula in practical calculations.

If we only require an expression for the frequency, and are not concerned with the instantaneous value of the terminal potential difference of the condenser, or with the current at any instant, the equation (18) can be reached in a more simple manner as follows[9]: Let us suppose that a condenser having a capacity C and dielectric conductance S, discharges with oscillations, through a coil having inductance L and resistance R. Damped oscillations will then be set up, and since the potential difference of the condenser terminals is a function of the time of the form $V\epsilon^{-\alpha t}\cos\beta t$, it can be represented as proportional to the real part of ϵ^{jPt}, where $P=\beta+j\alpha$, $j=\sqrt{-1}$, and $\beta=2\pi n$, n being the frequency. Hence, at the instant when the discharge current has a value I, the potential difference of the condenser terminals is $I(S+jPC)^{-1}$, and the potential fall down the coil must be $I(R+jPL)$. Since there is no impressed E.M.F. in the circuit, the sum of these quantities must be zero. Hence—

$$(R+jPL)+(S+jPC)^{-1}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Multiplying out and writing $2a$ for R/L, $2b$ for S/C, and adding $(a-b)^2$ to both sides, we have—

$$\{jP+(a+b)\}^2=j^2\left\{\frac{1}{CL}-(a-b)^2\right\} \quad . \quad . \quad . \quad (23)$$

Hence, solving this quadratic, we have—

$$P=\beta+j\alpha=j(a+b)\pm\sqrt{\frac{1}{CL}-(a-b)^2} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Accordingly equating real parts—

$$\beta=2\pi n=\pm\sqrt{\frac{1}{CL}-\left(\frac{R}{2L}-\frac{S}{2C}\right)^2} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

If S=O this gives us the formula (19) and since $\alpha=a+b$, it shows us that the ratio of two successive oscillations is $\epsilon^{-(a+b)T/2}$, where T is the complete time-period oscillation.

It is clear, therefore, that both resistance in the coil and conductance in the condenser dielectric have the effect of reducing the frequency or lengthening the time period, but that if R/L=S/C the time period is the same as if there were no dissipation of energy at all.

6. Experimental Confirmation of Theory—The Objective Representation of Electric Oscillations.—The predictions of Lord Kelvin and Von Helmholtz, that the discharge of a condenser may take place by a series of electric oscillations or alternating and decadent discharges, subsequently received abundant experimental confirmation.

The first to give this confirmation was B. W. Feddersen, who, in 1858 and 1859, published an account of his experiments on the examination of the spark of a Leyden jar by the aid of a rapidly revolving mirror (see *Poggendorff's Annalen der Chemie und Physik*, vol. 103, p. 69). Feddersen found that the image of the spark was not always drawn out into a uniform band of light when viewed by reflection in a rapidly revolving mirror, but when the resistance of the discharge circuit was low, this image was seen to be composed of a number of separated images, thus proving the existence of separate discharges or oscillations.

[9] See J. A. Fleming, *Proc. Phys. Soc. Lond.*, vol. 25, p. 217, 1913, "Some Oscillograms of Condenser Discharges and a Simple Theory of Coupled Oscillatory Circuits."

Paalzow also described, in 1861 and 1863, experiments with a vacuum tube, which proved that these intermittent discharges of a Leyden jar are alternately in opposite directions. He passed the discharge, or part of it, through a vacuum tube, and found that if the discharge circuit had a high resistance, the difference in the appearance of the glow at the two electrodes showed that the discharge was unidirectional. If, however, the resistance of the discharge circuit was low, then the identity in appearance showed that the discharge was bidirectional. Moreover, a magnet held near the tube then split the discharge into two lines of light, thus proving that the discharge was alternating (see *Poggendorff's Annalen der Chemie und Physik*, vol. 112, p. 567, and vol. 118, p. 178).

Many years later, Vernon Boys photographed the oscillatory spark of a Leyden jar by another ingenious method. He employed a series of lenses set in a rapidly revolving disc (see Fig. 18).[10] These lenses projected upon a photographic plate images of the spark of the jar. The lenses were set at various distances from the centre of the disc so that each lens formed its own separate curved image of the spark, which was circular in form.

The Leyden jar was replaced in some experiments by a condenser formed of a number of sheets of window glass with metal plates or coatings placed between,

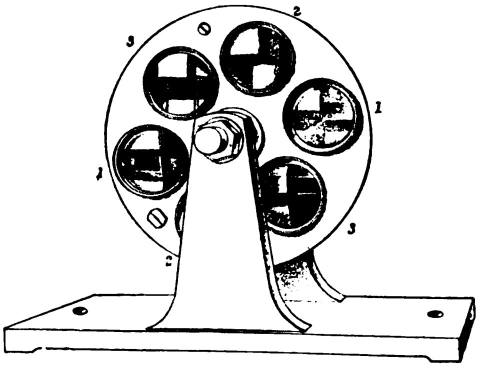


FIG. 18.—Prof. Boys' Revolving Lens Disc for Photographing Oscillatory Electric Sparks.

and was connected in series with a large inductance, so as to give to the circuit a somewhat low natural frequency.

The capacity of the condenser was measured, and the inductance also predetermined. The capacity used was about 0·1 of a microfarad. The inductance consisted of a large coil of insulated wire having an inductance of 0·026 of a henry. Hence the oscillation frequency was about 3300. The several images of the spark were projected by the revolving lenses upon a photographic plate and drawn out into segmental bands, broken up into dark and bright portions, corresponding to the electric oscillations. From the known speed of the lens disc, the time interval corresponding to each separate spark image could be calculated. One of the photographs is shown in Fig. 19. The photographs showed from 14 to 23 oscillations per spark, and the measured periodic time or frequency agreed very well with that calculated from the inductance and capacity.

Professor J. Trowbridge has also obtained some interesting photographs of oscillatory sparks taken from the discharge of a large glass plate condenser charged by means of a battery of 20,000 small lead storage cells. The battery was employed to charge the condenser plates in parallel, and then these last were changed by a commutator into series so as to add up the potentials. In this manner he obtained discharges representing a potential difference of 3 million

[10] See Vernon Boys, *Proc. Phys. Soc. Lond.*, November 1890, vol. xi. p. 1.

volts.[11] The sparks were 6 or 7 feet in length, and photographs of them showed distinctly their oscillatory character (see Fig. 20).

By using a large inductance, the frequency was reduced as low as 800. The frequency of the oscillatory spark represented in Fig. 20 is 5000.

Trowbridge found that with potentials of 3 million volts air at ordinary pressures became conducting, and he also showed by photographs that the discharges through air at this potential resembled miniature flashes of lightning, and were clearly oscillatory in character.

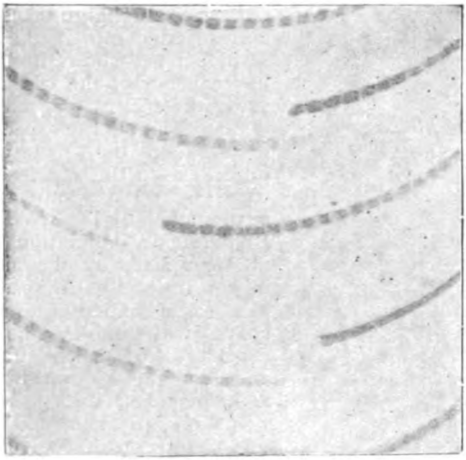


FIG. 19.—Photograph of an Oscillatory Electric Spark taken by Prof. Boys with a Revolving Lens.

Professor Trowbridge has also given in another place some beautiful reproductions of photographs of oscillatory sparks.[12] In these experiments a condenser was charged by an induction coil actuated by an alternator, and the discharge

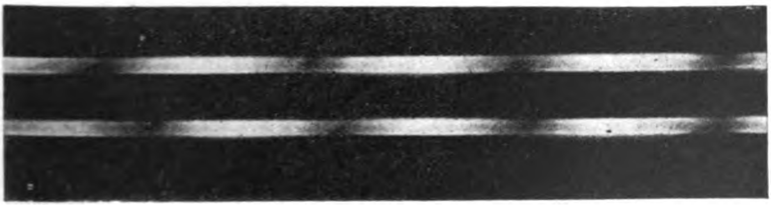


FIG. 20.—Photograph of Oscillatory Electric Sparks, taken by Prof. Trowbridge.

took place across a spark gap in a primary coil or circuit having inductance. This circuit acted inductively upon the two other circuits, also having inductance and capacity in them, and each also having a spark gap.

The images of sparks occurring at the three spark gaps were simultaneously photographed by being thrown on a sensitive plate after reflection from a revolving mirror. The spark images were therefore drawn out into bands of light (see Fig. 21), and these were serrated at the edges when the spark was oscillatory.

[11] See a paper read by Professor J. Trowbridge at a meeting of the American Academy of Arts and Sciences, Harvard University, Cambridge, U.S.A., or *Nature*, August 2, 1900, vol. 62, p. 325, "On Some Results Obtained with a Storage Battery of Twenty Thousand Cells."

[12] See *Phil. Mag.*, August 1894, ser. 5, vol. 38, p. 182, Plate VII.

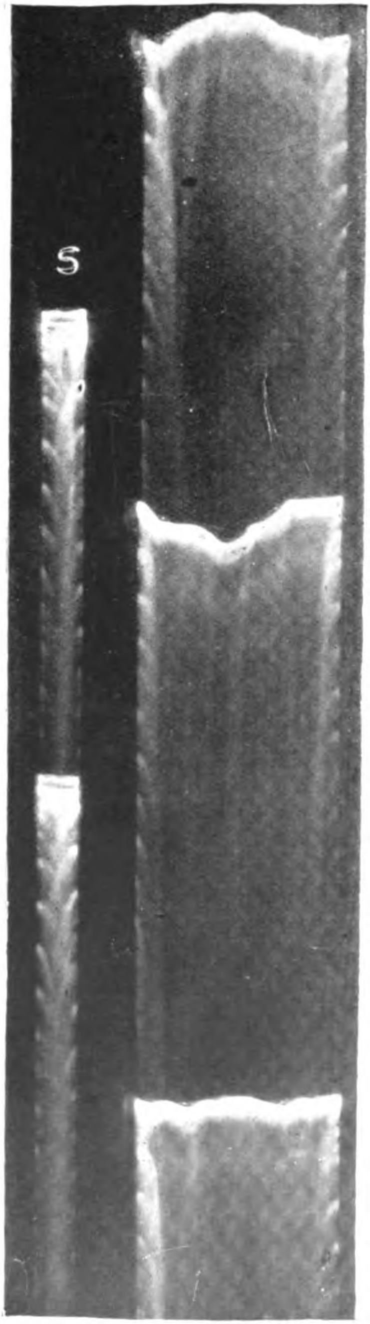


FIG. 21.—Photographs of Oscillatory Electric Sparks by Prof. Trowbridge, taken with a Revolving Mirror.

These researches clearly showed that even when the primary spark was not oscillatory it could yet give rise to an oscillatory secondary current in one of the adjacent circuits.

Another matter studied by Professor Trowbridge was the influence of the magnetic permeability of the material in and near the discharge circuit.

If the inductance coil through which the condenser discharge takes place has an iron core inserted into it, the resulting increase of inductance shows itself by the reduction in frequency of the oscillatory spark. Also since the magnetic hysteresis of the iron demands an energy expenditure, this damps out the oscillations more quickly than would otherwise be the case. This is well indicated by some photographs of oscillatory sparks taken by Dr E. W. Marchant in Lord Blythswood's laboratory at Renfrew. He photographed, by the aid of a revolving mirror, the oscillatory spark obtained by discharging a condenser formed of glass plates coated with tinfoil. The condenser had a capacity of 0·06 microfarad, and the resistance coil through which it was discharged an inductance of 0·005 henry. The frequency was therefore about 9000. The condenser was charged to 13,500 volts. The image of the spark in the revolving mirror is shown in Fig. 22.

A core of 550 iron wires No. 28 S.W.G. was then inserted in the inductance coil, and the spark again photographed. In this last case the frequency of the oscillations, as shown by the time-interval between the successive images, is markedly decreased (see Fig. 23). Also the decay of the oscillations is seen to be increased, thus showing the augmented damping due to the iron core.[13]

If the oscillations do not exceed a certain frequency, one of the simplest methods of photographing them and comparing the observed frequency with that calculated from the capacity and inductance, is the method adopted by Dr. A. Schuster and Dr. G. A. Hemsalech.[14]

In this case a circular sheet of photographic sensitive film is attached to the flat surface of a steel disc which revolves inside a closed box. The disc is capable of revolving at a speed of 120 turns per second, and as it has a diameter of about 33 cms., a point near the edge has a linear velocity of about 10,000 cms. per second, or 100,000 mms. per

[13] See a letter by Dr. E. W. Marchant, *Nature*, vol. 62, p. 413, August 30, 1900.

[14] See G. A. Hemsalech, *Journal de Physique*, February 1902, "La Constitution de l'étincelle électrique."

second. The box in which the disc is contained has a small slit opposite the periphery of the disc, and by means of a lens an image of another slit, illuminated by an electric spark behind it, can be thrown upon the sensitive film. When the spark is continuous, the photographic image on the film is a band of light, the length of which corresponds with the duration of the spark, but when the spark is oscillatory, the image is a series of separated images. As 1 mm. between the images corresponds to about 0·00001 of a second, we can determine from the angular separation of the images and the speed of the disc the frequency of the oscillations. In Fig. 25 is shown a photograph of the oscillatory spark taken by Dr. Hemsalech by this means. Fig. 24 shows the image on the plate when the disc is at rest, and Fig. 28 shows the image of the oscillatory spark

FIG. 22.—Coil without Iron Core.

FIG. 23.—Coil with Iron Core.

Photographs of Oscillatory Electric Sparks, taken with a Revolving Mirror by Prof. Marchant.

produced when a condenser consisting of eight large Leyden jars (capacity about 0·048 mfd.) was discharged through an inductance of 0·042 henry or 42,000,000 cms.[15]

The frequency is therefore about 3500 complete periods per second.

If the bobbin forming the inductance had an iron core 18 mms. in diameter inserted into it, the effect was to greatly reduce the number of oscillations in the train (see Fig. 26). This photograph shows clearly that the iron core absorbs some of the energy of the discharge and acts as an additional damping. As already stated, this is due to the magnetic hysteresis loss and to the energy loss due to the eddy electric currents set up in the core by the rapid oscillatory magnetization to which it is subjected. These photographs are interesting because

FIG. 24.

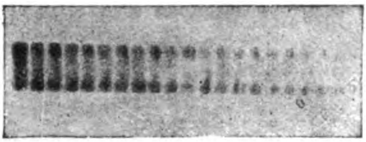


FIG. 25.

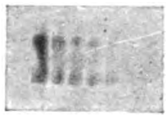


FIG. 26.

Photographs of an Oscillatory Electric Spark, by Dr. Hemsalech.

they reveal to us something of the mechanism of the discharge. By examining the image of the spark with a spectroscope, Dr. Schuster and Dr. Hemsalech have shown that in this case the first effect of the initial oscillation is to pierce the air between the discharge balls, or rather that the electric current constituting the first oscillation is carried by conduction through the air of the spark gap. This forms the so-called "pilot spark," which is well shown in certain photographs. The energy of this first oscillation volatilizes some of the metal of the spark balls and creates a supply of metallic vapour, which conducts the next oscillation, and thereafter each oscillation travels in or by the conducting metallic vapour produced by the preceding oscillation, and in turn creates a further supply. Hence the energy of the oscillatory discharge is chiefly expended in creating the metallic vapour between the electrodes whereby the discharge passes. Interesting questions therefore arise as to the resistance of the electric spark, and whether this resist-

15 See A. Schuster and G. A. Hemsalech, *Phil. Trans. Roy. Soc.*, 1899, vol. 193, p. 189. Also G. A. Hemsalech, *Comptes Rendus*, 1901, vol. 190, p. 898; vol. 132, p. 917.

ance remains constant during the whole period of a train of oscillations. We shall return to the consideration of this matter in connection with the damping of electrical oscillations in circuits containing a spark gap.[16] Meanwhile it is sufficient to say that the resistance of an oscillatory spark as used in wireless telegraphy is rarely more than the fraction of an ohm. It does not remain constant during the discharge, but increases towards the end of each train of oscillations. Generally speaking, it may be said that the larger the quantity of electricity which passes at each oscillation, the less is the equivalent spark resistance.

It is found, however, that the equivalent resistance of a single spark or single isolation group of oscillations is different, and greater than that of a closely recurring series of oscillatory electric discharges.

Whilst the above-described methods enable us to photograph and thus analyze an oscillatory discharge, there are other processes which enable us to observe visually the oscillations which compose the train, or at least some optical effects equivalent to them. Four such methods are known and used, viz. those depending on the use of an oscillograph, a Braun cathode ray tube, a Gehrcke oscillographic vacuum tube, and lastly a method which depends upon the effects of an air blast upon an oscillatory spark.

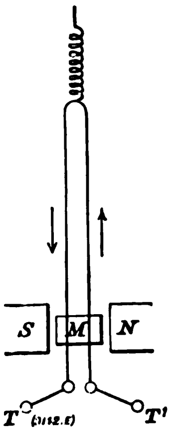


FIG. 27.—Diagram of Duddell Oscillograph.

The first of these methods with the oscillograph is only suitable for the objective representation of or for photographing oscillations of relatively low frequency, say, a few hundreds up to 1000 or 1200 per second.

An oscillograph is a type of galvanometer in which the movable part of the instrument, whether coil or needle, which is displaced when a current flows through it, has such a high natural time period of its own, from $\frac{1}{2000}$ to $\frac{1}{10000}$ of a second, that it can follow consecutively the fluctuations in the value of a periodic current passing through the instrument, when these are not too rapid.

In one form as constructed by Duddell, it consists of a loop of fine wire (see Fig. 27) placed in a strong magnetic field having a small mirror, M, resting on the two wires forming the loop. A ray of light from an arc lamp falls on this mirror, and is then again reflected from a larger mirror on to a screen or photographic film. When an alternating current is passed through the loop of wire, the two sides of the loop vibrate so that the attached mirror oscillates synchronously about a vertical axis. The second mirror is made to oscillate by a small motor synchronously about a horizontal axis, and the combined motions cause the ray of light to possess a double motion and to delineate on the screen a curve which reproduces the wave form of the alternating current in the wire loop of the oscillograph.

To adapt this appliance to delineate the discharge of a condenser, the author fixed on the shaft of an alternator a disc of insulating material, having on its edge brass sectors. Against this disc three brass wire brushes press, and the sectors are so arranged that as the disc revolves the middle brush is alternately connected first to one and then to the other of the outside brushes. If, then, a condenser, battery, and oscillograph loop are joined up as shown in Fig. 28, it will be evident that as the disc revolves the condenser is alternatively charged by the battery and discharged through the oscillograph. The number of sectors on the disc is made the same as the number of pairs of magnetic field poles of the alternator. The small synchronous motor of the oscillograph is then driven by the current of the alternator. Hence the ray of light reflected on to the screen of the oscillograph continually repeats the same motion, and a naturally non-repetitive process, like the discharge of a condenser, is made periodic, and therefore suitable for record by the oscillograph.

Photographs can then be taken showing the variation of the condenser discharge current for various capacities, inductances, and resistances in the discharge

[16] See Chap. III. of this treatise.

circuit. In the Pender Electrical Laboratory, University College, London, a number of such discharge curves have been photographed, using a paraffin paper condenser of capacity variable between 0·5 and 7·0 mfds., an inductance consisting of a long helix of copper wire of 31 millihenrys = (31×10^6) cms., and added non-inductive resistances of various values. The curves given in Plate I., Figs. 1 to 5 (see p. 96), are reproductions of these photographs. Curves 1 to 5, inclusive, are the discharge curves of various capacities from 4·0 to 0·25 mfd. through an inductance always equal to 31·5 millihenrys. In curves 6 to 10, inclusive, a capacity of 5·0 mfds. had non-inductive resistances varying from 4·4 to 112 ohms added in series with it and with the inductance coil, which itself had a resistance of 7 ohms and inductance of 31·5 millihenrys.

These photographs show in a striking manner the way in which the time period increases with the capacity. They also show how the introduction of resistance into the circuit damps out the oscillations. It should be noted that in some of the photographs the time interval allowed by the commutator for the discharge was not sufficient to take in all or nearly all the oscillations which would have taken place if circumstances had permitted.[17]

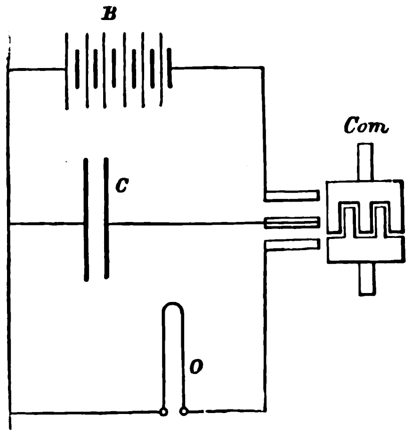


FIG. 28.—Arrangement of Condenser, *C*; Commutator, *Com*; battery, *B*; and oscillograph, *O*, for delineating Condenser Discharge Curves.

The resistance of the circuit at which the oscillations just become extinguished or dead-beat agrees well with that calculated from the formula (18), viz. 156 ohms.

Another method of objective representation is found in the use of a Braun cathode ray tube.[18] The tube is a form of high vacuum tube, having at one end a cathode from which cathode rays are projected (see Fig. 29). The tube has in it two baffle screens A with small holes in them, and on an enlarged anticathode end a screen, B, of phosphorescent material. When the tube is set in operation by a large electrostatic electrical machine, such as a Voss or Wimshurst, giving a unidirectional and continuous discharge, so that a continuous projection of cathode particles takes place from the cathode, we see on the screen a brilliant point of light due to the cathode ray phosphorescence. This ray is a flexible conductor. If, then, a pair of coils traversed by an electric oscillation are placed on either side of the neck of the tube, the cathode ray is deflected up and down by the alternating magnetic field of the coils, and the spot of light on the screen is expanded into a

[17] Some excellent photographic curves representing the damped oscillations of condenser discharges have also been taken by Professor E. Taylor Jones with a short-period electrometer used for determining the frequencies of slow electrical oscillations (see *Phil. Mag.*, vol. **14**, 6th series, August **1907**, p. **238**). The experimental results obtained by Professor Taylor Jones also agree with the Kelvin formula with considerable exactness. These are referred to in § **11**, Chap. III.

[18] See Professor F. Braun, *Wied. Ann. der Physik*, 1897, vol. 60, p. 552.

line of light. If this line of light is examined in a rotating mirror suitably placed, it can be expanded into a wavy decrescent line of the form of the lines in the photographs taken with the oscillograph. Although the plan succeeds in producing an objective representation of the discharge current, it is more troublesome to operate, and not so suitable for quantitative work as the method employing the oscillograph above described.

Professor F. Braun and Dr. J. Zenneck have pointed out that such a tube may be used to trace the forms of alternating current curves (see *Annalen der Physik*, 1902, vol. 9, p. 497); and Dr. W. Mansergh Varley has described the use of it in high frequency work.[19]

The arrangement used in connection with the Braun tube for delineating alternating current curves is shown in Fig. 29. For the optical delineation of

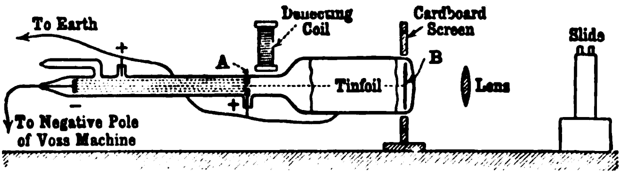


FIG. 29.—Method of employing a Braun Cathode Ray Tube to delineate Alternating Current Curves.

oscillatory discharges, Messrs. Varley and Murdoch recommend an electrostatic method of deflecting the cathode ray. In Fig. 30 a diagrammatic scheme of the apparatus is shown. The Braun tube T has its cathode terminal led to the negative pole of a Voss machine driven by a small electric motor. Two brass

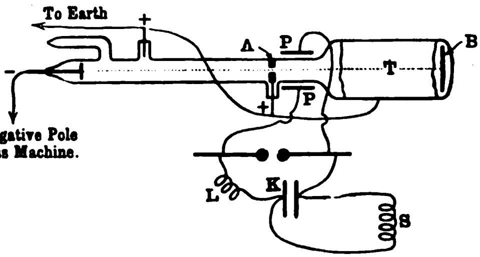


FIG. 30.—Method of employing the Braun Cathode Ray Tube with Electrostatic Deflection Plates for delineating Condenser Discharge Curves.

plates, P, P (see Fig. 30), are placed on either side of the tube just beyond the diaphragm in it, and these are connected with the spark balls of the oscillatory circuit containing a condenser, K, and an inductance, L. The plates P, P, were about $3\frac{1}{4}$ inches by $2\frac{1}{4}$ inches in size, and placed 3 inches apart. The capacity was 0·003 mfd., and the inductance about 1 henry, being the secondary circuit of a small transformer. On the phosphorescent screen B is seen a brilliant green spot of light when the cathode tube is in action, and this expands into a bright line when the condenser discharges take place, since the electrostatic field then produced between the plates P, P deflects the cathode ray up and down. If this line of light is examined in a revolving mirror, the usual form of discharge curve of a condenser is seen in it. In carrying out this experiment, the widened part of the cathode tube should be covered with tinfoil and earthed. An interesting set of

[19] See Dr. J. Mansergh Varley, *Phil. Mag.*, 1902, ser. 6, vol. 3, p. 500; and also Dr. Varley and Mr. W. H. F. Murdoch, *The Electrician*, 1905, vol. 55, p. 335, on "Some Applications of the Braun Cathode Ray Tube."

experiments was carried out in 1895 by Professor A. Hay, in which the discharge curve of a condenser was graphically delineated by a modification of the Joubert point-by-point method so much used in connection with alternating currents. For the details of these experiments, the reader is referred to the original paper in *The Electrician*, 1895, vol. 35, p. 840. The results confirmed experimentally the predictions of the theoretical formula for the frequency and strength of the discharge at various instants.

A very beautiful method of rendering the oscillations in an oscillatory spark visible has been employed by Lehmann, Klingelfuss,[20] Zehnder,[21] and Hemsalech.[22] Hemsalech's method has the great advantage of rendering the oscillations visible to the eye, whilst at the same time they can be photographed if necessary. The method is as follows:—

Two plates of thick copper, A and B (see Fig. 31), about 8 mms. in thickness, 8 or 10 cms. in length, and 4 or 5 cms. in width, have one pair of edges bevelled off, and these edges are set at a slight angle to one another. On the top of these plates are fixed two screws, *a* and *b*, by means of which are clamped two short thick platinum wires, the points of which project very slightly beyond the edges of the copper. Above this is fixed a glass tube through which a powerful blast of air

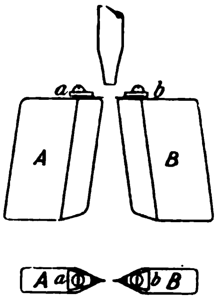


Fig. 31.

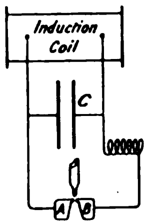


Fig. 32.

Figs. 29, 30, 31, *and* 32 *are, by kind permission of Dr. Hemsalech and M. Ch. Delagrave, taken from "La Science au XX^e Siecle."*

can be forced, the diameter of the jet being 3 mms. and the interval between the platinum points 3 or 4 mms. The jet of air should issue with the velocity of about 36 metres per second. The two plates are connected through an inductance coil and a condenser, C (Fig. 32), and with an induction coil which can make an oscillatory discharge across the platinum points connected with the two copper plates A and B. If the air blast is set in operation, then when the induction coil is set working it charges the condenser, which is discharged across the spark gap intermittently. This intermittent spark is an oscillatory discharge, but the oscillations are of course superimposed. When the air blast is started, the successive oscillatory discharges are separated from one another and move down between the edges of the copper plates, each successive discharge being represented by a bright band in the shape of an arrow-head, and the whole series of oscillations constitute one train, forming a band traversed with V-shaped bars of light, one below the other (see Figs. 33 and 34). This spectrum can be photographed and also observed by the eye. The method has the great advantage that we can observe the effect of varying the different factors in the discharge circuit. Thus, for instance, the introduction of any source of energy absorption into the circuit, such as the insertion of iron wires into the inductance coil, causes a diminution of the number of oscillations in a train, and therefore shortens the spectrum; and in

[20] Klingelfuss, *Ann. der Physik*, 1901, vol. v. p. 837.
[21] L. Zehnder, *Ann. der Physik*, 1902, vol. ix. p. 899.
[22] G. Hemsalech, *Comptes Rendus*, 1903 vol. 140, p. 1103.

the same way anything which causes the absorption of energy in the condenser produces an immediate effect upon the appearance of this drawn-out discharge. The method is particularly applicable for lecture illustration.

Another useful method of obtaining an objective representation of electric oscillations is by the use of an oscillograph vacuum tube invented by Dr. Gehrcke. This consists of a glass tube having in it two polished aluminium strips or wires (see Fig. 35), the strips being about 10 cms. long and 15 mms. wide, fixed to a platinum wire sealed through the glass and nearly meeting in the middle of the tube. The tube is exhausted of its air and then filled with nitrogen under a pressure of 8 mms. Under these circumstances, if a sufficiently large difference of potential is made between the electrodes, the glow light extends over both electrodes for certain distances proportional to their difference of potential. When such a tube is connected to the terminals of a condenser which is creating an oscillatory discharge, the length of the glow light on the aluminium strips or wires varies with every change of potential of the condenser terminals. If, then, the tube is examined in a revolving mirror, the successive images are separated out from one another into a number of bars of light, decreasing successively in length if it is a

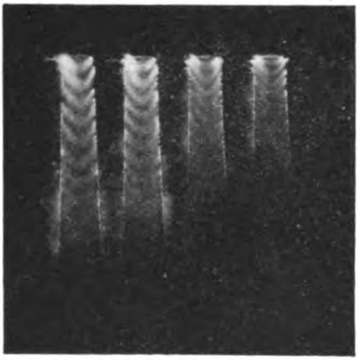


FIG. 33.

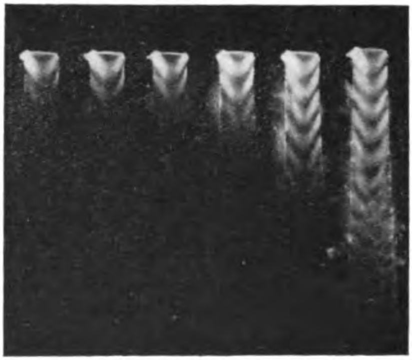


FIG. 34.

Photographs of Oscillatory Discharges taken by Dr. Hemsalech.

damped oscillation, or maintaining a uniform length if it is an undamped oscillation. Fig. 36 is reproduced from a photograph thus taken by Herr Hans Boas of a damped electric oscillation.

A modification of the tube, in which there are two anodes and one cathode, enables two photographs to be taken simultaneously. To observe the oscillations it is, of course, necessary to employ a mirror driven at a very high speed. A convenient arrangement is that of Hans Boas, in which a small continuous current motor driven at a very high speed has on its shaft a polished metal mirror, concave or plane, according to whether it is for eye observation or for photographs. The mirror reflects an image of the electrodes of the oscillograph tube on to the eye, or the photographic plate, and then at intervals, when a discharge takes place at the moment when the mirror is in the right position, the eye will perceive an image as in the photograph in Fig. 39, which consists of separated-out images of the discharges taking place with each oscillation. When photographed on a plate, the frequency of these oscillations can be determined if the number of revolutions of the mirror per second is known, and also the distance of the mirror from the plate.

Probably the most exact confirmation of the truth of the Kelvin formula (20) for the natural time period of a low resistance oscillatory circuit has been furnished by the measurements made by Glazebrook and Lodge on the oscillatory discharge

of an air condenser employed as a means of determining the value of "v," or the ratio of the electromagnetic and electrostatic units.

In the formula for the time period $T=2\pi\sqrt{CL}$, let us suppose that capacity C is measured in electrostatic units, and L in electromagnetic units. Then, since an

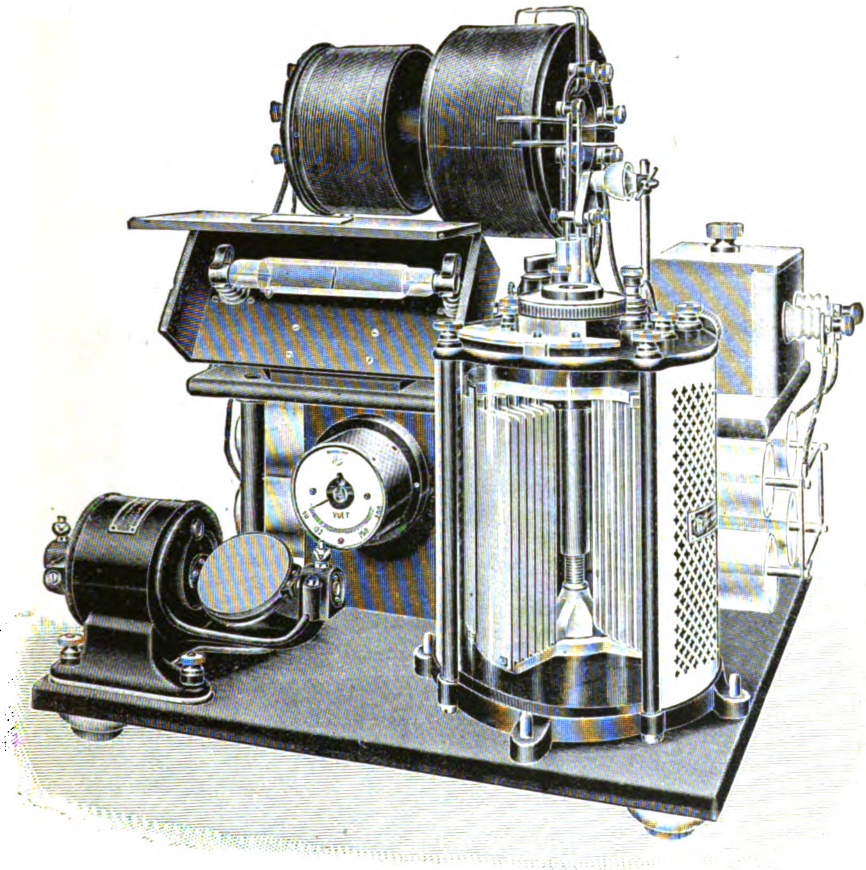


FIG. 35.—Apparatus for Using the Gehrcke Oscillograph Tube for Photographing Electric Oscillations. (Hans Boas.)

electromagnetic unit of capacity is v^2, or 9×10^{20} times larger than an electrostatic unit, we have to introduce a factor and write the formula in the form—

$$T=\frac{2\pi}{v}\sqrt{CL},$$

where C is capacity measured in electrostatic units, and L is inductance measured in centimetres or electromagnetic units. Glazebrook and Lodge used this expression to determine the value of v from measurements of T, C, and L (see *Cambridge Philosophical Transactions*, vol. 18, p. 136, 1900), and found that

$$v=3\cdot009\times10^{10}.$$

From numerous determinations of "v" by other methods, its numerical value is known to be very near 3×10^{10}. Hence the fact that the numerical values of T

determined by this last formula, when we are given the numerical values of C, L, and v, agree with the periodic time found by the measurement of photographs taken of the discharge spark of the circuit on a revolving photographic plate, affords strong proof of the accuracy of the formula.

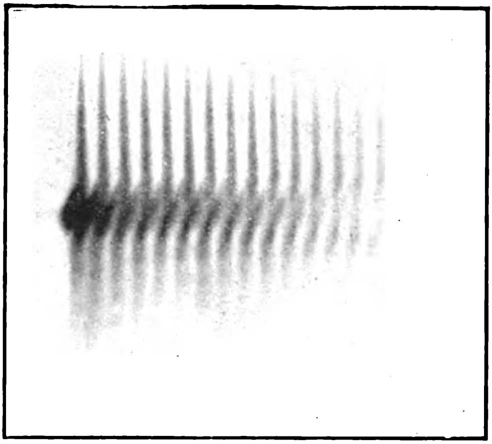


FIG. 36.—Photograph of a Damped Electric Oscillation taken wtih Gehrcke Oscillograph Tube. (Hans Boas.)

The oscillation frequency employed by Glazebrook and Lodge was comparatively low, viz. from 880 to 1600.

The whole question of the agreement between the observed values of the frequency in an oscillatory electric circuit, and that predicted by Kelvin's law as in formula (20) has been examined in a very thorough manner by Professors A.

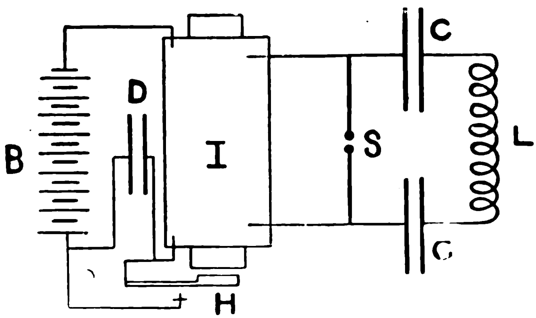


FIG. 37.—Diagrammatic Representation of the Arrangement of Apparatus for the Production of Damped Electric Oscillations. B, battery; I, induction coil; S, spark balls; C, C, condensers; L, inductance coil; H, hammer break; D, coil condenser.

Battelli and L. Magri.[23] They employed a revolving mirror driven by a steam or air turbine to photograph the spark. The image of a slit behind which an oscillatory spark was formed was reflected from the mirror and focussed on a photographic plate. From the intervals between the images as determined by the speed of the mirror the time period of oscillation was found, and this was compared with the time period calculated from the capacity and inductance of the circuit by

[23] See *Phil. Mag.*, 6th ser., vol. 5, pp. 1, 620, 1903.

the Kelvin formula, viz. $T=2\pi\sqrt{CL}$. They found in three sets of experiments a very good agreement as follows:—

Time period observed in microseconds or millionths of a second	53·76	3·024	1·212
Time period calculated	53·17	3·008	1·201

The frequencies used varied therefore from about 800,000 to 20,000.

The condenser employed was made of glass plates, the capacity being very carefully determined by an absolute method. The inductances used consisted of copper wire spirals wound on marble cylinders. The inductance of these spirals was calculated, and allowance made for the variation of inductance with frequency (see Chap. II.), and for the fact that the inductance of a spiral is greater for high frequency currents than for steady or low frequency currents.

The numerous measurements made by Battelli and Magri may be said to have established the agreement between theory and experiment within 1 or 2 per cent.

7. Apparatus for the Production of Damped Trains of Intermittent Electric Oscillations.—The usual method employed for the production of damped

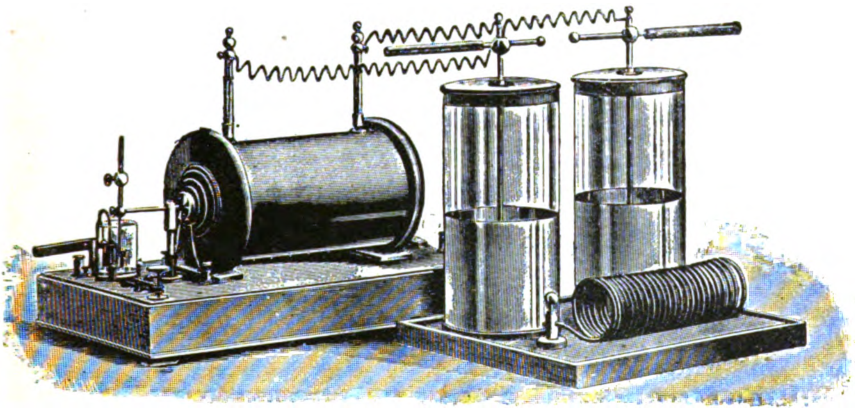


FIG. 38.—Perspective View of the Arrangement of Apparatus for the Production of Damped Electric Oscillations, consisting of an Induction Coil, Condensers (Leyden Jars), Spark Gap, and Inductance Spirals.

electric oscillations is the intermittent discharge of a condenser of some kind, the charge and discharge being repeated at regular and frequent intervals.

The arrangement consists of a condenser suitable for being charged to a high potential, which is then discharged through an inductance of low resistance, thus creating a train of oscillations, and this process is repeated several times in a second.

One of the simplest and most convenient arrangements consists in connecting to the secondary terminals of an induction coil a high tension condenser, such as a Leyden jar or jars, joined in series with an inductive resistance. The secondary terminals of the induction coil are provided with spark balls, or else connected to a separate ball-discharger, and the arrangement is as shown diagrammatically in Fig. 37, and in perspective in Fig. 38. When the induction coil is set in action, at each interruption of the primary current an electromotive force is created in the secondary circuit. This charges the condenser, and if the spark balls are placed at a suitable distance apart, easily found by trial, the electromotive force breaks down the insulation of the air between the spark balls when it reaches a certain value, and the charged condenser then discharges across the spark gap and creates electric oscillations in the inductance coil. This process is repeated at every interruption of the primary circuit of the coil, and if the adjustments are properly

made, it results in the production of a bright crackling spark between the spark balls, which is in fact a continuous series of oscillatory discharges with short intervals of time between them, corresponding to the groups of electric oscillations produced in the inductive circuit.

In place of an induction coil, any other type of generator of high electromotive force might be employed; such, for instance, as an electrostatic machine, a voltaic battery of a large number of cells, a continuous or alternating current dynamo, or an alternating current transformer. If, however, a voltaic battery, continuous current high tension dynamo, or alternating current transformer, is employed, the arrangement will not operate well unless some means are used to continually destroy or prevent the electric arc discharge which tends to be produced and maintained across the spark gap. The spark which occurs at this gap must consist wholly, or nearly entirely, of the discharge coming from the condenser, and not have superimposed on it any true electric arc discharge, either continuous or alternating, proceeding directly from the source of the electromotive force. We shall discuss in a later section the various devices for controlling the operation of the electric generator in this respect. In the majority of cases, the most convenient source of electromotive force is found to be either a large induction coil, the primary circuit of which is traversed by an interrupted continuous current or alternating current, or else the employment of some form of alternating current transformer.

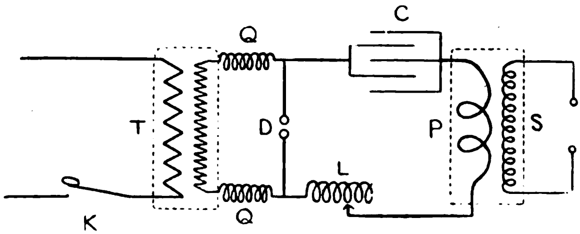


FIG. 39.—Arrangement of Apparatus for producing Damped High Frequency Electric Oscillations by means of an Alternating Current Transformer.

We proceed to consider in further detail the practical arrangements which have to be employed. It is essential that the source of electromotive force, whatever its nature, shall not only be able to create a large difference of potential between the surfaces of some form of condenser, but shall also be able to supply a certain minimum electric current. Hence, for many purposes, an electrostatic electrical machine would be unsuitable, because although capable of producing a large difference of potential, it acts like an electric generator of very high internal resistance, and therefore the current which can be obtained from it, that is, the rate of supply electricity, is very small. The employment of voltaic cells, or secondary batteries, as a source of electromotive force, presents many advantages, but the very large number of cells required and the expense of maintaining them in order renders this form of electromotor more suitable for special research purposes than for general use.

Professor Trowbridge has employed a battery of 20,000 small secondary cells, giving an electromotive force of 42,000 volts, in special researches on electric oscillations. For this purpose high potential continuous current dynamos have also been used, but although the difficulties involved in the commutation of these high potential continuous currents have been overcome, at least as far as the construction of continuous current dynamos up to 10,000 volts is concerned, yet the complications which are involved in the use of the continuous current do not compensate for the other advantages.

Hence practically we are limited at the present moment to one of two appliances as a source of high electromotive force for charging the necessary capacity, viz. either an induction coil or an alternating current transformer.

In the next place, we have to provide some form of condenser to receive and

store the energy. This must be one capable of being charged to a potential of 20,000 volts, or more, as otherwise the oscillations produced are very feeble. The condenser has to be placed in series with an adjustable spark gap and with an inductance which generally consists of the primary circuit of an air core transformer called an oscillation transformer.

In the next place there must be means, such as certain choking coils or inductances, for preventing the formation of an electric arc between the spark balls, and, lastly, a key for controlling the operation of the arrangement at pleasure. Accordingly there are seven elements in the complete oscillation-producing appliance, which are as follows :—

1. The induction coil transformer or source of electromotive force (T).
2. The condenser (C).
3. The discharger of spark balls (D).
4. The arc quenching inductances (Q).

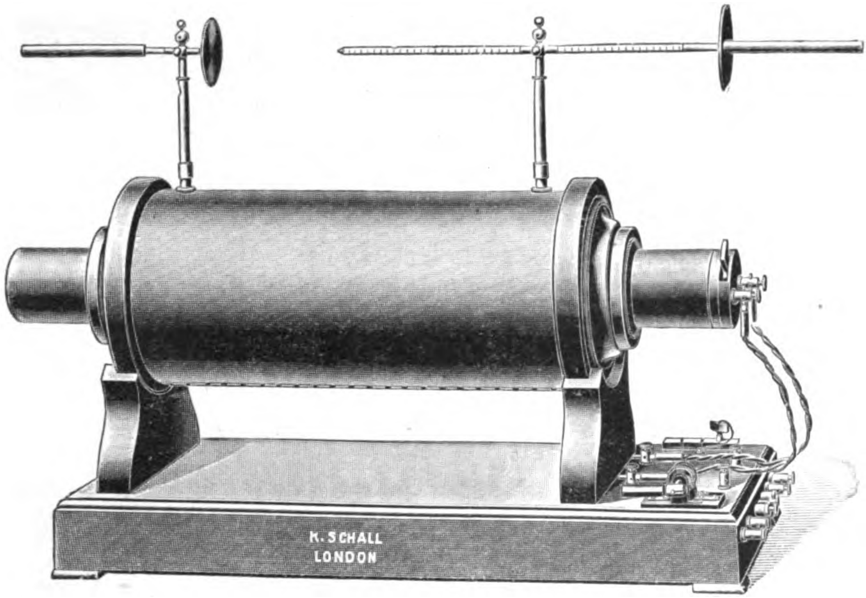


FIG. 40.—10-inch Spark Induction Coil.

5. The oscillation transformer (PS).
6. The adjustable inductance for varying the period (L).
7. The controller or key in the primary circuit of the coil or transformer (K).

These several elements have each to be considered separately with reference to their best practical forms for various purposes.

Diagrammatically, the complete appliance for producing trains of damped electric oscillations is as shown in Fig. 39, where the letters have reference to the parts or elements 1 to 7 as enumerated above.

When the key K is closed, and the apparatus in operation, we have trains of intermittent decadent electrical oscillations set up in the circuit CPL, and if the terminals of the secondary circuit S of theoscillation transformer are near together, we have high potential high frequency oscllatory sparks passing between them.

There are certain modifications of the above arrangement which will be considered later, but the above described apparatus in a typical form is generally called a Tesla apparatus for the production of high frequency electric currents.

8. Induction Coils for Creating Electric Oscillations.—It is not necessary to occupy space with any elementary explanation of the construction of the

induction coil. A coil very generally employed for the production of electric oscillations is that known as a 10-inch coil, that is, one which is capable of giving a 10-inch spark between pointed conductors in air at the ordinary pressure (see Fig. 40). The construction of a large induction coil is a matter requiring very great technical skill, and should not be attempted without considerable previous experience in the manufacture of smaller coils.[24] A coil of the above size usually has a primary circuit consisting of a length of 300 or 400 feet of insulated copper wire, No. 12 or No. 14 S.W.G. The secondary circuit would consist of a double silk-covered copper wire, No. 34 or No. 36 S.W.G., a length of 10 to 17 miles of wire being employed, according to the diameter of the wire selected. It is necessary to wind the secondary circuit of such a coil in a large number of flat sections, the sections being prepared separately, and each carefully insulated with paraffin wax and discs of shellaced paper, the coils being so wound in two layers that there are no joints between sections at the inside, but all soldered junctions are at the outside ends. A number of such sections, varying from 100 to 500, are employed in building up the secondary coil, and these are slipped on to a thick ebonite tube, in the interior of which is placed a primary circuit and the iron core.

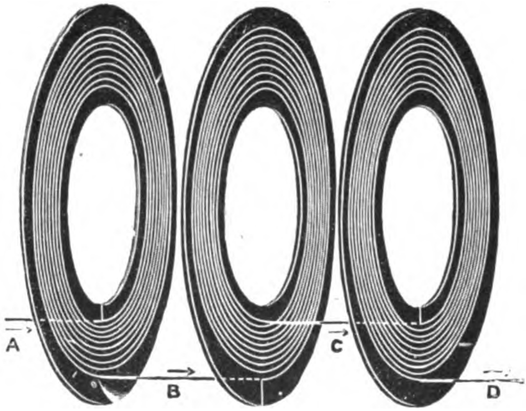


FIG. 41.—Method of building up the Secondary Circuit of an Induction Coil in Sections by Leslie Miller's mode of winding.

A special form of winding machine has been invented by Leslie Miller (British Pat. Spec., No. 5811 of 1903) for winding the flat sections of the secondary bobbin, so that no joints at all between sections are necessary, the secondary wire being continuous from end to end (see Fig. 41).

By this invention the secondaries of induction coils and transformers can be wound in a manner not hitherto accomplished. The secondary bobbins in induction coils, made to give from 10- to 18-inch sparks, are built up in the Miller process of 700 to 1200 separate single wire sections, with a disc of paper between each section, the wire being continued from one section to the other without any joint. The method of winding will be readily understood from the diagram in Fig. 41. For the sake of clearness, this diagram shows the sections widely separated from one another, whereas in reality they are closely compacted together.

The construction of the secondary circuit must be such that no parts of the econdary wire, which are at great differences of potential when the coil is in action, are near together, and one very important point is the construction of the

[24] Detailed instructions for the manufacture of large induction coils are given in a "Treatise on the Construction of Large Induction Coils," by A. T. Hare (Methuen & Co.). Particulars of many large coils are given in a treatise on "The Alternate Current Transformer," by J. A. Fleming, vol. ii. chap. 1 (Messrs. Benn Bros., 8 Bouverie Street, London, E.C.).

secondary in a sufficient number of flat sections. Another essential detail is the sufficient insulation of the secondary bobbin from the primary coil. With this object, in all large induction coils the primary circuit and its iron core are entirely enclosed in a stout ebonite tube, the walls of which must be at least half an inch in thickness, and it should preferably be overlaid with a layer of paraffin wax an inch in thickness. On the compound tube so formed the sections forming the secondary circuit of the coil are slipped. When the sections in the secondary circuit have all been joined up and the connections well insulated, the whole of the secondary circuit should be compressed and immersed in molten paraffin wax. This is best done by enclosing the secondary circuit in an iron box of the required size, which, after being closed, is heated and the air exhausted from it. Molten paraffin wax is then allowed to flow in under pressure and set solid. In this manner the entire secondary circuit is penetrated with paraffin wax, and the production of vacuous spaces as the wax cools is prevented. The silk-covered copper wire employed in winding the secondary should also be heated to a temperature above that of boiling water, previous to being immersed in paraffin wax, during the winding of the secondary sections. When completed, the secondary winding is enclosed in a cylinder of ebonite, and thick ebonite cheeks are fitted to the ebonite tube on which the secondary is wound. As the surface of ebonite deteriorates in insulating quality by exposure to light, it is better to enclose the completed induction coil in a wooden box, which is filled in solid with paraffin wax, the ends of the secondary circuit being brought out through thick ebonite tubes, which pass right down into the wax. Instrument makers are too prone to study external appearance in instrument making, and the ordinary type of induction coil, though very suitable for the lecture table, is not at all well adapted for practical use in connection with wireless telegraphy, when the coil has to be used in damp exposed places, such as in a lighthouse or on board ship.

The primary circuit of a 10-inch spark coil generally consists of 360 turns of No. 12 S.W.G. copper wire wound round an iron core consisting of a bundle of soft iron wires, 2 inches in diameter. It has resistance of about 0·46 ohm and an inductance of 0·02 henry. The secondary circuit of such a coil may consist of 17 miles of No. 34 S.W.G. copper wire, making about 50,000 turns. This coil would have a resistance at ordinary temperatures of about 6600 ohms, and when the iron core is in it an inductance of 460 henrys. The mutual induction between the primary and secondary circuits would be about 2·75 henrys, and with a primary current of 10 amperes the coil should give a 10-inch spark. A smaller coil giving a 6-inch spark would usually have a primary circuit with a resistance of 0·426 ohm and an inductance of 0·013 henry. The secondary circuit would be wound with No. 36 S.W.G. wire, which would have a resistance of 9750 ohms and an inductance of 234 henrys, the mutual inductance between the primary and secondary circuit being 1·5 henrys.

An important matter in connection with an induction coil to be used for creating electrical oscillations is to secure a sufficiently small resistance in the secondary circuit. The purpose for which the coil is employed is to charge a condenser of some kind.

If a constant electromotive force V is applied to the terminals of a condenser having a capacity C, the condenser being placed in series with a wire of resistance R, then the full difference of potential V is not created between the terminals of the condenser instantly, but the terminal potential difference rises up gradually and any time t seconds after the contact is made, an expression for its value, v, at that instant may be obtained, as follows :—

Let i be current at the time t in the inductionless resistance R in series with the condenser, then Ri is the fall of potential down this resistance. Also $C\frac{dv}{dt}$ is the current through the condenser and resistance. Hence we must have—

$$CR\frac{dv}{dt}+v=V \qquad (22)$$

The solution of this equation is—

$$v=V\left(1-\epsilon^{-\frac{t}{CR}}\right) \qquad (23)$$

In the above equation the letter ϵ stands for the number 2·71828, the base of the Napierian logarithms, and R for the resistance in megohms of the wire in series with the condenser, of which the capacity is C microfarads. This equation shows that the potential difference v of the terminals of the condenser does not instantly attain a value equal to that of the steady impressed electromotive force V, but that it rises up gradually. Thus, for instance, suppose that a condenser of 1 microfarad is being charged through a resistance of 1 megohm, by an impressed constant voltage of 100 volts, the equation shows that at the end of the first second after contact the terminal potential difference of the condenser will be only 63 volts, at the end of the second second 86 volts, and so on. The gradual increase in v with time is shown by the curve in Fig. 42. The equation indicates that only after an infinite time is the terminal potential difference v of the condenser plates equal to the impressed electromotive force V, viz. to 100 volts in this instance. Since, however, ϵ^{-10} is an exceedingly small number, in ten seconds the condenser would be practically charged with a voltage equal to 100 volts. The

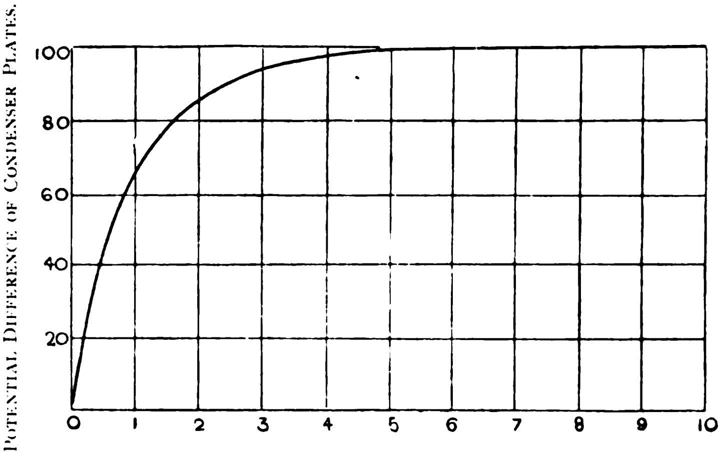


Fig. 42.—Curve showing the Gradual Rise in the Terminal Potential Difference of a Condenser with Time, under a Constant Impressed Electromotive Force of 100 volts, when a Condenser of 1 microfarad capacity is charged through a resistance of 1 megohm.

product CR in the above equation is called the *time-constant* of the condenser, and we may say that the condenser is practically charged after an interval of time equal to ten times the time-constant, counting from the moment of first contact between the condenser and the source of constant voltage. The time-constant is to be reckoned as the product of the capacity C in microfarads and the resistance of the charging circuit R in megohms. To take another illustration. Supposing we are charging a condenser having a capacity of $\frac{1}{10}$ of a microfarad through a resistance of 10,000 ohms. Since 10,000 ohms is equal to $\frac{1}{100}$ of a megohm, the time-constant would be equal to $\frac{1}{1000}$ second. Hence, in order fully to charge the above capacity through the above resistance, it is necessary that the contact between the source of voltage and the condenser should be maintained for at least $\frac{1}{100}$ part of a second.

We may put the equation (23) in a form more convenient for calculation.

We have $V - v = V\epsilon^{-\frac{t}{RC}}$ . . . . (24)

Hence $t = RC\{\log_\epsilon V - \log_\epsilon(V - v)\}$ . . . . (25)

or $t = 2{\cdot}3026\ RC\{\log_{10} V - \log_{10}(V - v)\}$ . . . . . (26)

This last expression can be employed to calculate the value of any of the four quantities v, R, C, or t, when three of them are given.

When an induction coil has its secondary terminals connected to a condenser, we may regard the electromotive force created in the secondary circuit as acting through the resistance of the secondary circuit to charge the condenser.

Hence, in order that the charging of the condenser may be achieved in the shortest possible time, it is desirable that the secondary circuit of the coil should have as low a resistance as possible, consistent with permissible cost of construction. This involves winding the secondary circuit with a rather thick wire. If, however, we employ a wire much larger in size than No. 34, or, at the most, No. 32, the bulk and the cost of the induction coil begin to rise very rapidly. Hence, as in all other departments of electrical construction, the details of the design are more or less a matter of compromise. Generally speaking, however, it may be said that the larger the capacity which is to be charged, the lower should be the resistance of the secondary circuit of the induction coil.

It is this fact which gives the alternating current transformer, as usually made, an advantage over the induction coil for the purposes considered, because a transformer is merely an induction coil specially constructed for a large power output, and having therefore a secondary circuit of relatively low resistance.

In coils intended for the production of electrical oscillations, and for wireless telegraphy, the preservation of high insulation in the secondary circuit is of great importance. The insulation is then subjected to strains far greater than when the coil is employed for Röntgen ray work or other similar purposes.

A large inductance coil is an expensive instrument, but it hardly ever retains for long its pristine powers of spark production. This is due to some degree of failure of internal insulation, or to surface leakage over ebonite surfaces outside, which have deteriorated in insulation power by exposure to light and air.

In those cases where portability is not an essential quality, an induction coil made with oil insulation may be used, and preserves its insulation better than one made in the usual way. If the coil is intended to be used with interrupted continuous primary currents, the iron core must be in the form of a straight bundle of iron, and not in the form of a closed circuit. Hence a so-called open magnetic circuit transformer or induction coil cannot be enclosed in an iron case. It can, however, be placed in a stoneware jar or vessel, and the whole coil can be immersed in insulating oil. For this purpose vaseline oil or heavy resin oil may be employed, provided it has been perfectly freed from water by heat. It is desirable to employ an oil with density greater than that of water, and to seal the jar as perfectly as possible. The secondary winding must be in sections as usual, but need not be impregnated with paraffin wax.

Induction coils intended for use on board ship for wireless telegraph purposes require especially good insulation, and should be so perfectly water-tight that the coil is not injured by even being put under water. If ebonite covering is used to enclose the coil, it should then be well varnished over with several coats of good waterproof varnish.

Otherwise the coil may be contained in a teak box filled in solid with paraffin wax and resin, in which case it can be screwed up against a bulkhead.

In the case of coils worked with an interrupted continuous primary current, it is necessary to place a condenser (called the primary condenser) across the point of rupture of the primary circuit, where the break spark occurs to reduce the spark and annul the magnetism of the core more suddenly.[25]

[25] For a theory of the action of the condenser, the reader may be referred to the author's "Treatise on the Alternate Current Transformer," vol. ii. p. 51, where it is suggested that the efficacy of the condenser may depend upon the demagnetizing action on the core of the electric oscillations set up in the circuit of the primary coil and condenser at the moment when the condenser is thrown into the circuit.

For another view of the action of the condenser, the reader is referred to a very interesting paper by Lord Rayleigh, in the *Philosophical Magazine* for December 1901, ser. vi., vol. 2. p. 581, "On the Induction Coil," in which the principal, if not the only, function of the condenser is shown to be that of quenching the spark or arc at the contact points when the primary circuit is opened.

Instrument makers generally determine by trial for each particular coil the proper size of condenser, and fix it in a box which supports the coil.

A better plan is to provide in a separate box a condenser divided into sections, the capacity of each section being marked on it, so that the capacity used may be varied. The condenser generally consists of sheets of well-baked and paraffined bank post paper, alternated with tinfoil sheets an inch narrower than the paper, but of the same length.

In the usual construction sheets of tinfoil are placed alternately with double or treble sheets of paraffined paper between them, and the sheets of tinfoil arranged to project out alternately on one side and the other. The odd and even sheets are then respectively clamped together.

The capacity of a condenser of this kind may be very roughly reckoned as equal to 0·01 mfd. per square foot of effective tinfoil surface.

Considerable difference of opinion exists between coil builders as to the capacity of the condenser suitable for use with a 10-inch coil. Some makers would use a primary condenser of 1·25 mfd. capacity; others one as small as 0·5 or even 0·32 mfd. for the above size of coil. Provided the capacity of the condenser is not too small, it may be varied within somewhat wide limits without objection, but if a platinum hammer break is employed, it is better to err in the direction of using too much rather than too little capacity. Even with a 6-inch coil having a hammer break, some makers provide a primary condenser of 8 mfd. capacity.

The question of the right primary capacity to employ with any given coil and break has been investigated by Dr. J. E. Ives; he observes that—

> "The *optimum capacity* of an induction coil is defined to be that capacity which, if placed across the break, will give the longest spark in the secondary circuit. It has also been found by experiment to be the least capacity that causes the sparking at the break to disappear—if not entirely to disappear, to become very small."[26]

Ives carried out experiments with a hand-worked mercury break in which the primary current was interrupted by raising an amalgamated copper wire out of mercury covered with water. He calls the copper wire the breaking pole, and found that the optimum capacity was much greater when the breaking pole is negative than when it is positive for the same current broken.

His conclusions are that in general the optimum capacity is proportional to a power of the primary current greater than the square, but less than the cube. It depends very much upon the resistance of the connections leading to the break and condenser, increasing with these connection resistances. It is also to some extent affected by the inductance of the primary circuit.

The capacity required is, however, in a considerable degree determined by the nature of the break employed. It has been shown that the more sudden the rupture of the primary circuit, the less the capacity necessary, and if that break is very sudden, then the addition of a condenser across the rupture point is not necessary.

Professor J. Trowbridge has described an effective form of quick motor break for large coils, in which the interruption is caused by withdrawing a stout platinum wire from a dilute solution of sulphuric acid, and by this means he increased the length of spark given by a coil originally provided with a hammer break and condenser from 15 to 30 inches by using the liquid break and no condenser.[27]

Lord Rayleigh has also shown that if the interruption of the primary circuit is extremely sudden, as when it is severed by a bullet from a gun, the primary condenser can be removed, and yet the sparks obtained from the secondary circuit are actually longer than those obtained with a condenser and the ordinary hammer break.[28]

[26] See "Contributions to the Study of the Induction Coil," by J. E. Ives, *Physical Review*, vol. xiv. No. 5, May-June 1902; also vol. xv. No. 1, July 1902. Also J. E. Ives, "On the Law of the Condenser in the Induction Coil," *Phil. Mag.*, October 1903, ser. vi., vol. 6, p. 411.

[27] See Prof. J. Trowbridge, "On the Induction Coil," *Phil. Mag.*, April 1902, ser. vi., vol. 3, p. 393.

[28] See Lord Rayleigh, "On the Induction Coil," *Phil. Mag.*, December 1901, ser. vi., vol. 2, p. 581.

In the use of the coil with any ordinary break, except the Wehnelt (see next section), a condenser of suitable capacity, joined across the break points, increases the secondary spark length. For additional information on this subject the reader is referred to the following papers :—

T. Mizuno, "On the Function of the Condenser in an Induction Coil," *Phil. Mag.*, 1898, vol. 45, p. 447.

K. R. Johnson, "On the Theory of the Condenser in an Induction Coil," *Phil. Mag.*, 1900, vol. 49, p. 216.

R. Beattie, "The Spark Length of an Induction Coil," *Phil. Mag.*, 1900, vol. 50, p. 139.

On the whole it cannot be said that the information is yet very precise on the subject of the size of a condenser or capacity to be used. It varies with many factors, and hence the necessity for providing the coil with a primary condenser of variable capacity for use in different experiments.

In induction coils by some makers, the primary circuit is wound in sections and the ends of each brought out in such a manner that the various sections can

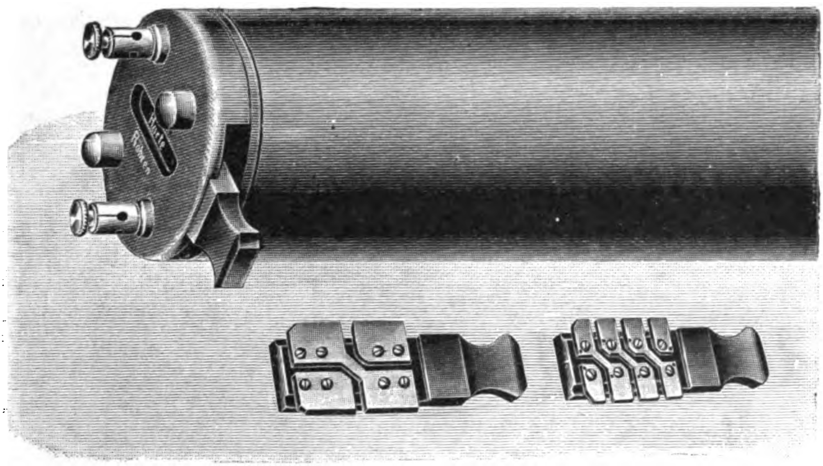


FIG. 43.—End of Primary Coil Tube of an Induction Coil, by K. Schall.

be joined in series or parallel, so as to vary the resistance and inductance of the coil, as well as the effective number of turns.

An ingenious arrangement of this kind is placed on coils by K. Schall, in which the various primary circuits have their ends connected to brass plates, and by sliding into a groove an ebonite piece with brass plates upon it these serve to effect the required arrangements and connection. The diagram in Fig. 43 shows the end of the ebonite tube containing the primary coils, and also the connecting plates which are slipped in to effect various combinations of the different primary circuits, so as to put them (1) all in series, (2) all in parallel, or (3) in series-parallel in various ways.

In making an estimate of the value of a coil for wireless telegraph purposes, or for the production of electric oscillations, the experimentalist should not be guided merely by external appearance or even by the length of spark given between pointed terminals in air.

The resistance of the secondary circuit should be ascertained, and inquiry made into the power of the coil to give a good oscillatory spark of at least 1 cm. in length when the secondary terminals are connected to a condenser having a capacity, say, of $\frac{1}{100}$ mfd.

A very fair way to judge the value of a coil for this particular purpose is to

ascertain what length of secondary spark it will give between brass balls 1 cm. in diameter when these balls are connected to the two poles of a glass plate condenser having a capacity of $\frac{1}{30}$ mfd. The spark should be at least 5 mms. in length.

A coil of the ordinary type, giving a 10-inch spark in air between pointed conductors, will not give much more than a 6- or 7-mm. spark, even if as much, when the secondary terminals are joined to the plates of glass condensers having a capacity of $\frac{1}{18}$ mfd.

When it is desired to obtain the advantages of a very low secondary resistance to charge large condensers, and thus obtain a longer oscillatory spark than can be obtained with one coil, two induction coils may be used with their secondary circuits joined in parallel and their primary coils joined in series. In this case only one hammer break is employed, and the condensers of both coils are joined in parallel across the break. This can always be done when the ends of the primary coil are accessible.

To aid the experimentalists in making these connections, we give below a diagram (see Fig. 44) which shows the usual mode of connecting up the various parts of an ordinary induction coil of the usual pattern with hammer break. This applies to the coils made by English makers such as Apps, Newton, Marconi's Wireless Telegraph Company, Ltd., and others who follow the same pattern. The

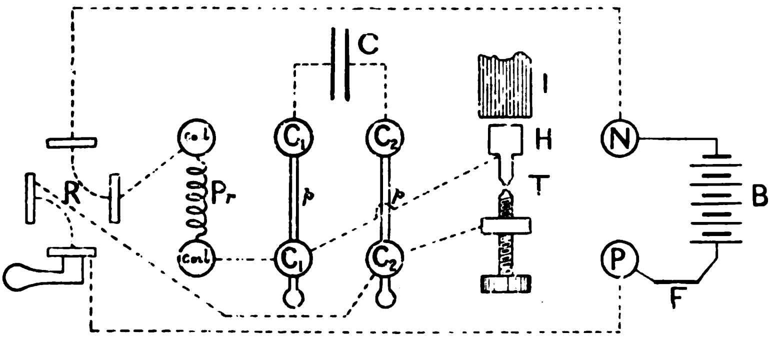


FIG. 44.—Diagram showing the Usual Connections of an Induction Coil.

diagram represents part of the baseboard of the coil, and the dotted lines show the wire connections which are made in the baseboard box.

The board generally has on it two terminals at one side marked P and N (see Fig. 44). To these the working battery is attached, a fuse wire, F, being interposed ; also, as above stated, an ammeter and a Morse key when the coil is used for wireless telegraphy. The end of the iron core of the coil is represented by I, and the hammer break by H. The platinum terminals between which the rupture of the primary circuit takes place are represented by T. On the other side are seen four terminals marked C_1, C_1, C_2, C_2. The two pairs C_1, C_1, and C_2, C_2 are generally connected by small brass pins, *p*, *p*, ending in ivory knobs. To one pair, C_1, C_2, are connected the plates of the primary condenser C. If the pins *p*, *p* are withdrawn, then the condenser C is isolated. Beyond are two other terminals marked *coil*. These are the ends of the primary coil of the induction coil. The current reverser is marked R. The connections under the base are denoted by dotted lines. If it is desired to work the coil with the usual hammer break, all that has to be done is to connect a working battery of the right size and number of cells to the terminals P and N, and then adjust the break and throw over the reverser handle to one side or the other.

If, however, it is desired to use some other break, then the pins *p*, *p* must be withdrawn and the required break connected in series with the battery used and with the primary coil. The terminals T must then be kept open by inserting a wooden wedge between them.

Separate wires must then be brought from the condenser terminals to the opposite sides of the particular break used, so that when the circuit is opened the condenser C is thrown across the break point. If coils have to be used in series, then the mode of arranging the connections can easily be worked out from the diagram of connections in Fig. 44.

The coils used for Röntgen ray work, and also for wireless telegraphy, are now often constructed without breaks and condensers on the same baseboard, and are intended to be used with some form of separate break (see section 9 of this chapter), and a separate condenser of a capacity suitable for the voltage and primary coil employed.

A practical precaution which it is advisable to adopt when working an ordinary pattern induction coil off secondary cells as a source of primary electromotive force is to insert a fuse wire in between the cells and the battery. If the hammer break sticks, as it often does, then the secondary cells send a large current through the contact, and this often welds the platinum contacts together. The use of a fuse wire or other form of cut-out may prevent damage to the break. It is always desirable to insert also an ammeter in the primary circuit, and also a voltmeter across the terminals of the battery to show the current and voltage acting on the primary circuit.

9. Interrupters for Induction Coils.—When a continuous current is employed to actuate an induction coil, it is, of course, necessary to interrupt the primary current periodically, in order to create an electromotive force in the secondary circuit. An important adjunct, therefore, of the induction coil is the interrupter or break for intermitting the primary current. We may divide interrupters into five classes:—

1. Hammer interrupters.
2. Dipper interrupters.
3. Motor interrupters.
4. Turbine or jet interrupters.
5. Electrolytic interrupters.

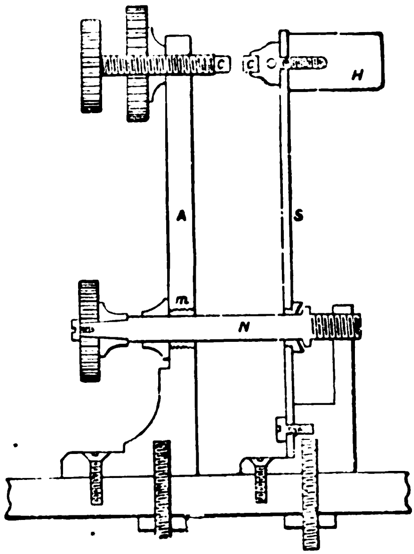


Fig. 45.—Apps' Hammer Break for the Induction Coil.

We have first the well-known *hammer interrupter*, which Continental writers generally attribute to Neef or Wagner.[20] In this interrupter the magnetization of the iron core of the coil is caused to attract a soft iron block fixed at the top of a brass spring, and by so doing to interrupt the primary circuit between two platinum contacts. Mr. Apps added an arrangement for pressing back the spring against the back contact, and the form of hammer break that is now generally employed is therefore called an Apps break (see Fig. 45).

As the 10-inch coil takes a current of 10 amperes at 16 volts when in operation, it requires very substantial platinum contacts to stand this current continuously without damage. The small platinum contacts that are generally put on these coils by most instrument makers are very soon worn out in practical wireless telegraphy work. If a hammer break is used at all, it is essential to make the contacts of substantial pieces of platinum, at least 6 mms. in diameter, and from time to time, as they get burnt away or roughened, they must be smoothed up with a fine file. It does not require much skill to keep the hammer contacts in good order and prevent them from sticking together and becoming damaged by the break spark.

By regulating the pressure of the spring against the back contact by means of

[20] Du Moncel states that MacGauley, of Dublin, independently invented the form of hammer break as now used. See J. A. Fleming, "The Alternate Current Transformer," vol. 2, chap. i.

the adjusting screw, the rate at which the break vibrates can be adjusted to make from 10 to 50 or 60 interruptions a second. The hammer break is usually operated by the magnetism of the iron core of the coil, but for some reasons it is better to separate the break from the coil altogether, and to work it by an independent electro-magnet, which, however, may be excited by a current from the same battery supplying the induction coil. For coils up to the 10-inch size the hammer break is sufficiently good when very rapid interruptions are not required. It is not in general practicable to work coils larger than the 10-inch size with a hammer break, as such a platinum contact becomes overheated and sticks if more than 10 amperes is passed through it. In the case of larger coils, we must therefore employ some form of interrupter in which mercury or a conducting liquid forms one of the contact surfaces. On account of its simplicity and ease of management, however, the hammer break is still much used in induction coils employed in wireless telegraphy.

The second class of interrupter is the self-acting or hand-worked *dipper break*, in which a platinum or steel pin is made to plunge in and out of mercury. This movement may be effected by the attraction of an iron armature, by an electromagnet, by the varying magnetism of the core of the coil, or it may be effected slowly by hand, or rapidly by an electric motor.

The mercury surface must be covered with water, alcohol, paraffin, or creosote oil, to prevent oxidation and to extinguish the break spark. The interruption of the primary current obtained by the mercury dipper break is more sudden than that obtained by the platinum contact in air, at least when the mercury is covered with oil, in consequence of the more rapid extinction of the spark; hence the sparks obtained from coils fitted with mercury dipper interrupters are generally from 20 to 30 per cent. longer than those obtained from the same coil under the same conditions with platinum contact interrupters. The mercury must be cleaned at regular intervals by emptying off the oil or alcohol and rinsing the metal well with clean water, and hence they require rather more attention than platinum interrupters. The mercury interrupter has, however, the advantage that the contact time during which the circuit is kept closed may be made longer than is the case with the hammer break. Also if fresh water is allowed to flow continuously over the mercury surface, it can be kept clean, and the break will then operate for considerable periods of time without attention.

The hammer or platinum contact interrupter will not work well with an electromotive force of more than 12 or 16 volts, because at higher electromotive forces the break spark prolongs the decadence of the primary current. Hence, if coils are worked on a 100-volt circuit or at higher voltage, some form of mercury break must be employed.

A third kind of interrupter is called a *motor interrupter*, and of these a large number have been invented in recent years. In this interrupter some form of a continuously rotating electric motor is employed to make and break a metal contact with mercury or other liquid. In one simple form the motor shaft carries an eccentric which intermittently dips a platinum point into mercury, or else a platinum horseshoe into two mercury surfaces, making in this manner an interruption of the primary circuit at one or two places. As a small motor can easily be run at 1200 revolutions per minute, or 20 per second, it is possible easily to secure in this manner a uniform rate of interruption of the primary current at the rate of about 20 per second. If, however, much higher speeds are employed, then the time of contact becomes abbreviated, and the power of the coil to charge the capacity is diminished.

A fourth class of interrupter is called a *turbine or mercury jet interrupter*. In this appliance a jet of mercury forced out of a small aperture by means of a centrifugal pump is made to squirt against a metal plate, and the jet is interrupted intermittently by means of a toothed wheel made of insulating material, rotated by the motor which drives the pump. Otherwise a revolving jet of mercury is made to impinge intermittently upon a fixed metal plate. The current supplying the coil passes through or along this jet of mercury, and is therefore rendered intermittent when the jet or metal plate revolves. The mercury is covered with paraffin oil or alcohol to preserve the mercury jet from oxidation.

In the case of this interrupter, the duration of the contacts, as well as a number of interruptions per second, is under control, and for this reason better results are probably obtained with it than with most other forms of break.

A description of an early form of turbine mercury break by Max Levy was given in the *Elektrotechnische Zeitschrift*, October 12, 1899, vol. 20, pp. 717 (see also *Science Abstracts*, vol. 3, p. 63, Abstract No. 165), as follows :—

A toothed wheel made of insulating material carries from 6 to 24 saw-shaped teeth, and can be made to rotate from 300 to 3000 times per minute by a motor. The teeth of this wheel interrupt a jet of mercury thrown by a centrifugal pump against a metal plate (see Fig. 46). Moreover, by raising or lowering the position of the interrupting wheel by a lever the duration of the contact can be varied, so that it is possible to regulate this period without disturbing the number of interruptions per second. The pump and wheel are contained in a vessel partly full of mercury overlaid with paraffin oil.

The sparks obtained from a coil worked with a turbine interrupter are much thicker and convey a greater electric quantity than the sparks made by hammer breaks. Also by means of the turbine break an induction coil can be worked with a higher voltage than is possible when using a hammer break, and the turbine break has also the advantage of being nearly noiseless in use. By properly adjusting the break, the appearance of the secondary sparks can be varied from the thin snappy sparks given by the hammer break to the thick flame-like arc sparks given by the electrolytic break. The turbine break can be adapted for any voltage from 12 to 250 volts, and the primary circuit cannot be closed before the interrupter is acting.

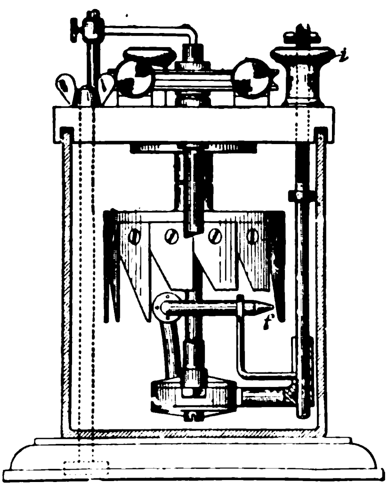


FIG. 46.—Max Levy Mercury Turbine Interrupter.

The chief drawback to its use is that the mercury has to be cleaned at intervals if the interrupter is much used. If alcohol is employed to cover the mercury the metal needs only to be rinsed under a water tap and afterwards dried with blotting-paper. When paraffin oil is used the cleaning is more troublesome, but is effected with the help of a few ounces of sulphuric acid. The mercury by use gets gradually resolved into a sort of black mud, consisting of globules of mercury intermingled with oil. If the mud is well shaken up with a little strong sulphuric acid this oily film is removed. The acid is then washed away with fresh water, and clean metallic mercury remains behind.

The motor driving the centrifugal pump can be wound for any voltage, and it is best to have it so arranged that this motor is worked by the same battery as that which supplies the primary circuit of the coil, the two circuits working parallel together. A rheostat can be added to the motor circuit to regulate the speed.

In Fig. 47 is shown a diagram of a good form of mercury jet break by Schall.

A centrifugal pump causes a revolving jet of mercury to impinge against a copper plate. The mercury is contained in a glass vessel, and is covered with paraffin oil, so that the jet of mercury takes place in oil. The duration of the contact can be varied by an adjusting lever which shifts the position of the copper plate.

The motor driving it can be wound for either high or low voltage, and in selecting a break of this kind it is necessary to determine the choice of voltage by the nature of the winding of the primary circuit of the induction coil used with it. Generally speaking, the maker of the coil specifies this to the purchaser.

The great trouble with all these mercury breaks when used with paraffin oil or

alcohol as a covering liquid is that the mercury is before long worked up into a sort of black mud and ceases to conduct. M. Béclère, of Paris, made an enormous improvement in substituting coal gas for the insulating liquid. A break of the type shown in Fig. 47 is employed, but instead of filling the space above the mercury with oil it is kept full of coal gas from a small gas-bag which just supplies the loss by leakage. When so modified, the break will work for hours and even months without the slightest attention, and a mercury coal-gas interrupter of this type is now essential in all prolonged experimental work with the induction coil used as a source of oscillations. An excellent form of coal-gas mercury break, made by Messrs. Watson & Sons, driven by a separate motor, is shown in Fig. 48.

Lastly, we have the *electrolytic interrupters*, which were first introduced by Dr. Wehnelt, of Charlottenburg, in the year 1899, and modified by subsequent inventors. In its original form it consists of a glass vessel filled with dilute sulphuric acid, consisting of one part of strong acid to five or else ten parts of water. This vessel contains two electrodes of very different sizes; one is a large lead plate, formed of a piece of sheet lead laid round the interior of the vessel, and the other is a

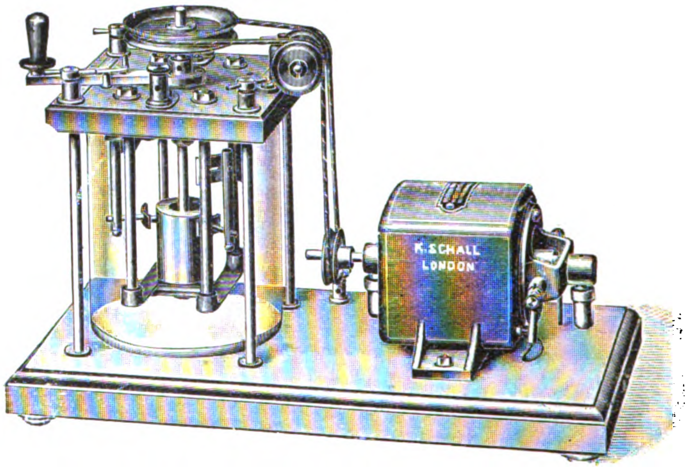


FIG. 47.—Mercury Turbine Interrupter. (Schall.)

short piece of platinum wire projecting from the end of a glass or porcelain tube (see Fig. 49). The smaller of these electrodes is made the positive, and the large one the negative. If this electrolytic cell is connected in series with the primary circuit of the induction coil (the condenser being cut out), and supplied with an electromotive force from 40 to 80 volts, an electrolytic action takes place which interrupts the current periodically. An enormous number of interruptions can, by suitable adjustment, be produced per second, and the appearance of the discharge from the secondary terminals of the coil, while using the Wehnelt break, more resembles an alternate current arc than the usual disruptive spark.[30]

At the time when the Wehnelt break was first introduced, great interest was excited in it, and the technical journals in 1899 were full of discussions as to the theory of its operation.[31]

The general facts concerning the Wehnelt break are that the electrolyte must

[30] See Dr. Wehnelt's article in the *Elektrotechnische Zeitschrift*, January 20, 1899.

[31] See *The Electrician*, 1899, vol. 42, pp. 721, 728, 731, 732, and 842; communications from Mr. Campbell Swinton, Prof. S. P. Thompson, Dr. Marchant, the author, and others. Also p. 864 of same volume, for a leader on the subject. Also p. 870, letters by M. Blonde and Prof. E. Thomson. See also *The Electrician*, 1899, vol. 43, p. 5, extract from a paper by P. Barry, *Comptes Rendus*, April 10, 1899. See also *The Electrical Review*, February 17, 1899, vol. 44, p. 235.

be dilute sulphuric acid in the proportion of one of acid to five or ten of water. The large lead plate must be the cathode, or negative pole, and the anode, or positive pole, must be a platinum wire about a millimetre in diameter, and projecting 1 or 2 mms. from the pointed end of a porcelain, glass, or other acid-proof insulating tube. The aperture through which the platinum wire works must be so tight that

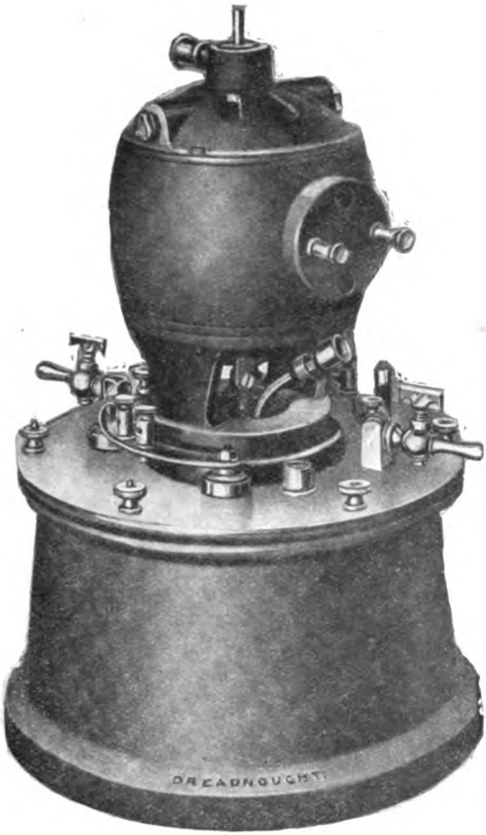


FIG. 48.—Coal-Gas Mercury Turbine Motor-driven Break for Induction Coil Working. (Watson & Sons.)

acid cannot enter, yet it is desirable that the platinum wire should be capable of being projected more or less from the aperture by means of an adjusting screw. The glass vessel which contains these two electrodes should be of considerable size, holding, say, a quart of fluid, and it is better to include this vessel in a larger outer one in which water can be placed to cool the electrolyte, as the latter gets very warm when the break is used continuously. If such an electrolytic cell has a continuous electromotive force applied to it tending to force a current through the electrolyte from the platinum wire to the lead plate, we can distinguish there

stages in its operation, which are determined by the electromotive force and the inductance in the circuit. First, if the electromotive force is below 16 or 20 volts, then ordinary and silent electrolysis of the liquid proceeds, bubbles of oxygen being liberated from the platinum wire and hydrogen set free against the lead plate. If the electromotive force is raised above 25 volts, then when there is no inductance in the circuit the continuous flow of current proceeds, but if the circuit of the electrolyte possesses a certain minimum inductance, the character of the current flow changes, and it becomes intermittent, and the cell acts as an interrupter, the current being interrupted from 100 to 2000 times per second, according to the electromotive force and the inductance of the circuit. Under these conditions the cell produces a rattling noise, and a luminous glow appears round the top of the platinum wire. Thus, in a particular case, with an inductance of 0·004 millihenry in the circuit of a Wehnelt break, no interruption of the circuit took place, but with 1 millihenry of inductance in the circuit and with an electromotive force of 48 volts, the current became intermittent at the rate of 930 per second, and by increasing the voltage to 120 volts the intermittency rose to 1850 a second.

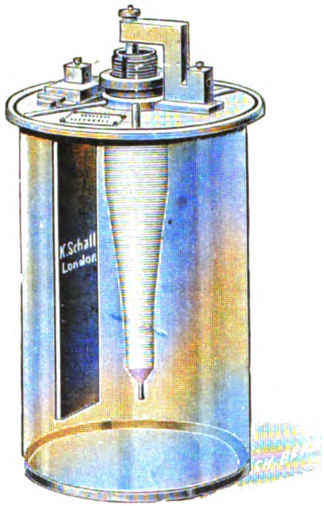


FIG. 49.—Wehnelt Electrolytic Break.

The Wehnelt break acts best as an interrupter with an electromotive force from 40 to 80 volts. At higher voltages a third stage sets in; the luminous glow round the platinum wire disappears, and it becomes surrounded with a layer of vapour, as observed by MM. Violle and Chassagny; the interruptions of current cease, and the platinum wire becomes red-hot. If there is no inductance in the circuit the interrupted stage never sets in at all, but the first stage passes directly into the third stage. In the first stage bubbles of oxygen rise steadily from the platinum wire, and in the interrupted stage they rise at longer intervals, but regularly. The cell will not, however, act as a break unless some inductance exists in the circuit.

In applying the Wehnelt break to the usual form of induction coil, the condenser and ordinary hammer break are cut out of circuit, and the Wehnelt break is placed in series with the primary coil. In some cases the inductance of the primary coil alone is sufficient to start the break in operation, but with voltages above 50 or 60 it is generally necessary to supplement the inductance of the primary coil by an additional external inductance coil. The best form of Wehnelt break for operating induction coils is the one with multiple anodes (see Fig. 50, also see remarks by Dr. Marchant, in *The Electrician*, 1899, vol. 42, p. 841), and when it has to be used for long periods the cathode may advantageously be formed of a spiral of lead pipe through which cold water is made to circulate.

When the Wehnelt break is applied to an ordinary 10-inch induction coil, and the inductance of the primary circuit and the electro-motive force varies until the break interrupts the current regularly, with a frequency of some hundreds a second, the character of the secondary discharge is entirely different from its appearance with the ordinary hammer break. The thin blue lightning-like sparks are then replaced by a thicker mobile flaming discharge, which resembles an alternating current arc, and when carefully examined or photographed is found to consist of a number of separate discharges superimposed upon one another in slightly different positions.

Although the Wehnelt break has some advantages in connection with the use of the induction coil for Röntgen ray work, its utility as far as regards the production of electric oscillations and its use in electric wave telegraphy is not by any

means so marked. It has already been explained that in order to charge a condenser of a given capacity at a constant voltage the electromotive force must be applied for a certain minimum time, which is determined by the value of the capacity of the condenser used, and the resistance of the secondary circuit of the induction coil.

If the coil is a 10-inch coil, and has a secondary resistance of, say, 6000 ohms, and if the capacity to be charged has a value, say, of $\frac{1}{30}$ mfd., then the time-constant of the circuit is $\frac{1}{5000}$ second. Therefore the electromotive force charging the condenser must be maintained for at least $\frac{1}{500}$ second, so that the condenser may become charged to the voltage which the coil is then producing.

In the induction coil the electromotive force generated in the secondary coil at the "break" of the primary current is higher than that at the "make," and the magnitude and duration of this electromotive force, other things being equal, depends upon the rate at which the magnetism of the iron core dies away. Its

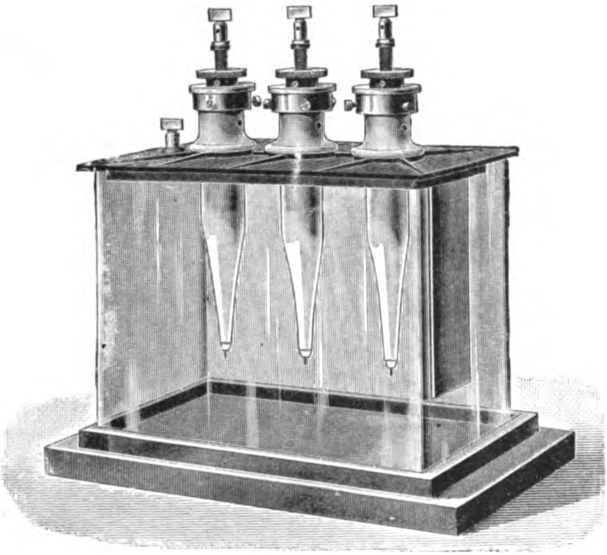


FIG. 50.—Wehnelt Electrolytic Interrupter with Multiple Anodes.

duration is shorter in proportion as the whole time occupied in the disappearance of the magnetism is less. The Wehnelt break does not increase the actual value or duration of the electromotive force in the secondary circuit, but it greatly increases the number of times per second this electromotive force is created. Accordingly, it increases the secondary discharge current, but not the secondary electromotive force. Hence, when employing an induction coil to create electric oscillations in an aerial wire, and therefore to send out trains of electric waves, the nature of the receiver or wave detector used determines whether the use of the Wehnelt break is an advantage or not. When using those types of wave detector which are influenced chiefly by the maximum value of the wave train, and not by the root-mean-square value, the increase in the number of wave trains per second produces no additional advantage.

On the other hand with most modern receivers, and with the aural or telephone method of reception explained in a later chapter, it is very essential to have in the spark transmitter a high and uniform spark frequency, which should preferably be about 500 per second. The frequency, moreover, must be perfectly under control. The Wehnelt break does not fulfil these requirements, and hence is of no use in

modern radiotelegraphy. There are, in addition, quite different methods of securing the necessary interruptions in the primary current of an induction coil depending on the discharge of energy from a condenser or an inductance coil which may be mentioned here. One of these is as follows :—

An electrolytic condenser of large capacity such as that made by Grisson is employed. It is constructed of plates of aluminium placed in a special electrolyte, like plates in a secondary cell. The alternate plates are connected together and form the two opposed surfaces of the condenser. If a current is passed in one direction through the cell from one set of plates to the other, a current flows for a short time, but is soon stopped if the E.M.F. does not much exceed 100 volts. This is due to the formation on the anode plates of a film of impervious or non-conducting aluminic hydroxide. If, however, the direction of the current is reversed, the cell again becomes conductive for a short time, and the impervious

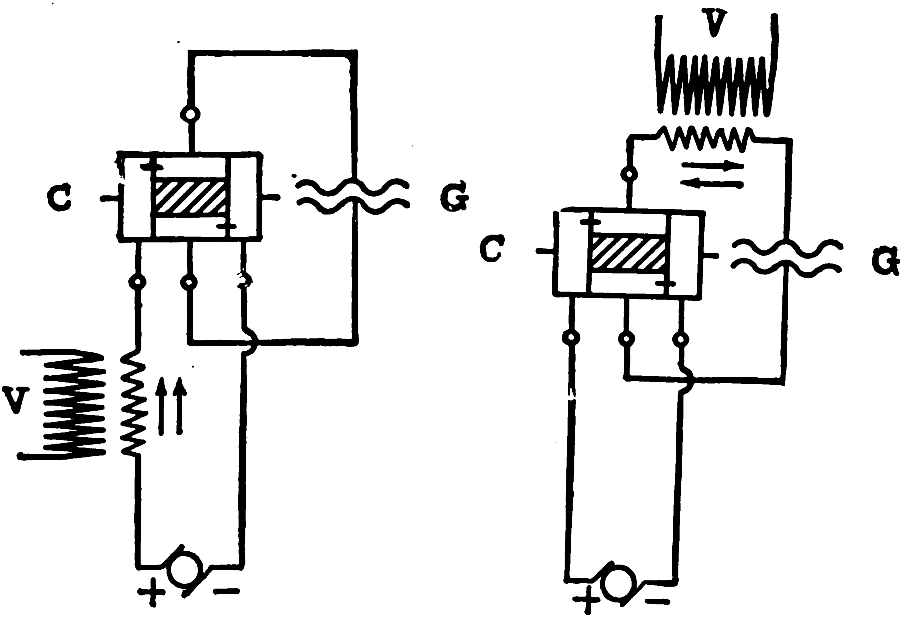
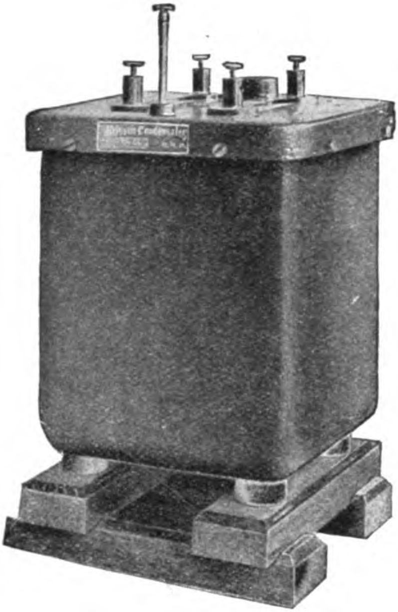


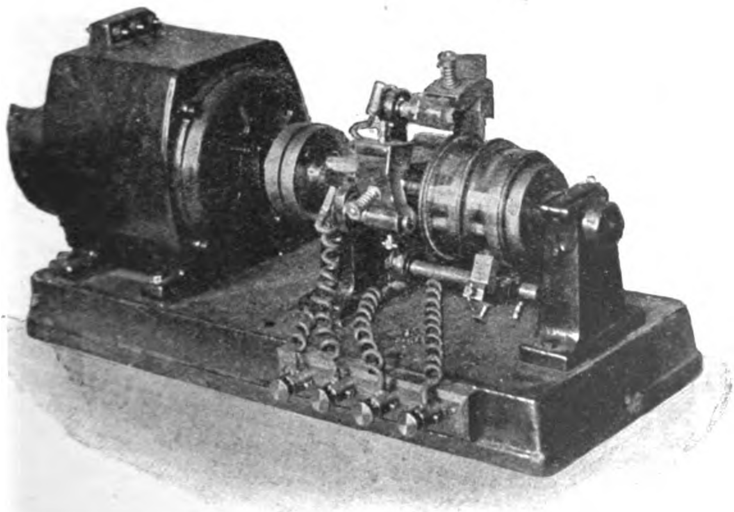
FIG. 51.—Arrangement of Grisson Electrolytic Condenser and Induction Coil.
G, electrolytic condenser; C, commutator; V, induction coil.

film is transferred to the other set of plates. Hence the cell acts like a condenser of large capacity, and one equivalent to 100 mfds. occupies a space of only 12 inches by 12 inches by 14 inches.

If a sufficiently large cell of this kind is connected in series with the primary circuit of an induction coil, and a steady E.M.F. of 100 volts or so applied to the terminals, a current flows through the primary for a short time. This current rises very quickly to a maximum value and then dies more slowly away. Hence it creates in the secondary circuit two electromotive forces of very unequal value and in opposite directions. Suppose, then, that by a special commutator the position of the plates of the electrolytic condenser is reversed, another brief current would flow through the primary coil in the same direction and another pair of secondary electromotive impulses be created. This reversal of the position of the plates of the condenser is affected by the use of a revolving motor-driven commutator. At the moment when the condenser is fully charged and the current has ceased in the primary coil, the condenser connection with the circuit can be



Grisson Electrolytic Condenser.



Grisson Commutator for use with Electrolytic Condenser.

FIG. 52.

broken *without spark*, and remade with the plates in a reversed direction. Hence the arrangement sends through the primary circuit of the induction coil a rapid series of "puffs" of electric current, and these create secondary electromotive forces which predominate in one direction. This gives us exactly the result we have in the coil as worked with the ordinary break, and does it without spark. A diagram of the connections is shown in Fig. 51, and the general appearance of the condenser and commutator in Fig. 52.

In those researches, in which very regular groups of oscillations must be produced, this apparatus offers a great advantage.

In another arrangement, also devised by Grisson, an induction coil with a special form of primary winding is employed. The primary coil has three terminals, one at each end and one in the centre of the winding. The centre terminal is connected to one pole of the battery, and the other two terminals are alternately connected to the other pole by means of a revolving commutator. The following operations then take place:—

(i.) The primary current flows through one-half of the primary coil and magnetizes the iron core.

(ii.) The current flows in opposite directions through the two halves of the primary winding, and the core has no resultant magnetization.

(iii.) The current flows through the other half of the primary winding, and the direction of the magnetization of the core is reversed.

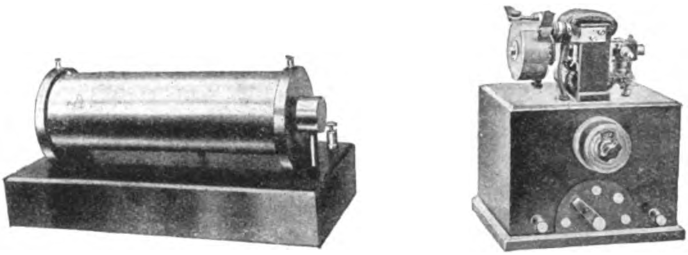


FIG. 53.—Wilson Induction Coil and Commutator.

(iv.) The current again splits and flows equally through both sections of the primary coil, and the core is not magnetized.

These operations are rapidly repeated in the same order. The result is, a series of secondary currents are induced, which are alternately in one direction and the other.

Condensers are placed across the break gaps to quench the spark and exalt the secondary electromotive forces as usual.

These changes of current direction are effected by means of a rotating commutator, consisting of metal segments let into the periphery of a disc of insulating material. Brushes press against this disc, and it is driven round by an electric motor actuated by the source of current supply.

Another rather different method of producing induction coil discharges is due to Professor E. Wilson and Mr. W. H. Wilson.[32]

In this method energy is taken from an alternating current or continuous current source of supply and stored in the form of a magnetic field by an inductance. It is then permitted to surge into a condenser, which forms with the inductance a low frequency oscillation circuit. When the energy is thus stored the condenser is bridged across by the primary of the induction coil, with which it forms a high frequency oscillation circuit. The energy is then transferred to the secondary circuit. These various changes of connection are made by a motor-driven commutator.

[32] See E. Wilson and W. H. Wilson on "A New Method for Producing High Tension Discharges," *The Electrician*, September 9, 1910.

A general view is shown in Fig. 53. The spark coil on the left hand is a 10-inch induction coil, wound with aluminium wire if light weight is desired, or copper strip for very low resistance and heavy discharges. The condenser is in the base of the coil. The motor-driven commutator is on the right, and in its box or base is the inductance. This last coil has several tappings so that it can be used with either 250, 100, or 12 volts—according to the supply of voltage available.

The following is a more detailed description of the mode of action as given by the inventor[33] :—

Fig. 54 is a diagram of the connections, in which 1 is the winding of the inductance or choking coil; 2 and 3 are the primary and the secondary windings respectively of the spark-coil; 4 is the condenser and 5 is the interrupter. The inductance (1) is supplied with a number of taps, corresponding to the terminals mentioned in connection with Fig. 53, one of which is connected to one pole of the supply source (6). The other pole of the supply is connected to a brush (*a*) bearing on the interrupter, which may now be briefly described. In the form shown it consists of a disc of ebonite which carries a metal segment and a complete metal ring connected to the segment. Three brushes (*a*, *b*, *c*) bear upon the cylinder. Brush *a* makes contact with the segment once in every revolution. Brush *b* is always in contact with the segment, since it bears upon the slip ring attached to the segment. Brush *c* is mounted on an adjustable arm, and makes contact with the segment once in a revolution. The junction of the inductance and condenser is connected to brush *b*. The junction of the inductance and the primary winding of the spark-coil is connected to brush *c*. It may be mentioned that at a speed of 3600 revolutions per minute the frequency of interruptions is sixty per second. This can be increased by providing more than one cycle of events per revolution of the motor.

We may now consider what happens at the epochs of "make" and "break." At "make" the brushes *a* and *b* are connected by the segment, and the supply source (6) is switched on to the system. A current rises in the inductance (1) and stores energy in its magnetic field. The condenser (4) is charged to the potential of the supply, the charging current passing through the primary winding of the spark-coil. At "break" the brush *a* severs contact with the segment, and is insulated. The system with energy stored in the inductance, and the condenser is now isolated. The condenser immediately parts with its energy by discharging it into the inductance. The condenser then begins to be charged in the reverse direction by the falling field of the inductance, and in a time depending upon the constants of the circuit the whole of the energy is transferred to the condenser, and it is fully charged. Brush *c* can be so adjusted that at this moment it begins to touch the segment of the contact-maker, and thereby short-circuits the inductance (1) The condenser then discharges with greater rapidity through the primary winding of the spark-coil, and the energy as received at the terminals of the secondary winding can be in the nature of practically a unidirectional discharge of the spark-coil if desired for X-ray work. This is rendered possible by the fact that the core can be closed, thereby reducing the leakage as between the primary and secondary windings. In other words, the first half wave of E.M.F. in the secondary is enormously greater than the second, and this latter is so small as to cause no appreciable current to pass as an inverse current through the tube. As for the inverse current at "make," this is due to the slowly changing current to the condenser, and is negligibly small even when supply voltages of 250 are employed. On such circuits as 250 volts no series resistances are necessary, and the efficiency is still maintained at a value of from 70 to 80 per cent. In other words, it is only necessary on such voltages as 100, 200, or 250, to have an ordinary twin flexible conductor, such as is used for a reading lamp.

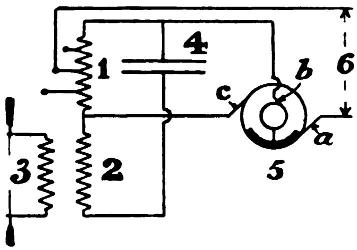


FIG. 54.—Diagram of Connections of Wilson Induction Coil and Commutator.

10. Low Frequency Alternators and Transformers for Generating Electric Oscillations.—When alternating current is available, an alternating current transformer is now always employed to produce electric oscillations in place of an induction coil operated by continuous currents. An ordinary induction coil can also be employed as an alternating current transformer, if its condenser and break is removed and the primary circuit supplied with an alternating current. The frequency of the primary current employed should not be less than 100 periods

[33] See *Proc. of Royal Society of Medicine*, April 1911.

per second, and it is now usual to work with a much higher frequency, say 500 periods per second.

If continuous current is available which can be drawn from supply mains, from a private electric lighting circuit, public town supply, or from ship lighting circuits on board vessels, then we may employ it to drive a motor generator, producing alternating current from a continuous current. One good plan for so doing is to use an ordinary four-pole continuous-current motor with an armature of the Gramme ring type and the usual commutator. From two points at the opposite ends of a diameter of the armature, connections are brought to two insulated slip rings, fixed on the shaft of the motor. When a continuous current is passed into the motor on the commutator side in the ordinary manner, it revolves, and we can draw, from brushes pressing against the slip rings, an alternating current, the effective voltage of which is, however, less than that of the continuous current supplying the motor. Thus, if the motor is driven by direct current at a pressure of 100 volts and makes 1200 revolutions per minute, we can, from the slip rings so connected, draw off an alternating current with an effective voltage of 70 volts and a frequency of 20 periods per second. By the use of a four-pole motor we can obtain a frequency of 100 when the motor is driven at a speed of 3000 R.P.M.

The alternating current so generated can be led to an alternating current transformer of the closed iron circuit type, and by means of it raised in pressure to 20,000 or 30,000 volts. Thus if the transformer has a transformation ratio of 400 to 1, we can by means of it produce an alternating current having an electromotive force of 28,000 volts from a continuous current supply at 100 volts. For laboratory purposes, a 3-kilowatt (kw.) continuous current motor, arranged as described, associated with a 2-kw. step-up transformer, constitutes a very convenient arrangement. In places where a continuous current cannot be obtained to drive the motor, it may be driven as a dynamo or alternator by means of a pulley and belt, by a small oil engine, thus making an arrangement which is independent of outside aid.

It is essential that the high voltage transformer should be an oil insulated transformer, if pressures are employed higher than 20,000 volts, as the oil prevents the formation of brush discharges inside the coils which would in time destroy the insulation.

When pressures higher than 30,000 volts have to be employed it is better to join a number of separate transformers in series. Thus, for instance, to obtain alternating current at a pressure of 120,000 volts, four transformers of 30,000 volts or six of 20,000 volts can be arranged with their secondary coils in series. In this case, all the transformers must be exceedingly well insulated by being placed on stands supported on strong porcelain oil insulators. In experimenting with alternating currents of very high pressures, supplied by transformers and alternators, and used to charge large condensers, the greatest precaution must be taken to avoid accidents or touching a high-tension wire, as the result would in all probability be fatal. The experimentalist should himself have control over the current exciting the transformers or exciting the alternator supplying them, and should disregard no precaution necessary to ensure safety. Means should also be taken to ascertain the frequency of the current. This can, of course, be done at once by counting the revolutions of the alternator or motor, but if the supply of alternating current is obtained from a distance, then in addition to the voltmeters and ammeters, necessary to show the current going into the transformers and the pressure at which it is supplied, a frequency meter ought to be provided.

One well-known form of frequency indicator is that due to Mr. Campbell, which was developed from a principle first suggested in 1889 by Professors Ayrton and Perry. This instrument depends upon the fact that if a light steel elastic strip is fixed over an alternating current electro-magnet, the strip will be set into strong vibration if the frequency of the alternating current agrees with the time period of vibration of the strip. In the Campbell Frequency Teller a steel strip is pushed forward through a clamp by means of a rack and pinion, the pinion carrying an indicating needle which moves over a scale. An alternating current

electro-magnet is placed under the strip, and the time period of vibration of the strip can be varied within certain limits by altering the length of the strip which protrudes beyond the clamp. As long as the time period of the spring is out of agreement with that of the magnet current the spring hardly vibrates at all, but if the pinion is turned until agreement is produced, the strip vibrates vigorously, and, striking against a contact point, makes a loud noise.

Many other forms of frequency teller have been developed for practical use in connection with transformer working, but they nearly all depend upon the principle above explained.

Another form of such resonance frequency teller or indicator is shown in Fig. 55. In this instrument a number of musical reeds such as are used in organ-pipes or harmoniums are fixed in a row. Each reed is tuned to a particular frequency, and the whole number comprise a range of frequency extending, say, from 80~ to 110~. These reeds are fixed in a frame, and opposite to them is fixed an electro-magnet, the coils of which are traversed by the current the frequency of which is to be measured. The reeds are made of steel spring, and the periodic magnetic field of the magnet sets them in motion. Hence if an alternating current of a certain frequency within this range is passed through the magnet coils, the corresponding reed is set in vibration, and its movement indicates the frequency.

FIG. 55.—Hartmann-Kempf Resonance Frequency Indicator.

A very compact form of apparatus for producing high-pressure high frequency oscillations, employing a motor and transformer as above described, was some years ago described by Professor Elihu Thomson. The following is a description of this apparatus:[34] A small continuous current electric motor has, in addition to its ordinary commutator, a pair of slip rings on the shaft and brushes pressing against them. When the motor is run in the usual way by continuous current, it produces an alternating current at the slip-ring brushes. A step-up transformer is connected to these brushes, and raises the pressure to 20,000 volts. The shaft of the motor drives also an insulating frame with metal contact pieces on it, the function of which is to connect together alternately, in series or parallel, a set of glass condenser plates, covered with tinfoil (see Fig. 56). These plates are charged once in each revolution with the secondary terminal voltage of the transformer, but the contact only endures for a short time, during which the potential has its maximum value. During the next part of a revolution, the condenser plates are insulated and connected in series and caused to discharge across a spark gap. By employing in this way eleven condenser plates, each one charged at 20,000 volts, a machine was constructed which gave 12-inch sparks in air, having all the properties of sparks from an electrostatic machine. These discharges, if the spark gap was small enough, would be oscillatory discharges.

The most convenient arrangement when large power is not required is to use a petrol engine direct coupled to a small rotary converter or continuous current dynamo, having two insulated slip rings on the shaft connected to opposite points on the winding of the continuous current armature. From these slip rings alternating currents can be drawn off and raised in voltage by a transformer.

For some years past the author has possessed in his laboratory such a transformer plant for producing high frequency oscillations. This consists of a four-pole continuous current motor driven at a speed of 1200 R.P.M.; the motor is provided, as described above, with a Gramme ring armature, and slip rings

[34] See *The Electrician*, 1889, vol. 43, p. 779; also *The Electrician*, vol. 44, p. 40. See Prof. Elihu Thomson on "Apparatus for Obtaining High Frequencies and Pressures."

connected to two opposite points on it, hence the machine produces alternating current at a frequency of 40. This is passed through two transformers arranged in cascade, the first of which steps up the voltage from about 70 to 400, and the second from 400 to about 24,000 volts. The secondary terminals in this last transformer are connected to a large glass plate condenser, the capacity of which can be varied between $\frac{1}{20}$ and $\frac{1}{300}$ mfd. The alternating current motor is 5-kw. size, and the two transformers 2-kw. size. The arrangement is capable of producing very powerful electric oscillations in suitably arranged circuits. A very compact plant can also be formed by employing a small oil engine to drive an alternator coupled to it by a belt and associating with the alternator, as described, a step-up high-tension transformer. This forms a portable arrangement for producing the necessary electric oscillations for wireless telegraphy, when the power required is beyond that capable of being given by an ordinary induction coil (see Fig. 57).

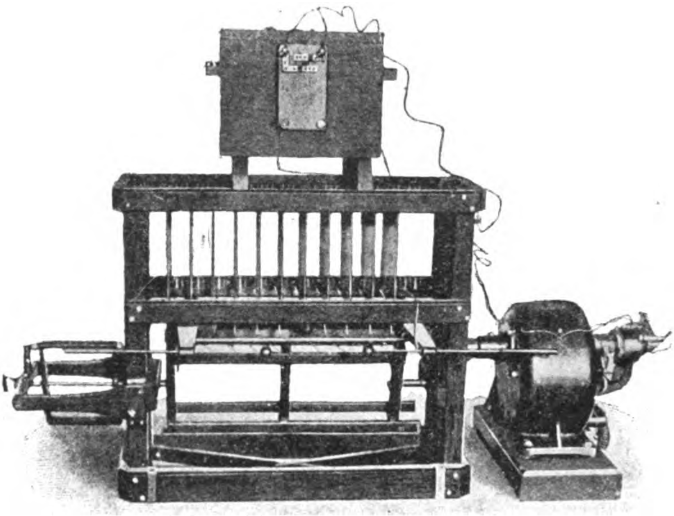


FIG. 56.—Apparatus for Producing High Tension and High Frequency Discharges (Elihu Thomson).

In Fig. 58 is shown in outline the disposition of apparatus necessary for such a transmitter for generating trains of controlled oscillations in an earthed antenna or aerial.

In modern alternator plants for wireless telegraphy operating on the damped wave train or spark system, it is now always the custom to employ an alternator having a frequency of 300 to 500 or perhaps 600. The reason for this will be explained more fully in a later chapter. It is dependent on the fact that the pitch of the note heard in telephonic receivers now used is determined by the spark frequency, and it is found most easy to hear these telephonic sounds when the spark frequency is about 500. Hence when an alternator is employed to charge condensers in a spark transmitter plant it is usual to employ such a frequency, and to drive the alternator by means of a directly coupled electric motor or by a steam or petrol engine.

11. Condensers for the Production of Electric Oscillations.—The next element to be considered is the condenser in which the electric charge is placed, the release of which produces the high frequency oscillations.

In this connection we need only consider the construction of condensers suitable for very high pressures. The properties of dielectrics will more particu-

arly be discussed in the next chapter, and we shall here merely discuss the structure of high-pressure condensers.

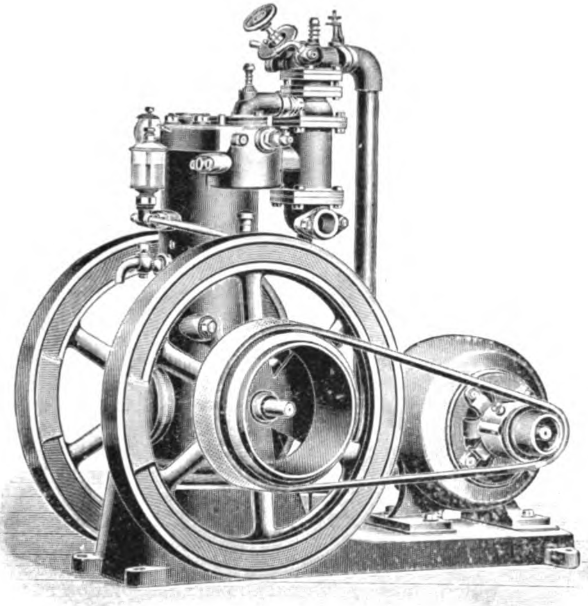


FIG. 57.—Belt-Coupled Oil Engine and Alternator for the Production of Electric Oscillations and High Frequency Currents.

A condenser essentially consists of a pair of conducting surfaces separated by a dielectric, and the familiar Leyden jar presents itself as an illustration. There

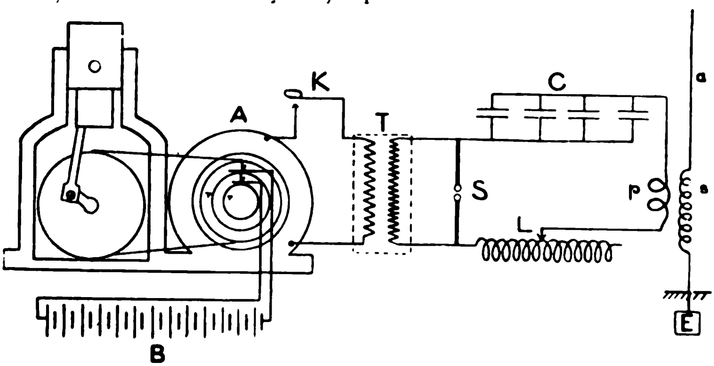


FIG. 58.—Arrangement of Power-Plant for the Production of Electric Oscillations and Electric Waves. O, oil engine; A, alternator; T, transformer; S, spark balls; C, condenser; L, induction; *p*, *s*, oscillation transformer; *a*, antenna or radiator; B, battery for exciting alternator fields; K, key.

are not many solid dielectrics which are capable of being used for charging voltages reckoned in thousands of volts, and the number available for condenser construction is still more limited when questions of cost and internal energy loss in the dielectric are considered.

Glass of certain compositions, ebonite, mica, and micanite, or mica sheets built up with shellac, almost exhaust the list of solid dielectrics suitable for very high pressures. On the other hand, compressed gases and also certain insulating liquids can be usefully employed as dielectrics in the construction of high-pressure condensers. Deferring for the present a further consideration of dielectric properties, it may be said that glass, micanite, and ebonite constitute almost the only available commercial solid dielectrics for condenser construction.

Of these, English flint glass is by far the best material to use, comparing either equal bulks or equal energy storing power, but it is brittle and liable to flaws. Its dielectric constant is high (from 5 to 10), but its dielectric strength is inferior to that of good ebonite or micanite. Ebonite has great advantages for certain quantitative work, as its dielectric constant is constant for a wide range of frequency. Micanite has greater dielectric strength than either glass or ebonite, but its dielectric constant varies considerably with frequency.

A condenser is constructed by applying sheets of flexible metal to the two surfaces of a sheet of dielectric. Usually tinfoil is put upon sheets of glass or ebonite or micanite, or the glass or ebonite may be silvered by an electrochemical process or metallic paint put upon it. Otherwise plates of thin zinc may be employed. The most usual practice is to stick tinfoil sheets upon glass with some adhesive such as shellac varnish, seccotine, or isinglass. In the construction of high-tension condensers, no adhesive containing water, such as gum or paste, should be employed, as the water cannot evaporate. A thin shellac varnish, made up with absolute alcohol or anhydrous methylated spirit or wood naphtha, answers well for glass. The tinfoil sheets must be made to adhere perfectly to the surface of the dielectric, and care taken to exclude air-bubbles. It is much more difficult to secure good adherence between tinfoil and ebonite, but the shellac solution answers well with micanite as the dielectric.

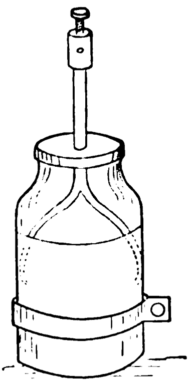


Fig. 59.—Leyden Jar with Spring Clips.

If glass is used it should be a good quality of flint glass, and should be absolutely free from bubbles. Any flaw of this kind is a weak place which sooner or later gives way.

In making an ordinary Leyden jar a considerable margin (at least 25 per cent. of the height) should be left uncovered with tinfoil, and this bare dielectric should be well varnished with anhydrous shellac varnish. The method of securing contact with the tinfoil surfaces is important. The outside coating of the jar should be embraced by a brass strap with a terminal and tightening screw (see Fig. 59), and the brass stem should end in a screw terminal, and should not have the ordinary chain, but be provided with spring extensions, which press tightly against the inner tinfoil surface. The object is to prevent any spark at these contact places, which would quickly pierce the glass. The jars so constructed can easily be joined in parallel or series by the aid of straps of thin sheet copper or stout copper wires.

The Leyden jar should always have its capacity measured and marked on it, expressed in fractions of a microfarad. Instrument makers still maintain the absurd custom of denominating Leyden jars as "pint size," "quart size," or "gallon size." The so-called pint size has a capacity of about $\frac{1}{700}$ microfarad, and the so-called gallon size about $\frac{1}{300}$ microfarad.

Glass Leyden jars, as usually made, will stand charging with 20,000 volts. Hence the energy-storing capacity of the "pint size" (being equal to $\frac{1}{2}CV^2$) is about 0·28 of a joule at this pressure, or nearly $\frac{1}{5}$ foot-pound. This is a very small storage compared with the over-all bulk of the jar.

A more satisfactory form of condenser for many purposes may be constructed by covering flat sheets of good flint glass with tinfoil on both sides. The tinfoil sheets should be cut 1 inch smaller each way than the glass plate. The glass should be carefully selected and free from bubbles or flaws, and about $\frac{1}{16}$ or $\frac{1}{8}$ inch (=3 mms.) in thickness.

The sharp edges should be taken off with an emery wheel. The tinfoil is then

stuck on with shellac varnish and the margin of the plate varnished. Each tinfoil sheet must have a wide tinfoil lug attached to it, and the lugs on opposite sides of

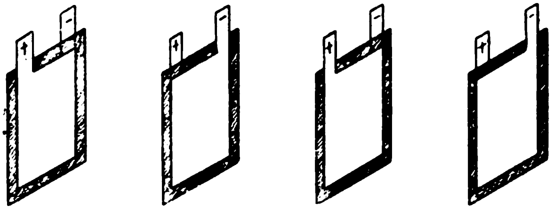


FIG. 60.—Diagram showing the Mode of Arranging the Coatings of Condenser Plates.

the same plate must be at opposite corners, but at adjacent corners of neighbouring plates. Plates should be prepared like right- and left-hand gloves, so that when piled one on the other the lugs on the adjacent condenser plates fall upon each other (see Fig. 60). In the diagram, for the sake of clearness, the plates are shown as widely separated. In actuality they are placed close together. The coated plates should, however, be prevented from coming into absolute contact by discs of card stuck on to the tinfoil by shellac varnish. A pile of any number of such sheets may be made, and when bound together with silk tape may be placed in a stoneware or ebonite box which is filled up with vaseline or double-boiled linseed oil. The oil prevents electric discharge over the edges of the plates. The positive lugs are then all connected to one terminal, and the negative lugs to another terminal placed on the lid of the box.

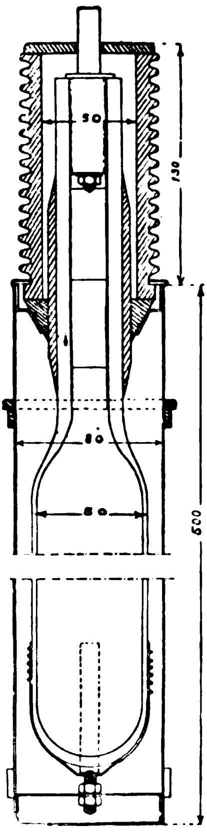


FIG. 61.—Section of Moscicki Condenser.

Glass-plate condensers of the above form can be made without oil insulation if the glass-plate margin beyond the tinfoil is large enough, but the use of oil is essential for very high-tension work.

In some cases glass tubes are employed coated partly outside and partly inside with tinfoil. Test tubes silvered inside and outside for half their height make very convenient small condensers.

Thin glass has a higher dielectric strength than thick glass, and hence nests of thin tubular glass condensers joined in series have often been employed.

Moscicki has suggested the use of glass tubes made thicker at the ends than in the middle, and coated within and without with a deposited silver film in the middle portion as a method of making condensers. (See *Engineering*, December 20, 1904, p. 865.)

An account of Moscicki's work will be found in *L'Éclairage Électrique* for October 1904 (vol. 41, p. 14); and also in the *Electrotechnische Zeitschrift*, Nos. 25 and 26, June 23 and 30, 1904. He came to the conclusion, as the author and others had done long previously, that glass was the most suitable dielectric for high-pressure condensers, and he employed it in the form of glass tubes 0·5 mm. thick; but these tubes were thickened up at the ends, as otherwise he found they were perforated at the edges of the coatings by a voltage which the central portions of the glass could easily sustain. These glass tubes are coated with tinfoil or silver by deposit, the foil being put on with turpentine, and air-bubbles carefully excluded.

A condenser for a power corresponding to 0·5 kilovolt amperes is made with five tubes of glass of which the diameter is 3 cms. and the thickness of wall 0·5 mm. These are contained in a cylinder of glass 47 cms. high and 9 cms. in diameter. The total weight varies from 3 to 3·5 kilogs. (7 to 8 lbs.). Such condensers will stand a working pressure of 20,000 volts. It is claimed for these cylindrical condensers that they can be operated at a higher voltage per millimetre of thickness of the glass than flat plate condensers, and do not fail or heat on continuous working, and that with an alternating current having a frequency of 50~ the dielectric loss or loss by surface discharge does not exceed 1 per cent. of the energy-storing capacity.

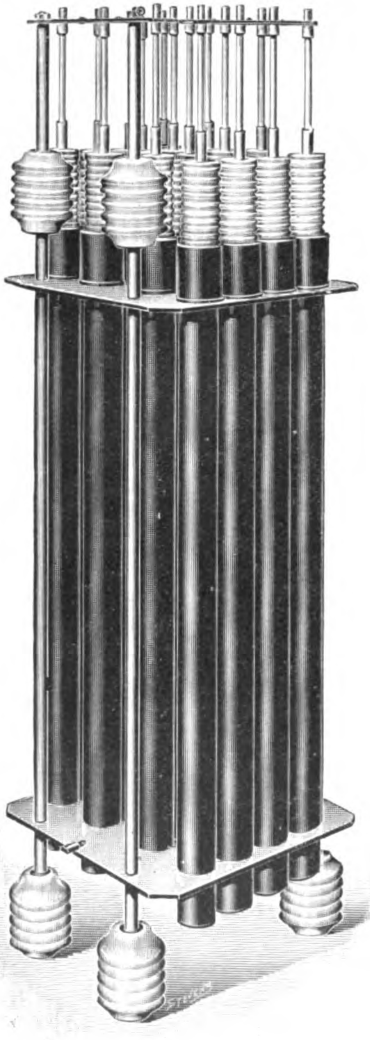


FIG. 62.—A Battery of Moscicki Condensers.

It is well known that in the absence of flaws a plate condenser or Leyden jar is most usually punctured by the electric strain at some place near the edge of the tinfoil where the electric density is greatest. Moscicki states that a glass condenser plate is more easily punctured at the edges of the tinfoil when it is immersed in insulating oil.

In the latest form the Moscicki condenser is a sort of narrow glass bottle with the neck thicker than the bulb (see Fig. 61). This is coated within and without with a chemically deposited film of silver overlaid with copper. The inner layer is connected to a well-insulated terminal. This tube is enclosed in a watertight metal tube, and the inner space filled in with glycerine and water. This acts as a cooling agent, and makes perfect contact between the outer metal case and the outer silver coating of the jar. Such tubes are then assembled in batteries of any required capacity (see Fig. 62). They are all tested to a very much higher voltage than that at which they are to be used.

For the construction of condensers intended for very high pressures, micanite sheets, $\frac{1}{10}$ inch or 2·5 mms. in thickness, may be employed as the dielectric. To these sheets of tinfoil 1 inch smaller each way may be affixed by means of shellac varnish, and the coated plates immersed in a stoneware or ebonite box, filled with double-boiled linseed oil. As this oil does not dissolve shellac, a wooden box, well coated in the interior and with all joints covered with shellaced paper, may be employed to hold the oil.

For quantitative purposes, condensers constructed of metal plates placed in paraffin oil are to be preferred, since the dielectric constant of paraffin oil is not,

like that of glass, a function of the frequency (see Chap. II.). If ebonite is used as the dielectric the difficulty is to make the tinfoil stick to the ebonite. The adhesive called siccotine, or else indiarubber solution, may be employed for this purpose. The author has, however, found that a better plan is to cut sheets of ordinary tin-plate in pairs with right- and left-handed lugs, and pile these together with sheets of ebonite interposed on the plan just described for making a glass-plate condenser. The pile of condenser plates must be strongly compressed, bound together with silk tape, and immersed in insulating oil.

In some cases condensers of adjustable capacity are required. If only small capacities are required, this may be provided in the form of an air condenser with flat plates, which can be moved to or from each other, or the plates may be immersed in some liquid dielectric, such as paraffin oil or turpentine.

A convenient form of sliding condenser consists of a thin-walled cylinder of ebonite, closed at the bottom and lined within up to an inch of the top with a closely fitting cylinder of metal. The outside of the cylinder of ebonite is also covered with a closely fitting cylinder of metal, and the arrangement resembles that called a dissected Leyden jar. By drawing the outside cylinder more or less off the ebonite one, the capacity is reduced, and the capacity corresponding to various positions of the outer cylinder can be marked on the ebonite.

Another form of condenser of adjustable capacity, suitable, however, only for a small range of variation and for a small capacity, is made as follows :—

In a glass jar or ebonite box are fitted a number of pairs of quadrant-shaped plates, one above the other. These resemble the fixed plates in a Kelvin multicellular electrostatic voltmeter. All these quadrant plates are connected together and to one terminal on the box. In the centre is a metal rod in pivot carrying a number of paddle-shaped metal plates which are spaced apart by the same distance as the fixed plates. The rod is so held that the plates on it are interspaced with the fixed plates. The box is filled with insulating oil. When the movable plates are turned by the rod so as to be quite within the fixed plates, they form with these last a condenser of which the oil is the dielectric. When they are turned so as to be quite apart from the fixed plates, the capacity is greatly reduced. If the rod carries a pointer moving over a scale, the scale can be calibrated to show the capacity of the two sets of plates with respect to each other for any required positions of the movable plates. The rod is, of course, connected by some form of spring or bearing contact with the second terminal of the instrument (see Fig. 63).

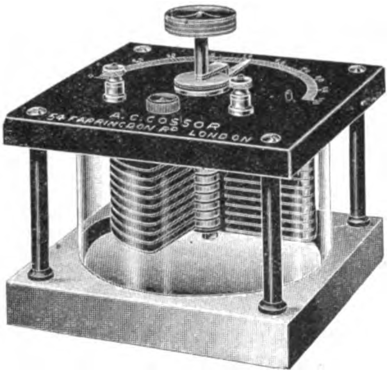


FIG. 63.—Variable Capacity Air or Oil Condenser.

In the construction or selection of condensers, especially those of large capacity for wireless telegraph purposes, we have to give due weight to various considerations. We have to consider questions of durability, energy dissipation, bulk, and cost. The ordinary Leyden jar is simple and not objectionable where small capacities alone are concerned, but its energy-storing capacity is small compared with its bulk, and its use is out of the question when large capacities such as 1 or 2 microfarads are concerned.

When large condensers have to be in continual use, the dielectric hysteresis becomes important, and also any tendency in the dielectric to "age" or become brittle by long use. Glass gives some trouble in this last respect. Ebonite is too costly to be used for large capacities, and micanite has too much dielectric hysteresis. Hence attention has been directed to the use of air as a dielectric.

Owing to the relatively small dielectric strength of air at normal pressures, we

are either obliged to use very large metal plates set far apart, or else to employ compressed air as the dielectric.

Since the dielectric strength of air at atmospheric pressure is very nearly 38,000 volts per centimetre (see Chap. II. § 6), and since a factor of safety of at least 5 or 6 should be used to avoid considerable energy loss by brush discharge, it is seen that if we wish to work an air condenser at a voltage of 100,000 volts, the plates must be at least 20 cms. apart. It will be shown in the next chapter that the capacity in microfarads of a parallel plate condenser of which the plate diameter is large compared with their distance apart can be very roughly calculated by the formula—

$$\textit{capacity in microfarads} = \frac{\textit{surface of plate in square centimetres}}{4\pi \times 9 \times 10^5 \times d}$$

where d is the distance of the parallel plates in centimetres. If, then, $d = 20$ cms., we should require a total positive or negative plate surface of nearly 226 million square cms., or 22,600 square metres, to obtain a capacity of 1 mfd. This means that two square plates, each having a side of 150 metres, or nearly 500 feet, placed 20 cms., or nearly 8 inches, apart in air at ordinary pressure, would have a capacity of 1 mfd., and would stand charging to a pressure of 100,000 or even 300,000 volts without sparking across. At 100,000 volts this condenser would store up 5000 joules of energy, or nearly 4000 foot-pounds, and would have a bulk of nearly 170,000 cubic feet. This is at the rate of 40 cubic feet per foot-pound of stored energy. A glass-plate condenser for the same capacity and voltage would not occupy one-hundredth part of the above volume.

The use of compressed air as a dielectric presents some advantages.

The dielectric strength increases almost proportionately to the pressure. Hence if, instead of employing air at atmospheric pressure as the dielectric, we compress it to 140 pounds on the square inch, it attains a dielectric strength far greater than that of glass. Also the dielectric constant is slightly increased.

Moreover, as R. A. Fessenden has shown, brush discharges are at high air pressures almost abolished. Accordingly an air condenser can be advantageously constructed with compressed air as dielectric.

Metal plates kept at a small distance apart are enclosed in a strong iron vessel in which air can be compressed under 10 or 12 atmospheres. Thus Fessenden states (see U.S.A. Patent, No. 793,777, applied for March 30, 1905, or *The Electrician*, 1905, vol. 55, p. 795) that in air at 175 pounds pressure per square inch metal plates 0·083 inch apart will withstand without sparking a voltage of 27,500 volts. At this rate an air condenser of 1 mfd. capacity to stand 100,000 volts could be contained in a space of 500 cubic feet, and would not exhibit energy loss by electric brush discharge or dielectric hysteresis to any sensible degree. It seems evident that the use of compressed air, or, better still, compressed nitrogen or carbonic dioxide, as a dielectric for condensers will be found to possess many advantages in constructing high voltage condensers at reasonable cost for wireless telegraph power stations. In large radiotelegraphic stations, where space can be obtained, air condensers consisting of plates of metal suspended in the atmosphere several inches apart have been employed, as mentioned in a later chapter.

12. Oscillation Transformers.—An essential part of the arrangements for producing trains of electric oscillations by condenser discharge is the inductive circuit which is placed in series with the condenser. This most frequently consists of one circuit of an air core transformer which is called an *oscillation transformer.*

Two circuits are associated together inductively by being wound over one another on some support, but at the same time well insulated from each other. One of these is called the primary and the other the secondary circuit. The primary circuit is placed in series with the condenser and the spark ball discharger, this constituting the circuit in which electric oscillations are set up by the discharge of the condenser. These oscillations induce other oscillations called secondary oscillations in the secondary circuit of the oscillation transformer, and if the secondary circuit has a larger number of turns than the primary circuit, the potential difference at the extremities of the circuit of the oscillation transformer will be greater than at the terminals of the primary circuit in a certain ratio.

The form which this oscillation transformer takes is dependent upon the purposes to which the apparatus is to be applied. One well-known form of oscillation transformer is called a Tesla Coil, and a description of this coil was given by Mr. Nikola Tesla in a lecture delivered some years ago before the Royal Institution in London (in February 1892) as follows[35] :—

"The coil consists of two spools of hard rubber, R, R (see Fig. 64), held apart at a distance of 10 cms. by bolts, *c*, and nuts, *n*, likewise of hard rubber. Each spool comprises a tube, T, of

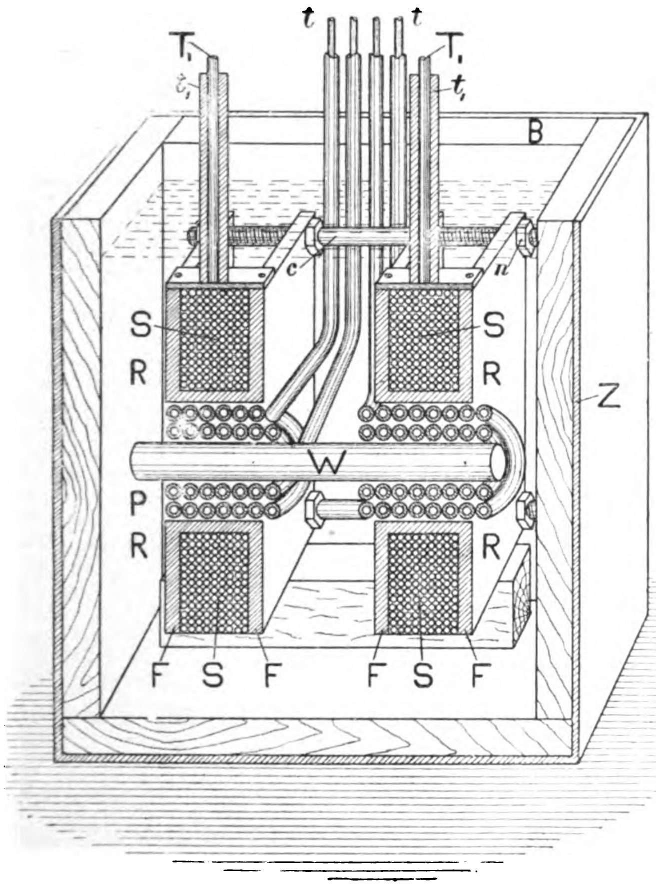


FIG. 64.—Tesla Oscillation Transformer. (*Sectional view.*)

approximately 8 cms. inside diameter and 3 mms. thick upon which are screwed two flanges, F, F, 24 cms. square. The secondary, S, S, of the best guttapercha-covered wire, has 26 layers, 10 turns in each, giving for each half a total of 260 turns. The two halfs are wound oppositely and connected in series, the connection between both being made over the primary. This disposition, besides being convenient, has the advantage that when the coil is well balanced—that is, when both of its terminals, T_1, T_1, are connected to bodies or devices of equal capacity—there is not much danger of breaking through to the primary, and the insulation between the primary and the secondary need not be thick. In using the coil, it is advisable to attach to *both* terminals devices of nearly equal capacity, as, when the capacity of the terminals is not equal, sparks will

[35] See Nikola Tesla, "Experiments with Alternate Currents of High Potential and High Frequency," *Journal of the Institution of Electrical Engineers*, 1892, vol. xxi. p. 62.

be apt to pass to the primary. To avoid this, the middle point of the secondary may be connected to the primary, but this is not always practicable.

"The primary P, P is wound in two parts, and oppositely, upon a wooden spool, W, and the four ends are led out of the oil through hard rubber tubes, t, t. The ends of the secondary T_1, T_1 are also led out of the oil through rubber tubes, t_1, t_1, of great thickness. The primary and secondary layers are insulated by cotton cloth, the thickness of the insulation, of course, bearing some proportion to the difference of potential between the turns of the different layers. Each half of the primary has four layers, 24 turns in each, this giving a total of 96 turns. When both the parts are connected in series, this gives a ratio of conversion of about 1 : 2·7, and with the primaries in multiple 1 : 5·4; but in operating with very rapidly alternating currents, this ratio does not convey even an approximate idea of the ratio of the E.M.F.'s in the primary and secondary circuits."

The coil is placed in a wood or ebonite box, which is filled with double-boiled linseed oil or highly insulating resin oil, or else with ordinary fluid high flash-point paraffin oil, and the coil when in place must be entirely covered by the oil. The coil is held in position in the oil on wooden supports, there being about 5 cms. thickness of oil all round. Where the oil is not specially needed, the space is filled with pieces of wood, and for this purpose principally the wooden box B surrounding the whole is used.

In oscillation transformers in which the primary circuit is traversed by the discharge of a condenser and a secondary circuit is inductively associated with it, this latter, if in many turns, becomes the seat of very high electromotive forces. In fact, differences of potential amounting to many hundreds of volts may exist between adjacent turns of the secondary. Hence, very good insulation is required, and it has been found that no form of secondary winding in which layers of wire are wound over one another or in which the different turns of the wire are in close contact will very long withstand the electric strain without failure of insulation.

Hence one great principle in the construction of a high potential high frequency induction coil or oscillation transformer is to wind the primary and secondary circuit in single layers, the turns not touching.

This may be achieved in the following manner. The primary circuit consists of a spiral of bare copper wire, 3 or 4 mms. in diameter, the spiral consisting, say, of 20 turns wound open fashion round a mandril 7 or 8 cms. in diameter. Within this spiral is placed an ebonite or glass tube, the walls of which are at least 3 mms. thick and the length 25 cms. or so. On this glass or ebonite tube is wound in one single layer a much finer silk-covered wire, say No. 26 S.W.G. size = 0·457 mm. diameter. The turns of this wire are prevented from touching each other by winding a paraffined silk thread in between them, or by winding the wire in a groove turned in the cylinder. This bobbin may be placed in a glass or ebonite box full of double-boiled linseed oil or vaseline oil free from water. The coil must be entirely immersed, and the ends of the primary and secondary wires must be brought out through glass or ebonite tubes which have their lower ends well under the oil.

When oscillatory discharges from a condenser or Leyden jar are passed through the thick spiral, we can obtain high potential high frequency discharges from the secondary circuit.

The following is a detailed description which has been given by Professor Elihu Thomson of two oscillation transformers of the above kind. One suitable for creating 30-inch sparks was made as follows[36]:—

The primary consisted of 10 turns of wire, made up of two No. 6 copper wires wound on a wooden frame. The wires were wound side by side in notches. This coil or mandril was 18 inches long and 15·5 inches in diameter. Its resistance was 0·0088 ohm and inductance 0·9976 millihenry.

The secondary consisted of 396 turns of insulated wire, No. 26 B. and S. gauge, wound as a single layer in notches on an ebonite frame, the wire turns being spaced apart so as to form a coil 18 inches in length. The diameter of the secondary was 12 inches, and it was placed inside the primary. The total length of secondary wire was 1250 feet and weight one pound. The resistance was 41·6 ohms and inductance 25·2 millihenrys. These coils were immersed and supported

[36] See Elihu Thomson, "On Apparatus for Obtaining High Frequencies and Pressures," *The Electrician*, November 3, 1899, vol. 44, p. 40.

concentrically in a vat of oil, and the secondary had its terminals carried out through glass tubes to spark balls.

Two condensers were provided for creating the primary discharges. They consisted of two boxes, each 7 inches by 15½ inches inside and 17½ inches deep. Each box contained 84 built-up mica sheets, 15 inches square and 0·075 inch thick; 42 of these were coated with tinfoil 10 inches by 11 inches in area. The effective coated surface of each plate was 110 inches, and the total surface 4510 square inches. The capacity of each condenser was 0·03 mfd. Hence the two boxes afforded a total capacity of 0·06 mfd. When these condensers were charged from the high-tension terminals of an alternating current transformer at a pressure of 20,000 volts, and discharged across an air gap through the primary circuit of the

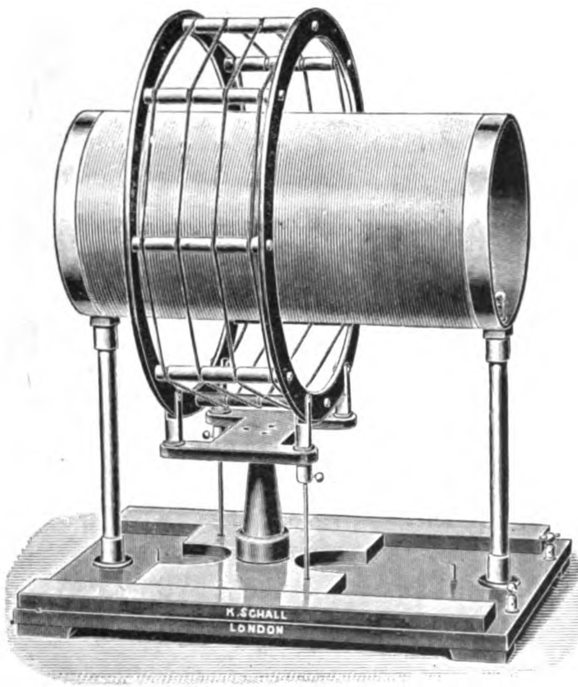


FIG. 65.—Oscillation Transformer, Primary and Secondary loosely coupled.

above-described oscillation transformer, oscillatory high frequency sparks 30 inches in length passed between the terminals of the secondary circuit.

An oscillation transformer giving 64-inch sparks was made as follows :—

The primary coil consisted of 15 turns of double No. 6 S.W.G. copper wire, the length being 85 feet of wire double wound in an open coil 28 inches in length and 22 inches in diameter. The resistance of this primary circuit was 0·0147 ohm and inductance 0·09 of a millihenry.

The secondary bobbin was 28 inches long and 17 inches in diameter, and was placed inside the primary coil. The wire was No. 26 S.W.G. size, about 2·25 lbs. in weight and 2600 feet in length, and wound in notches on an ebonite frame in 580 turns.

The associated condenser consisted of mica plates covered with tinfoil and arranged in oil in a box. Three such boxes were used, each having a capacity of 0·015 mfd., and having therefore a total capacity of 0·045 mfd. These were charged

by an alternating current transformer having a voltage of 30,000 and discharged through the primary of the above-described coil across an air-gap. A blast of air was kept blowing on the gaps during discharge to destroy the arc.

In some cases the use of vats of oil is objectionable, hence for moderate spark lengths it is desirable to dispense with oil insulation. In this case the secondary circuit must be wound on one layer on a glass or ebonite tube. If guttapercha covered wire is used, it must be covered with a layer of well shellaced tape to protect it from the action of light and air. This ebonite tube may be placed inside another tube, on the outside of which the primary coil is wound in a few open turns, or the primary may be placed inside the glass or ebonite tube on which the secondary is wound. In any case the ends of the secondary circuit must be

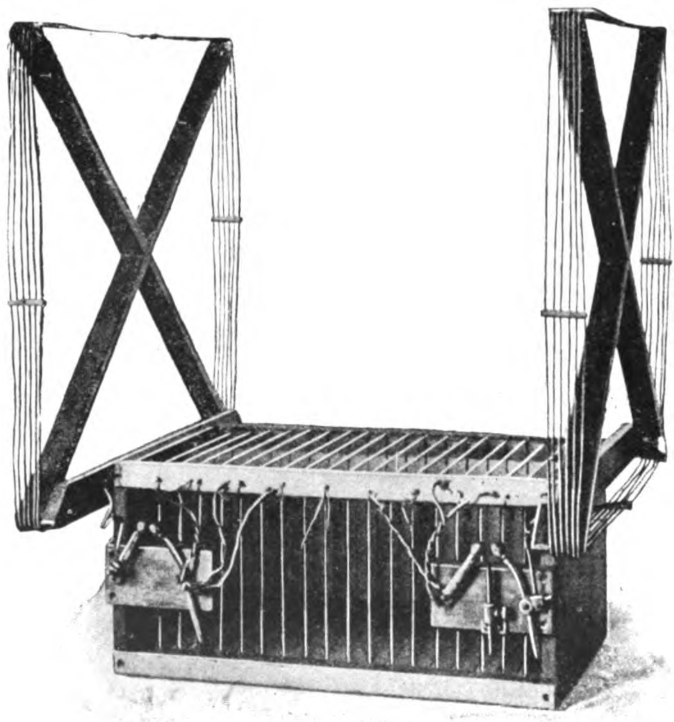


FIG. 66.—Oscillation Transformer, with Primary and Secondary Coils widely separated.

brought out at opposite ends of the tube in which or on which it is wound. In other cases the primary circuit forms a few open turns of much larger diameter than the secondary circuit (see Fig. 65). This last form is described as the "loose coupling" of the primary and secondary circuits, whereas when the primary and secondary are in close contact the arrangement is called "close coupling."

In any case the primary circuit should consist of a few turns of wire as openly spaced as possible, for the sake of making the inductance low.

Nothing is gained by using a primary circuit of many close turns, because the increase of inductive effect on the secondary, due to an increase in the number of primary turns, is almost exactly annulled by the decreased current through the primary, due to its own greater inductance. This matter will be considered more in detail in the next chapter.

Marconi devised for wireless telegraph purposes a form of oscillation trans-

former in which the two circuits are of heavily insulated wire, and are wound ove one another on a square or round wooden frame. The primary circuit consists of one or a few turns of a number of wires in parallel, and the secondary circuit of a few turns of wire in series.

In some cases where very loose coupling is required the primary and secondary circuits may consist of insulated wire wound on two square frames, the primary on one and the secondary on the other, and these frames may be placed a considerable distance apart (see Fig. 66).

The full theory of oscillation transformers cannot be given until the subject of electric resonance has been considered, but meanwhile it will be sufficient to caution the reader that the ratio in which a high frequency oscillation transformer transforms electric pressure is by no means in the ratio of the number of turns. The oscillation transformer must in some of its forms be considered as a transformer with very large magnetic leakage, hence only a small portion of the magnetic flux created by the primary circuit is linked with the secondary circuit.

In the chapter on wireless telegraphy receivers we shall consider the structure of certain peculiar forms of oscillation transformer which have been found to be of use in transforming the extra high frequency oscillations produced in wires by the impact on them of electric waves.

13. General Arrangement of Apparatus for Producing Electric Oscillations by Means of Condenser Discharges. Arc-stoppers and Dischargers.—Having considered the principal pieces of apparatus in detail, we may next discuss the general arrangements convenient for certain classes of work in connection with the production of trains of damped electric oscillations and electric waves.

When no very great power is required, say an expenditure up to 150 watts or so, the most simple and easily managed arrangement is a 10-inch induction coil worked off secondary cells, actuated either by a motor-driven mercury break or else an automatic hammer break. Since the above coil requires 10 amperes at 16 volts to work it well, it is best to work it with 8 to 10 storage cells, capable of giving 10 amperes for 4 or 5 hours. These cells are now made up in sets of 4 or 6 in celluloid boxes contained in a teak case. There should be a double pole cut-out or fuse wire inserted between the battery and coil, and also a double pole switch, so that if the hammer break sticks the cells will not be overworked. These cells can be charged from any continuous current lighting circuit through resistances or lamps.

The condenser attached to the secondary terminals of the coil may consist of Leyden jars or an ebonite or glass-plate condenser in oil.

The variable inductance used in series with it may be of the pattern shown in Fig. 12, § 5, of Chapter II. A special spark discharger with balls adjustable for distance by a fine screw is very convenient, and this should be contained in a wooden or metal box, so as to shut in the light of the spark and reduce the noise. The circuit of the condenser may also contain a Tesla coil or oscillation transformer, and we can then draw from the terminals of this coil high frequency high potential discharges, sparks, or brushes. An apparatus of this kind is much used for electro-medical work.

When a more powerful plant is required, then an alternating current transformer must be employed. This may be of any size from ½ kw. output upwards. A convenient size is a 2-kw. transformer, raising pressure from 140 to 20,000 volts, adapted for a frequency of 50. Associated with this is a motor-generator consisting of a four-pole continuous current motor with Gramme ring armature and slip rings on the shaft, as described in § 10 of this chapter. This machine may be of 3 kw. size, and if wound for 200 volts on the continuous current side and a speed of 1500 R.P.M., will give alternating current at a frequency of 50 and a voltage of 140 on the slip rings.

If a continuous current supply is not available to drive such a motor-generator, then a small oil engine may be coupled to it to drive it, or else the engine may be employed to drive a suitable small alternator as already described.

In those cases in which larger powers still are required, a plant consisting of an oil or steam engine driving directly or by a belt an alternator giving alternating current at 2000 volts may be arranged. The pressure of the current is then raised

by transformers to 20,000 or 30,000 volts. In this case the transformers should be oil-insulated transformers.

When low resistance transformers of large size are employed to charge condensers, it is necessary to destroy the alternating current arc which tends to form across the spark balls, and so stops the production of oscillations.

This may best be accomplished by means of a plan devised by the author.[37] In the primary circuit of the transformer T (see Fig. 67) are placed two choking coils, H_1, H_2, or inductances in series, each consisting of a long bobbin of wire standing on an insulated wooden slab. An iron core for each coil, E_1 and E_2, is provided, made of thin sheet-iron stampings like a transformer core, and it is in the shape of a letter E (see Fig. 67). If this E-shaped iron is let down into the coil it gives it a greatly increased inductance. In the wooden base there is a transverse piece of laminated iron which completes the magnetic circuit when the core is let right down. Two such choking coils are joined in series with each other and with the primary circuit of the transformer T. These choking coils can be short-circuited by keys, K_1 and K_2. The alternator A is run at a speed required to give the necessary normal primary current of the transformer, and both iron cores are let down into the choking coils. Then the secondary circuit of the transformer T is short-circuited, and also one of the choking coils H_2, by its appropriate key, K_2, and furthermore the core of the other choking coil H_1 is

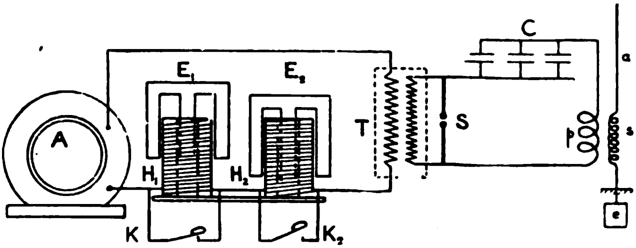


FIG. 67. — Arrangement of Apparatus for Producing Powerful Electric Oscillations. (Fleming.) A, alternator; T, transformer; H_1, H_2, choking coils; K_1, K_2, keys; C, condenser; *p*, *s*, oscillation transformer.

raised until the current flowing through it is not more than the full load current of the transformer T. In the next place the secondary terminals of the high-tension transformer are connected to a pair of spark balls, S, and to a condenser, C, and inductance in *p* series, which last may consist of the primary circuit of an oscillation transformer, *ps*, of any form.

If then the key K_2 is raised, and if the spark balls are adjusted at the proper distance, it will be found that no spark passes when the short-circuit key of the choker is up, but that a condenser discharge takes place when the key is depressed. The reason for this is that when both choking coils are operative the impedance is so great that no current can flow through them sufficient to create much secondary voltage in the high-tension transformer. If, however, one choker is short-circuited, then the impedance is so far reduced that the transformer receives current enough to create a secondary voltage. By adjusting the length of the spark gap and the position of the core of one of the chokers, it is possible to make this spark consist wholly of an oscillatory discharge of the condenser, and not have superimposed upon it any alternating current arc discharge directly due to the transformer. If this arc discharge is not suppressed there will be no true oscillatory discharge in the condenser circuit, or only a feeble one.

The reason for this is obvious. As long as the arc discharge continues, the secondary terminals of the transformer are reduced to nearly the same potential, or at most differ by a few hundred volts. It is not until the arc is stopped that the

[37] See British Patent Specification, No. 3481 of 1901, application of February 18, 1901; also United States Patent Specification, No. 758,004, application of April 8, 1901.

spark balls come up to a sufficient potential difference to give a fresh charge to the condenser, and by creating a discharge across the gap start into existence a fresh train of oscillation.

Various other plans have been suggested for destroying the arc discharge whilst permitting the condenser discharge to take place.

Tesla employed a powerful magnet placed with the direction of its magnet interpolar field transverse to the line joining the spark balls. The pointed field poles were covered with some non-conducting and non-inflammable material, such as mica or porcelain. This strong magnetic field blows out the arc just as in the ordinary electric tram-car controller. Another plan, due to Elihu Thomson, is to employ a powerful jet of air. The air blast is applied just between the spark balls, and blows away the arc but not the condenser spark.

A third plan, proposed by M. D'Arsonval, is to construct the discharger with the spark balls at the extremities of metallic arms. One of these is made to revolve at a high speed. Hence the arc, if formed, is broken as the balls separate. The condenser is then again charged, and discharges again as the balls pass each other, but the electric arc which forms at that instant is again destroyed as the balls move apart. A somewhat similar arrangement has been described by Robert Grison. A shaft has on it four arms of metal each ending in a ball. It is caused to revolve so that the balls at the arm extremities pass in their revolution between two other fixed balls, but just not touching them. The condenser in series with these two last balls is then discharged four times every revolution, but the arc which attempts to follow is at once extinguished. The high speed rotating dischargers of Mr. Marconi, which are described in detail in Chap. VII. § 18, effect still more perfectly an extinction of the arc discharge. A fourth plan is to employ a transformer, as made by Leslie Miller, with large magnetic leakage. Hence, as soon as the condenser is charged and discharges, and the true arc discharge created, the current given out by the secondary circuit of the transformer is greatly increased. This, owing to the construction of the transformer, causes so large a fall in potential between the terminals that the arc can no longer be maintained.

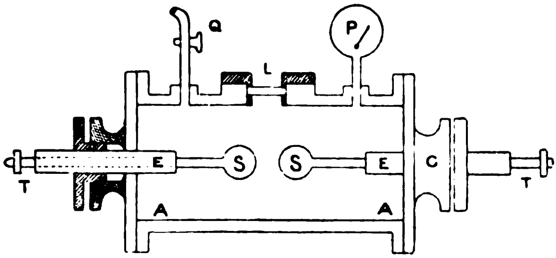


FIG. 68.—Silent Discharger. (Fleming.) S, S, spark balls; A, A, cast-iron case; L, peep-hole; P, pressure gauge; Q, air-pipe.

This extinction of the alternating current arc is facilitated by the employment of curved metallic horns instead of spark balls.

It is well known that if an alternating current arc is formed between such horns, the arc tends to rise up to the wider part of the gap. In so doing it gets stretched out and extinguished, and the process is assisted by the upward draught of air caused by the arc, and this can be furthermore helped by putting a non-conducting porcelain or stoneware chimney over the horns to help the draught action.

When employing only small powers, the spark discharger consists usually of two brass balls, 1 or 2 cms. in diameter, their distance being adjustable. The ordinary sliding rods terminate in brass balls, which are placed on induction coil secondary terminals, are quite suitable as a discharger for many experiments, and even for wireless telegraphy. For larger powers, balls of some more refractory material, such as cast iron, are better.

The distance of these discharge surfaces must be capable of accurate adjustment by means of a screw. As the noise of the oscillatory spark is very distressing when large powers are being employed, the author has devised a plan by which the discharger is contained in a cast-iron case with thick walls. A peep-hole

glazed with thick plate glass is provided, and also stuffing boxes or glands, through which are passed ebonite rods pierced with metal rods, by which the discharge is conveyed to balls fixed on the ends of the rods. The diagram in Fig. 68 shows such a silent discharger.

The discharger will only be silent if the iron case has very thick walls and is closed perfectly air-tight. It may also be arranged to contain compressed carbonic acid gas or nitrogen. If the spark is taken in compressed air or other gases, the spark length for any given voltage is almost inversely as the total pressure. Further reference to this matter is made in the next chapter.

One difficulty which presents itself when the spark is taken in a closed vessel full of air is the chemical production of oxides of nitrogen by the discharge. These vapours, being acid, cause a loss of insulation by condensing on the insulating supports. The difficulty is only slightly overcome by placing quicklime or caustic potash in the interior. A better plan is to fill the vessel once for all with nitrogen gas.

This can be prepared sufficiently pure for this purpose by burning pieces of phosphorus under a glass bell jar standing over water. When the phosphoric pentoxide produced has dissolved, the residual gas can be pumped into the spark-ball chamber, provided that the air has previously been exhausted from it. When once the spark box has been filled with nitrogen, it will not, if air-tight, require further attention for some time, and no production of oxides of nitrogen can take place.

Since the spark balls wear away with the discharge, it is necessary to make some arrangement for turning them round, so as to bring fresh surfaces continually in opposition.

The author has devised a special form of enclosed discharger with rotating balls, which can be worked in compressed gases. The detailed description of this discharger is given in a later chapter of this treatise.

In all experimental work in which an induction coil or transformer is employed to charge a condenser, subsequently discharged across a spark gap to produce damped oscillations, it is a great advantage to keep a steady blast of air impinging upon the balls and the gaps between them. Even with an induction coil there is a certain amount of arcing between the balls, that is to say, the discharge which takes place between them is not wholly due to energy coming out of the condenser, but is partly due to energy coming directly from the coil secondary circuit which takes the form of an arc.

The air blast extinguishes this arc as soon as it is formed, and it also keeps the balls cool, and so maintains at its highest value the spark potential difference corresponding to a given spark gap length. The blast of air can be conveniently provided by a Lennox blower, or rotary fan driven by a small electric motor, which can be actuated by any ordinary electric supply service. An air blast equal to a pressure of 16 or 20 inches of water can thus be obtained, and this is allowed to impinge on the air gap by means of a glass jet. In the case of larger transformer plants a higher air pressure is necessary to quench the arc. In all quantitative experiments an air blast thus used greatly assists in keeping the discharge current constant. We shall further discuss the action of dischargers and various types of them in a later chapter when dealing with radiotelegraphic appliances, and the reader is particularly referred to Chap. VII. § 18, for a description of Mr. Marconi's high-speed rotating dischargers for long-distance radiotelegraphy.

14. The Production of Undamped Oscillations by Means of the Electric Arc.—In connection with investigations on high frequency electric currents and electric oscillations, it was very early recognized that some means was required for producing undamped or persistent oscillations of a much higher frequency than those which can be conveniently and easily generated by high frequency alternators. Remarkable discoveries in connection with the continuous current electric arc have, however, provided one solution of the problem, and enabled us to produce undamped oscillations of a frequency and amplitude useful in radiotelegraphy.

They have at the same time given the means for accomplishing the important feat of transmitting not merely signals, but articulate speech, by means of electric

waves to a distance. A point of departure in this matter is a United States Patent Specification, No. 500,630, July 18, 1892, filed by Professor Elihu Thomson, describing a method for the production of undamped or persistent electric oscillations by the following means. One or two coils of large inductance, R (see Fig. 69), are placed in series with a spark gap across continuous current mains, and the spark gap, S, is shunted by a condenser, C, in series with another inductive circuit, L, which may be the primary coil of a high frequency transformer.

The continuous voltage may be supplied by a storage battery, B, or by a dynamo, but should be about 500 volts or more.

An air blast or magnetic field must be used to extinguish the continuous current arc.

The ninth claim of the specification reads as follows:—

"The method of obtaining from continuous currents or currents tending through self-induction or otherwise to remain unchanged or resist sudden changes of value, high frequency alternating currents of desired periodicity, consisting in bridging by determinate capacity of condenser and a determinate self-induction coil or circuit a spark gap in said continuous current circuit, said spark gap being adjusted and arranged so as to respond to the desired frequency substantially as set forth."

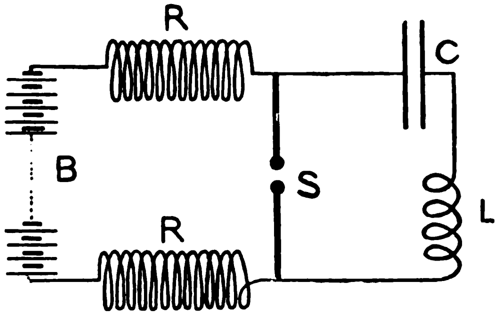


FIG. 69.—Elihu Thomson's Method for the Production of Continuous Trains of Electric Oscillations. B, battery; R, R, inductive resistances; S, spark balls; C, condenser; L, inductance.

In the above specification nothing is, however, said about the employment of a carbon arc instead of a spark gap, but Professor Elihu Thomson has informed the author he had observed the effect.

It is somewhat doubtful whether this particular arrangement did or can produce true undamped persistent oscillations. No proof of this was given in the specification. The only way in which it can be proved that any oscillations are persistent, is by causing them to induce high voltage oscillations of the same frequency in a secondary circuit containing a spark gap; and then examining the image of this induced spark in a rapidly revolving mirror. If the oscillations are persistent the image will be drawn out into an unbroken band of light.

In the absence of such evidence we cannot conclude that the oscillations given by Elihu Thomson's method are truly persistent, but at any rate credit should be given to him for an appreciation of the fact that a continuous electric current could be partly converted into high frequency oscillations by means of a condenser and inductance shunted across an arc or spark produced by a continuous current.[38]

At a later date, in 1906, Mr. S. G. Brown devised an arrangement somewhat similar, which also was claimed, though no proof was given, to produce undamped or persistent oscillations.[39]

The arrangement was as follows:—

A disc of metal, W (see Fig. 70), preferably of aluminium, is fixed to a shaft, and kept in slow rotation by an electric motor. Against the edge of this disc a copper block, C, rests, pressing lightly, and a direct current under a pressure of about 200 volts is passed through a resistance, R_1, and large inductance, L_1, and across the loose contact between the block and the disc. A condenser, K, and

[38] For a confirmation of this see the remarks made subsequently, in 1899, by Prof. Elihu Thomson in an address to the American Association for the Advancement of Science. *The Electrician*, September 22, 1899, vol. 43, p. 778. "The Field of Experimental Research."

[39] See S. G. Brown, *The Electrician*, vol. 58, p. 201. 1906, "On a Method of Producing Continuous High Frequency Electric Oscillations."

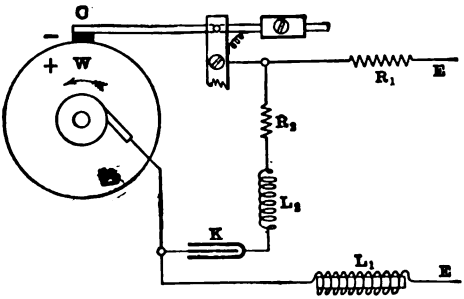
small inductance, L_2, in series are also joined as a shunt between the block and the disc. When the direct current passes, oscillations of high frequency are set up in this condenser circuit, and these can be transformed up or down by an oscillation transformer. We cannot conclude, however, without proof that this method produces continuous oscillations, and not a very rapid series of intermittent oscillations. The only convincing evidence that any method provides the means for the production of truly persistent undamped oscillations is afforded when an actual measurement of the logarithmic decrement shows it to have a zero value, and this has not been done either for Elihu Thomson's or for Brown's method.

In 1900 Mr. W. Duddell read an interesting paper before the Institution of Electrical Engineers of London, in which he showed that if a condenser of suitable capacity and an inductance are connected in series with their terminals attached to the carbons of a continuous current arc of certain length and current formed with *solid* carbons, the arc gives forth a musical note of high pitch.[40]

Hence an arc so shunted has from that time been called a *musical arc.*

The pitch of this note was found to vary with the capacity and inductance in the shunt, but these have to be both moderately large to bring the note within audible limits.

The same creation of oscillations in a condenser and inductive circuit is also observed in the case of a metallic arc, that is, an electric arc produced between metallic rods. By means of high tension continuous currents producing arc discharges between metallic surfaces in vacuum, MM. Simon and Reich state that they have been able to produce extremely strong oscillations in a condenser placed in an inductive shunt circuit connecting the two surfaces between which the arc discharge takes place.[41]



[*Reproduced by permission from " The Electrician."*

FIG. 70.

Mr. Duddell has given the following data for an open and enclosed carbon arc, which will serve as a guide in selecting a suitable capacity and inductance for producing the musical arc.

DATA FOR THE PRODUCTION OF MUSICAL ARCS.

	Open arc.	Enclosed arc.
Carbons, both solid	Conradty carbons	Electra carbons.
Diameter of carbons	9 mms.	13 mms.
Arc length	1·5 mm.	1 mm.
Arc current	3·5 amps	5 amps.
Resistance in series with the arc	42 ohms	28 ohms.
Induction of shunt across carbons	5·3 millihenrys	5·3 millihenrys.
Resistance of shunt	0·41 ohm	0·41 ohm.
Capacity of condenser	1·1 to 5·4 mfds.	1·1 to 5·4 mfds.
R.M.S. value of current through condenser	3 amps.	4·5 amps.

The production of this effect is, however, subject to certain conditions. The arc A must be formed by the electromotive force of a secondary battery or other steady generator, and a resistance, R (see Fig. 71), must be placed in series with it. The inductive resistance L placed as a shunt to the arc must be a low resistance—generally speaking, something less than 1 ohm. The condenser C employed should be one suitable for high potential, as although the impressed electromotive

[40] See W. Duddell, "On Rapid Variations in the Current through the Direct-Current Arc," *Journal of the Institution of Electrical Engineers*, 1900, vol. 30, p. 232. See also British Patent Specification, W. Duddell, No. 21,629 of 1900.

[41] See *La Revue Practique de l'Électricité*, April 20, 1904.

force on it is only 50 volts, the action of resonance (see Chap. III.) creates a potential difference between its plates, which at moments rises to several hundred volts, and hence a thin paper condenser may break down.

One explanation put forward as an explanation of this effect on its discovery was that it essentially depends upon the existence of a *negative resistance* in the arc, and that the frequency which can be obtained is limited by the arc itself. We shall present the outlines of this theory first, as proposed by Mr. Duddell and supported by some others.

Suppose a small instantaneous change, dV, is made in the potential difference of the electrodes, whether carbon or metal, between which the arc is formed, and let the corresponding small change in the current through the arc be denoted by dA. Also let the resistance of the inductance in series with the condenser be represented by r. The theory advocated by Mr. Duddell is that the conditions for the production of high frequency alternating currents or oscillations in the condenser circuit are that $\frac{dV}{dA}$ must be negative in sign, and must be numerically greater than r. A negative value of $\frac{dV}{dA}$ implies that the current through the arc must vary in the opposite sense to the potential difference, that is, the current must

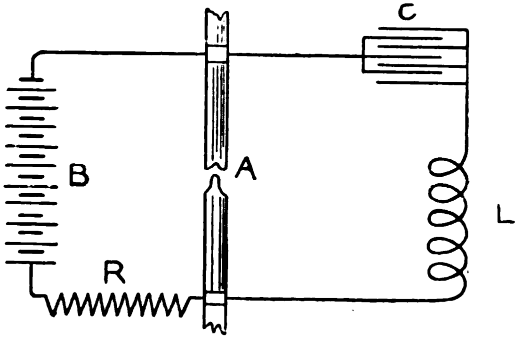


FIG. 71.—Arrangements for producing Duddell's Musical Arc. B, battery; R, resistance; A, carbon arc; C, condenser; L, inductance.

increase as the potential difference decreases, and *vice versâ*. Messrs. Frith and Rogers have experimentally determined the value of $\frac{dV}{dA}$ (which they call the resistance of the electric arc) for various arcs made with cored and solid carbons, and they found that whilst $\frac{dV}{dA}$ was always positive for cored carbons, it was negative when *both* carbons were solid and was as small as -2 ohms for a 4-ampere arc between solid carbons.[12] As the resistance of the inductive coil in series with the condenser can easily be made less than 2 ohms, the two criteria can be satisfied.

The operations taking place may be stated generally in the following manner. If a condenser in series with an inductance of low resistance is placed as a shunt across the arc, the first effect is to rob the arc of some current to charge the condenser. This action, however, does not decrease, but increases slightly the potential difference of the carbons. Hence the condenser continues to be charged. When the charge is complete, the current through the arc is again stationary, and the condenser at once begins to discharge back through the arc. This, however, increases the current and decreases the potential of the carbons, hence the action

[12] See *Proc. Phys. Soc. Lond.*, 1896, vol. xiv. p. 307; or *Phil. Mag.*, 1896, vol. 42, ser. v. p. 407.

proceeds until the condenser is discharged. The process then repeats itself regularly. The whole action is exactly analogous to that by which the resonance of the column of air in an organ pipe controls the operation of the jet of air issuing from the mouth of the pipe and impinging against the sharp edge of the upper lip, and so maintains the sound as long as the current of air is supplied. Mr. Duddell found that the direct current arc between *cored* carbons would not produce this effect. Also he found that in the case of arcs between metal surfaces the arc was even more readily extinguished by shunting the arc with a condenser than in the case of a solid carbon arc. He also found that there were limits to the production of the oscillatory currents by the carbon arc, but that it worked well as a transformer of continuous current to electric oscillations when the condenser and inductance were so adjusted that the frequency lay between 500 and 10,000.

The physical conditions to be fulfilled for this transformation to take place have also been set out mathematically by M. Janet as follows.[43]

Let C be the capacity of the condenser, L the inductance, and r the resistance of the coil in series with it placed as a shunt across the arc. Let R be the larger resistance placed in series with the arc, and E the electromotive force of the working battery. Let i be the instantaneous value of the current flowing through R, i_1 that through the arc, and i_2 that through the condenser. Then if an alternating current is set up and established in a permanent state in the condenser circuit it has a certain frequency, n. Let $p=2\pi n$ as usual.

Experiment shows that the current through the arc is also fluctuating, and consists of a periodic current superimposed on a steady current. Therefore the current coming out of the battery must be of the same nature. Let I_0 be the value of this steady current. Then—

$$i=I_0+I\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where I is the maximum value or amplitude of the periodic part of the current through the resistance R. It is also assumed that the frequency of the current through the condenser is the natural frequency of the condenser and inductive shunt. Therefore $n=\frac{1}{2\pi\sqrt{CL}}$ or $p^2=\frac{1}{CL}$. This circuit, therefore, acts as if it were non-inductive, since the relation between its inductance capacity and frequency is that under which the inductance annuls the capacity.

The main current thus consists of a continuous part and an alternating part. The current through the arc is of the same type, whereas the current through the condenser shunt circuit is purely alternating. If r is the resistance of the inductive shunt, and i_2 is the current through it, then, corresponding to the frequency $n=\frac{1}{2\pi\sqrt{CL}}$, the potential difference of the ends of the inductive shunt must be equal to ri_2, since then the inductance is annulled by the capacity. Hence this must be equal to the electromotive force represented by the periodic part of the main current, which is numerically equal to $RI\sin pt$. Accordingly we have—

$$i_2=\frac{R}{r}I\sin pt$$

Again, if v is the difference of potential of the carbons forming the arc at any instant, and E is the constant E.M.F. of the working battery, we have also—

$$v=E-Ri$$

or substituting the expression for i above given, we have—

$$v=E-RI_0-RI\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

But $$i_2=\frac{R}{r}I\sin pt$$

also the current i_1 through the arc is the sum of the current i through the main resistance and the condenser current i_2. Hence—

$$i_1=i+i_2$$

Therefore $$i_1=I_0+\frac{R+r}{r}I\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

[43] See P. Janet on "Duddell's Musical Arc," *Comptes Rendus*, 1902, vol. 134, p. 821.

Differentiate (28) and (29) with respect to time and take the quotient. We have then—

$$\frac{dv}{dt} = -\mathrm{R}\,Ip\cos pt \text{ and } \frac{di_1}{dt} = \frac{\mathrm{R}+r}{r}\,Ip\cos pt$$

Hence $$\frac{dv}{di_1} = -\frac{\mathrm{R}r}{\mathrm{R}+r} \qquad (30)$$

If R is large compared with r, the value of $\frac{dv}{di_1}$ approximates to r.

Hence the condition for the establishment of permanent oscillations in the condenser circuit having a frequency $n = \frac{1}{2\pi\sqrt{\mathrm{CL}}}$ is that the sign of $\frac{dv}{di_1}$ must be *negative*, and its value numerically equal to, or greater than, that of the ohmic resistance of the shunt circuit. Janet, therefore, arrives at a conclusion as to the essential conditions for the production of the musical arc which is identical with that reached by Duddell. The same result has been reached by Mr. Duddell in another manner by showing that with the above conditions (viz. $\frac{dv}{di_1}$ negative and equal to or greater than v) the energy wasted as heat in the inductive circuit is recouped during each half period by the energy given to it. Hence the oscillations are maintained.[44]

The high frequency oscillations so produced can, of course, be transformed up to higher potentials by using a Tesla coil or oscillation transformer, and placing the primary circuit in series with a condenser as a shunt across the arc.

We are thus able to cause a source of continuous current, such as a secondary battery or dynamo, to expend part of its energy in creating continuous or maintained electric oscillations of high frequency in a condenser and inductive circuit.

Several Italian physicists, however, disagree with Duddell and Janet as to the statement of the laws governing the frequency of the oscillations. Thus, A. Banti has asserted (*Elettricista*, January 12, 1903, vol. 12, p. 1) that with a condenser of 1 mfd. and an extremely small inductance (merely a connecting wire) a frequency of 120,000 may be obtained.[45] Banti says (*loc. cit.*) that the frequency is not the same when the inductance and capacity are varied inversely as one another. Thus, with an inductance of 0·048 henry and capacity of 1 mfd., the frequency is 13,000. With an inductance 0·012 h. and capacity 4 mfds. it is 8500, whilst with 0·003 h. and 16 mfds., it is 2750, whereas if the frequency of the oscillations was entirely determined by the formula $n = \frac{1}{2\pi\sqrt{\mathrm{CL}}}$, the frequency should have been in all cases the same, since the quantity $\sqrt{\mathrm{CL}}$ is preserved constant.

Duddell, however (see a letter in *The Electrician*, 1903, vol. 51, p. 902), has contended that since the value of $\frac{dv}{di_1}$ for the arc with solid carbons is not negative for frequencies as high as 100,000, oscillations cannot be then created in the shunt circuit, and that the statements made concerning very high frequencies are erroneous. In the same letter Duddell points out that, since the full expression for the frequency of the oscillations in an inductive circuit having capacity and resistance is given by $n = \frac{\sqrt{\frac{1}{\mathrm{CL}} - \frac{\mathrm{R}^2}{4\mathrm{L}^2}}}{2\pi}$, it follows that the frequency will be determined by the current through the arc, since the current is a function of the resistance of the arc. Numerical values are not, however, given in confirmation of this opinion.

The reader may be referred to the following sources for additional information :—

M. La Rosa, *Nuovo Cimento*, Jan. 1904, vol. 7, p. 5, "On Duddell's Currents." See also *Science Abstracts*, 1904, vol. 7, A., p. 456.

[44] See *loc. cit.*, "On Rapid Variations in the Current through the Direct Current Arc," Appendix II., *Journal of the Inst. Elec. Eng.*, vol. 30, p. 262.
[45] See also *Science Abstracts*, 1903, vol. 6, A, p. 387.

The above-named author states that his result shows that the actual condenser current is asymmetric. Its amplitude was determined by a Braun vacuum tube. He concludes that—

(i.) The amplitude of the oscillatory current is independent of the resistance of the shunt circuit until this reaches 2·5 ohms, when the oscillations cease.

(ii.) The change in amplitude with inductance does not follow any simple law.

(iii.) When the main current is small, the amplitude of the shunt current tends to vary inversely as the square root of the inductance, and inversely as the cube root of the capacity.

Corbino has studied the singing arc by stroboscopic methods.

See *Atti. dell' Assoc. Elettr. Ital.*, 1903, vol. 7, p. 369, also p. 597; or *Science Abstracts*, 1904, vol. 7, A, p. 537.

He says the current in the shunt circuit is not sinusoidal, and that this may be proved by using a Braun cathode ray vacuum tube. Hence, the formula $\frac{1}{2\pi\sqrt{CL}}$ is not strictly applicable for determining the frequency. Corbino deduces an equation for the shunt current i_2 in terms of the constants and the main current i as follows:—

$$L\frac{di_2}{dt}+\left(r-\frac{b}{(i-i_2)^2}\right)\frac{di_2}{dt}+\frac{1}{C}=0$$

A very full examination of this subject has been made by Maisel (*Physikalische Zeitschrift*, Sept. 1, 1904).[46] He contends that it has been shown by Wertheim Salomonson that a singing arc may produce oscillations having a frequency as high as 400,000, and that the latter observer has photographically registered a frequency as high as 135,000. Also Maisel says that Corbino has shown that the current in the condenser circuit is not sinusoidal, and not even symmetrical, and that the work of Salomonson, Ascoli, and Manzelti has shown that the frequency of the oscillations in the condenser circuit cannot be calculated by the simple formula $n=\frac{1}{2\pi\sqrt{CL}}$.

Maisel bases his views upon the theory of the electric arc developed by Mitkiewiez (see *Russian Journal of Physics and Chemistry*, 1903, pp. 507 and 675) and by J. Stark (*Ann. der Physik.*, 1903, vol. 12, p. 673). According to this theory (which, however, was originally suggested by the author of this book in 1899), the phenomena in the arc very much depend upon the thermal condition of the negative pole.[47] The discharge cannot pass if the temperature of the negative pole falls below a certain limit. If, then, we connect a condenser and inductance as a shunt across the arc, the first effect is to rob the arc of current. This causes a fall of temperature in the electrodes, and finally an extinction of the arc. If, however, the temperature of the negative terminal has not fallen below a certain point, the arc relights itself again as soon as the condenser is charged, and the condenser discharges through it. Maisel contends that stroboscopic observations have shown that this extinction of the arc takes place. He states that he has also produced the phenomenon of the singing arc with iron terminals and with mercury and carbon, as well as mercury and iron, and he gives diagrams of current curves taken with a Braun tube which show that the current variation is not sinoidal.

He contends that the sign of the slope of potential in the arc has no importance, and that the singing arc can be obtained with any electrodes and any frequency, and that this frequency cannot be calculated simply from the inductance and capacity in the shunt circuit. On the other hand, all Maisel's observations were made with a shunt circuit, having a capacity of 3·4 mfds. and an inductance

[46] See also *L'Éclairage Électrique*, 1904, vol. 41, p. 186, for a French epitome of Maisel's paper.

In *The Electrician*, vol. 51, p. 752, will be found a letter from I. Wertheim Salomonson referring to his paper in the *Proceedings of the Royal Academy of Amsterdam* on the effect of variation of current strength on the pitch of the note of the singing arc.

[47] See J. A. Fleming, *Proc. Roy. Soc. Lond.*, 1890, vol. 47, p. 118, "On Electric Discharge between Electrodes at Different Temperatures in Air and High Vacua"; also *Proc. Roy. Institution of Gt. Britain*, 1890, vol. xiii. p. 34, "Problems in the Physics of an Electric Lamp."

$\frac{1}{T} = \frac{5.033 \cdot 10^6}{\sqrt{2.4.34.10^6}} = 1480,3 = \frac{1}{T} = T = 0,0007$ 21

of $3 \cdot 4 \times 10^6$ cms. Hence the natural time period of the oscillating circuit was 0·0007 second, which is a frequency less than the value 10,000 given by Duddell as critical.

In a subsequent paper (*Physikalische Zeitschrift*, Jan. 15, 1905), S. Maisel attempts a general theory of the production of undamped trains of electrical oscillations. He assumes the possession of a conductor which rigidly obeys Ohm's law, but has the property that no current flows through it when the electromotive force falls below a certain value, and that the restoration of the current requires a high electromotive force. There are many forms of conductor which comply with these conditions, *e.g.*, a vacuum tube, the mercury vapour lamp, as well as the electric arc. The author works out a complete mathematical theory, and shows that when such a conductor is a shunt to a condenser in series with an inductance, a battery or source of steady E.M.F. will create oscillations in the condenser circuit.

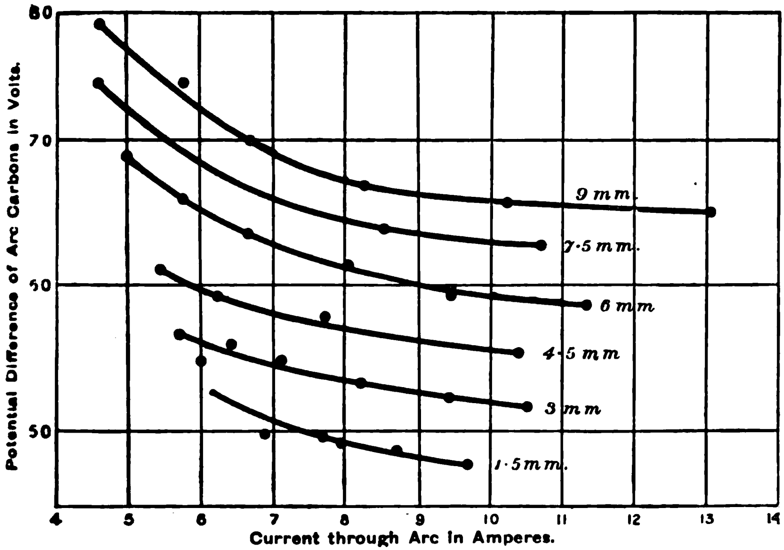


FIG. 72.—Static Characteristic Curves of a D.C. Arc between Carbon Electrodes in Air.

These different opinions are to some extent reconcilable when we consider the nature of the *characteristic curve* of a direct current arc. The above term is applied to a curve showing the relation between the current flowing through or out of any electrical appliance whether resistance, motor, lamp, dynamo, or anything else, and the terminal potential difference. Thus the characteristic curve of a resistance is a straight line, thus expressing graphically Ohm's law. The characteristic curve of a series-wound dynamo is a curve resembling the magnetization curve (B, H, curve) of iron. On the other hand, the characteristic curve of a direct current arc is a curve which, if plotted with a current as abscissa and potential difference of electrodes as ordinates, is a convex downward sloping curve, as in Fig. 72. If the currents are slowly increased, and the points on the curve represent the ratio of arc current to potential difference of the electrodes, the curve is called the *static characteristic*; but if the current runs rapidly through a repeated cycle of values as in the case of an alternating current, then the term of *dynamic characteristic* has been given by Professor H. Th. Simon to the closed curve representing the relation of current and potential differences.

In the case of the direct current (D.C.) arc formed with solid carbons, the

static characteristics for various arc lengths are curves similar to those depicted in Fig. 72. It will be noticed that the slope of the curve or the value of $\frac{dv}{di}$ is always negative, and that the value of $\frac{dv}{di}$, where i denotes the arc current at any instant and v the corresponding potential difference of the carbons, decreases as i increases. Hence $\frac{d^2v}{di^2}$ is positive because $\frac{dv}{di}$ changes from a somewhat large negative value to nearly zero.

It has, however, been shown by Mrs. Ayrton,[48] and by Professor H. Th. Simon,[49] that in the case of an alternating current arc the P.D. of the carbons corresponding to a given current is lower when the current is decreasing than when it is increasing. Hence, if we carry the current through the arc round a cycle of operations, increasing and decreasing, the corresponding *dynamic* characteristic is a closed loop. Simon has shown that the product area (A) and absolute temperature (T) of the crater of the arc determines the carbon P.D. necessary to produce a given current.[50] Taking this product AT as corresponding to a definite resistance of the arc, he showed that when heat is added from some outside source the characteristic is lowered or a smaller P.D. is required to produce a given current.

If then a continuous current arc is shunted by a condenser in series with an inductive circuit the following is a general description of the actions set up. Assume the condenser charged to the P.D. of the carbons, and that it is connected across the arc. The condenser begins to discharge. This increases the current through the arc and lowers the carbon P.D. in virtue of the negative slope of the characteristic curve. This, however, facilitates the discharge of the condenser. In virtue of the inductance of the condenser circuit this process continues, and the condenser is not only discharged, but charged up in the reverse direction. The current through the arc then begins to diminish, and this increases the P.D. of the carbons and facilitates the further charging of the condenser. The process is exactly analogous to that by which the steady jet of air from the mouth of an organ pipe sets up steady oscillations of air in the pipe, and these control the motion of the jet of air so as to cause it to play within or without the lip of the pipe, thus maintaining the oscillations. Returning then to the electric phenomenon, we note that since the static characteristic of the carbon arc in air is a curve, with greater slope downwards for small currents than for large, it follows that in the case of a large current arc (10 amperes and upwards) even large variations of the current will produce only small variations of P.D. between the carbons, but with small arc currents (1 or 2 amperes or less) then even small variations of the current will produce much larger variations in the P.D. of the carbons. If a condenser is shunted across a continuous current arc, and if oscillations are set up in the condenser circuit, we may regard the actual current through the arc as the sum of a constant unidirectional current I_0 and a periodic current, which under assumption of a sine variation may be represented by $I \sin pt$.

The P.D. of the carbons is therefore a function of $I_0 + I \sin pt$, and may be represented by $F(I_0 + I \sin pt)$. This P.D. may in turn be regarded as composed of a constant unidirectional part V_0 and a periodic part $V \sin(pt+\theta)$ differing in phase from the periodic part of the current. The amplitude V of the periodic part of the P.D. will depend upon the part of the characteristic curve at which we are working. If we are on the flat part of the curve characteristic, then variations of current through the arc will only be accompanied by small variations of arc P.D., and this implies also small power given to the condenser circuit.

If then we shunt an arc by a condenser, and gradually reduce the steady current through the arc, the variations of arc current produced by the condenser currents are accompanied by such large variations of arc P.D. that we can employ

48 "The Electric Arc," Mrs. H. Ayrton.

49 *Physical. Zeitschrift*, vi. p. 297, 1905. Also *Science Abstracts*, vol. 8, A, 1905, *abs.* 1465, "The Dynamics and Hysteresis of the Electric Arc."

50 "The Theory of the Singing Arc," H. T. Simon, *Physical. Zeitschrift*, vol. 7, p. 423, 1906, or *Science Abstracts*, vol. 9, A, 1906, *abs.* 1423.

a small capacity and yet obtain oscillations of considerable current amplitude in the condenser circuit. If, however, we employ a larger arc current, then the variations of potential are small, and we can only obtain sensible oscillations of current by the use of a condenser of relatively large capacity. The matter is not capable of being subjected to strict analytical treatment until we know the form of the function which connects the current and P.D. of the arc, but it is clear that for a given condenser the current in the condenser circuit will be increased by increasing the potential difference variation of the arc electrodes, and this is effected by working with small arc currents on a steep part of the characteristic curve. It is this, perhaps, which accounts for the difference of opinion between various observers as to the possible limits of frequency obtainable by the original Duddell method of shunting an ordinary carbon arc with a condenser. Those observers who used, say, a 10-ampere arc and a condenser having a capacity of 1 mfd. or so obtained only relatively low frequency oscillations, and could not obtain very high frequency because by using a condenser of small capacity the

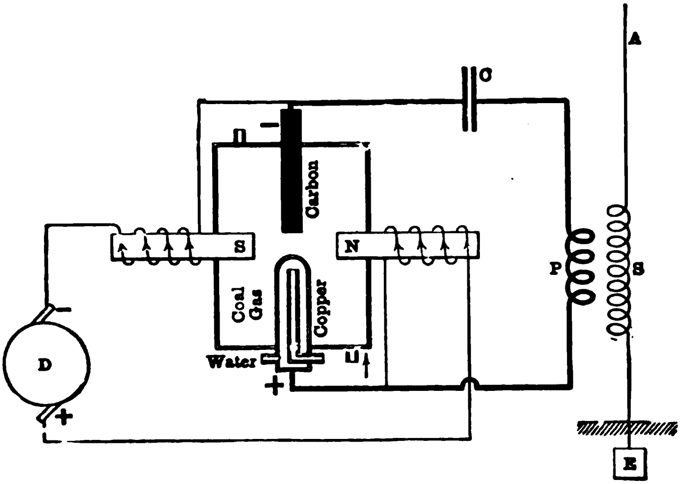


FIG. 73.—Poulsen's Arrangement for Producing Undamped Electric Oscillations.

variations of arc current were too small to convey sensible energy to the condenser circuit. Those, however, who employed a small capacity and small arc current were enabled to obtain oscillations of much higher frequency.

In 1903, however, V. Poulsen made known the interesting discovery, viz. that by employing a continuous current arc formed with carbon and cooled metal electrodes in an atmosphere of hydrogen or coal-gas or any hydrocarbon, it was possible to obtain in an inductive shunt circuit having a small capacity very high frequency vigorous oscillations, practically undamped.[51] He showed that when the oscillatory arc was placed in a magnetic field transverse to the arc the condenser plate potential difference was very greatly increased. He found that if the continuous current arc was formed inside a flame such as that of a spirit lamp it was possible to obtain oscillations of much higher frequency, by appropriately shunting the arc with a small capacity and large inductance, than with the ordinary Duddell musical arc. Hence Poulsen adopted the following arrangement. An electric arc is formed between the end of a thick carbon rod kept in slow rotation and a water-cooled copper rod, the latter forming the anode or

[51] See V. Poulsen, British Patent Specification, No. 15,599 of 1903; also *Transactions of the International Congress of Electricians at St. Louis*, vol. 2, p. 963, 1905; or *Science Abstracts*, vol. 8, A, *abs*. 1620, 1905; also *The Electrician*, vol. 58, p. 166, November 16, 1906. A report of a lecture given by Mr. V. Poulsen in the Queen's Hall, London.

positive pole of the arc (see Figs. 73, 74). This arc is created in a box kept full of coal-gas or vapour of hydrocarbon, with a continuous voltage of 400 to 500 volts. A variable resistance is placed in series with the arc and choking coils in both leads. The arc box is also perforated by two magnetic poles which produce a powerful field at right angles to the arc. The arc electrodes are connected outside the box by a condenser circuit consisting of a small capacity and a large inductance. The capacity may be something of the order of 0·004 of a microfarad and the inductance of the order of 100,000 cms. or 0·1 of a millihenry. Under these circumstances, when the arc is started, powerful oscillations, which are practically continuous or undamped, will be set up in the condenser circuit. Attention to several details is necessary to secure the best results. The carbon rod must have

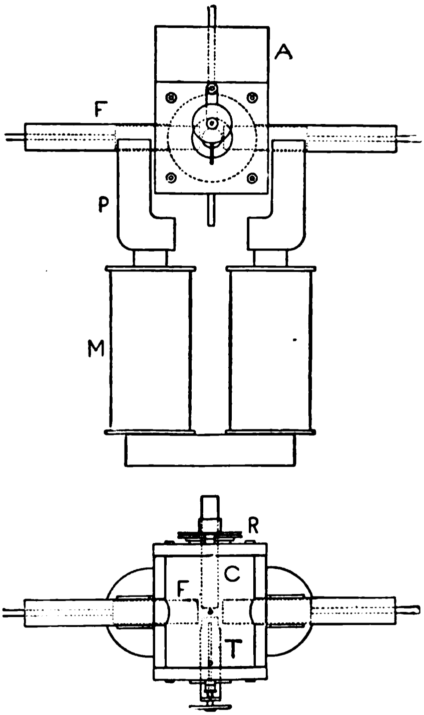


Fig. 74.—Poulsen's Arrangement for Producing Undamped Electric Oscillations.

a square, sharp edge, and the arc must spring from this edge to a copper nose on the end of the cold copper electrode. The carbon must be kept in a very slow, steady rotation. The magnetic field must be very strong, and the arc must have a certain length, best found by trial. The arc box must be kept cool, and also the copper electrode, by circulating water. In the case of portable apparatus Poulsen uses air cooling, the arc being contained in a box with metal flanges and the hydrocarbon vapour being formed by dropping alcohol or petrol into the box (see Fig. 75).

It has been shown by the author that even then the oscillations are not quite continuous.[52] The oscillating arc tends to break up into a series of intermittent

[52] See J. A. Fleming, "On the Poulsen Arc as a Means of Generating Undamped Oscillations," *Phil. Mag.*, August 1907; also *Proc. Phys. Soc. Lond.*, vol. 20, 1907; also "Recent Advances in Electric Wave Telegraphy," a discourse at the Royal Institution. See *The Electrician*, May 31, June 7, 14, 21, 1907.

discharges, and it is somewhat difficult to obtain absolutely unbroken undamped oscillations. Some interesting investigations were carried out in the author's laboratory in 1907 by Mr. W. L. Upson, on the characteristic curves of electric arcs between various electrodes and in different gases.[53] These experiments showed that for an arc taken between a carbon (negative) and a water-cooled copper (positive) electrode the characteristic curve is much steeper at and about the same arc current, than for the carbon-carbon arc in air (see Fig. 76). Hence it is clear that one element in Poulsen's discovery is the effect of hydrogen or hydrocarbon vapour in steepening the characteristic curve of the direct current arc. The reason for this has not yet been fully explained.

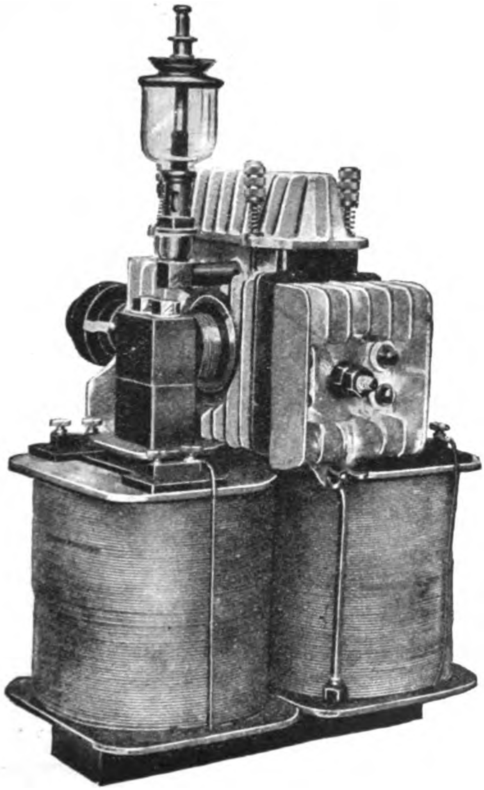


FIG. 75.—Poulsen's Arc Apparatus for Producing Undamped Electric Oscillations.

Poulsen immediately applied the above method for producing undamped electric oscillations in radiotelegraphy, with the co-operation of P. O. Pedersen.[54]

We shall return again, in Chap. VIII. on radiotelegraphic stations, to the consideration of the practical use of Poulsen's discovery and apparatus in wireless telegraphy and telephony.

[53] See W. L. Upson, "Observations on the Electric Arc," *Proc. Phys. Soc. Lond.*, vol. 20, 1907, or *Phil. Mag.*, July 1907. Also J. A. Fleming, "Some Observations on the Poulsen Arc as a Means of Obtaining Continuous Electric Oscillations," *Phil. Mag.*, August 1907, series vi., vol. 14, p. 254.

[54] See V. Poulsen, "A Method for Producing Undamped Electric Oscillations and its Employment in Wireless Telegraphy," *The Electrician*, vol. 58, p. 166, 1906. In this article a number of diagrams are given showing the type of receiving circuit used.

Meanwhile the reader's attention may be drawn to one or two other points in connection with the production of electric oscillations by the arc.

Much light has been thrown on the nature of the phenomenon by the careful researches of Professor H. Th. Simon.[55] He has studied by means of the oscillograph and Braun vacuum tube the form of the current curves in the two circuits: the main or arc circuit, and the shunt or oscillatory circuits. He shows that the arc current consists of a sinoidal current superimposed upon a steady current, and that the current in the shunt circuit is nearly a sinoidal current. It follows from this that the arc current increases and decreases periodically. Since the main current keeps constant, it follows that the current through the arc is increasing when that into the condenser is decreasing, and *vice versâ*.

The form of the oscillograms and characteristic curve of the arc when the current is periodic is well shown by some oscillograms and characteristic curves taken by Professor J. T. Morris with alternating current arcs under various conditions in air and in coal-gas.[56]

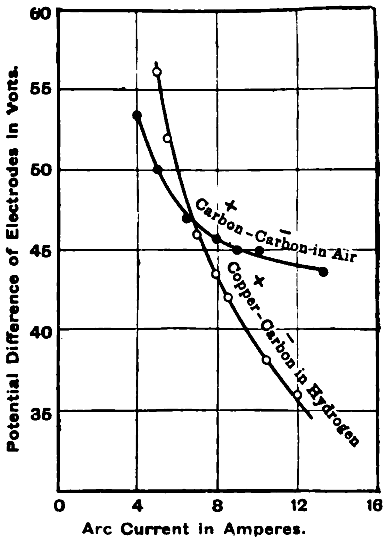


FIG. 76.—Diagram showing Results of Upson's Experiments on Characteristic Curves of Arcs.

Thus in Figs. 77, 78 are shown the oscillograms of an alternating current arc in air, formed with 440 and 110 volts respectively, at a frequency of 60. In the case of Fig. 77 the wave of current has been shifted through 180° to keep well separate the curves of voltage and current. It will be noticed that the current remains practically constant for quite a sensible time during the semi-period, and that during that time the P.D. of the carbons rises rapidly, but falls again quickly. The P.D. then remains nearly zero for some time as the current reverses sign. These values of P.D. (in volts) and current (in amperes) are set off in cyclical curves in Figs. 79 and 80, which show well the cycle of changes of current and P.D. in an alternating current arc, and the difference between the arc in air and in coal-gas, both with and without a transverse magnetic field across the arc. The effect of the coal-gas is seen in the much more sudden and greater rise of the curve as the current passes through zero, and also in the much steeper, falling characteristic as the current increases to its maximum.

The reasons, then, for the peculiar form of the closed characteristic curve for an alternating current arc are to be found in the phenomena of the arc and of gaseous conductors generally. If we apply a steady potential difference to two arc electrodes immersed in a gas, there is a great resistance to the passage of electricity, which chiefly resides at the negative electrode, and can be enormously reduced by heating that electrode.

Again, a gaseous conductor does not obey Ohm's law. Its conductivity is not constant, but is a function of the voltage, and in general there is a constant value, called the saturation current, which the current cannot exceed no matter what the voltage. If we consider the instant when the current through the arc is zero, the conductivity of the arc or interelectrode vapour is then very small, and the electrode

[55] See H. T. Simon, "The Dynamics and Hysteresis of the Electric Arc," *Phys. Zeitschrift*, vol. 6, p. 297, 1905, or *Science Abstracts*, vol. 8, A, *abs.* 1465; also "Theory of the Singing Arc," *Phys. Zeitschrift*, vol. 7, p. 433, 1906, or *Science Abstracts*, vol. 9, A, *abs.* 1423.

[56] See J. T. Morris, "Note on an Oscillographic Study of Low Frequency Oscillating Arcs," *Electrical Review*, August 9, 1907. A paper read before the British Association at Leicester, 1907.

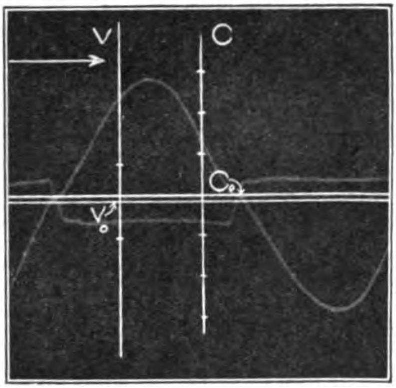


FIG. 77.—Oscillograms of 440-volt Alternating Arc in Air. The Current Curve is the square-shouldered curve, and is reversed or shifted through 180°. (J. T. Morris.)

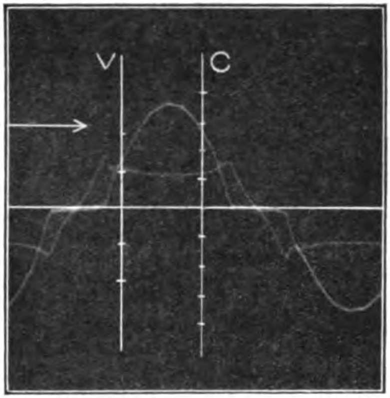


FIG. 78.—Oscillograms of 110-volt Alternating Arc in Air. The Current Curve is the square-shouldered curve. (J. T. Morris.)

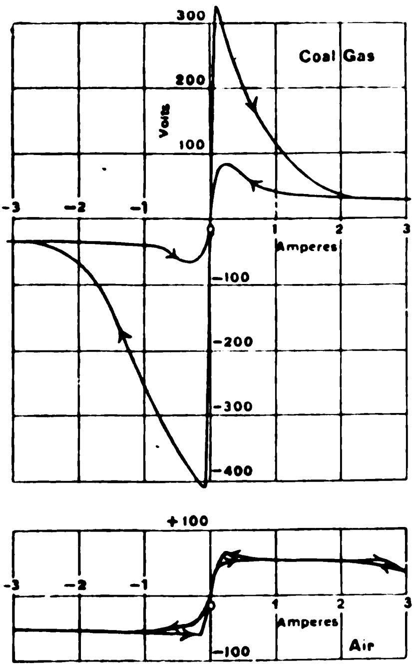


FIG. 79.—Cyclical or Dynamic Characteristic Curve of a 440-volt Arc in Coal-Gas (Upper Curve) and Air (Lower Curve) without Transverse Magnetic Field. (J. T. Morris.)

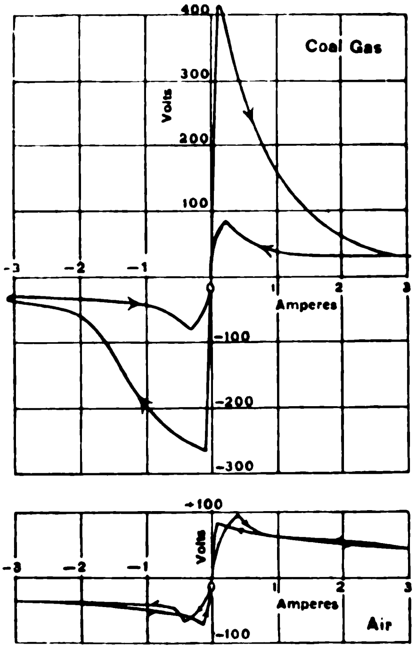


FIG. 80.—Cyclical or Dynamic Characteristic Curve of a 440-volt Arc in Coal-Gas (Upper Curve) and Air (Lower Curve) with Transverse Magnetic Field. (J. T. Morris.)

P.D. therefore rises to its full value. If, however, any of the gas is ionized by any cause, then these ions are moved by the electric force and a current begins to flow. As the current increases the conductivity of the gas increases, and the electrode or arc P.D. falls. When the arc current has reached its maximum value and commences to decrease, the electrodes still remain hot and the interelectrode conductivity remains good; therefore the arc P.D. keeps low until the current is nearly zero again, when the cooling of the electrodes causes a slight rise in current to be followed by a fall to zero, and then reversal and rapid increase in the negative direction. The useful effect of the coal-gas or hydrogen is largely due to its cooling effect, for as soon as the arc current falls to zero the hydrogen or coal-gas cools the electrodes and so promotes a rapid increase in arc resistance, and therefore a quick and large rise in P.D. between the electrodes. The transverse magnetic field helps to extinguish the arc more quickly. We thus find an explanation of the peculiar form of the cyclical characteristic curve. These cyclical or dynamic characteristics of the alternating current arc have been compared by H. Th. Simon with the magnetic hysteresis loops of iron and the static characteristics with the ordinary non-cyclical magnetization curve.

If a condenser is shunted across the arc it is then clear that oscillations once started will tend to persist. For the initial connection of the condenser robs the arc of some current, and this reduction of current at once increases the electrode or arc P.D. which is in the direction required to continue the charging. When the condenser is fully charged the arc current becomes again constant and rises to its steady value. This is accompanied by a fall in arc P.D. and the condenser then discharges through the arc, thus increasing the arc current still more and further decreasing the arc P.D. The changes in arc P.D. are then always automatically made in the direction necessary to give and take energy from the condenser periodically, and the process is self-sustaining. The process is greatly facilitated by immersing the arc in an atmosphere which does not act chemically on the electrodes and at the same time cools them. It is therefore assisted by surrounding the electrodes with an atmosphere of hydrogen or hydrocarbon, and also helped by replacing the negative carbon by a water-cooled copper cathode. It is also aided by a magnetic field placed transversely to the arc, because this tends to rupture the arc very suddenly and hence brings into play the inductances of the condenser and main circuit to increase the electromotive force in the circuit which charges the condenser. Hence it follows that the root-mean-square value of the P.D. of the terminals of the condenser as measured by an electrostatic voltmeter may be many times greater than the P.D. of the arc electrodes as measured by a direct current voltmeter. If V_0 is the last voltage and V_1 is the R.M.S. value of the true periodic or alternating P.D. at the terminals of the condenser, which may be for the moment assumed to have a sinoidal form and frequency $n=\frac{p}{2\pi}$, then the reading V of an electrostatic voltmeter placed across the terminals of the condenser would be—

$$\sqrt{\frac{1}{T}\int_0^T (V+\sqrt{2}V_1 \sin pt)^2 dt} = \sqrt{V_0^2+V_1^2}$$

Hence, $$V^2 = V_0^2 + V_1^2, \quad \text{and} \quad V_1 = \sqrt{V^2 - V_0^2}$$

Thus, if $V_0 = 400$ volts, V may be as much as 1600 volts.

It is found by experience that in using the carbon-copper arc in coal-gas to produce vigorous undamped oscillations, the capacity in the shunt circuit should be small and the inductance large. Thus, when using an 8-ampere arc formed with 400 volts the capacity can conveniently be about 0·005 mfd., or 4000 to 5000 cms., in electrostatic measure, and the inductance about 20 times as great, viz. 100,000 cms. in electromagnetic measure.

The general conditions which have to be complied with to obtain by this method powerful oscillations of a frequency high enough to be of use in radio-telegraphy, viz. of the order of 10^6, are as follows:—

The numerical value of the capacity C_{ES} when reckoned in electrostatic units

must be small compared with the numerical value of the inductance L_{EM} reckoned in electromagnetic units, and the capacity and inductance must have such values that the quantity $\frac{3 \times 10^{10}}{2\pi\sqrt{C_{ES} \times L_{EM}}}$ which measures the frequency must be of the order of 10^6.

The arc used should be a high potential small current arc, and used under such conditions that it is worked at a steep part of the characteristic curve. It is therefore essential to work the arc in a non-oxygenic atmosphere, and experience shows that the best results are obtained with a hard carbon anode and cooled copper anode in an atmosphere of hydrocarbon gas.

Amongst other investigations on the subject those of L. W. Austin[57] may be mentioned. He experimented with electric arcs formed between various materials, such as solid carbon, cored carbon, graphite, and metals, and found that high frequency oscillations could be produced with graphite electrodes, the frequency being of the order of 100,000 and upwards when the arc was formed in air. He found, as Poulsen and others had previously done, that vigorous high frequency oscillations can more easily be formed by an arc with copper-carbon electrodes placed in a gas flame or in an atmosphere of hydrogen; the arc in hydrogen or coal-gas enabling very large quantities of energy to be given up to the oscillating circuit without extinguishing the arc. Austin also found he could produce similar effects with the arc formed in steam and in compressed air.

A point of importance in connection with this subject is whether the oscillations produced in the shunt circuit are controlled as to frequency solely by the inductance and capacity of that circuit—in other words, whether the oscillations are free or forced.

It appears from Austin's experiments that there is always a fundamental oscillation, the frequency n of which is not far from that given by Kelvin's law, viz. $n = \frac{1}{2\pi\sqrt{CL}}$; but that there are higher harmonics as well, and also that the fundamental frequency is to some extent a function of the arc length and arc currents. Also he agrees with the author that for high frequencies there is a tendency for the oscillations to become discontinuous and break up into separate trains of oscillations.

In addition to the above types of arc generator there are some modifications of more recent invention.

The arc generator of M. Moretti has been employed with success in the production of practically continuous oscillations for wireless telephony. In this appliance a direct current at a voltage of 500 volts is used to create an electric arc between two copper electrodes. The negative is a solid copper rod, and the positive is a hollow copper tube through which water is made to flow at a regulated speed.

The electrodes are placed vertically with the negative uppermost. The arc is formed between the copper negative electrode and the water. The arc is supplied with continuous current through a pair of choking coils, and is shunted as in the Duddell arc by a condenser in series with an inductance. When this is adjusted a high frequency alternating current is created in the condenser circuits.

A satisfactory explanation of its operation has not yet been given. It has been assumed that the arc striking the water evaporates it, and that a series of small explosions ensues which result in a regular charge and discharge of the condenser.

It seems more probable, however, that the vaporization of the water includes the arc in an atmosphere of steam, hydrogen, and oxygen, and that the result is to bring about the same increase of steepness of the characteristic curve as in the case of the arc between copper and carbon electrodes in coal-gas or hydrogen.

This form of arc generator has been used for wireless telephony by Professor J. Vanni (see Chap. X.). Another form of arc generator for the production of continuous oscillations is due to the author.[58] In this the arc is formed between a

[57] L. W. Austin on the "Production of High Frequency Oscillations from the Electric Arc," *Bulletin of the U.S.A. Bureau of Standards*, vol. 3, No. 2, Washington, 1907.

[58] J. A. Fleming, British Patent Specification, No. 3963 of 1914. Feb. 16.

carbon negative electrode, and a copper closed or nearly closed tubular positive electrode fitting over it. The carbon is immersed in a vessel of heavy mineral or vegetable oil.

The construction will be understood from Fig. 81.

In a metal vessel, V, full of the oil is placed a metal plate which carries one or more insulated sockets in which are held solid carbon rods. These rods are just so long that their tops are exposed about 1 or 2 centimetres above the surface of the oil.

The copper electrodes, D, D, are in the form of copper cylinders closed at the top, all but a small hole or holes, *h*, and carried on the ends of steel rods. These are put over the carbons like extinguishers on the tops of candles. The carbon electrodes, C, are carried on an insulated plate which is so held and moved by a screw, S, that these copper cylinders can be simultaneously all lifted a little way above the carbons. The copper-carbon electrodes are joined in series, so that when supplied with continuous current at 400 or 500 volts all the copper electrodes are positive. The arc forms between the tip of the carbon and the underneath side of the copper top of the covering cylinder. The heat volatilizes the oil, and the chamber soon becomes full of vapour which is not oxidizing.

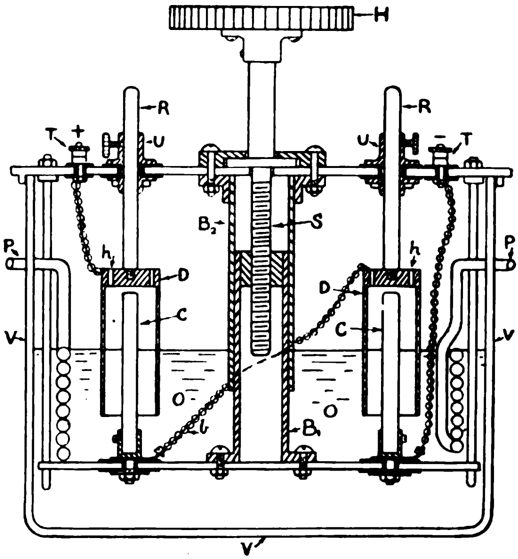


FIG. 81.—Arc Generator (Fleming) for Creating Continuous Electric Oscillations.

The small holes in the top of the copper allow the escape of this vapour.

The copper cylinders can be all moved up or down by the screw together, and each can also be adjusted separately. The carbon rods tend to increase in length with use by the deposit of carbon on them.

If this series of arcs is shunted by a condenser and inductance we can produce in the circuit high frequency oscillations which are undamped or persistent.

If necessary a coil of lead pipe can be included in the vessel of oil and water circulated through it to cool the oil.

The arrangement is very easily worked, and two or three of such arcs in series can be operated off a 220-volt direct current supply or six off a 400-volt supply with some adjustable resistance in series. The arc current should be kept small, viz. about two or three amperes.

An alternating current in the condenser circuit can be obtained having a frequency of one million or more by proper adjustment of the capacity and inductance.

15. Methods for the Production of Closely Sequent Trains of Damped Oscillations.—In addition to the above described methods for the production of true persistent oscillations by the high frequency alternator and the electric arc there are other methods by means of which oscillations almost equivalent to undamped trains may be produced by means of condenser discharges. In the ordinary use of the induction coil operated with a hammer or mercury break the number of interruptions of the primary circuit may be about 50 per second, but when an alternator is substituted for the induction coil the number of alterations

may be increased to 500 or more per second. If we suppose that a condenser is charged by such a coil or alternator, and that 500 trains of damped oscillations are thus created per second, and that each train contains 20 complete oscillations, and that the oscillation frequency is 10^6, we see at once that the time during which oscillations are actually taking place is only one-hundredth part of the whole time during which the process continues. It is obvious, therefore, that much may be done by closing up the intervals between the trains of oscillations, so as to occupy more of the whole time with oscillations. There are several methods by which this can be done.

The first of these is by the use of a short-spark gap and induction coil. If a coil has its secondary terminals connected by a condenser of no very large capacity, say by a Leyden jar of 0·004 mfd. capacity, and the spark gap across the secondary is made short, say 1 mm., then several discharges of the jar take place at each interruption of the primary circuit of the coil. This can be proved by the aid of the author's spark counter described in § 15, Chap. II. The explanation is that as the electromotive force rises up in the secondary circuit of the coil, when it reaches the value corresponding to the length of the short gap it causes a discharge, but since the electromotive force continues to exist it again repeats the charging, and hence 3, 4, or 5 or more sparks may occur at each interruption of the primary. It must, therefore, not be assumed that when we have 500 alternations in the primary circuit per second we have the same number of secondary sparks. There may be many more sparks than alternations of the primary current. It is, however, necessary to insert in the primary circuit of the transformer a large inductance which serves to arrest the current which would otherwise start an arc across the balls immediately the spark takes place. By the use of a suitable inductance in series with the primary of a coil actuated from a public electric supply of alternating current having a frequency of 40, Professor Q. Majorana, in 1904, succeeded in obtaining a series of 10,000 short sparks per second for the purposes of wireless telephony.[59] This phenomenon of multiple sparks per interruption had previously been noticed by H. Abraham (see *Bulletin Soc. Franc. de Phys.*, May 5, 1899) and by A. Blondel (see British Patent Specification, No. 21,919 of Dec. 3, 1899), but had not previously been utilized. Majorana states that blowing air or carbonic acid on the spark gap serves to keep down the temperature of the balls and maintain a steady state.

Such a multiple spark examined in a revolving mirror presents itself as an unbroken band of light, unless the speed of the mirror is very large.

A very similar multiplication of discharges can take place when a mercury lamp or bulb full of mercury vapour is substituted for a spark gap.

Mr P. Cooper-Hewitt, in investigations connected with the production of a mercury-vapour incandescent lamp, found that a column of mercury vapour has electrical properties very similar to that of the electric arc between solid carbons. If a glass tube is provided with mercury electrodes connected by sealed-in platinum wires with a circuit, and if the tube is highly exhausted of air so as to contain only mercury vapour, it is found that this vapour becomes electrically conductive when a continuous voltage is supplied to the ends of the tube which exceed a certain limit.[60]

The tube when cold offers a high resistance, and this appears to reside chiefly at the negative mercury electrode. If, however, a high voltage is momentarily applied, the resistance falls, and a moderate voltage of 50 or 100 volts will then maintain a current of several amperes through the tube, provided the tube has a sufficient diameter. When a certain current passes, the mercury vapour glows brilliantly with a bright greenish light. The efficiency of the device as a source of light is high. A tube taking 3 amperes at 60 volts will emit a light of 360 candles, and has therefore an efficiency of 0·5 watt per candle.

[59] See *The Electrician*, vol. 53, p. 991, 1904, October 7; also *Elektrotechnische Zeitschrift*, vol. 25, p. 943, 1904.

[60] A general description of the phenomena connected with the arc discharge in mercury vapour has been given by Mr H. P. Wills, in a paper on the "Conduction of Electricity in Mercury Vapour," in the *Physical Review* for August 1904, vol. xix. p. 65; see also a paper by P. C. Hewitt, *Electrical World and Engineer* of New York, April 27, 1901, p. 679.

If a mercury vapour lamp has its terminals shunted by a condenser in series with an inductive resistance, and a high voltage is applied to the terminals of the tube, the result is to excite electrical oscillations in the condenser circuit, including the condenser, inductance, and tube. Assuming the voltage to be alternating, the operations are as follows :—

As the voltage rises from zero the condenser becomes charged, but the mercury vapour tube does not conduct. At a certain critical voltage the resistance of the mercury vapour suddenly disappears or falls greatly, and a current passes through it. The condenser then discharges through this low resistance with oscillations, and when the voltage again falls below a certain value, the mercury

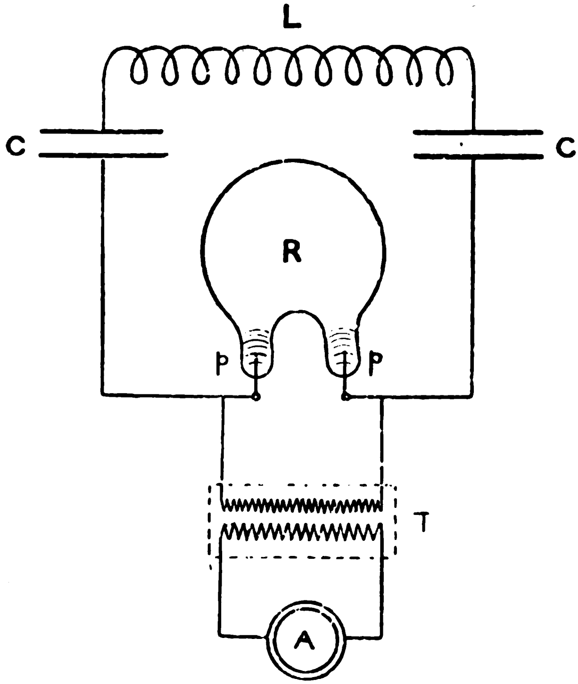


FIG. 82.—Mode of employing the Cooper-Hewitt Mercury Vapour Current Interrupter in place of a Spark Gap. R, vacuous glass bulb containing mercury vapour; *p*, *p*, mercury electrodes; A, alternator; T, transformer; C, C, condensers; L, inductance.

vapour ceases to be a good conductor, and remains of high resistance until the voltage rises again and the process repeats itself. Owing to this high initial resistance, it requires about 5000 volts alternating to maintain a current of 2 amperes through a tube which will take the same current at 100 volts continuous.[61] Based on these facts, Mr Cooper-Hewitt has devised a mercury vapour current interrupter, as follows :—

A large glass bulb about 8 or 10 inches in diameter has a pair of tubular extensions with platinum wires sealed in at the bottom. These tubes are partly filled with mercury (see Fig. 82). The globe is exhausted of air, and contains only mercury vapour. It may be put in a vessel of oil to keep it cool.

The platinum terminals are connected to a high vo'tage low frequency circuit,

[61] See *Science Abstracts*, 1904, vol. 7, A, p. 347; also British Patent Specification, P. Cooper-Hewitt, No. 9206 of 1903; also *Electrical World* of New York, Feb. 21, 1903, vol. 41, p. 316, and *Electrical Review* of New York, Feb. 21, 1903, vol. 42, p. 264.

such as the secondary terminals of a 20,000-volt transformer, and the terminals are also shunted by a condenser and an inductance, which may be the primary circuit of an oscillation or high frequency induction coil. When the low frequency voltage is turned on, the mercury vapour between the electrodes is a non-conductor until the voltage reaches a certain high value, say 10,000 or 15,000 volts. The bulb then suddenly becomes a good conductor due to the disappearance of the cathode resistance. Then the condenser discharges with oscillations and the voltage drops. At a certain low voltage, which can be adjusted, the cathode resistance again reappears and the bulb ceases to conduct. Hence it acts like a spark gap, but with much greater regularity. The behaviour of the interrupter as a substitute for a spark gap in producing the oscillatory discharge of a condenser has been investigated by Professor G. W. Pierce, of Harvard University. He employed one of the double-pool mercury type of Cooper-Hewitt bulbs, and considered its application especially to wireless telegraphy.[62] He operated with alternating currents, and found that several discharges may occur within a single half period of the transformer current. Thus with 15,000 volts, and a capacity of 0·117 mfd., one or two discharges per half period were obtained.

With a small capacity of about 0·001 mfd. and the same voltage over 200 discharges per half period, that is, in $\frac{1}{120}$ of a second, were produced. The complete discharges were separated only by $\frac{1}{100000}$ of a second.

The discharges of this particular interrupter always began at 7070 volts, and the condenser was left charged at about 1600 volts, sometimes positive and sometimes negative.

It was found that the resistance of the interrupter decreased with increasing capacity (C) and decreasing inductance (L) in the oscillatory circuit, and varied from 0·127 ohm for $L = 0{\cdot}000011$ henry and $C = 0{\cdot}117$ mfd., to 0·598 ohm for $L = 0{\cdot}00142$ henry and $C = 0{\cdot}073$ mfd.

The mercury interrupter seems to act, therefore, as a very low resistance air gap, but with much greater uniformity. On the other hand, attempts to use it for large powers have not been very successful.

A method of very practical utility for the production of rapidly repeated trains of feebly damped oscillations has been developed out of a discovery made by M. Wien in 1906, on the damping of short sparks.[63] Wien observed that when a condenser discharge takes place between good conducting metallic surfaces, placed very near together so as to produce short spark, the oscillations are very quickly damped out and the spark killed.

If two flat copper plates are placed with their surfaces parallel and not more than a quarter of a millimetre apart, then if a condenser or Leyden jar is connected in series with this flat plate spark gap, and the two plates connected to the secondary terminals of an induction coil in action, the condenser discharge takes the form of a very rapid series of highly damped discharges which are called *quenched sparks.* The cooling action of the flat copper plates tends to prevent the formation of any true arc discharge even if the source of high potential is a transformer or continuous current dynamo. These quenched sparks may occur at the rate of several hundred or even thousand per second.

A form of quenched spark discharger which deserves mention here is that of E. Leon Chaffee, which consists of two flat surfaces of aluminium and copper, which are placed a fraction of a millimetre apart in an atmosphere of moist hydrogen.[64] The surfaces are included in a metal box, provided with radiator flanges to keep it cool (see Fig. 83). If a condenser charged by a dynamo giving an E.M.F. of 500 volts is continually discharged across this gap, it will produce a very rapid series of quenched sparks, and these can be caused to create trains of feebly damped oscillations almost in close sequence by the arrangement of circuits shown in Fig. 84.

If the inductance in series with the condenser is the primary coil of an air core transformer, then at each spark between the plates there is a sudden discharge of

[62] See G. W. Pierce, "On the Cooper-Hewitt Mercury Interrupter," *Proc. Amer. Acad. of Science*, 1904, vol. 39, No. 18, p. 389. Also *Science Abstracts*, vol. 7, A, p. 346.
[63] See M. Wien, *Physikalische Zeitschrift*, No. 23, December 1906, p. 872.
[64] E. L. Chaffee, *Proc. American Acad.*, "Arts and Sciences," vol. 47, p. 265, 1911.

the condenser through the primary and, therefore, the production of a secondary oscillation in the secondary circuit. The primary oscillation is very nearly dead-beat as the spark is so quickly extinguished, but in the secondary circuit we obtain a series of feebly damped trains of oscillations very rapidly following each other.

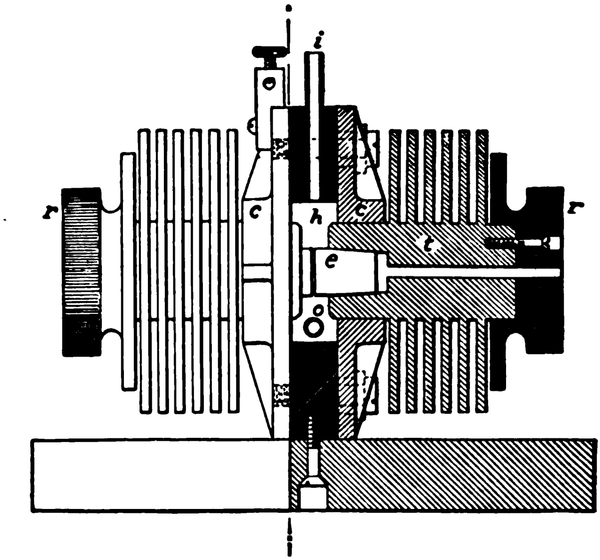


Fig. 83.—Section of Chaffee Quenched Spark Discharger for Producing Highly Damped Discharges.

The practical forms of this discharger, and others like it, called quenched spark dischargers, will be considered in Chap. VII. of this book.

Another form of such discharger due to E. von Lepel consists of two metal plates, with surfaces very near together, but separated by a ring cut out of a sheet of paper.

In these quenched spark dischargers there is, however, nothing to regulate the

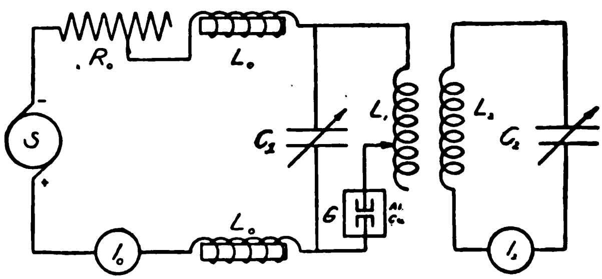


Fig. 84.—Scheme of Circuits Used with Chaffee Quenched Spark Discharger.

rate at which the sparks take place, and it may be very irregular, and hence the feebly damped trains of oscillations produced in the secondary circuit of the oscillation transformer will be irregular as to intervals and intensity.

Hence inventors have endeavoured to supply means of creating a nearly or else quite closely sequent trains of feebly damped discharges following each other

closely. One such method is due to R. C. Galletti.[65] His arrangement is shown diagrammatically in Fig. 85. Current is taken from the supply mains of a direct current system at high voltage, say 40,000 volts generated by high tension C.C. dynamos in series. These mains are represented in Fig. 85 by the lines marked P and N. Across these there are a series of inductionless resistances, R_1, R_2, R_3, etc., in series with condensers, C_1, C_2, C_3, etc., and all in series with one common condenser, C_0. Each condenser is shunted by a spark gap, T_1, T_2, T_3, etc., and all these gaps are in series with an inductance, L. The condensers, therefore, dis-

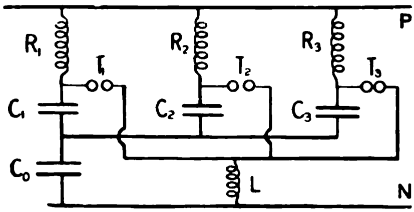


FIG. 85.—Galletti's Arrangement of Circuits for the Production of Close-Order Dischargers.

charge across their respective spark gaps and all through the common inductance, L. The presence of the common condenser, C_0, however, causes the condensers to discharge in sequence and not simultaneously. Hence the inductance L is always being traversed by oscillations which, although not undamped, are yet practically uninterrupted. Galletti has been able to transform in this way many kilowatts of power into the form of oscillatory discharges taking place 10,000 times a second.

Further discussion of methods for the production of closely sequent discharges is left for Chap. VII. of this book dealing with radiotelegraphic appliances.

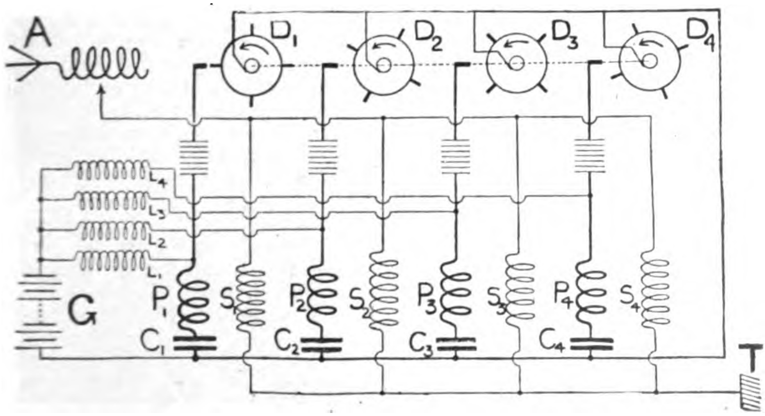


FIG. 86.—Marconi Multiple Discharger for the Production of Close-Order Dischargers.

In Galletti's arrangement there is nothing, however, to cause the successive trains of oscillations to follow each other without discontinuity or in step with each other.

Senatore Marconi has, however, described an ingenious method which is based upon the use of a number of his rotating disc-dischargers. These dischargers are

[65] See British Patent Specification, No. 15,497 of 1910.—"Improvements in or relating to the Production of Continuous Wave Trains by Means of Primary Spark Circuits."

described in detail in Chap. VII. § 18. Suffice it to say that he employs a rapidly rotating disc with studs or pins on its periphery, which rotates between fixed studs so that a condenser circuit is closed, and a discharge started at regular intervals several times during each revolution of the disc.

Suppose, then, that a number of these studded discs are arranged on a shaft insulated from each other, but so set relatively to each other that the rotating discs all closed separate but identical oscillation circuits successively and at intervals of time corresponding to one complete period of oscillation of the condenser circuits.

Then if the secondary oscillations are received in a common circuit as shown in Fig. 86, it is clear that the different damped trains of oscillations can be made to so overlap that the joint effect is a practically continuous or undamped oscillation, as shown in Fig. 87.

This system of production of undamped waves is particularly applicable in the case of the very long waves used in long-distance radiotelegraphy. In this case waves from 5 to 10 miles in wave length are used which require a frequency of 40,000 to 20,000. If, then, there were 10 such discs, each having 40 studs, and the whole revolved 3000 R.P.M., or 50 times a second, we should have the interval between two discharges of the separate condenser circuits equal to $\frac{1}{20000}$ of a second. This would enable practically undamped waves of 10 miles long to be created.

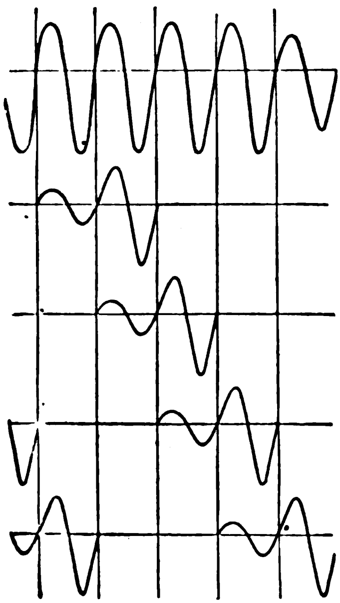


FIG. 87.

16. Frequency-Changing by Static Transformers.—In addition to the methods of producing undamped high frequency currents by the direct use of extra high frequency alternators, or the method of frequency-raising employed in the Goldschmidt or Béthenod alternators, another method has been developed in which an alternator of moderately high frequency, say 10,000, is used, and this frequency is raised by the use of associated static or stationary alternating current transformers. Since the difficulties of constructing an alternator of such moderate frequency as 10,000 are very much less than those involved in making one for 40,000 or 100,000, this frequency-raising by static transformers has been much studied. It has been developed out of an arrangement described by M. Maurice Joly in 1911.[66] Suppose that two iron cored transformers have their primaries joined in series, and their secondaries in opposition to each other (see Fig. 88). If the transformers are identical in every way then no current will flow in the secondary circuit, because the two electromotive forces balance each other.

Suppose that one of these transformers has its core of such size and wound with such ampere turns that it is magnetically saturated, whilst the other is very far from being saturated, when the primary current has attained, say, half its maximum amplitude during the period. Then the wave forms of the two secondary electromotive forces will be very different.

As soon as the core reaches magnetic saturation there can be no further increase in the secondary electromotive force because the latter depends upon the rate of change of the magnetic flux in the core. Hence for the nearly saturated core the curve of secondary E.M.F. will be a flat-topped curve *b*, whilst for the non-saturated core it will be a peaky curve *c* as shown in Fig. 89, where the two dotted curves represent the wave forms of the magnetic flux in each transformer

[66] See *La Lumière Électrique*, vol. 14, p. 195, 1911.

respectively. If, then, the primaries are joined in series and in opposition, and if the change-ratio of each transformer is adjusted to give equal opposed secondary E.M.F.'s, they will conspire to produce a triple frequency E.M.F. *d* in the secondary circuit, and this may be exalted by suitably tuning that circuit with a

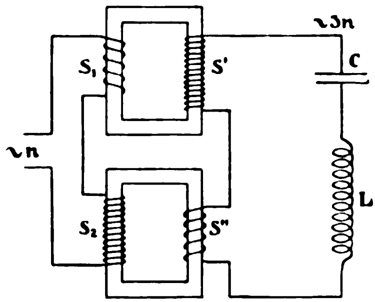


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FIG. 88.—Joly Method of Tripling Frequency by Coupled Static Transformers.

condenser and inductance so as to make it respond to this frequency. The reason will easily be seen from the curves in Fig. 98*a*.

Another method was suggested by Epstein in 1902 and worked out by Joly and Vallauri in 1911.[67] In this case each transformer has on it a tertiary circuit in which a continuous current is made to flow. The primaries, P_1, P_2, are connected up in opposition (see Fig. 90) and the secondaries, S_1, S_2, also in series and tuned

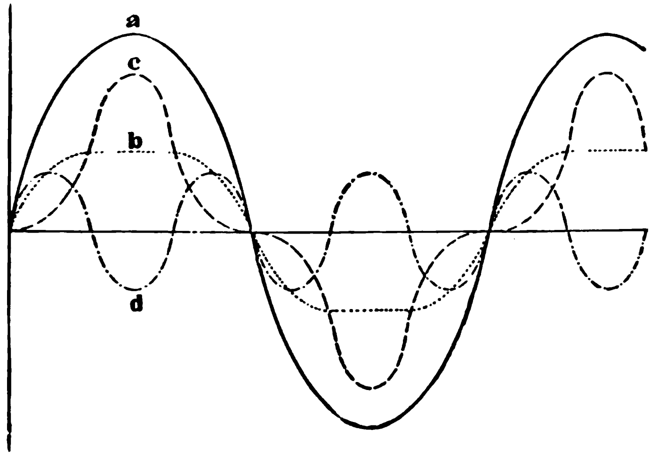

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FIG. 89.—Production of a Triple Frequency Static Transformer.

by a condenser C. The battery B supplies the continuous current in the tertiary circuits.

The operation is as follows: The continuous current is so adjusted that taken alone it would magnetize the core to a point near to the knee of the magnetization curve. Hence when the alternating magnetizing force is superimposed the wave form of the flux in each core is a curve which is flat in one half period and peaky in the

[67] See *The Electrician*, vol. 70, p. 97, 1912.

other half period as shown by the curves *a* and *b* in Fig. 92, because the alternating current magnetizing force is in each semi-period added to or subtracted from the constant direct current magnetizing force represented by the ordinates of the horizontal lines in the diagram. Therefore when these two flux changes act together they produce a double frequency flux variation as shown in the curve *c* in Fig. 91. The wave forms of the separate electromotive forces due to these flux changes in each secondary and of the resultant double frequency E.M.F. are shown in Fig. 92.

The diagrams in Figs. 88 to 92 are taken by kind permission from a paper on radio-frequency changers by Dr. Alfred N. Goldsmith in the *Proceedings of the Institute of Radio-Engineers* of New York (vol. iii. p. 55, March 1915), to which the reader is referred for further information. Another article on "Static Transformers for Frequency Changing," by Mr. A. M. Taylor (see *Journal of the Institution of Electrical Engineers* of London, vol. 52, p. 700, 1914, or *The Electrician*, vol. 73, p. 170, 1914), furnishes an account of a method devised by him for obtaining a triple frequency from a 3-phase current as follows :—

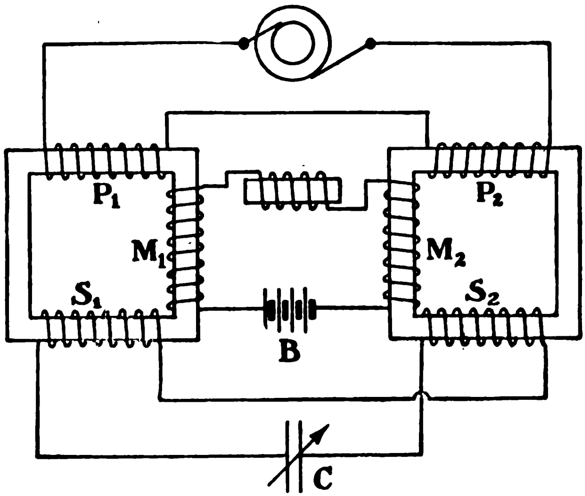

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FIG. 90.—Joly Method of Frequency Changing by Static Transformers.

Three choking coils are placed in the three phases of a 3-phase supply, the circuits being all connected with a primary wound on a common transformer (see Fig. 93).

The secondary of this transformer will then furnish a triple frequency single-phase current.

The common transformer is so wound that its core is magnetically unsaturated. The operation of Taylor's frequency changer may be thus explained :—

Consider a single transformer and choker (see Fig. 94), the transformer having an unsaturated core and the choker a saturated core. If we then consider what will happen if an alternating E.M.F. is supplied to the transformer primary in series with the choker, it will be seen that the saturated choker core will supply a back E.M.F., as long as the current is less than the value required for saturation. Hence the current through the coil is small until it reaches a certain value during the phase. It then rushes up and hence gives a wave form with pronounced peak. Accordingly the current obtained from the secondary of the transformer with an unsaturated core consists of a to-and-fro flow or complete wave of lesser period than the primary, and also of a silent period between these secondary waves. Hence as a single-phase arrange-

ment it is not practicable. If, however, 3-phase currents are employed and the secondary currents received into a common secondary circuit (see Fig.95), then the primary currents of the transformer are displaced so that the resultant secondary current is a complete alternating current of three times the frequency of the primary 3-phase current.

By this ingenious arrangement a 3-phase current of frequency 25 can be transformed into a single-phase current of frequency 75 suitable for lighting purposes.

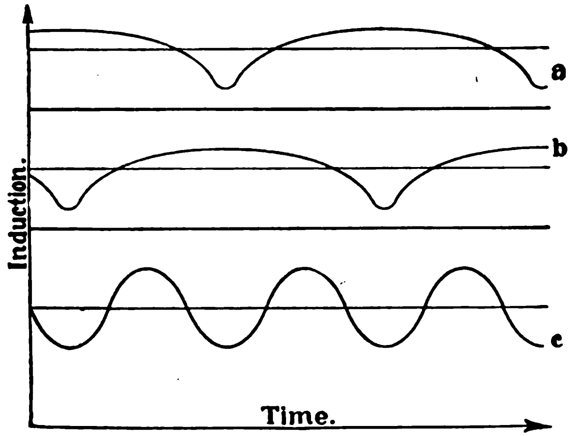

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FIG. 91.—Double Frequency Flux Variation Produced by Transformers arranged as in Fig. 99.

Taylor also proposed by the use of 9 phases and 9 chokers to create a nine-fold frequency current, using a polyphase alternator of suitable wave form to compress each secondary alternation into a period of $\frac{1}{9}$ of that of the primary.

This method of frequency-changing by static transformers is thoroughly practicable and has come into some use in connection with wireless telegraphy.

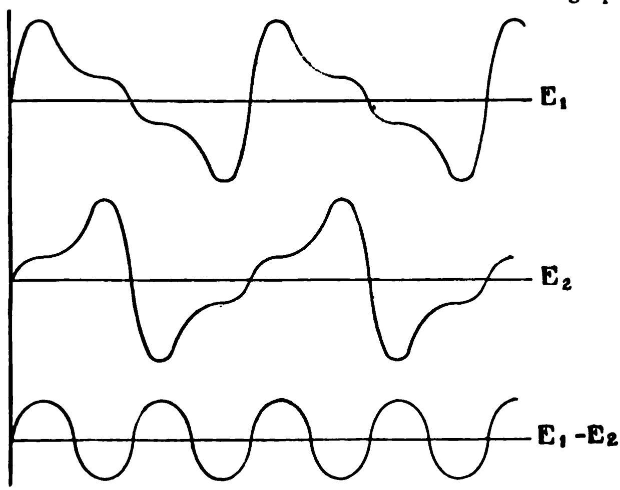

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FIG. 92.—Double Frequency E.M.F. Produced by Transformers arranged as in Fig. 99.

The loss of energy in the core of the transformer can be made small by working at a very low flux density, whilst that in the choking coils is not excessive owing to the frequency in their circuits being only $\frac{1}{3}$ or $\frac{1}{9}$ of that in the transformer.

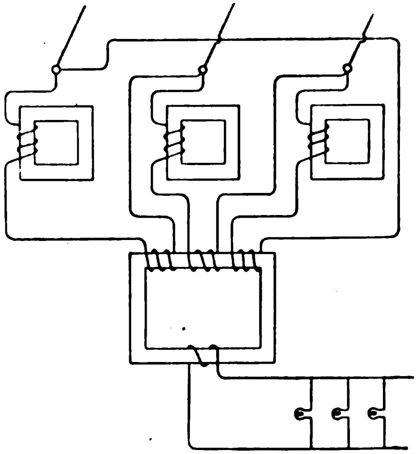

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FIG. 93.—Taylor's Arrangement of Static Transformers for Tripling Frequency.

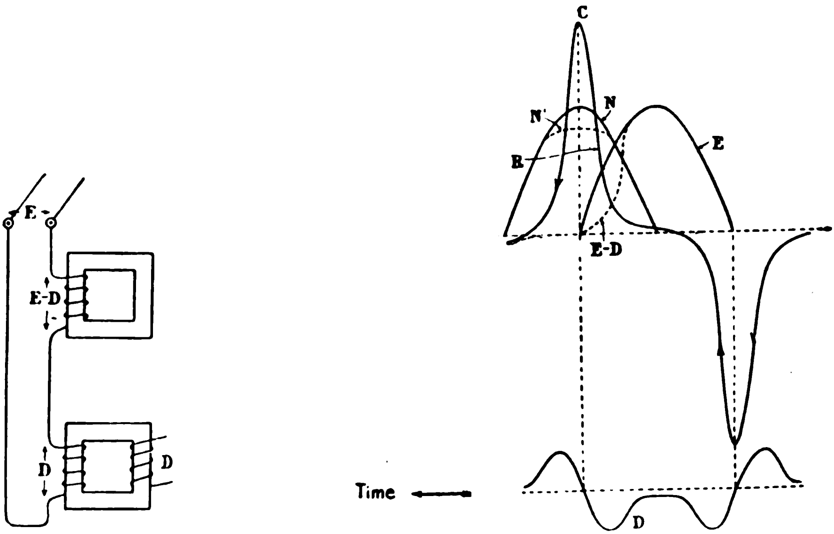

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FIG. 94.

By the above methods we can obtain from alternators giving a frequency, say, of 10,000 currents of frequency sufficient for radio-telephony, viz. 30,000 or more.

A special method of producing undamped electric oscillations by means of an appliance called a three-electrode valve is mentioned in the last chapter of this book on Radio-telephony.

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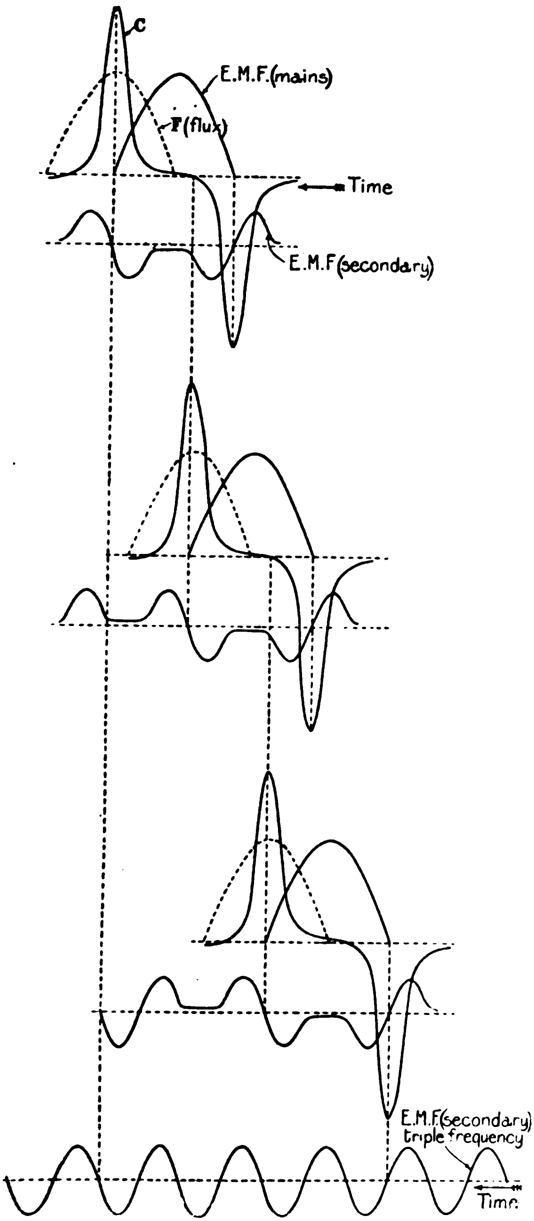

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95.—Diagram showing the operation of Taylor's Method for Tripling Frequency by Static Transformers.

CHAPTER II

HIGH FREQUENCY ELECTRIC MEASUREMENTS

1. The Essential Difference between High and Low Frequency Electric Measurement. High Frequency Electric Resistance.—The measurement of high frequency electric currents and potentials and other specific qualities of electric conductors and insulators, when subjected to the action of electric oscillations, to a considerable extent calls for the employment of special instruments and methods. The processes and means used for the measurement of low frequency alternating electric currents and potentials are not always applicable or correct if applied in high frequency measurements. The cardinal reason for the difference between the two cases is to be found in the fact that a high frequency current does not penetrate into the interior of a thick solid metallic conductor of good conductivity, but is a surface effect. Furthermore, inductances and condensers act towards high frequency currents in a manner quite different from that in which they act towards continuous or low frequency currents. A coil of wire of many turns may act as an almost complete barrier to electric oscillations and, on the other hand, a condenser which, when interposed in a circuit, will either prevent or reduce the flow of a continuous or low frequency current may actually increase the current if inserted in a high frequency circuit.

As we are much concerned when dealing with electric oscillations with the resistance, inductance, and capacity of circuits in which rapidly reversed electromotive forces exist, it is necessary to consider in the first place the manner in which these qualities are affected by the frequency.

Every electric circuit consists of a so-called conductor, immersed in an insulating material or non conductor. When traversed by an alternating current there are five qualities of the circuit to be considered :—

(i.) The *resistance*, or reciprocally the *conductance* of the conductor.

(ii.) The *inductance* of the conductor, depending on its geometrical form, material, and the nature of the surrounding insulator.

(iii.) The *capacity* of the conductor, depending on its position with regard to the return circuit and other circuits, and on the nature (*dielectric constant*) of the insulator surrounding it.

(iv.) The *dielectric conductance*, or reciprocally the *insulation resistance* of the insulator.

(v.) The *energy dissipating power*, due to causes other than conductance (such as the *dielectric hysteresis*) which exist in the insulator or dielectric.[1]

The resistance of a circuit may be defined as that quality of it in virtue of which energy is dissipated as heat when a current flows through it. The ordinary volume resistivity is the resistance per unit cube, *i.e.*, of one centimetre cube under uniform electric current flow between opposed faces.

The resistance under the action of uniform current flow may be called the steady resistance and will be denoted by R.

The power dissipated as heat in a conductor of steady resistance R when a uniform unidirectional current A is flowing through it is measured by A^2R, and the resistance R may therefore be defined as the quotient of the total energy dissipation per second, viz., A^2R, by the square of the current A.

[1] Under this heading we must also include such sources of energy dissipation as brush discharges through the air over the surface of the dielectric or between conductors.

In this case the current is uniformly distributed over the cross-section of the conductor, that is, has uniform current density. If, however, the current density is non-uniform over the cross-section, we have to define the resistance as follows :—

Let the conductor have a cross-section of any shape and a distribution of current density, c, over it in any manner. Then let $dx\,dy$ be an element of area of the cross-section the co-ordinates of which are x and y (see Fig. 1).

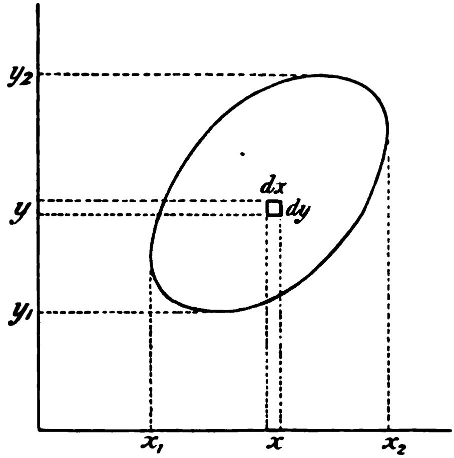


FIG. 1.

The current through the area is $c\,dx\,dy$, and if ρ is the resistivity of the material of the conductor supposed constant, then $\int_{x_1}^{x_2}\int_{y_1}^{y_2} \rho c^2\,dx\,dy$ is the expression for the total heat generated per second in unit length of the conductor.

Also the expression $\int_{x_1}^{x_2}\int_{y_1}^{y_2} c\,dx\,dy$ is the value of the total current through the conductor, x_1, x_2, y_1, and y_2 being certain limits of the area of cross-section. It is always possible to find a quantity R′ such that—

$$R'\left\{\int_{x_1}^{x_2}\int_{y_1}^{y_2} c\,dx\,dy\right\}^2 = \int_{x_1}^{x_2}\int_{y_1}^{y_2} \rho\,c^2\,dx\,dy \quad . \quad . \quad . \quad . \quad (1)$$

and this quantity R′ may be called the resistance for non-uniform current density over the cross-section.

It can be proved by a simple application of the Calculus of Variations that under the condition that the total current is constant, R′ has a minimum value R when c is constant. In other words, that the steady resistance is the minimum resistance. In the case of high frequency currents the distribution of the current is non-uniform over the cross-section, and R′ may then be called the high frequency resistance. It is easy to show that the resistance for non-uniform current density is greater than the resistance for uniform current density without the application of any mathematics. Imagine that the cross-section of the conductor, supposed square, is divided up into elements of area, and that each filamentary conductor into which we may suppose the whole conductor thus divided has the same resistance, and carries the same fraction of the total current. Then we have equal currents density as indicated by the uniform shading in Fig. 2. Imagine

that the current is removed from one filament and added to that in another. Then the total current is not altered, but the heat generated in the first-named filament becomes zero, and in the other four times what it was before. Accordingly, the total heat generation is increased, although the total current is not altered. The resistance of the conductor as above defined is therefore increased by any change in the distribution of the current which makes its density non-uniform over the cross-section. Moreover, by this mode of viewing the phenomena it is easy to see that any distribution of current density which is non-symmetrical round the periphery of the conductor is also a cause of increased resistance as compared with that corresponding to a symmetrical distribution of current density. Accordingly, not only is the resistance of a straight solid conductor greater for high frequency oscillations than for continuous currents, but the resistance of a spiral or helix of wire for high frequency currents is greater than that of the same wire when stretched out straight, because in the first case the current density is greater at the periphery of the wire than at the centre, and in the second case the peripheral distribution is non-uniform owing to the fact that the external distribution of field is non-uniform, being greater on the interior parts of the solenoid than on the outside.

In dealing with high frequency alternating currents, we are presented in a marked degree with the phenomenon of *skin or surface concentration* of the current.

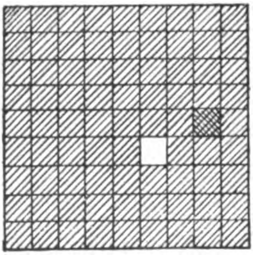


FIG. 2.

When a conductor is acted upon by an alternating electromotive force, the current does not spring into existence at all parts of the cross-section of the conductor instantly, but is created first at the surface and diffuses inwards.[2] The mathematical law according to which this diffusion takes place is the same as that which controls the penetration of magnetic flux into an iron bar, when it is exposed to a magnetizing force, by being surrounded by a coil through which a current is passed.

The propagation of magnetic flux through ferromagnetic substance is effected by a process which is in every way analogous to the diffusion of liquids into one another, or to the transference of temperature through a conductor—that is, as Lord Kelvin has called it, to the thermometric conductivity.[3] These two last-named processes are mathematically described by differential equations of the same form as those which determine the propagation of magnetic flux, or of an electric current into a conductor.

In the case of magnetic flux, this rate of diffusion, as Mr. Oliver Heaviside has shown, is inversely as the electric conductivity and inversely as the magnetic permeability of the material.

Consider the case of a cylinder of iron placed parallel to the lines of flux in a uniform magnetic field, say in the interior of a long solenoid traversed by a current. If we suppose the iron suddenly introduced into the uniform field, the magnetic flux seems to penetrate into it through its surface, and, so to speak, soaks more or less slowly into the mass.

A very elegant demonstration of this fact was afforded by experiments described by Dr. J. Hopkinson and Professor E. Wilson some years ago.[4] It was then experimentally proved that the application of a magnetizing force to a cylinder of

[2] See Stefan, *Sitzungsberichte der Wiener Akad. der Wissenschaft*, 1887, vol. 95, part ii. p. 917; also Oliver Heaviside, "Electromagnetic Theory," vol. i. p. 345, *et seq.*

[3] If v is the temperature at any point having abscissa x in a thin rod which has a thermal conductivity, k, and a thermal capacity, c, then the well-known equation of Fourier which determines the temperature at any place and time is $k\frac{d^2v}{dx^2}=c\frac{dv}{dt}$, assuming the constancy of k. An identical differential equation expresses the law of diffusion of liquids or gases and the propagation of electric potential along a submarine cable as pointed out by Lord Kelvin. See *Report of the British Association*, 1888, p. 571.

[4] See J. Hopkinson, "Propagation of Magnetization in Iron," *Journal of the Institution of Electrical Engineers*, 1895, vol. 23, p. 194.

iron resulted in the slow propagation of the magnetic flux into the iron from the surface inwards, and it was pointed out that the time required to establish the practically steady or uniform state of flux in the iron varies as the square of the diameter of the cylinder. Hence it follows that if the cylinder of iron, or any other conductor, is placed in a rapidly alternating magnetic field, the magnetic flux never quite penetrates to the centre of the mass of metal if its diameter exceeds a certain value. The alteration of magnetic force results in the flux, so to speak, being recalled before it has time to establish itself throughout the whole mass of the metal. A similar effect can take place with heat. If, for instance, a poker is placed in the fire, the outer surface heats up first, and after a certain time all parts of the cross-section of the poker where it is exposed to the heat come to very nearly the same temperature. If it is then removed, the outer surface cools first, but after a time it gets cool all through. If it is heated and cooled alternately and rapidly, it will be hotter on the surface than in the middle. The heat will not have time to go far in before it is compelled to return.

Similarly, if an electromotive force acts upon a conductor, the current begins at the surface and soaks inwards. If, therefore, the electromotive force is periodic or alternating, the current more or less is confined to the outer skin or surface of the conductor, and the higher the frequency the less does it penetrate. In the case of very high frequency currents, if the conductor is a fairly good conductor the current exists in a mere surface layer or skin. Accordingly the resistance R′ measured as above defined may have a much greater numerical value in the case of high frequency alternating currents than in the case of steady or non-periodic currents.

We have, therefore, to distinguish between the resistance to steady currents, the resistance to low frequency alternating currents, the resistance to very high frequency currents, and a fourth case presents itself when we consider damped high frequency oscillations.

The full mathematical discussion of the subject would occupy too much space. We can only indicate the mode of treatment in outline, and refer the reader to various sources for additional information.

If there be any conductive medium having resistance per unit volume ρ and in which currents are being established under the action of electromotive force, then we have to consider the following variables :—

(i.) The electric current density with rectangular components u, v, w at any point in the medium.

(ii.) The electric force with components X, Y, Z.

(iii.) The magnetic force with components α, β, γ.

(iv.) The magnetic flux density or induction with components a, b, c.

The relations between these quantities will be more fully discussed in Chap. V. They are expressed in the equations $a=\mu\alpha$, $X=\rho u$, and two similar equations in b, β, Y and v, and c, γ, Z and w, where μ is the magnetic permeability. We have then two other sets of three important equations which are called the Maxwellian equations, viz.—

$$-\mu\frac{d\alpha}{dt}=\rho\left(\frac{dw}{dy}-\frac{dv}{dz}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$4\pi u=\frac{d\gamma}{dy}-\frac{d\beta}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and four other symmetrical equations in u, v, w, α, β, γ, which will be further discussed in Chap. V. They express in symbolic form Faraday's law of induction, and the fact that the line integral of magnetic force round an element of area is equal to 4π times the current through the area. Differentiating the last equation with respect to time, and remembering that there can be no concentration of current at any point, which is expressed by the relation—

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0$$

we reach without difficulty an equation of the form—

$$\frac{4\pi\mu}{\rho} \cdot \frac{du}{dt} = \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and two similar equations in v and w (see Jeans, *Electricity and Magnetism*, § 535, p. 466).

These equations are identical with those first given by Fourier for the diffusion of heat into a body, and they show that the establishment of electric currents in a conductor obeys the same law of diffusion, and that it begins at the surface of a conductor and soaks inwards by a process analogous to the conduction of heat. The solution of these equations in the electrical case has been considered by Maxwell (see *Electricity and Magnetism*, vol. ii. § 690), by Oliver Heaviside (see *Electrical Papers*, vol. ii. p. 64), and by Lord Kelvin (see *Mathematical and Physical Papers*, vol. iii. p. 491), and more recently by Dr. A. Russell (see *Phil. Mag.*, April 1909, p. 524).

To apply them to the case of a straight wire of cylindrical form with circular cross-section we may take the axis of the wire as the z-axis and then the second differential coefficient of u with respect to z becomes zero. Also if r is the radius of the circular cross-section then $r^2 = x^2 + y^2$ and $rdr = xdx + ydy$.

We can then transform the variables and easily prove that

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr}$$

On the assumption that u is a simple sine function of the time in which u varies as the real part of ϵ^{jpt} equation (4) becomes—

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \left(\frac{4\pi\mu}{\rho} jp\right)u = 0 \quad . \quad . \quad .$$

Since then $du/dt = jpu$.

This last equation is a form of Bessel's Equation, viz.—

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + a^2u = 0$$

The solution of the above equation cannot be found in finite terms, but is expressed by the Bessel's Series—

$$u = 1 - \frac{(ar)^2}{2^2} + \frac{(ar)^4}{2^2 \cdot 4^2} - \frac{(ar)^6}{2^2 \cdot 4^2 \cdot 6^2} + \text{etc.}$$

This series is denoted by $J_0(ar)$ and is called a Bessel's function of the Zeroth order.

In our case $a = \sqrt{-j\frac{4\pi\mu p}{\rho}} = j - 1\sqrt{\frac{2\pi\mu p}{\rho}}$ since $(j-1)^2 = -2j$.

It is clear from the definition of resistance given above that the high frequency resistance per unit of length of the wire (R′) is given by the expression

$$R' = \frac{\rho\int_0^{r'} u^2 r dr}{\int_0^{r'} u r dr}$$

where r' is the radius of cross-section of the wire and u is the current density

The steady current resistance R per unit of length is $\rho/\pi r^2$.

Hence we have—

$$\frac{R'}{R} = \pi r^2 \cdot \frac{\int_0^{r'} \{J_0(ar)\}^2 r dr}{\int_0^{r'} J_0(ar) r dr}$$

The above integral is difficult to evaluate.

Lord Kelvin reduced it to the form[5]—

$$\frac{R'}{R}=\frac{q}{2}\cdot\frac{ber.\ q\ bei.'\ q-bei.\ q\ ber.'\ q}{(ber.'\ q)^2+(bei.'\ q)^2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where the accents denote differential coëfficients, and *ber.* q and *bei.* q stand for the following series :—

$$ber.\ q=1-\frac{q^4}{2^2.\ 4^2}+\frac{q^8}{2^2.\ 4^2.\ 6^2.\ 8^2}-\text{etc.}$$

$$bei.\ q-\frac{q^2}{2^2}-\frac{q^6}{2^2.\ 4^2.\ 6^2}+\text{etc.}$$

The symbol q denotes $\pi d\sqrt{\frac{2n}{\rho}}$

where d is the diameter of the wire, ρ the resistivity of the material, and n the frequency of the oscillations.

In Dr. Russell's discussion of the same problem he denotes the function *ber.* q *bei.'* $q-$ *bei.* q *ber.'* q by W (q), and the function $(ber.'\ q)^2+(bei.'\ q)^2$ by Y (q). So that the ratio of the alternating to the steady current resistance in this notation is given by the formula—

$$\frac{R'}{R}=\frac{q}{2}\cdot\frac{W(q)}{Y(q)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The numerical values of the ratio for various values of q were calculated for Lord Kelvin by Dr. Magnus Maclean, and are given in the table below.

VALUES OF $\frac{R'}{R}=\frac{q}{2}\frac{ber.\ q\ bei.'\ q-bei.\ q\ ber.'\ q}{(ber.'\ q)^2+(bei.'\ q)^2}$

q	$\frac{R'}{R}$	q	$\frac{R'}{R}$
0·0	1·0000	4·5	1·8628
0·5	1·0000	5·0	2·0430
1·0	1·0001	5·5	2·2190
1·5	1·0258	6·0	2·3937
2·0	1·0805	8·0	3·0956
2·5	1·1747	10·0	3·7940
3·0	1·3180	15·0	5·5732
3·5	1·4920	20·0	7·3250
4·0	1·6778		

To use this table we proceed as follows :—

Consider a copper wire having a diameter, say, of 0·3 cm. and therefore a circumference $c=\pi d$ of 1·24 cm. Then for copper $\rho=1700$ (nearly), and if we take $n=10^5$ we have—

$$q=\pi d\sqrt{\frac{2n}{\rho}}=1\cdot24\sqrt{\frac{2\times10^5}{1700}}=13\cdot5\ \text{(nearly)}.$$

By interpolation we find that, corresponding to this value of q, we have $\frac{R'}{R}=5\cdot04$ (nearly), or the high frequency resistance is more than 5 times the steady resistance.

Lord Rayleigh also extended a formula, first given by Maxwell, for calculating

[5] See *Journal of the Institution of Electrical Engineers* of London, 1889, vol. 18, p. 35, Lord Kelvin, "Presidential Address on Ether, Electricity, and Ponderable Matter."

the resistance of nearly straight conductors for alternating currents of low and high frequency when the variation follows a simple sine law.[6]

Let R be the resistance to steady currents of any straight conductor of circular cross-section, let l be its length, and μ the magnetic permeability of the material of the wire. Lord Rayleigh showed that the resistance R′ to alternating currents in terms of R and the constants can be expressed by the series—

$$R' = R\left(1 + \frac{1}{12} \cdot \frac{p^2l^2\mu^2}{R^2} - \frac{1}{180} \cdot \frac{p^4l^4\mu^4}{R^4} + \text{etc.}\right)$$

where $p = 2\pi$ times the frequency n.

When dealing with non-magnetic material such as copper, for which $\mu = 1$, we have—

$$R' = R\left(1 + \frac{1}{12} \cdot \frac{p^2l^2}{R^2} - \frac{1}{180} \cdot \frac{p^4l^4}{R^4} + \text{etc.}\right)$$

If the conductor is a circular-sectioned uniform wire of length l and diameter d made of a material of resistivity ρ, then $R = \frac{4\rho l}{\pi d^2}$. Hence $\frac{l}{R} = \frac{\pi d^2}{4\rho}$ and $\frac{pl}{R} = \frac{n\pi^2 d^2}{2\rho}$.

Accordingly we then have—

$$R' = R\left(1 + \frac{n^2\pi^4 d^4}{48\rho^2} - \frac{n^4\pi^8 d^8}{2880\rho^4} + \frac{n^6\pi^{12} d^{12}}{58647\rho^6} - \text{etc.}\right) \quad . \quad . \quad . \quad (7)$$

Let the quantity $\frac{n\pi^2 d^2}{\rho}$ be denoted by h so that h is the product of the square of the circumference c of the round wire, the frequency and the specific conductivity or $h = \frac{nc^2}{\rho}$. The above expression (7) may then be written in the form—

$$R' = R\left(1 + \frac{h^2}{48} - \frac{h^4}{2880} + \frac{h^6}{58647} - \text{etc.}\right) \quad . \quad . \quad . \quad . \quad (8)$$

This formula for the ratio of the high frequency to the steady resistance is applicable *when h is less than unity.* The variable being, not n, d, or ρ taken alone, but the above quantity $h = \frac{nc^2}{\rho}$. It is clear that $2h$ is the same quantity as that which Lord Kelvin denotes by q^2.

The above series is, however, not suitable for the calculation of $\frac{R'}{R}$ when the value of h is greater than about 5, as it is too slowly convergent.

To meet this last case, Lord Rayleigh shows (*loc. cit.*) that the value for R′ for very high frequencies is given by the expression—

$$R' = \sqrt{\tfrac{1}{2} p l \mu R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If S is the cross-sectional area of the conductor, then—

$$R = \frac{\rho l}{S}$$

where ρ is the resistivity or the ordinary steady specific resistance.

If the conductor has a circular section of diameter d, $S = \frac{\pi d^2}{4}$, we have—

$$R' = R\sqrt{\frac{p\mu S}{2\rho}}$$

or

$$R' = R\sqrt{\frac{p\mu\pi d^2}{8\rho}}$$

Furthermore, if the material of which the conductor is made is non-magnetic, then $\mu = 1$, and if it is of copper, then $\rho = 1640$ at ordinary temperatures. Hence,

[6] See Lord Rayleigh on "The Self-induction and Resistance of Straight Conductors," *Philosophical Magazine*, ser. v., May 1886, vol. 21, p. 381; also J. A. Fleming, "The Alternate Current Transformer," vol. i. p. 294.

writing as above, h for $\frac{nc^2}{\rho}$ where $c=\pi d=$ the circumference of the wire, Lord Rayleigh's formula takes the form—

$$\frac{R'}{R}=\tfrac{1}{2}\sqrt{h}=\frac{1}{2\sqrt{2}}q=0\cdot3536q \quad . \quad . \quad . \quad . \quad . \quad .. \quad (10)$$

where q is the expression $\sqrt{\frac{2nc^2}{\rho}}$ as used in Lord Kelvin's formula.

If we plot the values of $\frac{R'}{R}$ given by Lord Kelvin's formula in the above table in terms of q, we find they lie nearly on a straight line from $q=3$ to $q=20$ and upwards, but for $q=0$ to $q=1$ the value of $\frac{R'}{R}$ is nearly unity. If we plot the values of $\frac{R'}{R}$ as given by Lord Rayleigh's formula (10) we find they also lie on a straight line, inclined at a very small angle to the Kelvin line between the limits $q=3$, $q=20$ and upwards. Within these limits the vertical distance between the two is nearly 0·3. Hence we have the following practical rule for the high frequency resistance of round wires. Calculate for the given wire the value of q or of h. If q lies between 3 and 20, or h between 5 and 200 and upwards, then the ratio of the high frequency to the steady resistance is *very nearly* given by the semi-empirical formula—

$$\frac{R'}{R}=\frac{\sqrt{h}}{2}+0\cdot3=0\cdot3536q+0\cdot3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If, however, q is less than 3 or h is small, we must employ the series given by Lord Rayleigh as in formula (8) above. Thus, for example, if we have a copper wire of circular section 1 cm. in circumference, or about $\frac{1}{8}$ of an inch in diameter, and consider a frequency as low as $n=1000$, then $h=\frac{1000}{1640}=0\cdot625$ (nearly).

Since this is less than unity, the formula (8) must be used, and we have—

$$\frac{R'}{R}=1+\frac{1}{123}-\frac{1}{18874}=1\cdot008$$

Hence the resistance to currents of this frequency is now 1 per cent. greater than its steady resistance. If, however, the frequency $n=10^6$, then we should have $h=\frac{10^6}{1640}$ or nearly 625. Since this is vastly greater than unity we must now employ the formula (10), and since $\frac{1}{2}\sqrt{h}=12\cdot5$ we have—

$$\frac{R'}{R}=12\cdot5$$

or the high frequency resistance for currents of the above frequency is 12·5 times the steady resistance.

If the copper wire was only 1 mm. in circumference, equal to $\frac{1}{80}$ inch in diameter, then for a frequency of 10^6 we should have $h=\frac{100}{16}=6\frac{1}{4}$ and $\sqrt{h}=2\cdot5$, and we should have $\frac{R'}{R}=1\cdot25$, or the high frequency resistance would be 25 per cent. in excess of the steady resistance. If the wire were still smaller, say 0·1 mm. in circumference, or about $\frac{1}{800}$ inch in diameter, then h would be only $\frac{1}{16}$, and the formula (10) is no longer applicable, but we must employ (8), which gives us—

$$\frac{R'}{R}=1+\frac{1}{48}\left(\frac{1}{16}\right)^2-\frac{1}{2880}\left(\frac{1}{16}\right)^4, \text{ etc.}$$

$$=1+\frac{1}{12{,}300}$$

Accordingly, for such small-sized wires there is no difference between the high

frequency resistance and the steady resistance. In practice we may consider that for a wire as small as No. 40 S.W.G. this equality exists.

It should be noted that all the above formulæ are only applicable to straight or slightly curved round-sectioned copper or other non-magnetic wires.

A very full discussion of the formulæ for the effective resistance and inductance of circular wires has been given by Dr. A. Russell.[7] He considers the case of a concentric main consisting of one tube within another which becomes that of a simple straight solid round wire when the inner tube is solid and the outer tube of infinite radius.

Dr. A. Russell has given a formula for the ratio of $\frac{R'}{R}$ for the solid round wires exact for values of q greater than about 6, as follows :—

$$\frac{R'}{R}=\frac{q}{2}\left\{\frac{1}{\sqrt{2}}+\frac{1}{2q}+\frac{3}{8\sqrt{2}q^2}-\frac{1}{2\sqrt{2}q^4}\right\} \qquad (12)$$

$$=0{\cdot}3536q+0{\cdot}25+\frac{0{\cdot}265}{q^2}-\frac{0{\cdot}35}{q^4}. \qquad (13)$$

where q has the same meaning as in Kelvin's formula. When q has a value of 100 or upwards Dr. Russell's formula reduces to $0{\cdot}3536q+0{\cdot}25$, which is nearly identical with the formula (11).

Dr. Russell's formula for the resistance of a straight circular-sectioned non-magnetic wire can easily be put into the form—

$$\frac{R'}{R}=\frac{\sqrt{h}}{2}+\frac{1}{4}+\frac{3}{32\sqrt{h}}-\frac{1}{16h\sqrt{h}} \qquad (14)$$

where $2h=q^2$, and if we compare this with the formula (10) given by Lord Rayleigh, viz. $\frac{R'}{R}=\frac{\sqrt{h}}{2}$, it will be seen that for large values of h the first two terms of the Russell formula are sufficient to give a close value for $\frac{R'}{R}$, and also that the formula (11) given by the author is then almost equivalent. In general, for values of h above 10 we may, for most practical cases, employ the formula—

$$\frac{R'}{R}=\frac{\sqrt{h}}{2}+0{\cdot}25 \qquad (15)$$

The above formula is of very great use in calculating quickly the high frequency resistance R′ of round solid copper wires of diameter d cms., provided that the quantity $\frac{n\pi^2d^2}{1600}$ has a value much greater than unity, and also that the wire in question is sufficiently far from all other parts of its circuit, so that there is no disturbance of the uniform peripheral distribution of the current in it. Thus the resistance of a No. 16 S.W.G. copper wire 0·160 cm. in diameter for oscillations of frequency 10^6 is 7·4 times greater than its ordinary or steady current resistance.

Again, consider the case of a copper rod or circular section and 1 cm. in diameter. Let the frequency of the oscillations be 10^6. We have then in the above formula (15) to put $d=1$ and $n=10^6$. Then $\sqrt{n}=10^3$ and $R'=40R$ nearly. Hence, for this frequency a thick copper rod may have an effective resistance 40 times its steady resistance. This rule, however, cannot be applied to stranded conductors, as then the current is more or less independently started in each separate strand, and the alternating resistance will be less than that given by the above formulæ for solid conductors.

We see, therefore, that in the case of thick solid wires a serious error may be committed if we neglect the difference between the high frequency alternating current resistance and the steady resistance in calculations connected with electric oscillations.

There is, moreover, a small additional increase if the high frequency currents

[7] See Dr. A. Russell, "The Effective Resistance and Inductance of a Concentric Main and Methods of Computing the *Ber.* and *Bei.* and Allied Functions," *Phil. Mag.*, April 1909, p. 524.

consist of trains of *damped oscillations.* This case has been considered by Dr. E. H. Barton.[8] He takes the damped oscillation to be represented by an expression of the form—

$$i = A\epsilon^{-kpt}\cos pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where ϵ is the base of Napierian logarithms and k the damping factor.

In the case of simple harmonic motion, all the quantities vary as ϵ^{jpt} where $j=\sqrt{-1}$, but for damped oscillatory motion they vary as $\epsilon^{(j-k)pt}$. If we take R″ to represent the resistance of the conductor to damped oscillations, and R as before for the steady resistance, then Dr. Barton's formula for R″ in terms of R is—

$$R'' = R\left\{1 + \frac{1+k^2}{12}\cdot\frac{p^2l^2\mu^2}{R^2} + \frac{k(1-k^2)}{24}\cdot\frac{p^3l^3\mu^3}{R^3} - \frac{1-2k^2-3k^4}{180}\cdot\frac{p^4l^4\mu^4}{R^4}\right\}$$

If we put $k=0$ in the above expression, it becomes Lord Rayleigh's formula.

If we make p very large in the above expression, and write s for $\sqrt{1+k^2}$, and $\cos\theta$ for k, or $\cos\frac{\theta}{2}=\sqrt{\frac{\cos\theta+1}{2}}=\sqrt{\frac{s+k}{2s}}$, we get

$$R'' = R\left(\sqrt{\frac{l\mu ps^2}{R}}\right)\cos\frac{\theta}{2}$$

but since $R=\frac{4\rho l}{\pi d^2}$, we have—

$$R'' = R\sqrt{\frac{p\mu\pi d^2}{8\rho}}(s\sqrt{s+k}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The expression (17) only differs from that given by Lord Rayleigh by the factor $s\sqrt{s+k}$. When $k=0$, $s=1$, and the factor becomes unity.

The product kp is the same as that which in a previous section (see § 2, Chap. I.) we have called a, and is equal to $\frac{R'}{2L}$ for the circuit considered. Hence $k=\frac{R}{2Lp}$, or k is half the ratio of resistance to reactance. Also since $n\delta=a$ (see equation 3, Chap. I. § 1), where δ is the logarithmic decrement, we have $\delta=2\pi k$.

Since in all cases likely to arise in practice $\frac{\delta}{2\pi}$ is a small quantity of the order of 1 per cent., the correcting factor $s\sqrt{s+k}$ is also nearly unity.

The ratio of R″ to R′ is therefore generally near unity, although the ratio of R′ to R may be very large.

It is only when the semi-period decrement $\delta/2$ reaches a value of about 0·2 that the difference between R″ and R′ becomes important.

When we are dealing with magnetic metals this concentration of an alternating current at the surface of a conductor, or so-called *skin, effect* is very marked, even for quite low frequencies.

In the case of an infinite flat plate of thickness $2h$, traversed by an alternating current of frequency n, in a direction parallel to the plane of the plate, Sir J. J. Thomson has shown that the current amplitude decreases from the surface inwards in geometrical progression as the distance from the surface increases in arithmetic progression. Also if x be the distance of any point from the surface, the rate at which the maximum values of the alternating current at successive points, taken inwards from the surface, decay is determined by a decay factor $2\mu\sqrt{\frac{\mu n}{\rho}}$, where μ is the magnetic permeability, ρ the electric resistivity, and n is the frequency.

[8] See Dr. E. H. Barton, "On the Equivalent Resistance and Inductance of a Wire to an Oscillatory Discharge," *Proc. Phys. Soc. Lond.*, 1899, vol. 16, p. 409, or *Phil. Mag.*, 1899, ser. v. vol. 47, p. 433.

If we consider a plate iron for which $\rho = 10^4$ in C.G.S. units, μ = say 1000, and adopt a frequency of $100 = n$, then the decay factor is nearly 20. Hence, at a depth of 0·5 mm. from the surface, the maximum value of the current during its period will be only $\frac{1}{\epsilon} = 0{\cdot}368$ of its value at the surface, and for other depths as follows :—

Distance in millimetres of point from surface of plate.	Maximum value of the alternating current at that point expressed as percentage of the maximum value at the surface.
At surface	100·0
0·5 below	36·8
1·0 ,,	13·5
2·0 ,,	1·8
3·0 ,,	0·25

The corresponding percentage values for copper would be about 13 times greater.

Hence, in the case of iron, when employing alternating currents of a frequency of 100, the current practically penetrates only about 2 mm. into the surface. In the case of copper the practical penetration would be about 26 mm.

If, however, instead of employing alternating currents of a frequency of 100, we are dealing with electrical oscillations having a frequency expressed in millions, then the "skin" or used portion of the metallic circuit may be less than $\frac{1}{100}$ mm. in thickness.

Accordingly the specific resistance of the material of which the discharge circuit is made becomes of little consequence; the whole effects are determined by the frequency and inductance, which latter in turn depends upon the geometric form of the circuit. An experimental proof of the above statement can be obtained by the use of the author's cymometer (see Chap. VI.).

Sir J. J. Thomson has calculated (see "Recent Researches in Electricity and Magnetism," p. 281) that for electrical oscillations having a frequency of 10^6 the thickness of the conducting skin for soft iron is about $\frac{1}{250}$ mm., and for copper about $\frac{1}{15}$ mm. In these cases there is a concentration of the current at the surface, and the outer layers of the metal are for a short time carrying current at a current density which would suffice to melt the conductor if that current density was the same at all parts of the section.

It is important to notice, however, that the formulæ which have just been given for the resistance of a wire to high frequency currents only apply to wires which are straight or bent into curves with radius of curvature large compared with the diameter of the wire. The expressions given by Lord Rayleigh (see equations (8), (9), and (10)) do not apply to spirals which are formed of turns of wire close together or wound on mandrils of small diameter compared with the diameter of the wire. The reason for this is as follows: When a wire, say, of circular cross-section, conveys a current there is a magnetic field not only outside the wire but within the wire. If the field is alternating, we must think of this interior field as composed of self-closed lines of magnetic flux which are expanding and contracting rapidly. Thus they pulsate in and out of the wire, and create electromotive forces in the wire parallel to the axis. If the closed interior lines of magnetic flux are symmetrically placed with regard to the central axis of the wire, these longitudinal electromotive forces balance each other. If, however, the interior field is not symmetrical, then there is a tendency to produce eddy currents in the wire, due to unbalanced interior electromotive forces, and these dissipate energy. Hence the energy dissipation for a given field, that is, for a given current, is greater in the case of an unsymmetrical interior pulsating magnetic field, and the wire accordingly has a greater equivalent or effective resistance. Not only, therefore, has the wire a greater effective resistance due to the concentration of the current, that is, the field, at the surface of the wire, but it may have an increase on this increase in resistance if that current distribution is unsymmetrical with regard to the axis of symmetry of the wire. This can be prevented by constructing the wire of fine insulated wires stranded together in a certain manner.

To secure this result the stranding must be so carried out that each wire occupies in turn the same relative position in the conductor. It is not sufficient merely to twist fine insulated wires together. In this case, the exterior field of each wire will induce eddy currents in the neighbouring wires, which dissipates energy, and is equivalent to an increase in resistance. On this point see § 16 at end of this chapter.

The cases in which we can predetermine by calculation the high frequency resistance of conductors used in radiotelegraphy are comparatively few.

Hence we are obliged to resort to actual measurements to obtain the high frequency resistance of samples of stranded cables, or metal strips, used as the circuits of radiotelegraphic apparatus.

The following apparatus was therefore devised by the author for experimentally measuring the ratio of high frequency resistance R′ to steady current resistance R. Two glass tubes, each about 75 cms. long and 3 cms. in diameter, have an expansion at the upper end and a curved bend and expansion at the lower end (see Fig. 3). The ends are provided with I.R. corks perforated by thick rods of

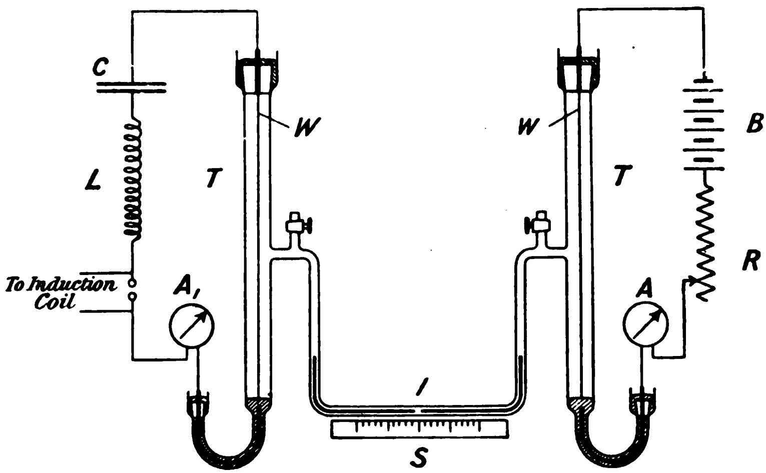


FIG. 3.—Apparatus for Determining the High Frequency Resistance of Wires. (Fleming.)

copper, and the lower bends are filled with mercury. The upper corks are also made air-tight with mercury or oil.

These tubes have side tubes blown on, by means of which they are connected by an inverted syphon of barometer tube which contains coloured water and an air-bubble in the centre to detect its displacement. This arrangement constitutes a differential air thermometer with two tubular bulbs. In these tubes are placed two identical wires, which are fastened to the copper rods passing through the corks at the upper ends, and dip into the mercury in the bends at the lower ends. It is convenient to keep these wires in a truly axial position by one or two discs of thin mica. Suppose, then, that we pass the same electric currents through these wires in series. Both are heated and heat the air in the tubes, but if everything is symmetrical and the tubes are equally heated the bubble is not displaced. To attain this balance, however, the whole apparatus has to be placed in an enclosure, and the position of the bubble observed through a window.

If, then, we pass electric oscillations through one wire, and a steady current through the other, it is possible to adjust the steady current until the heat produced by it in one wire balances the heat produced by the oscillations in the other wire. To do this the currents have to be passed for some time, so that the

thermal condition may become constant. Assuming, however, that this is the case, we have the following state of affairs. In one wire which has a resistance R, we have a steady current A producing heat at a rate of A^2R. In the other wire of high frequency resistance R′ we have oscillations of mean-square value A_1^2, producing heat at a rate A_1^2R'. Since the sources of loss are the same in both cases when the final steady thermal state is reached, we have $A^2R = A_1^2R'$ or $\frac{R'}{R} = \frac{A^2}{A_1^2}$. The measurements of the mean-square value of the high frequency current must be made with a thermoelectric ammeter, such as that devised by the author (see Fig. 32 of this chapter), which is correct for high frequency measurement. There are certain points which must receive attention if accuracy in the measurement is to be attained.

First, the spark gap must be kept as short as possible, not more than 1 mm., and preferably less, in length, in order to reduce the decrement of the oscillations as much as possible. The resistance of the wire for damped oscillations is greater than its resistance to undamped oscillations of the same frequency in the ratio of $s\sqrt{s+k}$ to 1, see formula (17), where $s = \sqrt{1+k^2}$ and $k = \frac{\delta}{2\pi}$, δ being the decrement. Accordingly if δ is kept down as low as 0·08, the correcting factor will be equal to 1·007, or the resistance to damped oscillations will be only 0·7 per cent. greater than the resistance to undamped. If, however, the spark gap is large, then this correction will be large also. Again, it is necessary to employ an air blast impinging on the spark gap to keep the oscillatory current constant, and without this no good results can be obtained.[9] In general it will not be necessary to make any correction for the heat produced by the low frequency charging current of the condenser, which also passes through the wire, as the heat so produced is negligible in comparison with that produced by the oscillations.

It is necessary, however, to eliminate certain differences due to want of symmetry in the two wires by passing the oscillations first through one wire and then through the other, keeping the oscillatory current the same in the two cases, but taking the mean of the continuous currents required to effect a thermal balance in the two experiments. The results of a number of such measurements on copper wires, No. 14, No. 16, No. 36 in size, are collected in the Table below, which were made in the author's laboratory, and the values of their high frequency resistance, calculated by the Russell or Kelvin formulæ, are given for comparison to show how well the measured H.F. resistance agrees with the value calculated to these formulæ.

The same apparatus can be employed to examine the high frequency resistance of spiral wires, and the results of one such measurement are given for a spiral of No. 16 S.W.G. copper wire of 2·57 turns per cm. in the table. The result is to confirm the theoretical prediction that the high frequency resistance of a spiral wire is greater than that of the same wire stretched out straight.

One of the first to investigate this matter experimentally was F. Dolezalek,[10] who measured inductances by the bridge method of certain spirals, and found that at various frequencies between 591 and 2286, the resistance was increased and the inductance diminished compared with their steady current values. He suggested that if the coils were made of insulated wires 0·1 mm. in diameter = No. 40 S.W.G., bunched or stranded together, the effect would be annulled. The increase noticed by Dolezalek was not the difference between the value of the resistance for steady currents and that for alternating currents, but the increase over the latter due to the coiling in a spiral.

The problem was then treated mathematically by M. Wien[11] and by A

[9] See J. A. Fleming and H. W. Richardson, "On the Effect of an Air Blast upon the Spark Discharge of an Induction Coil," *Phil. Mag.*, May 1909, or *Proc. Phys. Soc. Lond.*, vol. 21, p. 1909.

[10] See F. Dolezalek, *Annalen der Physik*, vol. 12, p. 1142, 1903, "Ueber Präzisionsnormale der Selbstinduktion"; also *Science Abstracts*, 1904, B, *abs.* 488.

[11] M. Wien, *Annalen der Physik*, vol. 14, p. 1, 1904.

MEASUREMENTS OF HIGH FREQUENCY RESISTANCE OF VARIOUS WIRES AND CABLES.

Conductor tested.	Value of $\sqrt{h}=\pi d\sqrt{\frac{n}{\rho}}$	Frequency n	R.M.S. value in amperes of high frequency current $=A'$	Mean value of continuous currents required to balance A' thermally $=A$	Ratio of the currents $\frac{A}{A'}$	Measured ratio of the resistances $\frac{R}{R}=\left(\frac{A}{A'}\right)^2$	Calculated value of the ratio of the resistances $\frac{R'}{R}$	Remarks.
Bare copper wire, No. 14 S.W.G. Diam. $d=0\cdot203$ cm.	11·33 14·68	535,000 900,000	6·2 6·2	15·0 17·08	2·42 2·75	5·85 7·59	5·92 7·60	Calculated by the first three terms of the Russell formula, taking $\rho=1700$.
Bare copper wire, No. 16 S.W.G. Diam. $d=0\cdot1626$ cm.	8·54 8·59	475,000 485,000	7·09 7·89	15·15 16·82	2·14 2·14	4·57 4·57	4·53 4·56	Ditto.
Bare copper wire, No. 36 S.W.G. Diam. $d=0\cdot0193$ cm.	1·05 1·38	510,000 880,000	1·92 1·92	1·93 1·98	1·005 1·03	1·01 1·06	1·02 1·07	Calculated value obtained by Lord Kelvin's formula.
Bare copper 2/16 cable or antenna wire	—	470,000 785,000	8·1 6·56	19·26 16·73	2·38 2·55	5·66 6·50	—	—
Bare copper cable 7/22 stranded	—	465,000 774,000	8·08 6·25	16·58 15·41	2·05 2·47	4·20 6·10	—	—
Silk-covered, 19/36 Each strand insulated	—	496,000 496,000	8·46 11·00	9·93 11·70	1·18 1·06	1·39 1·12	—	—
Bare copper strip, 1·82 cms. wide 0·0147 cm. thick	—	470,000 586,000	10·60 7·57	15·92 11·28	1·5 1·5	2·25 2·25	—	—
Spiral of bare No. 16 copper wire, 142·5 turns, length $=55\cdot4$ cms. turns per cm. $=2\cdot57$	—	470,000 475,000	6·16 5·90	14·61 14·2	2·37 2·4	5·62 5·76	—	Ratio of resistance of spiral to that of same wire stretched out straight for $n=450{,}000$ is 1·25.
Bare German silver wire No. 17 S.W.G. Diam. $d=0\cdot1422$ cm.	1·86 1·89	463,000 475,000	7·96 6·87	9·20 7·76	1·16 1·5	1·32 1·33	1·20 1·21	Calculated value obtained by Lord Kelvin's formula, taking $\rho=26{,}600$.

Sommerfeld,[12] who discussed Dolezalek's results and gave formulæ for calculating the increase in resistance of a solenoid due to the spiralization.

If we denote by R″ the resistance of a spiral of wire the steady current resistance of which is R, and if the resistance to alternating currents of the same frequency if the wire stretched out straight is denoted by R′, then the formulæ of Kelvin and Rayleigh, as above given, are the values of $\frac{R'}{R}$. Sommerfeld gave a formula for $\frac{R''}{R}$ when the frequency is very high, which, in the notation used above, is equivalent to—

$$\frac{R''}{R}=2\sqrt{\pi}\tfrac{1}{2}\sqrt{h}=2\sqrt{\pi}\frac{R'}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where h has the same meaning as in Rayleigh's formula (10). The constant $2\sqrt{\pi}=3{\cdot}54$, and hence this formula makes the ratio of the resistance of the spiral to that of the same wire stretched out straight always 3·54 for very high frequencies and independent of the diameter of the wire or number of turns per unit of length. This does not agree with the experimental results of T. P. Black.[13] He employed two equal wires, one coiled in a spiral, and the other stretched out straight enclosed in tubes like thermometer bulbs. He measured the relative rise in temperature and heat produced in these two wires when the same high frequency current, with frequency varying between 10^6 and 5×10^6, was sent through both wires.

He found that for the spirals used the ratio $\frac{R''}{R'}$ had values between 1·20 and 1·89, or not much more than half that predicted by the formula of Sommerfeld.

A theoretical discussion and experimental examination of this question of spiral resistance has been made by L. Cohen.[14] To enable the matter to be treated analytically the spiral was assumed to be made of square-sectioned wire, wound in one layer of closely compacted turns. The solenoid is assumed to be of considerable length so that the magnetic field within it is constant and equal to 4π times the current-turns per unit of length. He gave a general formula for the increase in resistance ΔR of one turn of the solenoid over and above the steady current resistance R which is due to frequency and spiralization as follows :—

$$\Delta R=128N^2p^2\pi d\, r\sigma\sum_{x=1}^{x=\infty}\left(\frac{1}{a_x x^4(a_x^2+\beta_x^2)}\right) \quad . \quad . \quad . \quad . \quad (19)$$

where N = number of turns of solenoid per unit of length, $p=2\pi$ times the frequency n, d = diameter of the wire, r = interior radius of solenoid, σ = specific conductivity, and $x=1, 3, 5, 7$, etc.

$$\text{and} \qquad \left.\begin{aligned} a^2&=\frac{m^2+\sqrt{m^4+16\pi^2\sigma^2p^2}}{2} \\ \beta^2&=\frac{-m^2+\sqrt{m^4+16\pi^2\sigma^2p^2}}{2}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$\text{where} \qquad m=\frac{x\pi}{d}$$

When the frequency is very high (say 10^6), the value of $16\pi^2\sigma^2p^2$ is much greater than that of m^4, and we may then take $a^2=\beta^2=2\pi\sigma p$, and the series part of the formula (19) for ΔR will reduce to—

$$\frac{1}{4\pi\sigma p\sqrt{2\pi\sigma p}}\left(1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\text{etc.}\right)$$

[12] A. Sommerfeld, *Annalen der Physik*, vol. 15, p. 193, 1905, "Ueber das Wechselfeld und den Wechselstromwiderstand von Spulen und Rollen."

[13] T. P. Black, *Annalen der Physik*, vol. 19, p. 157, 1906, "Ueber den Widerstand von Spulen für schnelle Elektrische Schwingungen."

[14] L. Cohen, *Bulletin of the Bureau of Standards*, Washington, U.S.A., vol. 4, No. 1, "The Influence of Frequency on the Resistance and Inductance of Solenoidal Coils."

The sum of the series in the brackets is very nearly equal to 1·2, and hence to a close approximation we may write—

$$\Delta R = 38{\cdot}4 N^2 d\, r \sqrt{n\rho} \quad . \quad . \quad . \quad . \quad (21)$$

where ρ = specific resistance, d = diameter of the wire, n = frequency, r = interior radius of the solenoid, and N = number of turns per unit of length.

Since the length of 1 turn of the wire is $2\pi r$, and the steady resistance per unit of length is $\frac{4\rho}{\pi d^2}$, we have for the total high frequency resistance per unit of length of the wire expression—

$$\frac{4\rho}{\pi d^2} + \frac{38{\cdot}4 N^2 d\, r \sqrt{n\rho}}{2\pi r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and hence we have a final formula—

$$\frac{R''}{R} = 1 + 4{\cdot}8 N^2 d^3 \sqrt{\frac{n}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

which gives us the high frequency resistance of such a single layer spiral. Suppose, for example, that a spiral is made of copper wire 2 mm. in diameter, and wound in 4 turns per cm. on a mandril 5 cms. in diameter, and subjected to oscillations of a frequency of 10^6, then we have $\rho = 1600$, $n = 10^6$, $d = 0{\cdot}2$, N = 4, $r = 5$, and—

$$\frac{R''}{R} = 1 + 15{\cdot}36 = 16{\cdot}36$$

Accordingly for this spiral and this frequency the high frequency resistance is 16·36 times the steady resistance. If the spiral were stretched out straight, then, by Dr. Russell's formula, the ratio of resistance to steady resistance would be—

$$\frac{R'}{R} = \tfrac{1}{2}\sqrt{h} + 0{\cdot}25 = 8{\cdot}1$$

Hence we have $$\frac{R''}{R'} = \frac{16{\cdot}36}{8{\cdot}1} = 2{\cdot}02 \text{ (nearly)}$$

which is not far from the ratio found experimentally by T. P. Black, and at any rate much nearer than the ratio given by the formula of Sommerfeld.

We shall, therefore, not be far wrong in saying that when a wire of anything like No. 16 or No. 18 S.W.G. is coiled into a spiral of one single layer of turns nearly in contact, the actual high frequency resistance of any considerable length of this spiral for a frequency of about 10^6 may be about double that of the same wire stretched out straight, and that this increase is due to the displacement of the interior magnetic field in the mass of the wire. It can be prevented by constructing the wire of many strands of very fine insulated wire bunched together. When the frequency is very high Cohen shows that the ratio of the resistance of the spiral to that of the same wire stretched out straight is given by the formula—

$$\frac{R''}{R'} = \frac{8N^2 d^2}{\pi}\left(1 + \frac{1}{9} + \frac{1}{25} + \text{etc.}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$= \frac{9{\cdot}6 N^2 d^2}{\pi} \text{ (nearly)}$$

$$= 3{\cdot}06 N^2 d^2 \text{ (nearly)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Hence, if the turns are closely adjacent, Nd = 1, and $\frac{R''}{R'}$ may be as great as 3.

It is to be noticed, however, that Cohen's formula has been deduced on the assumption that there is only *one* layer of wire in closely adjacent turns, and hence does not apply if Nd is very different from unity.

2. Inductance of Conductors for Various Frequencies.—The inductance of an electric conductor may be defined to be that quality of it in virtue of which energy in a magnetic form is stored up in connection with the circuit when a current is flowing in it. Thus, if at any instant there is a current i in a circuit, the magnetic energy associated with it is represented by $\frac{1}{2}Li^2$, where L is the inductance of the conductor. It will be seen, therefore, that L and i enter into

the expression for the magnetic energy of a current, just as mass, M, and velocity, v, enter into the expression $\frac{1}{2}Mv^2$ for the kinetic energy of a moving body.

It follows from this that the rate at which $\frac{1}{2}Li^2$ is changing with time is the rate at which magnetic energy is being stored up in the circuit, and must therefore be equal to the product of the current and impressed electromotive force, *less* the rate at which energy is being dissipated as heat.

Now $$\frac{d}{dt}(\tfrac{1}{2}Li^2) = \frac{d}{di}(\tfrac{1}{2}Li^2)\frac{di}{dt} = Li\frac{di}{dt}$$

Hence if E is the instantaneous impressed electromotive force, we must have—

$$Ei - Ri^2 = Li\frac{di}{dt}$$

or $$L\frac{di}{dt} + Ri = E$$

or $$\frac{d}{dt}(Li) + Ri = E$$

Therefore, by Faraday's law of induction, the quantity Li must represent the total flux due to the current itself, which is linked with the circuit. Accordingly we arrive at a second definition of inductance, L, which is that the inductance of a circuit is the total self-linked magnetic flux when unit current flows through the circuit.

The practical unit of inductance is called the *henry*, and one henry is defined to be the inductance of a circuit which has linked with itself a total magnetic flux of one *weber* (10^8 lines or 100,000 *kilolines*) when a current of one ampere flows through it.

The dimensions of an inductance on the electro-magnetic system of measurement is a *length*. Hence the absolute unit of inductance in the electro-magnetic system of measurement and in the centimetre, gramme, second system (C.G.S.) is *one centimetre. One henry* is equal to 10^9 cms., and hence *one millihenry* is 10^6 cms., and *one microhenry* is 1000 cms.

We shall chiefly be concerned in this treatise with small inductances which it is convenient to measure in absolute units, viz. in *centimetres*, or else in *millihenrys*, or in *microhenrys*.

Another way of regarding the subject is as follows: We may think of the current in a conductor as made up of a large number of filamentary currents flowing in the same direction. These similarly directed currents attract one another. Hence to separate them all to an infinite distance, and, so to speak, take the main current to pieces, requires an expenditure of energy.

The energy which must be expended to do this is the equivalent of the kinetic energy possessed by the whole original current, and this is therefore called the potential of the current on itself. If we consider two elements of length of the circuit, viz. ds and ds', which make an angle θ with each other, and are situated at a distance r, then it can be shown that the expression $\frac{ds\,ds'}{r}\cos\theta$ represents the potential energy of these elements when each is traversed by unit current. Hence to obtain the whole potential energy of the circuit with respect to itself, which is the same thing as the inductance, we have to calculate the value of the double integral $L = \int\int \frac{ds\,ds'}{r}\cos\theta$ for every pair of elements. The proof of this formula (due to Neumann) is given in every standard treatise on electricity and magnetism; *e.g.*, Maxwell's "Electricity and Magnetism," 2nd ed. vol. ii. § 423 and § 524; also Deschanel's "Nat. Phil.," part iii., rewritten by Everett, p. 194, § 263.

In its application we have to take into account the surface or skin distribution of high frequency currents already explained. Hence when we are dealing with steady or low frequency alternating currents, the current may be considered to be uniformly distributed over all parts of the cross-section of the conductor, and the inductance calculated on this assumption is called the ordinary or low frequency inductance. If, however, we are concerned with high frequency currents, then the

current is wholly concentrated on the surface or on the skin, and the inductance calculated on this distribution is called the high frequency inductance. This last is always less than the low frequency inductance.

Lord Rayleigh has given a formula for the relation between the two inductances for certain forms of nearly straight conductors.[15]

If L is the low frequency or steady current inductance of a conductor of length l, and L′ is the inductance for alternating currents of simple sine form and frequency, $n=\frac{p}{2\pi}$, then Lord Rayleigh shows (*loc. cit.*) that—

$$L'=l\left\{A+\mu\left(\frac{1}{2}-\frac{1}{48}\cdot\frac{p^2l^2\mu^2}{R^2}+\frac{13}{8640}\cdot\frac{p^4l^4\mu^4}{R^4},\text{ etc.}\right)\right\} \quad . \quad . \quad . \quad (26)$$

where μ is the permeability of the material and R is the steady current resistance.

In the above formula A is a constant depending upon the position of the return conductor.

Lord Rayleigh also shows that when the frequency is *very* high, the above expression takes the form—

$$L'=l\left(A+\sqrt{\frac{\mu R}{2pl}}\right)$$

or

$$L'=lA+\frac{R'}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

since by (9) we have $R'=\sqrt{\frac{1}{2}pl\mu R}$.

When the frequency is infinite, the value of L′ tends to a limit, lA. Hence the constant A is the inductance of the conductor per unit of length for infinitely great frequency.

On the other hand, if we put $n=0$ or $p=0$ in the expression (26) above, we have the value of the inductance (L) for steady or non-periodic currents. Hence—

$$L=l(A+\tfrac{1}{2}\mu)$$

or

$$A=\left(\frac{L}{l}-\frac{\mu}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Finally, if we write L_∞ for the inductance at infinite frequency and L′ for the inductance at a high frequency, n, we have—

$$L'=L_\infty+\frac{R'}{2\pi n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

If ρ is the resistivity of the material, then $R=\frac{\rho l}{s}$, where s is the cross-section of the conductor. Hence from (27) and (28) we have—

$$L'=l\left(\frac{L}{l}-\frac{\mu}{2}+\sqrt{\frac{\mu\rho}{2ps}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Furthermore, if the material is non-magnetic, $\mu=1$, and then—

$$L'=L-l\left(\frac{1}{2}-\sqrt{\frac{\rho}{4\pi ns}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

If the section of the conductor is circular and of diameter d, then $s=\frac{\pi d^2}{4}$, and—

$$L'=L-\frac{l}{2}+\frac{l}{\pi d}\sqrt{\frac{\rho}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The formulæ (30), (31), and (32) afford means for calculating the inductance L′ of a nearly straight circular-sectioned wire of length l, diameter d, and resistivity ρ, for high frequency currents of frequency n, when we know the low frequency inductance L.

[15] See Lord Rayleigh, "On the Self-induction and Resistance of Straight Conductor," *Phil. Mag.*, May 1886, ser. v. vol. 21, p. 381.

The formula (29) gives L' in terms of the inductance L_∞ for an infinite frequency.

It is sometimes convenient to calculate L' from (29) and sometimes from (32).

If we use copper wire, $\rho = 1640$ or $\sqrt{\rho}$ is nearly 40, and then—

$$L' = L - l\left(\frac{1}{2} - \frac{40}{\pi d\sqrt{n}}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Suppose, as before, that $d = 1$ and $n = 10^6$, then the quantity in the bracket is equal to [illegible], or nearly 0·5. Hence, in this case, if we deduct half the length of the wire in centimetres from the value of the steady current inductance L, we have the high frequency inductance L'.

For such circuits as are usually employed in high frequency work the difference between the two inductances is only at most a few per cent., whereas the ratio between the two resistances may be very large.

Dr. Barton has also considered the question of the inductance of a conductor under damped high frequency electric oscillations.[16] Taking into account the decay factor, he shows that the inductance L'' for simple periodic but decadent oscillations of frequency, n, is to the steady inductance L in the ratio given by—

$$L'' = l\left\{A + \mu\left(\frac{1}{2} + \frac{k}{6}\cdot\frac{pl\mu}{R} - \frac{1-3k^2}{48}\cdot\frac{p^2l^2\mu^2}{R^2} - \frac{k(1-k^2)}{45}\cdot\frac{p^3l^3\mu^3}{R^3}, \text{ etc.}\right)\right\} \quad . \quad (34)$$

where k is the damping factor, and A, as before, has the value $\left(\frac{L}{l} - \frac{\mu}{2}\right)$. If we put $k = 0$ we have Lord Rayleigh's expression (26).

If p is very large the above expression reduces to—

$$L'' = l\left(A + \sqrt{\frac{R\mu s}{lp}}\cos\frac{\theta}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where $\cos\theta = k$ and $s = \sqrt{1+k^2}$ as before.

If $k = 0$, the above expression (35) becomes identified with (27), that given by Lord Rayleigh; since, then, $\cos\frac{\theta}{2} = \sqrt{\frac{s+k}{2s}}$. As already shown, $A = \frac{L}{l} - \frac{\mu}{2}$ and $R = \frac{\rho l}{S}$. Hence we can write the above expression (35) in the case of non-magnetic materials in the form—

$$L'' = L - l\left\{\frac{1}{2} - \sqrt{\frac{\rho}{4\pi n S}(s+k)}\right\}. \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If the wire is circular-sectioned, $S = \frac{\pi d^2}{4}$, and we have—

$$L'' = L - \frac{2}{l} + \frac{l}{\pi d}\sqrt{\frac{\rho}{n}(s+k)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

This last equation also becomes identical with Lord Rayleigh's expression (32) when $k = 0$.

Taking the two formulæ for the high frequency resistance R' and the high frequency inductance L', viz.—

$$R' = \sqrt{\tfrac{1}{2}pl\mu R}, \text{ and } L' = l\left(\frac{L}{l} - \frac{\mu}{2} + \sqrt{\frac{\mu R}{2pl}}\right)$$

we eliminate μ and l and arrive at—

$$\frac{Lp - L'p}{R' - R} = \frac{R'}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Hence, as we increase the frequency from zero to a very high value, the decrement of the reactance is to the increment of resistance in the ratio of the high frequency to the steady or zero frequency resistance. Again, if we take the same

[16] See Dr. E. H. Barton, "On the Equivalent Resistance and Inductance of a Wire to an Oscillatory Discharge," *Proc. Phys. Soc. Lond.*, vol. xvi. p. 409, or *Phil. Mag.*, 1899, ser. v. vol. 47, p. 436.

two formulæ for the high frequency resistance R′ and inductance L′, we can deduce an expression for the high frequency impedance $\sqrt{R'^2+p^2L'^2}$ which is often required.

For $$R' = \sqrt{\tfrac{1}{2}pl\mu R}$$

and $$L' = l\left(\frac{L}{l} - \frac{\mu}{2} + \sqrt{\frac{\mu R}{2pl}}\right)$$

If for the sake of simplicity we put $\mu=1$, and consider only a non-magnetic circuit, then it is easy to show that—

$$R'^2+p^2L'^2 = R^2+p^2L^2+\frac{R'^4-R^4}{R^2}-2\frac{R'}{R}(R'-R)(R'+pL) \quad . \quad . \quad . \quad (39)$$

If R′ = R the right-hand side reduces to its first two terms, as it should do. Accordingly, the high frequency impedance greatly exceeds the impedance for steady sinoidal low frequency currents.

3. **Predetermination of Inductance for Certain Standard Forms of Circuit.**—There are certain forms of circuit for which we can predetermine the inductance by calculation. Fortunately, this can be easily accomplished for one very simple form of circuit, viz. a rectangle formed of round wire, the diameter of the wire being small compared with either dimension of the rectangle. As this

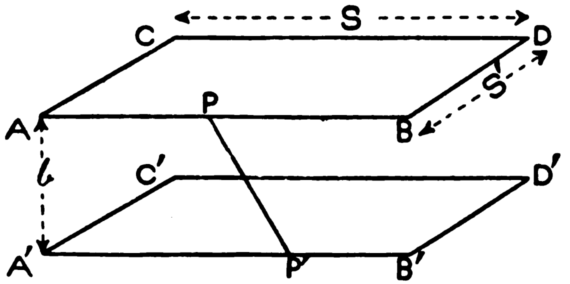


FIG. 4.—Mutual Inductance of Two Parallel Rectangular Circuits.

calculation illustrates very well the principles on which inductance generally is calculated, we shall give it in full, and then deduce certain consequences.

Suppose that two rectangles of the same size, made of infinitely fine wire, were placed with sides parallel to one another. If both are traversed in the same direction by unit current, we can calculate the potential energy M of the system by Neumann's formula, $M=\iint\frac{dx\,dx'}{r}\cos\theta$, where dx and dx' are two elements of length in the conductors, r their distance, and θ the angle between them.

In the case of the two rectangles, θ is either 0 or $\frac{\pi}{2}$, and hence $\cos\theta$ is either 1 or 0. We have then simply to take all possible pairs of elements in the two circuits, divide the product by their distance, and sum up all these quantities. Let ABCD, A′B′C′D′ (Fig. 4) be the two rectangles. Let the distance between their planes be b, and let the length of the sides AB, A′B′, CD, C′D′ be denoted by S, and that of AC, A′C′, BD, B′D′ be denoted by S′.

We consider first a pair of elements in the sides AB, A′B′ situated at P and P′. Let AP $=x$ and A′P′ $=x'$, and the length of these elements be dx and dx'. Then for this pair we have $r=\sqrt{(x-x')^2+b^2}$ and $\cos\theta=1$.

Hence $$M = \int_0^s \int_0^s \frac{dx\,dx'}{\sqrt{(x-x')^2+b^2}} \quad . \quad . \quad . \quad . \quad . \quad . \tag{40}$$

Now $$\int_0^s \frac{dx}{\sqrt{(x-x')^2+b^2}} = \log_0^s \left\{ (x-x') + \sqrt{(x-x')^2+b^2} \right\}$$

$$= \log \frac{S-x'+\sqrt{(S-x')^2+b^2}}{-x'+\sqrt{x'^2+b^2}} \quad . \quad . \quad . \quad . \quad . \tag{41}$$

Again, it is easily proved that—

$$\int \log \{S-x+\sqrt{(S-x)^2+b^2}\}\,dx$$

$$= -(S-x)\log\{S-x+\sqrt{(S-x)^2+b^2}\}+\sqrt{(S-x)^2+b^2} \quad . \quad . \quad . \quad . \tag{42}$$

Hence $$\int_0^s \log \frac{(S-x')+\sqrt{(S-x')^2+b^2}}{-x'+\sqrt{x'^2+b^2}}\,dx'$$

$$= S \log \frac{S+\sqrt{S^2+b^2}}{-S+\sqrt{S^2+b^2}} - 2\sqrt{S^2+b^2}+2b \quad . \quad . \quad . \quad . \tag{43}$$

But $$\frac{S+\sqrt{S^2+b^2}}{-S+\sqrt{S^2+b^2}} = \left(\frac{S+\sqrt{S^2+b^2}}{b}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \tag{44}$$

therefore $$\int_0^s \log \frac{(S-x')+\sqrt{(S-x')^2+b^2}}{-x'+\sqrt{x'^2+b^2}}\,dx'$$

$$= 2\left(S \log \frac{S+\sqrt{S^2+b^2}}{b} - \sqrt{S^2+b^2}+b\right) \quad . \quad . \quad . \quad . \tag{45}$$

This last expression (45) is the potential of AB on A′B′. To obtain that of AB on C′D′ we have to change b into $\sqrt{S'^2+b^2}$, and then prefix a negative sign, since then the currents are in opposite directions. We obtain thus the expression—

$$-2\left(S \log \frac{S+\sqrt{S^2+S'^2+b^2}}{\sqrt{S'^2+b^2}} - \sqrt{S^2+S'^2+b^2}+\sqrt{S'^2+b^2}\right) \quad . \quad . \quad . \tag{46}$$

Adding together (45) and (46), and doubling the sum, gives us the whole potential of the two pairs of sides of length S.

To obtain the potential of the sides of length S′ we exchange the position of S and S′ in the above final expression, and finally we obtain an expression for the whole potential energy M of the two rectangles as follows:—

$$\left\{ M = 4\ S \log \frac{(S+\sqrt{S^2+b^2})\sqrt{S'^2+b^2}}{(S+\sqrt{S^2+S'^2+b^2})b} + S' \log \frac{(S'+\sqrt{S'^2+b^2})\sqrt{S^2+b^2}}{(S'+\sqrt{S'^2+S^2+b^2})b} \right.$$

$$\left. + 2\sqrt{S^2+S'^2+b^2} - 2\sqrt{S'^2+b^2} - 2\sqrt{S^2+b^2}+2b \right\} \quad . \quad . \tag{47}$$

To obtain the inductance of the rectangle we have to consider that b is small compared with S or S′, and we have then to substitute for b the *geometric mean distance* of all the filaments of current composing the actual current in the wire.[17] As the case considered is that of a wire of circular cross-section and a surface distribution of current, we have to take for b the geometric mean distance of all points on the circumference of a circle. This, as Maxwell shows, is a length equal to the radius $\frac{d}{2}$ of the wire section.

Making this alteration in the formula (47), we have as an expression for the high frequency inductance of the rectangle the expression—

$$L' = 4\left\{ (S+S') \log_e \frac{4SS'}{d} - S \log_e (S+\sqrt{S^2+S'^2}) \right.$$

$$\left. - S' \log_e (S'+\sqrt{S^2+S'^2}) + 2\sqrt{S^2+S'^2} - 2(S+S') \right\} \quad . \quad . \quad . \tag{48}$$

[17] For the definition of the term *geometric mean distance* (G.M.D.), see Maxwell's "Treatise on Electricity and Magnetism," 2nd ed. vol. ii. p. 298.

If we call the sides of the rectangle A and B and the diagonal $D=\sqrt{A^2+B^2}$, and use ordinary logarithms, we can write the above formula in the form most suitable for calculation, as follows :—

$$L'=9\cdot2104\left\{(A+B)\log_{10}\frac{4AB}{d}-A\log_{10}(A+D)-B\log_{10}(B+D)-\frac{A+B-D}{1\cdot1513}\right\} \quad (49)$$

Let us then consider some special cases. If $S=S'$, the rectangle becomes a square, and the high frequency inductance of a square circuit of side S is—

$$L'=8S\left\{\log_e\frac{4S}{d}-\log_e(1+\sqrt{2})+\sqrt{2}-2\right\} \quad (50)$$

This can be thrown into the form—

$$L'=2l\left(\log_e\frac{4l}{d}-2\cdot853\right) \quad (51)$$

where l is the perimeter of the square and d the diameter of the wire of which it is made. If we use ordinary logarithms, the formula becomes—

$$L'=2l\left(2\cdot3026\log_{10}\frac{4l}{d}-2\cdot853\right) \quad (52)$$

or if S is the side of the square in centimetres—

$$L'=8S\left(2\cdot3026\log_{10}\frac{16S}{d}-2\cdot853\right) \quad (53)$$

Again, if in formula (48) we put S′ very small compared with S, that is, consider $\frac{S'}{S}$ can be neglected in comparison with unity, we have—

$$L'=4S\log_e\frac{2S'}{d} \quad (54)$$

This is the expression for the high frequency inductance of a pair of round parallel wires, each of length S, separated by a distance S′, each wire having a diameter d. Let l stand for the united length (lead and return) of the two wires, each having a length $\frac{l}{2}$, and let D be their distance apart; we can put the above formula in the form—

$$L'=2l\left(\log_e\frac{2D}{d}\right) \quad (55)$$

or, using ordinary logarithms—

$$L'=2l\left(2\cdot3026\log_{10}\frac{2D}{d}\right) \quad (56)$$

This agrees with a formula given by Lord Rayleigh, and also by Maxwell, with the difference that they consider the low frequency inductance, and we are considering the high frequency inductance.

The two formulæ for the inductance for infinite frequency, viz.—

$$L'=2l\left(2\cdot3026\log_{10}\frac{4l}{d}-2\cdot853\right) \text{ for a square,}$$

$$\text{and } L'=2l\left(2\cdot3026\log_{10}\frac{2D}{d}\right) \text{ for a pair of parallel wires,}$$

the parallel wires being not too near and short-circuited at the far end are of great use in practice, because these circuits can easily be formed of copper wire and their dimensions accurately measured, and then the inductance for high frequency currents calculated by the above formulæ from the dimensions. Again, if we take the expression (45) for the potential of two filamentary currents at a distance b, and put $\frac{d}{2}$ or b and l for S, we have the expression—

$$L'=2l\left(\log_e\frac{4l}{d}-1\right) \quad (57)$$

which gives us the high frequency inductance for a circular-sectioned straight wire of length l and diameter d, the return being at an infinite distance. Also we require occasionally the value of the inductance of such a wire bent into the form of the circle, the radius of this circle being large compared with the diameter of the wire.

It is not difficult to show that the high frequency inductance L′ of this circular and circular-sectioned wire of diameter d and perimeter l is given by—

$$L' = 2l\left(\log_e \frac{4l}{d} - 2 \cdot 45\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

$$\text{or} \quad L' = 2l\left(2 \cdot 3026 \log_{10} \frac{4l}{d} - 2 \cdot 45\right) \quad . \quad . \quad . \quad . \quad . \quad (59)$$

The proof of the above formula is given in the author's "Handbook for the Electrical Laboratory and Testing Room," vol. ii. p. 174.

It should be noted that in all these formulæ for high frequency inductance, (48) to (59), we have calculated really the inductance for *infinite* frequency (L_∞), and to obtain the true inductance for a frequency n, viz. L′, accurately, it is necessary to add to L_∞ the quantity $\frac{R'}{2\pi n}$, R′ being the high frequency resistance calculated by Lord Rayleigh's formula (9) corresponding to the frequency n considered. Generally speaking, the term $\frac{R'}{2\pi n}$ will be small compared with L_∞; hence no great inaccuracy is committed by taking L_∞ to represent the high frequency inductance.

In practical work we very frequently desire to predetermine approximately the inductance of a solenoid or spiral of one or more layers of wire, the turns being closely or not very closely packed. A very useful formula for this purpose has been given by Dr. A. Russell for the inductance (for steady currents) of a spiral of length l and mean diameter D having N turns per unit of length.[18] It is as follows:—

$$L = (\pi DN)^2 l \left\{1 - \frac{4}{3\pi}\frac{D}{l} + \frac{1}{8}\left(\frac{D}{l}\right)^2 - \frac{1}{64}\left(\frac{D}{l}\right)^4\right\} \quad . \quad . \quad . \quad . \quad (60)$$

In the above formula, if D and l are measured in centimetres and N in turns per centimetre, then L is expressed in centimetres or C.G.S. electro-magnetic units of inductance. If we reckon D and l in inches and N in turns per inch, the formula becomes—

$$L = (\pi DN)^2 \frac{l}{40}\left\{100 - 42 \cdot 4\left(\frac{D}{l}\right) + 12 \cdot 5\left(\frac{D}{l}\right)^2 - 1 \cdot 56\left(\frac{D}{l}\right)^4\right\} \quad . \quad . \quad . \quad (61)$$

and still gives the inductance in centimetres.

The above formula applies only to steady or very low frequency alternating currents.

In applying it to high frequency currents it needs a correction which has been supplied by L. Cohen (see *Bulletin of the Bureau of Standards*, U.S.A., vol. 4, No. 1), in a "Memoir on the Influence of Frequency on the Resistance and Inductance of Solenoidal Coils."

The inductance of a helix may be considered to be partly due to the magnetic flux which is linked with the turns of the spiral, and partly to flux which exists within the material of the windings. The former part, for an infinitely long spiral, is given by the expression $(\pi DN)^2 l$, or by the complete Russell formula for a spiral not infinitely long. The part due to flux within the material of the windings is less in the case of high frequency alternating currents than in the case of steady currents. Hence the inductance for high frequency currents is less than that due to steady currents. The diminution expressed as a percentage is, however, always very much less than the increase of the resistance expressed as a percentage for the same spiral and the same frequency.

[18] Dr. A. Russell, "On the Magnetic Field and Inductance Coefficients of Circular, Cylindrical, and Helical Currents," *Phil. Mag.*, April 1907.

The total inductance of a spiral is therefore made up of two parts, one due to the interior magnetic flux linked with the turns of the spiral, and the other due to the flux within the material of the spiral. In the case of continuous or steady currents these two components of the inductance for an infinite or very long spiral have values respectively equal to $4\pi^2N^2r^2l$ and $\frac{4}{3}\pi^2N^2d(4r+d)l$, where l is the length of the solenoid, r its internal radius, d is the diameter of the wire of which it is made, and N the number of turns of wire per unit of length (see Oliver Heaviside, "Collected Electrical Papers," vol. i. p. 356). Hence the whole inductance L for steady currents of a long solenoid formed of one layer of wire is given by—

$$L=4\pi^2N^2r^2l\left(1+\frac{2}{3}\frac{d}{r}+\frac{1}{6}\frac{d^2}{r^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

the quantity in the bracket being a correcting factor for the finite diameter of the wire.

In the case of alternating currents of very high frequency (say, 10^6 or so) L. Cohen (*loc. cit.*) shows that the component of the inductance due to flux within the material of the spiral, assumed to be a spiral of one single layer of closely compacted turns of square wire, is expressed by the function—

$$64N^3rdl\sum_{x=1}^{x=\infty}\frac{1}{x^2\alpha_x}\left(\frac{1}{\alpha_x^2+\beta_x^2}+1\right) \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where the letters have the signification given on p. 112.

If the frequency is very high, then $\alpha^2=\beta^2=2\pi\sigma p$, and is a large quantity. Therefore $1/2\alpha^2$ can be neglected in comparison with unity. Hence, as in the case of the similar formula for the resistance increase, we can say that the inductance due to the magnetic flux within the material is then equal to—

$$\frac{64N^3rdl\sqrt{\rho}}{2\pi\sqrt{n}}1{\cdot}2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

and the whole inductance of the solenoid of one layer of closely adjacent turns is—

$$L''=4\pi^2N^2r^2l+\frac{64N^3rdl\sqrt{\rho}}{2\pi\sqrt{n}}1{\cdot}2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

$$=4\pi^2N^2r^2l\left(1+0{\cdot}31\frac{d}{r}\frac{N\sqrt{\rho}}{\sqrt{n}}\right) \quad . \quad . \quad . \quad . \quad . \quad (66)$$

This value of L″ is always rather less than the value of L for the same spiral.

Hence the diminution in inductance of a long solenoid of one layer due to frequency and spiralization expressed as a percentage of the steady inductance is given by—

$$\frac{100(L-L'')}{L}=\frac{100\left(1-\frac{N}{2}\sqrt{\frac{\rho}{n}}\right)}{1+\frac{3}{2}\frac{r}{d}} \text{ (nearly)} \quad . \quad . \quad . \quad . \quad . \quad (67)$$

Thus, for instance, if $N=4$, $\rho=1600$, $n=10^6$, $r=5$, $d=0{\cdot}2$, we have $\frac{100(L-L'')}{L}=2{\cdot}4$, or about $2\frac{1}{2}$ per cent.

If we require the inductance of a fairly long coil or solenoid of length l cms. consisting of wire of diameter d cms. wound in one single layer of N turns per cm. on a former of diameter D cms., D being smaller than l, we can obtain it by multiplying the quantity $(\pi DN)^2l$ by two factors, one the quantity within the second pair of brackets in formula (60), which corrects for finite length or ratio D/l, and the second, which is the quantity in the brackets in formula (66), which corrects for frequency. These formulæ have, however, been obtained on the assumption that the wire is rectangular in section and fills up the whole space with only infinitely thin insulation between the turns. Since in general the wire is

round wire of diameter d and insulated up to a diameter d_1 we have to apply a correction ΔL which is subtracted, which corrects for this ratio. It has been shown by E. B. Rosa of the Washington Bureau of Standards that $\Delta L = 2\pi DNl(A+B)$, where A and B are constants depending respectively upon the ratio d/d_1 and Nl which are given in the following tables :—

d/d_1	A	d/d_1	A
1·00	+0·5568	0·55	−0·0410
0·95	+0·5055	0·50	−0·1363
0·90	+0·4515	0·45	−0·2416
0·85	+0·3943	0·40	−0·3594
0·80	+0·3337	0·35	−0·4928
0·75	+0·2691	0·30	−0·6471
0·70	+0·2001	0·25	−0·8294
0·65	+0·1261	0·20	−1·0526
0·60	+0·0460	0·15	−1·3404
		0·10	−1·7457

Nl	B	Nl	B
1	+0·0000	40	+0·3148
2	+0·1137	50	+0·3186
3	+0·1663	60	+0·3216
4	+0·1973	80	+0·3257
6	+0·2529	100	+0·3280
8	+0·2532	200	+0·3328
10	+0·2664	300	+0·3343
15	+0·2857	400	+0·3351
20	+0·2974	1000	+0·3365
30	+0·3083		

As an example take the following case. Let the coil consist of wire 2 mms. in diameter wound in 4 turns per cm. on a former 10 cms. in diameter and 100 cms. long. Then $d=0·2$, $N=4$, $D=10$, $l=100$, and $d_1=0·25$. Hence $d/d_1=0·8$, $Nl=400$. Therefore $A=0·3337$, $B=0·3351$, $A+B=0·6688$. Also $(\pi DN)^2 l=1577536$ and $2\pi DNl(A+B)=17800$. Now $D/l=0·1$ and hence the correction for dimension ratio given in formula (62) is equal to a factor of 0·9587. For high frequency the correction for field within the wire is very small, and hence the high frequency inductance of the above coil is nearly $1577536 \times 0·959 - 17800 = 1495057$, or very nearly 1·5 millihenrys.

A very commonly used form of inductive circuit in radiotelegraphy is a spiral of bare or insulated wire wound on a cylinder, the diameter of which is not small compared with its length, in a single layer, or else formed into a flat spiral of a single layer.

It is necessary then to consider the predetermination of the inductance of single-layer spirals of various forms and types.

Maxwell showed that the inductance L of a circular conductor of n turns, having a rectangular section of radial depth c, and axial breadth b, and mean radius r, is approximately given by—

$$L = 4\pi r n^2 \left\{ \log \frac{8r}{R} - 2 \right\} \quad . \quad . \quad . \quad . \quad . \quad (68)$$

where R is the geometric mean distance of the section from itself, that is, the geometric mean of the distances between all possible pairs of elements of area into which we can divide the total cross-section. The value of $\log_e R$ for a rectangle is—

$$\log_e R = \log \sqrt{b^2+c^2} - \frac{1}{6}\frac{b^2}{c^2}\log\sqrt{1+\frac{c^2}{b^2}} - \frac{1}{6}\frac{c^2}{b^2}\log\sqrt{1+\frac{b^2}{c^2}}$$

$$+\frac{2}{3}\frac{b}{c}\tan^{-1}\frac{c}{b}+\frac{2}{3}\frac{c}{b}\tan^{-1}\frac{b}{c}-\frac{25}{12} \quad . \quad . \quad . \quad . \quad . \quad (69)$$

It will be seen that the expression remains the same if b and c are interchanged. Accordingly for the same mean radius and turns a coil of breadth b and depth c has the same inductance as a coil of breadth c and depth b (see Fig. 5). Hence for the same mean radius and number of turns a flat helical spiral of one layer has the same inductance as a cylindrical helix of one layer. Numerous formulæ have been given for the inductance of single layer spirals having various dimension ratios. The expression given by Maxwell was shown by Weinstein to be not quite correct, and he gave another which was subsequently simplified by Stefan as follows :—

$$L = 4\pi r n^2 \left\{ \left(1+\frac{3b^2+c^2}{96r^2}\right)\log\frac{8r}{\sqrt{b^2+c^2}} - C_1 + \frac{b^2}{16r^2}C_2 \right\} (70)$$

where L is the inductance for steady currents, r is the mean radius of the coil, b is the breadth and c the depth of the section (rectangular), and C_1 and C_2 are constants which are functions of $\frac{b}{c}$ or $\frac{c}{b}$, which have been tabulated as follows :—

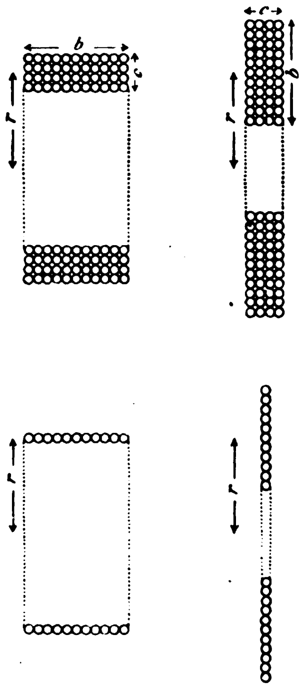


FIG. 5.—Circular Inductance Coils.

$\frac{b}{c}$ or $\frac{c}{b}$	C_1	C_2	$\frac{b}{c}$ or $\frac{c}{b}$	C_1	C_2
0·00	0·50000	0·1250	0·55	0·80815	0·3437
0·05	0·54899	0·1269	0·60	0·81823	0·3839
0·10	0·59243	0·1325	0·65	0·82648	0·4274
0·15	0·63102	0·1418	0·70	0·83311	0·4739
0·20	0·66520	0·1548	0·75	0·83831	0·5234
0·25	0·69532	0·1714	0·80	0·84225	0·5760
0·30	0 72172	0·1916	0·85	0·84509	0·6317
0·35	0·74469	0·2152	0·90	0·86697	0·6902
0·40	0·76454	0·2423	0·95	0·84801	0·7518
0·45	0·78154	0·2728	1·00	0·84834	0·8162
0·50	0·79600	0·3066			

The above formula is, however, obtained on the supposition that the wire is square-sectioned wire with infinitely thin insulation, and packed so as to fill up the whole of the rectangular space $b \times c$. As, however, the wire used is generally round wire with thick insulation, and does not fill up the whole space, three corrections to the above formula have to be made, which are all additive and may

be combined into a single correction ΔL, so that the actual inductance is $L+\Delta L$, where ΔL has the value—

$$\Delta L = 4\pi r n\left(\log\frac{d_1}{d} + 0{\cdot}13806 + C\right) \quad . \quad . \quad . \quad . \quad . \quad (71)$$

The first term takes account of the fact that the wire is round and of diameter d and insulated up to a diameter d_1. The second term reduces from square to round section, and the third term C takes account of the difference in the mutual inductance of the various terms when the wire is of round section from that when it is of square section and having no insulation. Maxwell considered that this term C was constant and had a value $-0{\cdot}01971$, but Rosa has shown in *Bulletin* No. 3, p. 37, of the Bulletins issued by the Bureau of Standards, Washington, that C is a function of the number of layers and windings as follows:—

Turns.	Layers.	C.
2	1	0·006528
3	1	0·009045
4	2	0·01691
1	1	0·01035
8	2	0·01335
10	1	0·01276
20	1	0·01357
16	4	0·01512
100	10	0·01713
400	20 × 20	0·01764
1000	50 × 20	0·01778
Infinite	...	0·01806

The above formula and correction enables us to calculate the inductance of cylindrical or flat spiral coils, provided that the breadth b or depth c are small compared with the mean radius r.

Thus, for instance, we can employ the above formulæ to predetermine the inductance of a flat spiral of one layer and 10 turns with mean radius 10 cms., the turns being closely adjacent, and made of round copper wire insulated up to a diameter of 5 mms., the ratio $\frac{d_1}{d}$ being 2·6.

Then we have—

$$r=10,\ b=0{\cdot}5,\ c=5{\cdot}0,\ b^2+c^2=25{\cdot}25,\ \sqrt{b^2+c^2}=5{\cdot}025,\ 8r=80,\ \frac{8r}{\sqrt{b^2+c^2}}=\frac{80}{5{\cdot}025}$$

$$=15{\cdot}92,\ \log_e 15{\cdot}92=2{\cdot}7676,\ 4\pi n^2 r=12566,\ 1+\frac{3b^2+c^2}{96r^2}=1{\cdot}0026.$$

Then $L = 12566\,(1{\cdot}0026\times 2{\cdot}7676 - 0{\cdot}59243 + 0{\cdot}1325\times 0{\cdot}00016)$
$= 12566\times 2{\cdot}18236 = 27424$ cms.

also $\Delta L = 1256{\cdot}6\,(\log_e 2{\cdot}6 + 0{\cdot}13806 + 0{\cdot}01276) = 1445$

and $L+\Delta L = 28869$ cms.

In order to check the accuracy of this predetermination, a flat spiral was made by winding I.R. covered $\frac{7}{22}$ wire on a flat board, as shown in Fig. 6. The total length of wire used was 628·3 cms. or 21 feet. The outside turn was 25 cms. in diameter and the inside 15 cms.

The inductance was measured with the cymometer, as follows: An oil condenser of capacity 0·00129 mfd. was joined up in series with a rectangle of wire of calculated inductance 5000 cms. The oscillation constant of this circuit was then $\sqrt{5000\times 0{\cdot}00129}=2{\cdot}54$. Two similar spirals, made as above described, were then inserted in series in this oscillatory circuit, and the corrected oscillation constant

found to be 8·97. Hence the inductance L of the spiral is obtained from the equation—

$$(5000+2L)\times 0{\cdot}00129=(8{\cdot}97)^2=80{\cdot}46$$

Hence the inductance of each spiral is—

$$\tfrac{1}{2}\times(62372-5000)=28{,}686 \text{ cms.}$$

Accordingly, theory predicts the inductance to be 28,869 cms., and experiment finds it to be 28,686, a difference of about one-half of 1 per cent.

Another test was made with a coil of 15 turns of $\frac{3}{22}$ wire insulated up to a diameter of 0·43 cm. Equal lengths of wire were wound up respectively into a flat helix of turns, and also into a cylindrical coil of 15 turns, the mean radius in both cases being nearly 7·25 cms. and the turns closely adjacent. We have then $r=7{\cdot}25$ cms., $n=15$ and $b=0{\cdot}43$, $c=6{\cdot}5$ for flat spiral, and $b=6{\cdot}5$, $c=0{\cdot}43$ for cylindrical coil.

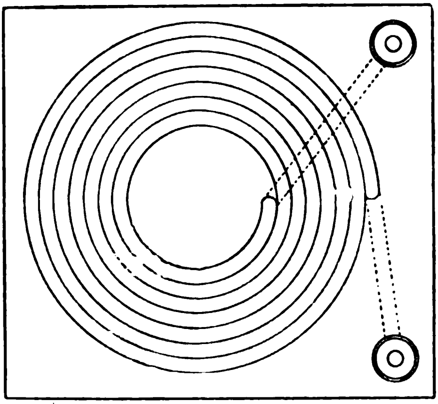


FIG. 6.—Flat Spiral Inductance Coil.

Hence $\quad r^2=52{\cdot}56$

$$\sqrt{b^2+c^2}=6{\cdot}5$$

$$\log_e\frac{8r}{\sqrt{b^2+c^2}}=2{\cdot}188$$

$$4\pi n^2 r=20{,}488$$

Also for the ratio $\frac{b}{c}=\frac{c}{b}=\frac{1}{15}$ we have—

$$C_1=0{\cdot}562 \text{ and } C_2=0{\cdot}132$$

Again, $1+\frac{3b^2+c^2}{96r^2}=1{\cdot}009$ for the flat spiral and 1·025 for the cylindrical coil, whilst $\frac{b^2}{16r^2}=0{\cdot}00024$ for the flat spiral and 0·05 for the cylindrical.

Hence $\quad L=33723$ cms. for the flat spiral

and $\quad L=34584$ for the cylindrical coil.

The correction ΔL is the same in both cases. We have—

$$d_1=0{\cdot}43,\ d=0{\cdot}15,\ \frac{d_1}{d}=3,\ \log_e\frac{d_1}{d}=1{\cdot}09852,\ 4\pi nr=1366.$$

Therefore, $\quad \Delta L=1366\,(1{\cdot}09852+0{\cdot}13806\times 0{\cdot}01351)=1366\times 1{\cdot}25=1707$

Accordingly the predetermined inductances are 35,430 cms. for the flat spiral and 36,291 cms. for the cylindrical coil.

The actual values measured as above described by the cymometer were 36,085 cms. for the flat spiral and 37,340 cms. for the cylindrical coil.

The difference in the predetermined values for the flat spiral and the solenoid of same mean radius and number of turns is due to the fact that the Stefan formula is not quite symmetrical in b and c, and hence does not give exact values unless b and c are both small compared with r.

The difference in the measured values is also due to a small difference, about 2 per cent., in the mean radii of the flat spiral and solenoidal coil.

The agreement, however, is sufficiently close to show that for approximate predeterminations of the inductance of flat spirals the Stefan formula can be employed with certain restrictions. Several other tests have been made confirming this conclusion.

For instance, a flat spiral of 17 turns was made by winding up on a board insulated copper wire, No. 18 S.W.G. size. The diameter of the bare wire was 0·1219 cm. $=d$, and the diameter over the insulation was 0·44 cm. $=d_1$. The mean

radius of the spiral was 7·75 cms. $=r$, and the depth of the winding was 7·5 cms. $=c$, and width was 0·44 cm. $=b$. Hence from Stefan's formula (70) we have for the inductance L, L = 44,350 cms. Maxwell's correction for diameter $\Delta L = 2380$ cms. Hence the total calculated inductance was 46,730 cms. The inductance of this spiral was measured by a high frequency method (see next section) at a frequency of one million, and also by the Anderson Bridge method at lower frequencies of 1000 and 5000.

The results were as follows :—

Inductance at 10^6	= 43,646	cms.
1000	= 45,500	,,
5000	= 47,100	,,

The mean of the last two low frequency measurements is 46,300, which agrees fairly well with the calculated value, viz. 46,730 cms. The high frequency value is less than the low frequency value as it should be.

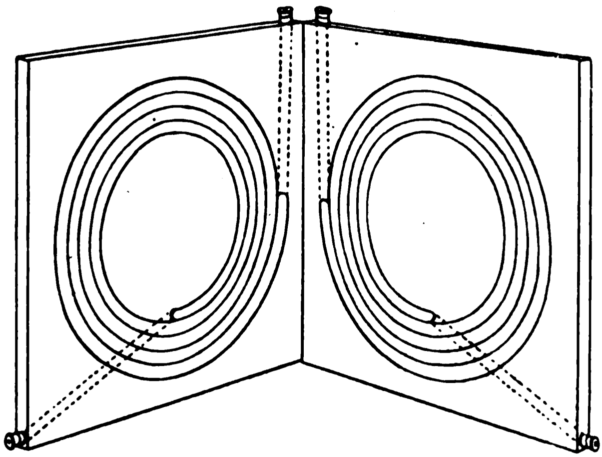


FIG. 7.—Double Flat Spiral Variable Inductance.

These results show that the Stefan formula may be trusted within the limits of its range to give an approximately correct result for such flat or short cylindrical coils as are used in radiotelegraphy.

As regards circular coils wound with n total turns of wire in a single layer, the mean radius of each circular turn being r cms. and the over all breadth or length of the coil being b cms., such that b/r is small compared with unity, Lord Rayleigh gave in 1881 a convenient formula for the inductance as follows :—

$$L = 4\pi r n^2 \left\{ \log_e \frac{8r}{b} - \frac{1}{2} + \frac{b^2}{32r^2}\left(\log_e \frac{8r}{b} + \frac{1}{4} \right) \right\}$$

If the wire is circular and insulated or the turns separated we have to apply the correction $\Delta L = 4\pi r n(A+B)$, the constants being taken from the two Tables on p. 122. If the total $A+B$ is positive in sign it has to be subtracted, and if negative added to the value of L given by the Rayleigh formula to obtain the true inductance.

Very convenient forms of variable inductance without sliding contacts can be made with flat spiral coil. For instance, two such spirals may be mounted on hinged boards, so as to come more or less into apposition to each other. The two spirals can be joined up in series by a flexible connection so that when the boards are shut up like a book, the currents in the two spirals oppose each other,

and the joint effect is a minimum inductance (see Fig. 7). When, however, the boards are opened out the inductances of the two coils are added and the joint inductance is a maximum. We can, therefore, by opening the hinged boards, more or less adjust the inductance in about the ratio of 8 : 1 without altering the total resistance or introducing a rubbing contact.

We can also mount four spirals on two boards pivoted at the centre, so as to cross each other and join up these four coils, so that when the upper board is turned round through 180° the inductance varies from a minimum to a maximum value over a considerable range (see Fig. 8). Another convenient form of variable inductance consists of a circular or cylindrical coil in the interior of which is a coil wound on another concentric cylinder or circle. The two coils are joined in series, and the inner coil can be turned round an axis so that its current coincides with or else opposes in direction that in the fixed coil. The inductance can thus

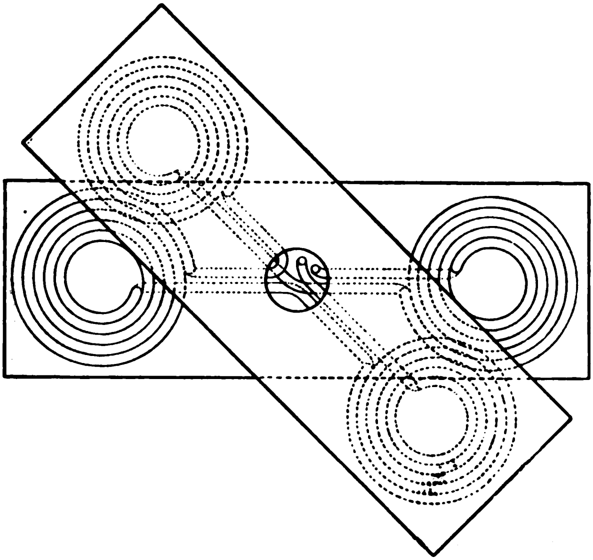


FIG. 8.—Continuously Variable Inductance.

be varied continuously. Such arrangements are useful in tuning coils for radio-telegraphic receivers and transmitters.

4. The Practical Measurements of Small Inductances.—The inductances with which we are concerned in practical high frequency work are generally small, that is, do not exceed a few thousand centimetres or a few microhenrys. Hence, methods are required for quickly and accurately determining the value of such inductances. There is no occasion to occupy space with a discussion of all the numerous methods which have been proposed for measuring inductance. For this information the reader must be referred to text-books on electrical measurements. The author has, however, worked out in detail one very convenient method for measuring small inductances, which has been found to be most useful for this purpose.[19]

The inductive coil L, R (see Fig. 9), or circuit of which the inductance L is to be measured, is connected to a Wheatstone's Bridge, P, Q, S, in the usual way,

[19] See J. A. Fleming and W. C. Clinton, "On the Measurement of Small Inductances and Capacities," *Phil. Mag.*, May **1903**, ser. **6**, vol. **5**, p. **492**, or *Proc. Phys. Soc. Lond.*, vol. 18, p. **386**. Also J. A. Fleming, "Note on the Measurement of Small Inductances and Capacities," *Phil. Mag.*, May **1904**, ser. 6, vol. **7**, p. **586**.

and a plug resistance box, capable of giving a high resistance, r, is placed in the bridge circuit, together with a condenser, C, as described by Professor Anderson.[20]

The condenser may consist of one or more Leyden jars, or preferably a condenser made of sheets of ebonite coated with tinfoil placed in oil, and the capacity of this condenser must be very accurately determined by the method given in § 7 of this chapter. In the bridge circuit must be arranged a galvanometer, G, exchangeable with a telephone, T, and in the battery circuit (see Fig. 2) a "buzzer," Z, or device for interrupting the circuit about 250 or 300 times a second. This buzzer consists of a thin plate of iron placed over an electro-magnet. There is a platinum-tipped contact point above the plate, arranged like that of an electric bell, so that when the magnet is energized by a couple of secondary cells the plate vibrates rapidly. A second platinum contact is arranged on the plate, so as to interrupt the battery circuit of the bridge. The buzzer is best contained in a sound-proof box. The first step is to balance the resistance of the inductive

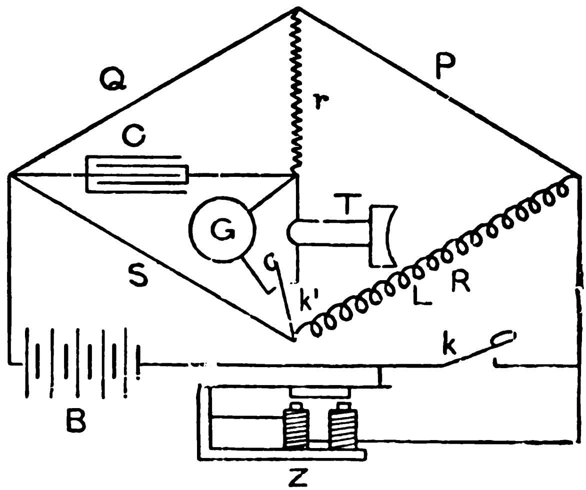


FIG. 9.—Anderson-Fleming Bridge Method of Measuring Inductance.

coil L, R on the bridge for steady currents, using the galvanometer, G, as a detector, or else the buzzer and telephone in series may be put in the place of the galvanometer. If the resistance of the inductive coil is very low, it may be increased by adding a non-inductive resistance to it. The buzzer is next put in the battery circuit, and the telephone in the bridge circuit, and the high resistance r in the bridge circuit altered until the telephone gives no sound. If the observer has an acute ear, or obtains the assistance of some one who has, it is possible to do this with such an accuracy that a variation of 1 per cent. or less in the resistance r is detectable.

It can then be shown that the inductance L of the coil measured in henrys is given by the formula given by Anderson (*loc. cit.*), viz.—

$$L = \frac{C}{10^6}\{r(R + S) + RQ\} \quad . \quad . \quad . \quad . \quad . \quad (72)$$

where C is the capacity of the condenser in microfarads, and R is the whole resistance in the arm of the bridge which contains the inductive circuit. Since P : Q = R : S when the bridge is balanced for steady currents, and since the

[20] See A. Anderson, "On a Method of Measuring Inductance," *Phil. Mag.*, 1891, vol. 31, p. 329; or *The Electrician*, vol. 27, p. 10; or J. A. Fleming, "Handbook for the Electrical Laboratory and Testing Room," vol. ii. p. 192.

balance is not upset by the adjustment or introduction of the resistance r, we can write the above formula in the form $L = \frac{C}{10^6}S\left(r + r\frac{P}{Q} + P\right)$, which gives the inductance in henrys, or $L = 10^3CS\left(r + r\frac{P}{Q} + P\right)$, giving the inductance in centimetres, the last being rather more convenient for most calculations.

The above-described method has been much used and tested by the author and his assistants in the last few years, and found to afford an excellent means for measuring inductances as small as 5 or 10 microhenrys (mhys.). It requires no apparatus that is not found or can be easily made in any electrical laboratory. The result gives us L, or the low frequency inductance of the coil or circuit. If, however, this is made of round-sectioned copper wire, the high frequency resistance, and therefore inductance, can easily be calculated from the formula already given.

As an example of the method, we may give the following instances of two measurements of inductances, one small and one very large.

The first case is that of a long helix of insulated wire, consisting of a single layer of 5000 closely adjacent turns wound on a wooden circular-sectioned rod, the mean diameter of a circular turn being 4·096 cms., and the length of the helix 200·3 cms. By the formula for the inductance of such a helix already given, we have $L = (\pi DN')(\pi DN)$, and since $N = 5000$, $D = 4·096$, and $N' = \frac{5000}{200·3}$, we have in this a calculated value $L = 20·6 \times 10^6$ cms.

This helix was connected to a bridge, and had its inductance measured with a telephone and buzzer as above described. The values of the various bridge arms, bridge resistance, and the capacity were as follows :—

$P = 100$ ohms, $Q = 100$ ohms, $R = 152$ ohms, $S = 152$ ohms, $r = 217 \pm 1$ ohm, $C = 0·256$ mfd.

Hence $L = 256 \times 152 \quad (217 + 217\frac{100}{100} + 100) = 20·8 \times 10^6$ cms. The agreement is fairly close.

The second case is that of a round-sectioned copper wire 0·164 cm. in diameter, laid round a room in the form of a square, the side of which was 607·1 cms., the ends being brought to the middle of one side and connected to the bridge. By the formula (53) on p. 119 for the inductance of such a square, we have—

$$L = 8S\left(2·3026 \log_{10}\frac{16S}{d} - 2·6\right). \quad . \quad . \quad . \quad . \quad . \quad (73)$$

We take 2·6 as the constant instead of 2·853, because in the measurement here made we are concerned with a low frequency inductance, and the larger value of the constant concerns the high frequency inductance.

Hence, substituting the measured values $S = 607·1$ cms. and $d = 0·1994$ cm., we have $L = 39,726$ cms.

The inductance was then measured as above, using a bridge and a condenser consisting of two Leyden jars having a total capacity together of 0·002783 mfd.

The following were the values of the bridge arms and bridge resistance :—

$P = 10$ ohms, $Q = 1000$ ohms, $R = 1·46$ ohms, $S = 146$ ohms, $r = 92 \pm 0·5$ ohm, $C = 0·002783$ mfd.

$$\text{Hence } L = 1000 \times 0·002783 \times 146(92 + 0·92 + 10)$$
$$= 41,816 \text{ cms., or } 41·816 \text{ mhys.}$$

The value calculated from first principles is 39·7 mhys., or less by 2·5 per cent. than the observed value.

The capacity of the condenser used was not probably ascertained with certainty to less than 2 per cent. Hence for such a small inductance the agreement is fairly good.

The same method is applicable to the measurement of small mutual inductances. If two coils are placed with axes in one line, they exert on each other a mutual inductance, and the current in one when varying produces an induced current in the other. The mutual inductance or coefficient of mutual inductance, M, is defined to be the numerical value of the total magnetic flux which is linked with both coils when unit electric current flows in them.

Hence, if we join both the coils in series, and call L and N the inductance or self-induction of each, and M the mutual inductance or coefficient of mutual induction, then the total flux linked with the circuit when unit current flows in it is either $L+2M+N$ or $L-2M+N$, according as the currents flow the same way or the opposite way in the two coils. Accordingly, if we join up two such coils of one circuit, and measure the inductance of the circuit, first with the coils joined up to add, and secondly with the coils joined so as to oppose their respective fields, and call L_1 and L_2 the apparent inductances, we have—

$$\left.\begin{aligned} L_1 &= L+2M+N \\ L_2 &= L-2M+N \end{aligned}\right\} \qquad (74)$$

whence $$M=\frac{L_1-L_2}{4}$$

and $$L+N=\frac{L_1+L_2}{2}$$

If, then, we measure L or N separately, we have all three coefficients.

As an instance of such a measurement, we give the following :—

Two equal square coils, each consisting of 8 turns of wire, the side of each square being 64·5 cms., were placed parallel to each other and at a little distance. The inductance was then measured.

(i.) Of each coil separately . . . . $=L$ and N
(ii.) Of both coils in series, but far apart and with planes at right angles . . . . $=L+N$
(iii.) Of both coils so joined in series to add their fields . . $=L+2M+N$
(iv.) Of both coils so joined in series as to oppose their fields $=L-2M+N$

The values were found by the bridge method (Anderson-Fleming method) just described, with the telephone and buzzer at a frequency of 256 or so. The results were—

$$L=116{,}200 \text{ cms.},\ N=116{,}200 \text{ cms.}$$
$$L+N=234{,}600 \text{ cms.}$$
$$L_1=L+2M+N=287{,}800 \text{ cms.}$$
$$L_2=L-2M+N=180{,}700 \text{ cms.}$$

From the last two observations we find $M=26{,}775$ cms., and $L+N=234{,}200$, which agrees very well with the direct measurement of $L+N=234{,}600$, and fairly with that of the sum of L and N separately, which is 232,400 cms.

The quantity $\dfrac{M}{\sqrt{LN}}=k$ is called the *coefficient of coupling*, and in the above case $k=\frac{26775}{116200}=0{\cdot}23$. Hence the coupling would be described as *fairly close*, because it is greater than 0·1.

The above method is easily applied to determine the mutual inductance of two coils at any moderate distance from each other, and thus to set out a curve showing the variation of mutual inductance of the coils with that distance.

Methods have been devised by the author for measuring directly the high frequency inductance and coefficient of coupling for high frequency currents of coils by means of a special instrument called a *cymometer*, to which further reference will be made later on.

This instrument enables us to determine the frequency in an oscillating circuit (see Chap. VI.).

Deferring for the present a detailed description of the appliance, we may note that since the frequency of an oscillating circuit is given by the formula—

$$n=\frac{5\times10^6}{\sqrt{CL'}}$$

where C is the capacity of the condenser in it in microfarads, and L' the high frequency inductance in centimetres, we can determine L' if we know C and n. The cymometer enables us to measure the frequency n, and then, assuming the capacity of the condenser used can be measured independently, we calculate L' by the formula—

$$L' = \frac{25 \times 10^{12}}{Cn^2}$$

where C is measured in microfarads and L' is given in centimetres.

The process of measurement consists in placing the coil, the inductance of which is required, in series with a spark gap and a condenser, say a Leyden jar, of known capacity, and by means of an induction coil exciting electric oscillations in the circuit. The frequency of these oscillations being measured by the cymometer or other means, we have the value of n, and therefore of L'.

In one form of cymometer the measurement actually made is the wave length of a stationary electric oscillation set up on a long helix of wire. The velocity with which this wave travels along the helix can be determined from the calculated inductance and measured capacity of the spiral per unit of length. For the particular helix with which the measurements below given were made, this velocity was found to be 175×10^6 cms. per second. The process of measurement consists in attaching the helix either directly or with the interposition of a small air condenser to an oscillation circuit constructed with a known capacity and with the inductance to be determined, and then adjusting a sliding metal saddle on the helix in such a position that when the saddle is connected to earth the section of the helix between it and the oscillatory circuit is one complete wave length of a stationary electric wave on the helix. The quotient of wave velocity along the helix by this stationary wave length then gives us the frequency n of the oscillatory circuit (see Chap. IV. § 1).

The self and mutual inductances of an oscillation transformer were measured for a frequency $2 \cdot 5 \times 10^6$ as follows: The primary coil consisted of one turn of stranded copper wire nearly 1 metre in length bent into the form of a square. Its inductance, L, was determined by finding the oscillation frequency as above described, when this coil was associated with a condenser having a capacity of 0·005835 mfd. to form an oscillatory circuit. The wave length on the cymometer helix was found to be 71 cms., and hence the frequency was $\frac{175 \times 10^6}{17}$, and this must be equal to $\frac{5 \times 10^6}{\sqrt{0 \cdot 005835 L'}}$, where L' is the inductance of the primary coil in test. Hence $L' = 719$ cms.

In the same way the total inductance of the primary and secondary was determined for the two modes of connection and found to be—

$$\begin{aligned} & L_1 = L + 2M + N = 57{,}933 \text{ cms.} \\ & L_2 = L - 2M + N = 45{,}384 \text{ cms.} \\ \text{whence} \quad & M = 3137 \text{ cms.} \\ \text{and} \quad & L + N = 51{,}658 \text{ cms.} \end{aligned}$$

Deducting the separately measured primary inductance, viz. 719 cms., from $L + N = 51{,}658$ cms., we have the secondary inductance 50,940 cms., or nearly 51,000 cms.

A separate and independent measurement of the inductance N of the secondary circuit gave the value $N = 52{,}600$.

The difference between 52,600 and 51,000 is about 3 per cent., but the length of the stationary wave on the helix is hardly certain to 1 per cent., and the inductance varies as the square of the wave length on the helix. Hence the percentage error of the wave length is doubled in calculating the inductance.

From the above figures we find the *coefficient of coupling* $k = \frac{M}{\sqrt{LN}}$ for this transformer to be—

$$k = \frac{3137}{\sqrt{719 \times 51{,}000}} = 0 \cdot 52 \text{ (nearly).}$$

Hence the coupling is *close*, because k has a value greater than about 0·1.

Another confirmation of the accuracy of this last method was obtained by

measuring the inductance of a single copper wire 0·1994 cm. in diameter bent into the form of a square having a side of length 607·1 cms. The frequency used was about 10^6. Associating this square inductance with a capacity of 0·00146 mfd., the cymometer determined the frequency of the oscillations set up in this circuit to be $\frac{175 \times 10^6}{264}$, and this by the general formula, viz. $n = \frac{5 \times 10^6}{\sqrt{CL}}$, gives us a value for L of 39,970 cms. as the inductance of the square. The inductance calculated from the length of side of square $= S$ and diameter of wire $= d$ by the formula—

$$L = 8S\left(2 \cdot 3026 \log_{10} \frac{16S}{d} - 2 \cdot 85\right)$$

is 38,562 cms. Hence the two are in very fair agreement.

For additional information on the measurement of small inductances by means of electric oscillations the reader is referred to a paper by Mr. H. H. Taylor, in the *Physical Review* for October 1904, vol. xix. p. 273. Taylor employed a resonance method in which the inductance to be measured has its value deter-

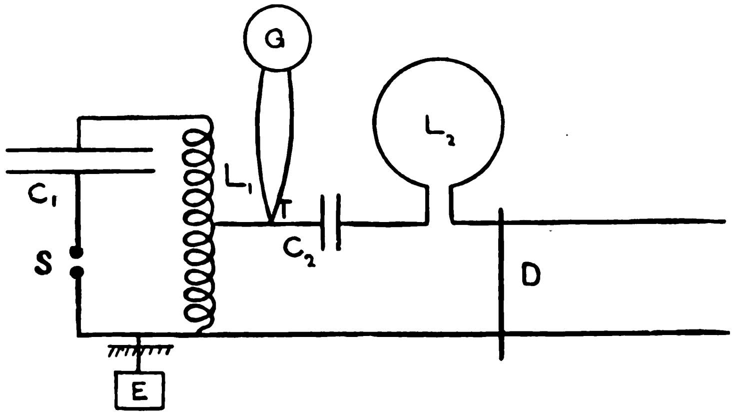


FIG. 10.—Taylor's Method of Measuring High Frequency Inductance.

mined by substituting for it an equivalent inductance obtained by sliding a slider along two parallel wires. The inductance per unit of length of the parallel wires can be calculated, and hence if the effective length of the parallel wires is altered by moving the slider, the addition to their inductance becomes known.

The arrangement is shown in Fig. 10. An oscillating circuit is set up consisting of a capacity, C_1; an inductance, L_1, which is preferably variable; and a spark gap, S. One point on this circuit is earthed at E. To two adjacent points on L_1 near the earthed end a pair of parallel wires are connected, and in the run of one of these is inserted a condenser, C_2, and the inductance, L_2, to be measured. A slider, D, can be moved along the parallel wires. The measurement consists in moving D to two positions, one with the inductance L_2 short-circuited, and adjusting the position of D so that the maximum current flows in the parallel wires as shown by the maximum deflection produced on a galvanometer, G, when connected with a delicate thermoelectric junction, T, attached to some point on the parallel wires.

This method is simple, and seems capable of considerable accuracy. It can be checked by using for L_2 a single wire bent into the form of a circle or square. It has the advantage that no special apparatus is necessary. The only limitation is that the method is not applicable to inductances whose resistances vary so

widely as to affect seriously the period of the auxiliary circuit unless a compensating inductionless resistance is inserted to swamp any difference in the resistances of the inductances compared.

For a description of another form of direct-reading cymometer devised by the author for making high frequency measurements of capacity and inductance, the reader is referred to Chap. VI. § 15 of this treatise.

5. Inductance Coils of Variable Inductance.—In practical work on electric oscillations or Hertzian wave telegraphy, we often require to insert in circuits inductances which can be varied gradually or by steps. Arrangements for effecting this are called inductance boxes or sliding inductances. In some cases the change in inductance must be gradual and not accompanied by any change in the resistance, in other cases a slight change in resistance is not of moment. For varying the inductance of a circuit within certain narrow limits, without making any break in the circuit or change in its resistance, a very convenient arrangement is one introduced by the author, called an *accordion coil* or *concertina coil*, from its rough resemblance to these musical instruments.

On a tube of vulcanized fibre is placed a couple of rings of wood, one of them fixed at the end of the tube and the other sliding on the tube (Fig. 11). This last ring can be clamped by a screw in any position. The rings are connected by a spiral wire of brass or hard drawn copper, which is covered with india-rubber or otherwise insulated. When the rings are near together this wire is arranged in a close spiral with the turns in one layer and touching. When the rings are moved far apart the turns of the wire are widely separated, and the inductance has then a minimum value.

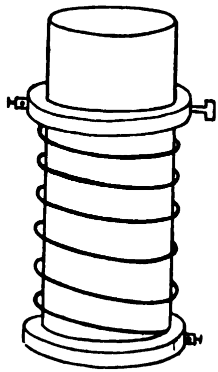


FIG. 11.—Variable Accordion Coil Inductance. (Fleming.)

By sliding the movable ring to various positions, the inductance can be given any value within certain limits. When a small accompanying change of resistance does not matter, the following arrangement due to the author is effective.[21]

On a boxwood cylinder, about 10 cms. in diameter and 100 cms. in length, a screw groove is cut, the grooves being separated by at least 5 mms. This cylinder is mounted with brass end plates and held in bearings. A winch handle serves to rotate it (see Fig. 12).

In the groove is wound a bare thick copper wire, say No. 12 or No. 14 S.W.G., and the ends of the wire are soldered or screwed to the end plates on this cylinder. Against one end plate a spring contact with terminal on it presses.

Parallel with the cylinder is fixed a thick brass rod, and on this travels a contact piece, the end of which makes contact with the copper wire. A weight on this contact serves to keep a good electrical connection. When the cylinder is turned, the contact piece slides along and interposes a variable number of turns of the wire between the end contact on the cylinder and the moving contact on the wire. Hence the inductance between these points can be varied.

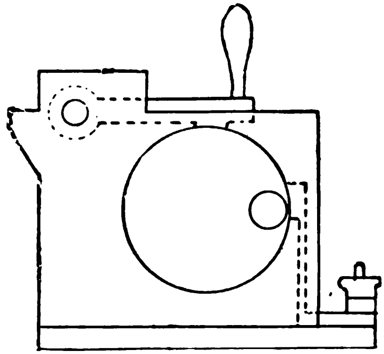
In employing such an inductance with high frequency currents, it should be noted that there is always a certain dielectric current between the turns of wire, which acts to diminish the effective inductance, and it must not be assumed that such a bare spiral inductance has exactly the same inductance for high frequency currents as for low frequency currents, apart altogether from the variation of distribution of current over the cross-section of the wire. The inductance for high frequency currents will always be less by a somewhat uncertain amount owing to this dielectric current between the turns.

When a small variation is required, a couple of bare wires may be stretched parallel to each other, and a sliding metal connecting piece laid across them and moved along. The same remarks, however, as above apply in this case. The

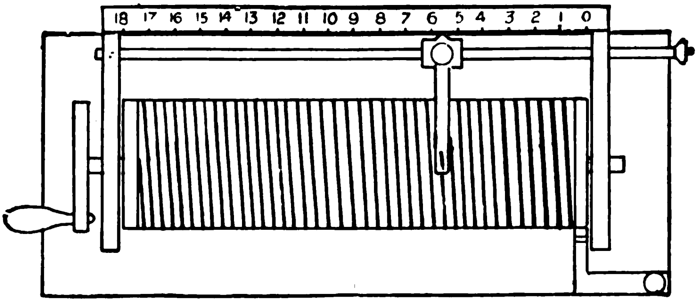
[21] See J. A. Fleming, "On a Standard of Small Inductance," *Phil. Mag.*, May 1904, p. 592.

dielectric current across from wire to wire prevents us from determining exactly, either from this calculated value or a low frequency measurement, the true inductance when high frequency currents are employed with it. Nevertheless, when the inductance is not required to be known very accurately the arrangement is convenient.

In the case of larger inductances it is convenient to be provided with a number of glass or ebonite rods, on which is wound silk-covered copper wire in one layer, the turns close together. The length of the rod must be at least 20 times, and



End Elevation.



Plan.

Fig. 12.—Variable Inductance Coil. (Fleming.)

preferably 50 times, its diameter, and then the inductance can be approximately calculated from the Russell formula—

$$L=(\pi DN')^2 l\left\{1-\frac{4}{3\pi}\frac{D}{l}+\frac{1}{8}\left(\frac{D}{l}\right)-\frac{1}{64}\left(\frac{D}{l}\right)^4\right\} \quad . \quad . \quad . \quad . \quad (75)$$

where N is the total number of turns, l the length, D the diameter, S the cross-section of the rod, and N′ the number of turns per unit of length of the helix, all measurements being in centimetres or square centimetres.

When the dimension ratio is at least 50 : 1, the inductance predetermined by the simple formula $L=(\pi DN')^2 l$ will not differ from the actual inductance by more than 2 or 3 per cent., as shown by the comparison between the so calculated value of a coil used by the author, and its inductance repeatedly measured by the bridge (Anderson-Fleming) method.

RESULTS OF INDUCTANCE MEASUREMENTS OF A LONG COIL, HAVING A DIMENSION RATIO OF 50 : 1.

P.	Q.	R.	S.	r.	C. in mfds.	L. observed in cms.	
100	1,000	152·26	1,522·2	4,260	0·00272	19,900,000	A
100	1,000	152·31	1,523·1	7,675	0·00149	19,400,000	A
100	1,000	151·1	1,511	4,400±50	0·00272	20,300,000	B
1,000	10,000	151·5	1,515	3,350±50	0·00272	19,200,000	B
100	10,000	151·5	15,150	365±5	0·00272	19,300,000	B
1,000	1,000	152	152	24,200±100	0·00272	20,100,000	B
100	1,000	151·4	1,514	4,400±50	0·00272	20,300,000	B
1,000	10,000	151·4	1,514	3,330±20	0 00272	19,200,000	B
10	1,000	151·7	15,170	485±5	0·00272	20,600,000	B
100	10,000	151·7	15,170	365±5	0·00272	19,300,000	B
100	100	152	152	217±1	0·256	20,800,000	B

Mean of A readings = $19{\cdot}7 \times 10^6$ cms.
Mean of B readings = $19{\cdot}9 \times 10^6$ cms.
Value calculated from the formula $L = (\pi DN)(\pi DN') = 20{\cdot}6 \times 10^6$ cms.

In connection with the question of standards of small inductance it should be pointed out that Mr. A. Campbell has advocated the use of standards of mutual inductance for the following reasons [22] :—

(*a*) The absolute values can be calculated with much more certainty from the geometrical dimensions, since the formulæ for mutual inductance are of high theoretical accuracy, while those for self-inductance are much less satisfactory.[23]

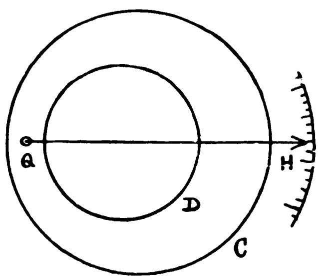


FIG. 13.

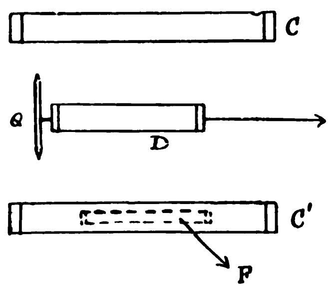


FIG. 14.

Scheme of Circuits of Campbell Inductometer.

(*b*) Unless the conductors are highly stranded, the current distribution varies with frequency, and in general the self-inductance will also vary. By keeping the two circuits at a relatively large distance from one another the mutual inductance is practically free from this effect.

(*c*) The effects of distributed capacity are probably less in mutual than in self-inductances. In all cases the distributed capacity of one of the two coils can be made very small by sufficiently decreasing the number of turns in it (or opening them apart) while increasing the number in the other coil to keep the M constant.

When the mutual inductance is of the variable type, it can always be designed so that its value can be varied continuously from *negative to positive through zero.*

[22] "On the Use of Variable Mutual Inductances," A. Campbell, *Proc. Phys. Soc. Lond.*, vol. xxi. p. 69, 1908.

[23] For example see Rosa, *Bull. Bur. Stands.*, vol. ii. p. 161, 1906; and Strasser, *Ann. der Phys.*, vol. xviii. p. 763, 1905.

This is a very great advantage, for with variable self-inductance standards the impossibility of reaching a zero value is a distinct drawback.

He has therefore designed a variable inductance made as follows, the description being taken, by permission, verbatim from his paper (*loc. cit.*).

The general arrangement of the apparatus is shown diagrammatically in Figs. 13 and 14, which are plan and side view respectively. The primary circuit consists of two equal coaxial coils C and C′ (see Fig. 14), which are connected in series, their windings being in the same direction of turning. The secondary consists of the coils D and F in series. Of these coils, D is movable, being mounted on an eccentric axis Q so as to be free to turn in a plane parallel to those of C and C′ and midway between them. Rigidly connected with the movable coil is a pointer H, which moves over a circular scale of about 180° in extent and graduated to read directly (see also Fig. 19). The coil F is subdivided into ten sections, which are in series, each of them being 0·1 millihenry, and their junctions are brought to a set of separate terminals or studs with a turning head. The range of the moving coil extends from − 0·002 to + 0·11 millihenry. This gives a continuous range from 0 up to 1 millihenry, readable near zero to 0·02 microhenry, to 1 part in 500 at 0·1 millihenry, and to 1 in 5000 at 1 millihenry. The subdivision of the coil F is easily carried out by the following artifice. The coil is wound with uniformly stranded wire of ten insulated strands, all the strands are connected in series, and the whole adjusted to give 1 millihenry. If the stranding has been properly done, it will be found that no one of the sections differs from its neighbours by more than 1 part in 1000, and each is 0·1 millihenry. The placing of the movable secondary coil midway between the planes of the two primary ones ensures that small axial displacements shall have very little effect on the mutual inductance.

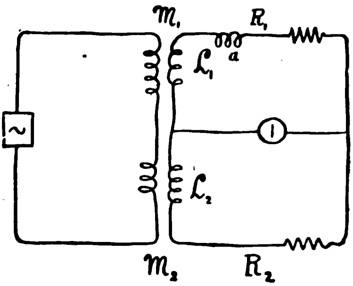


FIG. 15.

The equality of any pair of sections can be tested by connecting them in series with their windings in opposition in circuit with a ballistic galvanometer and reversing the current in the primary. It should be noticed here that, if a primary coil has any number of secondary circuits, the mutual inductance to all the secondaries in series is equal to the algebraic sum of their separate mutual inductances (+ or − according to the direction of the winding). Owing to this very important property we can build up and step down in the values as easily as if we were dealing with resistances, and there is the further simplification that we can subtract as well as add the values. The marking of the scale and the setting of the coil F are done by comparison with a fixed standard mutual inductance of the kind devised by Mr. Campbell.[24] The comparison may be made by Maxwell's method, but using a sensitive ballistic galvanometer or a vibration galvanometer (see Fig. 21) as detector. When a vibration galvanometer is used as in Fig. 15, it should be remembered that, for a balance, *two* conditions must be satisfied, viz.—

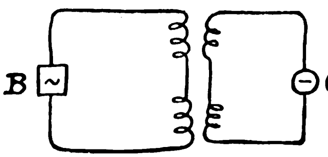


FIG. 16.

$$\frac{M_1}{M_2} = \frac{R_1}{R_2}$$

and $$\frac{L_1}{L_2} = \frac{R_1}{R_2}$$

[24] *Phys. Soc.*, May 1907; *Phil. Mag.* [6], vol. xiv. p. 494, Oct. 1907. Also see *Proc. Roy. Soc. A.*, vol. lxxix. p. 428, June 5, 1907.

where R_1 and R_2 include the resistances of the secondary coils. In order that the second condition may hold, it is necessary to introduce into one of the secondary branches a coil a whose self-inductance can be continuously varied; by alternate adjustments of $\frac{R_1}{R_2}$ and the self-inductance of this coil, a balance is easily obtained. The fact that R_1 and R_2 are partly of copper coils is apt to introduce some inaccuracy. The copper resistance, however, can usually be largely swamped without losing too much of the sensitivity.

Any unknown mutual inductance, whose value lies within the range of the variable standard, can be at once determined by connecting the primaries of the unknown and the variable in series to B (Fig. 16), a source of alternating or intermittent current, while the secondaries, with their windings in opposition, are connected in series to a vibration galvanometer G. The variable M is then adjusted to bring the deflection to zero, and the reading gives directly the value of the unknown M. This is an extremely simple method, as it involves no knowledge of any resistances. A ballistic galvanometer and a commutated current may be used. This method does not apply to mutual inductances higher than the maximum value of the variable standard. The standard of mutual inductance can then be used to measure a self-inductance as follows:—

In Fig. 17 let a variable mutual inductance M whose *primary* includes the subdivided coil be connected into a Wheatstone's network, as shown, along with a self-inductance L_2. Let the resistances of the arms be P, Q, R, and S respectively, the self-inductance of the arm P being L_1 (the secondary coils of M) and that of Q being L_2. Let H be a source of periodic current, and G a vibration galvanometer tuned to resonance with it, so that we may take the wave form of the currents to be a sine curve. Let the instantaneous potentials of the three upper corners be v_1, 0, and v_2 respectively, and the instantaneous values of the currents into the upper corner be i_1, i_2, and i as marked.

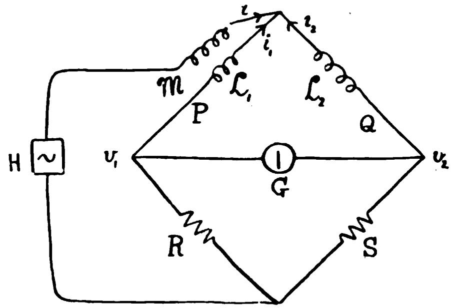


FIG. 17.

Let $p = 2\pi n$, where n is the frequency, and for convenience of writing let $p\sqrt{-1}$ be denoted by a, so that $a^2 = -p^2$. The mutual inductance M may be made positive or negative according to the way in which the coils are connected; and in all that follows we might write $\pm$ M for M throughout. When the galvanometer shows a balance, $v_1 = v_2$, and the instantaneous value of the current through G is zero.

Also
$$i = -i_1 - i_2$$

Accordingly we may write—
$$(P + L_1 a)i_1 - Mai = (Q + L_2 a)i_2$$
therefore
$$[P + (L_1 + M)a]i_1 = [Q + (L_2 - M)a]i_2$$
also
$$Ri_1 = Si_2$$
Hence
$$S[P + (L_1 + M)a] = R[Q + (L_2 - M)a]$$

Equating the real and imaginary parts each to zero, we have—
$$SP = QR \qquad (76)$$
and
$$S(L_1 + M) = R(L_2 - M) \qquad (77)$$

Exactly the same equations hold when the positions of the source and the galvanometer are interchanged.

The most useful case is when the non-inductive arms are made equal, *i.e.* $S = R$; then (76) and (77) become—
$$P = Q$$
and
$$L_2 - L_1 = 2M \qquad (78)$$

This case gives an extremely convenient way of measuring small self-inductances, which is done as follows:—

The arrangement is shown in Fig. 18. The non-inductive arms are equal (R, R). In the arm AB there is the secondary coil *a* of self-inductance L in series with a practically non-inductive rheostat *r*. In the arm AC is placed a "balancing" coil *b* also of self-inductance L and of resistance equal to or slightly greater than that of *b*. By adjusting *r* the bridge will balance when $M = 0$. The small self-inductance N to be measured is now inserted in series with coil *b* in arm AC, and a balance obtained by altering *r* and M. Then, by (78), $N = 2M$. Thus N is found directly from the reading of M, and the range of values that can be measured runs from 0 up to twice the highest reading of the variable mutual inductance. [For values of N above this range the more general case (equations (76) and (77)) may be used.] The L's of the coils *a* and *b* should be adjusted to equality once for all by putting M at zero and setting one of the coils. An exact setting is convenient, but not necessary, for if L_a and L_b differ slightly, they can be balanced (without N) by a small reading M_0. If M be the reading for balance when N is inserted, then $N = 2(M - M_0)$.[25]

Even if *a* and *b* are well matched, it is always well to begin by reading their difference, if any.

It will be noticed that the method is really a differential one; when N is introduced into the arm AC no alteration has to be made in the other arms except to increase the resistance of AB by an amount equal to the resistance of the coil N. But although it has *all the advantages of differential measurement, the reading can be made to give* N *directly* without having to take a difference at all. This is due to the use of the inductive balancing coil *b*.

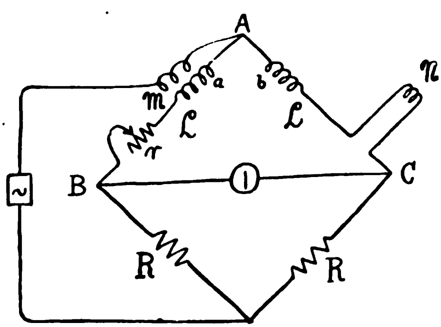


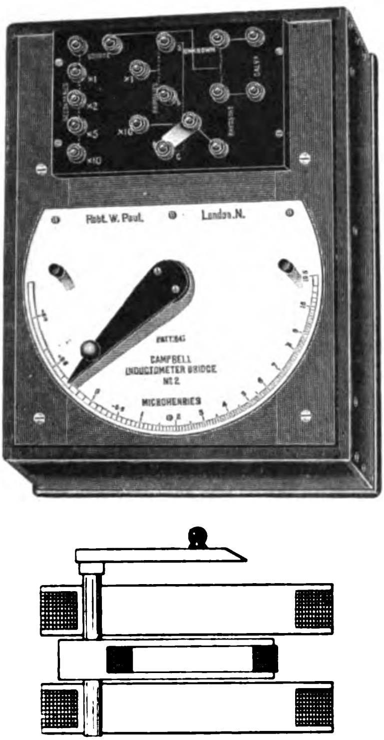
FIG. 18.

The method has the advantage that it does not require the knowledge of the absolute value of any resistance. The non-inductive bridge arms must be equal; to check the equality they can be interchanged. For the non-inductive adjustable resistance *r* it is best to employ a special rheostat consisting of two slightly flattened thin wires running parallel to one another at a few millimetres' distance, with a sliding contact piece across them to complete the circuit. The inductance of such a rheostat can be approximately calculated, and may thus be allowed for when measuring very small self-inductances. The inductance of the part added to compensate for the introduction of N has merely to be subtracted from the result.

In practice the method proves very convenient; with the variable mutual inductance described above, self-inductances of any value from 0·1 microhenry up to 2000 microhenrys can be measured directly without the bridge being altered in any way except in the rheostat *r*. In a later model the whole scale of the movable coil corresponds to 20 microhenrys, and at this value it can be read to 1 or 2 in 1000—at 200 microhenrys to 1 or 2 in 10,000 All the resistances of the coils are very low, and the sensitivity can be considerably increased by using M. Wien's method of connecting the vibration galvanometer to the bridge by means of a transformer of suitable ratio $\left(\frac{n_1}{n_2} \text{ small}\right)$. The method will also give the difference between two unknown self-inductances introduced into AB and AC.

[25] If coil *b* be made non-inductive we revert to Maxwell's method of comparing the M of a pair of coils with the L of one of them. Equation (77) then reduces to $\frac{L}{M} = -\left(1 + \frac{R}{S}\right)$. Maxwell, "Elect. and Mag.," 2nd edit. vol. ii, § 756.

In the practical instrument (see Fig. 19) Mr. Campbell combines the ratio arms and variable mutual inductance connected as in Fig. 20, which represents the instrument in the form made by R. W. Paul. If an inductance N is to be measured it is joined into the bridge as shown in Fig. 20. The bridge must be supplied with alternating currents from some alternator or vibrator giving a pure sine curve wave form, and the galvanometer must be a vibration galvanometer. This last instrument consists in one form of a coil through which the alternating current passes. In front of this coil is a small needle of soft iron which is attached to the centre of a stretched wire. The soft iron needle is directed by a magnetic field which can be varied in strength so as to give the soft iron needle a variable frequency of vibration. If this rate of vibration is adjusted to be the same as that of the alternating current, then when even a feeble alternating current passes through the coil it will set the needle in vibration. The needle carries a mirror, and a ray of light thrown on it is reflected to a screen where it is drawn out into a band of light when the needle vibrates. Hence if the spot of light is not drawn out this indicates the absence of an alternating current in the coil. The vibration galvanometer serves, therefore, as a means of indicating



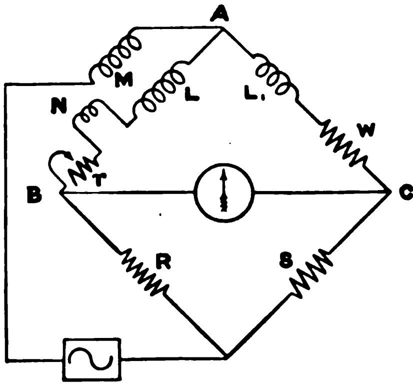


FIG. 19. FIG. 20.

External appearance and arrangement of Mutually Inductive Coils of Campbell Inductometer as made by R. W. Paul.

the fulfilment of any conditions in an arrangement of apparatus used with alternating currents which produces zero currents in a certain branch. A somewhat different form of vibration galvanometer, in which the coil itself vibrates, has been designed by A. Campbell and is made by R. W. Paul, and is illustrated in Fig. 21. The following description of it is given by Paul :--

A small moving coil is supported in the narrow air-gap of a permanent magnet by an upper and lower suspension of phosphor bronze strip. A tension can be applied to the upper suspension through a spring, by rotating a milled head. The effective length of the upper suspension can also be varied by raising or lowering a bridge-piece, which engages with the suspension. This adjustment is effected by rotating a knurled head, which is attached to a coarse threaded screw.

To tune the instrument, a small alternating current is sent through the moving

coil from the source of supply. This current may have a value of about 20 microamperes. The bridge-piece is then traversed until the galvanometer spot widens out into a long band, and the tensioning head is adjusted until the band attains its maximum width. The galvanometer is thus brought into mechanical resonance with the supply frequency, and, when in this condition, is very insensitive to harmonics. This fact simplifies considerably the theory of the A.C. bridge.

The coil is fitted with a plane mirror, and a convex lens is fixed in the cover. The instrument covers a frequency range from about 70 to 400; it can also be tuned for lower frequencies by attaching one of the small weights provided to the moving coil. With the smaller of these in position, the frequency can be reduced to 50 or less; the larger is used to reduce the frequency to between 30 and 10 per second. A sensitivity of about 34 mms. per microampere, with a scale distance of 1 metre, is obtained when the instrument is tuned to a frequency of 100. At higher frequencies the sensitivity is proportionately reduced.

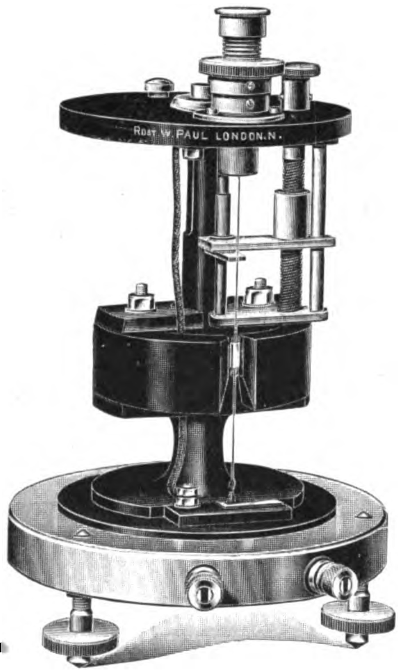


FIG. 21.—Campbell Vibration Galvanometer made by R. W. Paul.

For the measurement of small inductances such as are used in radiotelegraphic work, a very useful outfit comprises a small alternator giving alternating currents of simple sine wave form and variable frequency, and a vibration galvanometer and standard of mutual inductance as above described.

The process of determining the inductance of a coil N (see Fig. 20) is then as follows:—

Balancing coils having self-inductances $\frac{1}{9}$ and $\frac{1}{99}$ of that of the primary are required. They are connected up as shown in Fig. 20, where L_1 represents one of the balancing coils, r a non-inductive rheostat, R, S the ratio coils, W a small resistance coil of any type, and N the inductance under test.

To obtain the multiplying ratio of 10, the balancing coil $\frac{L}{9}$ should be used, and the ratio $\frac{R}{S}$ should be equal to 9. To obtain the multiplying power of 100, the balancing coil $\frac{1}{99}$ is used, and the ratio $\frac{R}{S}$ should then be 99. With the latter arrangements, measurements up to 100 millihenrys can be made.

Balance, with the leads to N short-circuited at their outer end. The resistance W should then be set so that the initial reading of r is well in excess of the probable resistance of N. Note the readings M_0 and r_0 of the inductometer and rheostat respectively.

Introduce N into the circuit, and balance afresh by adjusting M and r. Call the new readings M_1 and r_1.

$$\text{Then the self-inductance of N} = \frac{R+S}{S}(M_1 - M_0)$$

$$\text{Effective resistance of N} = r - r_1$$

6. Electrical Properties of Dielectrics. Dielectric Strength.—We have next to consider the special properties of dielectrics, especially those which are important in connection with high frequency phenomena.

When a dielectric or insulator is subjected to electric force, it has produced in it electric strain or electric displacement, just as a ferromagnetic body, when submitted to magnetic force, has the state called magnetization produced in it. There is, however, a great physical difference between the two phenomena. If the electric force rises beyond a certain limit, the dielectric is mechanically ruptured or destroyed at some place, and this is accompanied by a transformation of some, at least, of the potential energy of the electric strain into heat and light or mechanical motion. In the case of liquids and gases, the wound so created is self-healing, and the dielectric restores itself at that point to the original state as soon as the electric force is diminished. In solid this, however, is not done, so that the result of the operation is to leave a puncture, or hole. The electric force corresponding to which this rupture or puncture takes place is called the *dielectric strength* of the insulator, and is measured in absolute units of electric force, or in its equivalent in volts or kilovolts per centimetre. It is convenient sometimes to state it in *volts per millimetre*, since the thickness of layers of dielectric used is generally expressed in millimetres. Since one electrostatic unit of potential in C.G.S. measure is equal to 300 volts, we convert kilovolts per centimetre into its equivalent electric force expressed in electrostatic units by multiplying by 3·333.

This dielectric strength depends upon (1) the thickness of the dielectric, thin layers being apparently stronger than thick; (2) it varies with the form of the conducting surfaces opposed, and (3) with the manner in which the electric force is applied, that is, whether gradually, suddenly, steadily, or periodically varying.

According to the investigations of C. Baur, every dielectric, whatever its thickness, requires a certain voltage to puncture it, which is proportional to $t^{\frac{2}{3}}$, where t is the thickness.

Hence, if V is the puncture voltage—

$$V = Ct^{\frac{2}{3}}$$

where C is some constant.[26]

The above formula may be put in the form—

$$\frac{V}{t} = \frac{C}{\sqrt[3]{t}}$$

Hence the dielectric strength $\left(\frac{V}{t}\right)$ should vary inversely as the cube root of the thickness. Therefore, according to this formula, to puncture a sheet of dielectric 9 mms. thick would require only half the voltage per millimetre that is necessary to puncture a sheet of the same dielectric 1 mm. thick. In other words, a thin sheet of any dielectric is relatively stronger than a thick one of the same material.

This rule, however, must be accepted with great limitations. The puncture voltage is very largely determined by the state of the surface of the dielectric. Nevertheless, the above statement holds good approximately for a large number of solid, and liquid, and gaseous dielectrics.

A very extensive set of experiments on dielectric strength has been described by Mr. T. Gray.[27] He used alternating electromotive forces of simple sinoidal form, and a frequency of 133 periods per second. The discharges were taken between the curved surfaces of two polished discs of copper, which were portions of spheres 70 cms. in diameter, all edges being rounded. He tested the dielectric strength of air, various oils, and solid dielectrics, and states the results in *kilovolts per centimetre.*

His experiments support the experience that, generally speaking, the *apparent dielectric strength* of a thin layer of a dielectric is greater than that of a thicker one.

Gray found that rupture voltage of a sheet of dielectric under an alternating electromotive force of simple sine form is identical with that due to a steady electromotive force having the same value as the maximum of the alternating force. Hence, in stating the dielectric strength in kilovolts per centimetre, the

[26] See *The Electrician*, 1901, vol. 47, p. 758; or *Science Abstracts*, vol. iv. p. 1064.
[27] See *Physical Review*, vol. vii. p. 199.

values given below are those corresponding to *the maximum value* of the alternating electromotive force employed, this maximum being calculated from the root-mean-square (R.M.S.) value observed on the voltmeter at the moment of rupture.

In the case of the alternating electromotive force used by him, this maximum value was equal to the R.M.S. value multiplied by 1·312. His results for air confirm those of previous observers. Lord Kelvin established long ago the fact that the electric force required to produce a very short spark in air between slightly rounded metallic surfaces was greater than that required to produce a longer one.[28] Mr. Gray's results for the dielectric strength of air are as follows :—

Air at Normal Pressure and Temperature.

Thickness of layer of air in centimetres.	Dielectric strength in kilovolts per centimetre.
0·02	57·5
0·04	52·5
0·06	49·5
0·08	46·2
0·10	43·6
0·20	37·8
0·40	34·5
0·60	32·7
0·80	31·1
1·0	29·8
1·20	28·8
1·40	28·8
1·60	27·4

Hence to produce a spark 1 cm. in length in air requires about 30,000 volts.

The apparent dielectric strength of air decreases, therefore, slightly with increasing thickness, and, according to Mr. Gray, ultimately it reaches some value not far from 24 kilovolts per centimetre, or 80 C.G.S. units of electric force in electrostatic measure. On this matter, however, the reader is referred to some remarks on a later page (p. 146) concerning the true dielectric strength of air.

Similar results were obtained by Gray in the case of glass. Employing a variety of glass called *crystal glass*, used for lantern slides, he found the dielectric strength for various thicknesses to be as follows :—

Crystal Glass.

Thickness in centimetres.	Dielectric strength in kilovolts per centimetre.
0·1	285
0·2	253
0·3	224
0·4	200
0·5	183
0·6	168

For window glass 0·2 cm. thick, he found the dielectric strength to be 160 kilovolts per centimetre.

He also made tests with sheet ebonite, india-rubber, mica, and micanite, with results as follows :—

Ebonite.

Thickness in centimetres.	Dielectric strength in kilovolts per centimetre.
0·093	538
0·186	434

India-rubber Sheets.

0·135	476
0·270	318

[28] See Lord Kelvin, "Reprint of Papers on Electrostatics and Magnetism," p. 258, or *Proc. Roy. Soc.*, vol. x. p. 326, February 23, April 12, 1860, "Measurement of the electromotive force required to produce a spark in air between parallel metal plates at different distances."

Mica.

Thickness in centimetres.	Dielectric strength in kilovolts per centimetre.
0·001	2000
0·010	1150
0·02	950
0·05	750
0·10	610

Micanite.

0·05 } 0·10 }	400

Paper of various kinds impregnated with paraffin wax possessed dielectric strengths as follows :—

Material.	Thickness in centimetres.	Dielectric strength in kilovolts per centimetre.
Thin printers' paper	0·012	400
Tissue paper	0·009	510
Manilla paper	0·018	430
American linen paper	0·013	640
Typewriter linen paper	0·014	540

Fuller board, a kind of vulcanized fibre, showed a dielectric strength of 205, 192, and 169 kilovolts per centimetre of thickness of 0·05, 0·1, and 0·2 cm.

Oils of various kinds were tested in layers having thickness from 4 to 8 mms., and the following values for the dielectric strength were found, though somewhat variable :—

Oils.

	Dielectric strength in kilovolts per centimetre.
Light mineral lubricating oil	48
Sperm oil	52
Vaseline oil	60
Cotton-seed oil	67
Olive oil	70
Linseed (raw) oil	83
,, (boiled) oil	85

Other observations by the same author seemed to show a decrease in dielectric strength with thickness in the case of oils. Thus, for instance, he found for vaseline oil the following values of the dielectric strength :—

Vaseline Oil.

Thickness.	Dielectric strength in kilovolts per centimetre.
8 mm.	91
1 ,,	131

Paraffin Oil.

Sp. gr. 0·28. Varied between 64 and 101 kilovolts per centimetre.

Experiments by other observers substantially confirm the above results. T. W. Edmondson has measured the dielectric strength of air, and finds that his observations agree fairly well with the formula—

$$V^2 = at + bt^2$$

where t is the spark length or thickness in millimetres of the layer of air ruptured,

and V is the spark potential in (C.G.S.) electrostatic units, whilst a and b are constants varying with the diameter of the spark balls as follows [29] :—

Diameter of spark balls.	a	b
0·5 cm.	235·13	82·25
1·0 ,,	186·35	99·42
2·0 ,,	144·41	114·49
3·0 ,,	49·41	144·71

If we reckon the spark length in centimetres and spark potential in kilovolts, Edmondson's formula reduces to the following form :—

Apparent dielectric strength of air in kilovolts per centimetre $=3\sqrt{b+\frac{a}{10t}}$ where t is the thickness in centimetres. This gives a dielectric strength of 33 $\frac{\text{kv.}}{\text{cm.}}$ for 1 cm. thick, which agrees fairly well with observations by Baur, Gray, and others, and it shows that the apparent dielectric strength decreases with increasing thickness, and finally reaches a limit $3\sqrt{b}$. The formula, however, must not be extrapolated beyond the limits of observations.

Edmondson also gives a series of useful curves for the dielectric strength of various oils, all of which show a slight increase of dielectric strength with decrease of thickness of film punctured.

When using as discharge surfaces brass balls 2·6 cms. in diameter, and within the limits 2 to 10 mms. for sparking distance, a simple linear formula for the spark potential can be conveniently employed, viz.—

$$V = 10{\cdot}2t + 7{\cdot}07$$

where V is the spark potential in electrostatic (C.G.S.) units and t is the spark length in millimetres in air at normal pressure and temperature.[30] This is transformed into measurement in volts by multiplication by 300.

Hence—

Spark voltage in air at normal pressure = 2121 + (3060 × *spark length in millimetres*)

or $\left.\begin{array}{r}\textit{Apparent dielectric strength of}\\ \textit{air in kilovolts per centimetre}\end{array}\right\} = 30{\cdot}6 + \dfrac{21}{\textit{spark length in millimetres}}$

M. O'Gorman has given values for the dielectric strength of certain insulating materials used in cable manufacture as follows [31] :—

Material.	Dielectric strength in kilovolts per centimetre.
Gutta-percha	109
Paraffin wax (solid)	130 to 270
,, (melted)	56
Vaseline	91
Resin oil	279 to 1350

There are so many circumstances which cause variation in the dielectric strength of insulators that the figures given by different observers are not in very close agreement. C. Baur has given the results of some measurements on various dielectrics as follows [32] :—

[29] See *Physical Review*, 1898, vol. vi. p. 65.

[30] The above formula embodies results obtained in the physical laboratory of University College, London.

[31] See *Journal of the Institution of Electrical Engineers*, vol. 30, p. 680, Appendix VIII., O'Gorman, "Insulation on Cables."

[32] See *The Electrician*, 1901, vol. 47, p. 758, or *Science Abstracts*, vol. iv. p. 1067.

Dielectric.	Dielectric strength in kilovolts per centimetre.	
Dry air	33	
Vulcanized india-rubber	100	
Mica	580	
Empire cloth	125	These are various fibrous materials impregnated with oils or resins.
Fuller board	190	
Impregnated jute	220	

The practical conclusion to be drawn from the above-described experiments is that in air at ordinary pressure and temperature, and for metallic spark balls a few centimetres in diameter, electric sparks pass and rupture the air when the electric force in the gap between the balls varies from 4500 to 3000 volts per millimetre of spark length, as the spark length increases from 1 mm. in length and upwards.

To create a spark in air of 1 cm. in length between such surfaces requires a steady voltage of about 30,000 volts, or an alternating voltage of sinoidal form having an effective or R.M.S. value of nearly 21,000 volts, and at the same rate for greater spark lengths.

The whole subject of the dielectric strength of air has been carefully rediscussed by Dr. A. Russell (see *Proc. Phys. Soc. Lond.*, November 1905) in a valuable paper. He points out that the results of various observations with different-sized discharge balls differ considerably. It is a well-known fact, as first shown by C. F. Varley in 1871 (*Proc. Roy. Soc.*, January 12, 1871), that there is a minimum sparking potential in air and other gases below which it is impossible to obtain a discharge. For air at normal pressure and temperature this is not far from 790 volts (see the Hon. R. J. Strutt, "On the Least Potential Difference Required to Produce Discharge through Various Gases," *Phil. Trans. Roy. Soc. Lond.*, 1899-1900, vol. 193A, p. 377). Hence, if there is a potential difference, V kilovolts, between two metal balls, we may say that the effective potential difference is in fact (V − 0·79) kilovolts. Kirchhoff published, in 1860, a valuable paper in *Crelle's Journal*, entitled, "Über die Vertheilung der Elektricität auf Zwei leitenden Kugeln," in which he shows how to express the maximum electric force in the form of an infinite series. Dr. Russell has provided a simple proof of Kirchhoff's formula by the method of electric images.

When the discharge balls are of equal size and at equal and opposite potentials, $+\frac{V}{2}$ and $-\frac{V}{2}$, the shortest distance between them being x, Dr. Russell shows that the maximum electric force is expressed by $\frac{Vf}{x}$, where V is the potential difference of the balls, and f is a function of their diameter $2a$ and distance x. He gives the following values of f for various values of $\frac{x}{a}$:—

$\frac{x}{a}$	f	$\frac{x}{a}$	f
0·0	1·000	1·5	1·559
0·1	1·034	2·0	1·770
0·2	1·068	3·0	2·215
0·3	1·103	4·0	2·678
0·4	1·138	5·0	3·151
0·5	1·174	6·0	3·631
0·6	1·209	7·0	4·117
0·7	1·246	8·0	4·601
0·8	1·283	9·0	5·095
0·9	1·321	10·0	5·586
1·0	1·359	100·0	50·51
		1000·0	500·50

If, then, V is measured in kilovolts, the true dielectric strength of air is given by the maximum value of the electric force, viz. R_{max} ; and—

$$R_{max} = \frac{(V - 0{\cdot}79)}{x} f \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

The fraction 0·79 is the value in kilovolts of the potential difference, which must be exceeded before any spark begins, and the quantity f in the above expression (tabulated above) is a factor by which the average effective kilovolts per centimetre must be multiplied to give the maximum electric force. From a discussion of various results by different experimentalists, Dr. Russell shows that for air at normal pressure and temperature *the true dielectric strength* lies between 38 and 39 kilovolts per centimetre, or in electrostatic units to a force of 127. The true dielectric strength of air is therefore expressed by a number about one-third larger than the average kilovolts per centimetre for sparks 1 cm. in length taken between balls 1 or 2 cms. in diameter.

In the construction of high-tension condensers a liberal margin should be allowed as a *factor of safety*, and the working pressure should not be more than a quarter of the rupture voltage.

Thus, in the case of glass, Gray's experiments show that for a thickness of 2 to 3 mms. the dielectric strength is from 253 to 224 kilovolts per centimetre. This means that a voltage of 62,000 volts will pierce a plate of glass 2 mms. thick. If we construct a condenser of glass plates 0·1 inch or 2·5 mms. thick, the safe working voltage would be about one-third of the above breaking voltage, viz. 20,000 volts, equivalent to a 6- or 7-mm. spark in air.

The above is in accordance with practical experience. Ebonite and mica or micanite have undoubtedly greater dielectric strength than glass. Ebonite is about twice, and mica is about three times, as strong : whilst micanite, which consists of plates of mica stuck together with shellac, has a still greater dielectric strength.

Many circumstances, however, contribute to affect the dielectric strength. J. Kiessling and B. Walter have called attention to the fact that if a tube of dielectric is partly immersed in oil and electric stress applied to the material it punctures at the surface of the oil.[33] In the same way, if a drop of melted paraffin wax is placed on a sheet of glass, and this is afterwards submitted to electric strain between electrodes, the puncture takes place at the edges of the wax. If, however, a needle prick is made in the wax, puncture will more readily occur through this hole. Plates of ebonite coated with tinfoil on both sides and placed in oil for use as high-tension condensers are generally found to puncture near the edges of the tinfoil if an excessive voltage is used. The electric force has the highest value at the edges of the metal plates, and the puncturing is determined, not by the mean but by the maximum electric force acting on the dielectric.

Thus a sheet of ebonite 4·2 mms. thick withstood a voltage equal to a 50-cm. spark in air. This is equivalent to a dielectric strength of 3000 kilovolts per centimetre. If, however, a drop of wax was placed on the surface, the ebonite gave way under a stress of about half the above value. If a needle hole was made in the wax the ebonite was pierced at that spot by a force of 600 kilovolts per centimetre.

Hence the authors conclude that any scratches or flaws on the surface of a sheet of dielectric greatly reduce its strength. They state that bubbles in glass, as long as they do not open upon the surface, do not bestow particular weakness at that point. These experiments show that in the case of sheets of dielectric to be used for making high-tension condensers it is important to avoid the slightest pricking or cracking of the surface.

In the case of gases, pressure exercises a most marked effect on their dielectric strength.

Wolf has given a formula for the electric force in electrostatic units required to create a discharge in air under a pressure of P atmospheres between metal balls 10 cms. in diameter.[34] If E is this electric force, then—

$$E = 107P + 39 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

[33] *Ann. der Physik*, June 4, 1903, vol. 11, p. 570, or *Science Abstracts*, vol. vii. A, p. 603.
[34] *Wied. Ann.*, 37, 1889, p. 306.

Thus if P=1, then E=146 E.S. units, or 146×300=43,800 volts per centimetre. Accordingly the dielectric strength at normal pressure, according to Wolf, is 43·8 kilovolts per centimetre, which is higher than the value obtained by other observers.

Formula (80) is said to hold good up to 5 atmospheres. Hence, if P=5, then E=574, and this corresponds to a dielectric strength of 172·2 kilovolts per centimetre. The dielectric strength is thus nearly proportional to the pressure, and the potential difference required to produce a spark between rounded metallic surfaces varies almost as the distance between them and as the pressure, *i.e.* upon the mass of gas lying between the electrodes.

In forming oscillating circuits by joining in series a spark gap, condenser, and inductance, it is always prudent to consider what spark length is permissible,

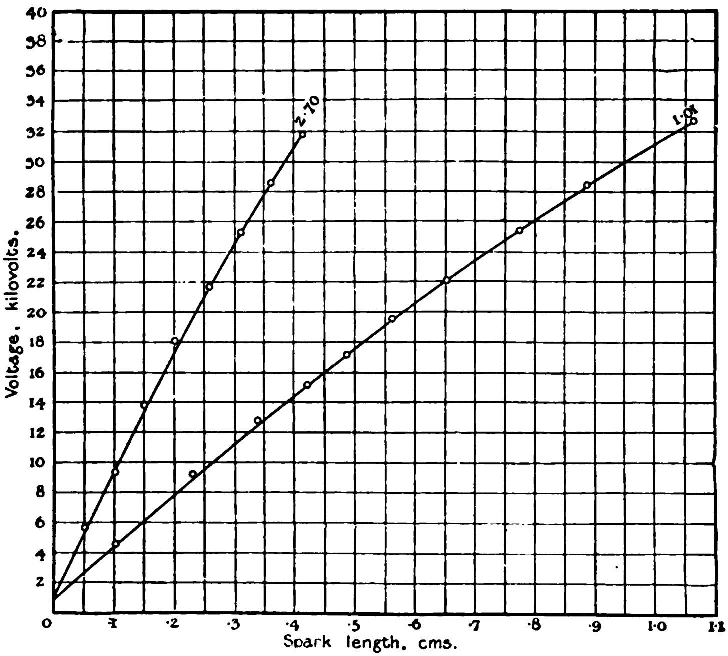


Fig. 22.—Spark Voltages for 2·54 cms. Spark Balls with Alternating Voltage.

having regard to the thickness and nature of the dielectric used. Few glass Leyden jars will bear more than 20,000 volts without risk of puncture. Hence this corresponds to a spark length of 7 or 8 mms. in air. If, then, glass-plate condensers or Leyden jars are used and larger spark gaps are required, the jars must be placed in series in sufficient number to bear the strain. Thus, if the capacity of a single jar is required, but a spark length of 1·5 cms., four jars should be arranged, two in parallel and two in series, and so on.

A very useful series of experiments has been carried out by Mr. E. A. Watson on the dielectric strength of compressed air.[35] He employed spark balls of various sizes and various spark lengths, and measured the spark voltages in dry air of various pressures with alternating and also with direct currents. The two curves in Fig. 22 show the variation of spark voltage (alternating) with spark length for spark balls 2·54 cms. (=1 inch) in diameter in air under 1·01 atmo-

[35] *Journal of the Institution of Electrical Engineers*, vol. 43, p. 113, 1909.

sphere or normal pressure, and air at 2·7 atmospheres pressure. It will be seen that the spark voltage is more than doubled.

In Fig. 23 are shown curves for the same sized balls taken with alternating voltages and for pressures from 3·25 to 7·36 atmospheres.

The lines in Fig. 24 show the spark voltages for various spark lengths with the same sized balls but with direct current.

His results show that the dielectric strength in kilovolts per cm. varies almost proportionally to the pressure, and more precisely may be expressed by the formula—

Dielectric strength = 20 + 25·6 times pressure in atmospheres . . . (81)

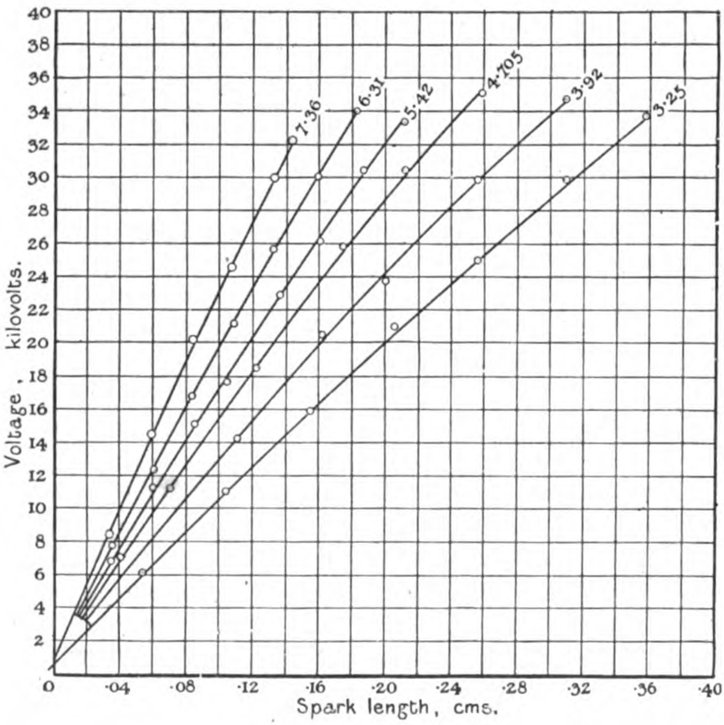


FIG. 23.—Spark Voltages for 2·54 cms. Spark Balls with Alternating Voltage.

Thus for 10 atmospheres it is 276.

The dielectric strength of various oils is an important practical matter. In investigations on high frequency currents the type of condenser most convenient for the purpose consists of metal plates placed in oil, the oil forming the dielectric, and in practical radiotelegraphy when the dielectric used is glass, the glass plates having metal plates applied to both surfaces, the whole arrangement has to be immersed in oil to prevent glow discharges, or sparking over the glass margin.

Since oils differ very much in dielectric strength, and as this quality is greatly dependent on temperature, and on the presence of moisture in the oil, oils to be used for this purpose should be carefully tested for dielectric strength. This is done in the following manner. The oil to be tested should be warmed up to 70° Fahr., and placed in a perfectly dry glass beaker. A bar of ebonite which can rest across the vessel carries two rods to which metal balls 1 centimetre in diameter are attached, these balls being so set that their nearest surfaces are 2 mms.

from each other. The balls must be well below the surface of the oil. The balls are then connected to a high-tension electrostatic voltmeter suitable for measuring voltages up to 30,000 or 40,000 volts, and are also connected to the secondary terminals of a transformer which can give these voltages when fed with alternating current of simple sine form on its primary side. The secondary voltage is best regulated by inserting a variable rheostat or choker in the primary circuit. The voltage is then slowly raised until a spark passes through the oil, when the voltmeter must be read. It is not well to employ points as spark surfaces, because the breakdown voltage is greatly affected by the sharpness of these points, and it is not easy to define this sharpness so as to make the conditions of the experiment

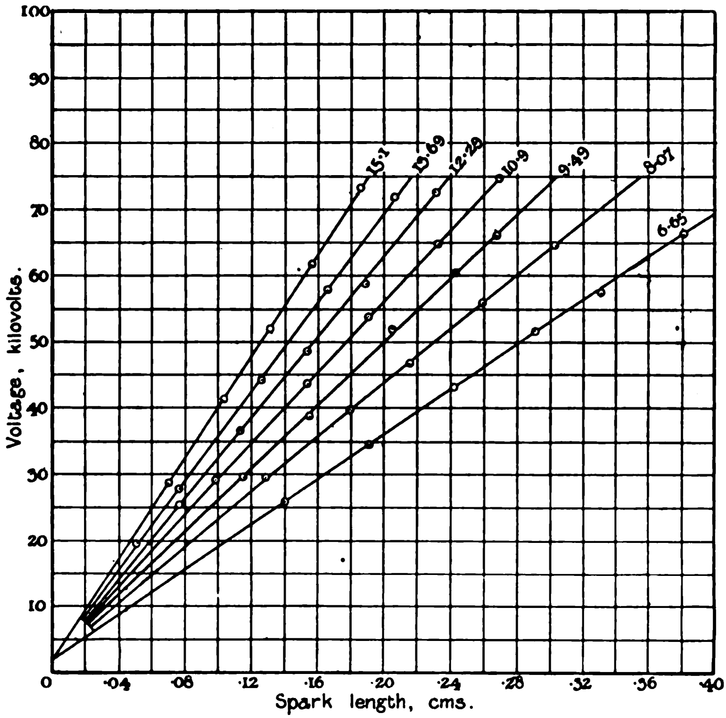


FIG. 24.—Spark Voltages for 2·54 cms. Spark Balls with Direct Currrent.

definite. On the other hand, with small spheres as spark surfaces the electric field is definite and predeterminable in all cases.

7. The Practical Measurement of the Capacity of Conductors.—If there be any two conductors, and these are respectively charged with equal quantities of electricity of opposite sign, and if a difference of potential having a value of one unit is created between them, then the quantity of electricity or the charge on either of the conductors is a measure of their *capacity* with respect to each other. If any body is charged to unit potential with respect to the earth, and all other conductors are removed to a very great distance, the charge on the conductor in question is a measure of its *capacity with respect to the earth.* The quantity of electricity which will raise the body to unit potential above the earth depends on its form and position and upon a quality of the surrounding insulator called its *dielectric constant.* We may define the dielectric constant as follows :—

If electric force acts upon a dielectric it produces in it electric displacement.

If a uniform electric force acts upon a dielectric and produces in it uniform electric strain or displacement, the numerical ratio of the displacement through unit area to the force, or of the electric strain to the electric stress, is called the *dielectric constant* of this insulator. The name is not well chosen, because the so-called constant is far from being constant, but varies with temperature, voltage, and frequency, and it would be better to coin another name.[36] The dielectric constant bears the same relation to electric strain and stress or electric force and displacement that magnetic susceptibility bears to magnetic force and magnetization. The dielectric constant may otherwise be defined as the number which expresses the ratio in which the capacity of any air condenser is increased if the air is wholly replaced by the dielectric in question.

If C is the capacity of any air condenser when its plates are charged to a potential difference, V, then if Q represents the quantity of electricity stored in the condenser, we have—

$$Q=CV$$

Hence $C=\frac{Q}{V}$, and we may define the capacity of a conductor as the ratio of its charge to its potential. If the air is wholly replaced by some other insulator and capacity becomes K times C, or KC, then K is the dielectric constant of the insulator.

The process of determining the dielectric constant generally consists in measuring the capacity of some form of air condenser and then measuring it again, when for air we have substituted the insulator in question.

It was in this manner, and by the increase in capacity so observed, that Faraday made the first measurements of dielectric constant.[37]

It is not necessary here to consider all the numerous methods for determining dielectric constants which have been proposed, nor the whole of the processes by which electric capacity can be determined. These are explained in text-books on physics and electrical measurement.

It is, however, desirable to explain rather fully one method of measuring small capacities at low or moderate frequencies, which the author, in conjunction with Professor W. C. Clinton, has perfected, as it affords a means of making many of the capacity measurements which are required in connection with high frequency electric current investigation or Hertzian wave telegraphy.

If we charge an insulated conductor to a potential V, and measure the charge Q so given, then the ratio $\frac{Q}{V}$, when Q and V are measured in consistent units, gives us the capacity of the conductor.

If that capacity is small, we may repeat the charging n times a second and measure the quantity nQ. Suppose, then, we discharge this quantity nQ in one second through a galvanometer. It is equivalent to a current nQ in its effect on the instrument. Hence, if we have the means to continue this process uniformly, and can calibrate the galvanometer, we have all the information necessary for measuring the capacity.

Many methods have been suggested for conducting the above operation, but there are practical difficulties in it which have only been overcome by the invention of a thoroughly effective rotating commutator, designed to effect this process of charging a conductor with a known potential, then sending the charge through a galvanometer, and repeating the process uniformly a known number of times per second.[38]

The details of this commutator are shown in Fig. 25.

The instrument consists of a continuous current electric motor of $\frac{1}{6}$ h.p., but

[36] The term *permittance* has been employed by Mr. Oliver Heaviside to signify that which is generally called capacity, and the word *permittivity* to denote the same quality which the terms dielectric constant, or specific inductance capacity, are generally used to express.

[37] See Faraday's "Experimental Researches in Electricity and Magnetism," vol. i. ser. xi. § 1187.

[38] See J. A. Fleming and W. C. Clinton, "On the Measurement of Small Capacities and Inductances," *Proc. Phys. Soc. Lond.*, **1903**, vol. **18**, p. **386**; also *Phil. Mag.*, May **1903**, vol. v. ser. 6, p. **493**.

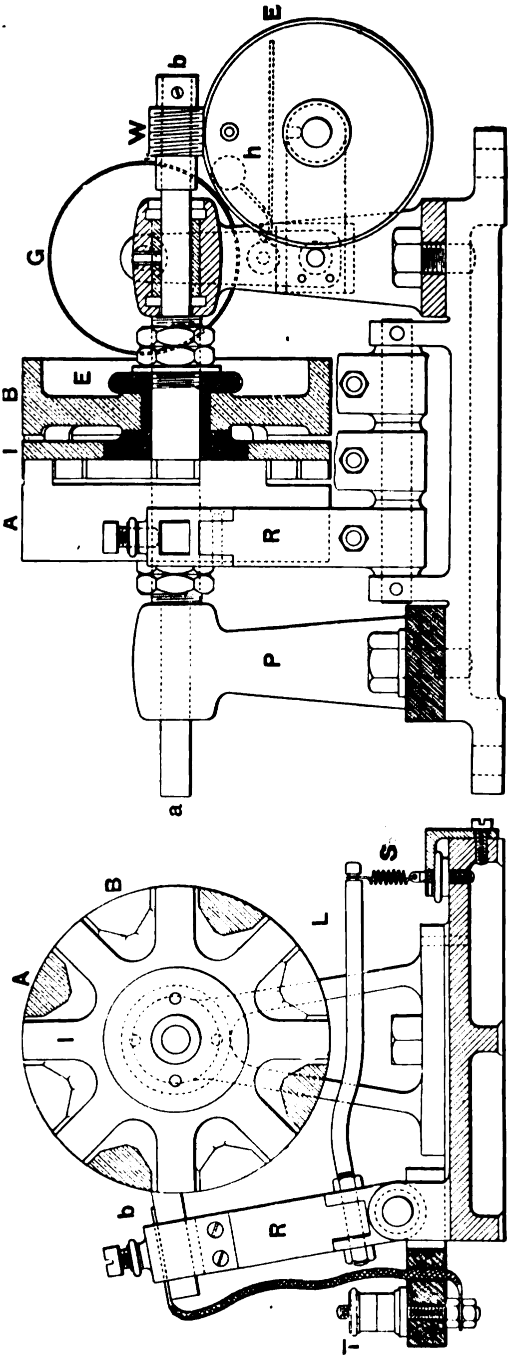


FIG. 25.—Fleming and Clinton Revolving Commutator for the Determination of Small Capacities.

for certain purposes, and where very small capacities have to be measured, it is preferable to employ a motor of $\frac{1}{2}$ h.p. This motor (not shown in the diagram) is bolted down upon a baseboard, and has connected with it a starting and regulating resistance. The motor is preferably 100 or 200 volt shunt-wound motor. To the shaft of this motor is connected by a flexible coupling the commutating arrangement (shown in the diagram in Fig. 25), the function of which is to charge the capacity or condenser to a given voltage, and then discharge it through a galvanometer, repeating this process four times in each revolution of the motor. This commutator is fixed on a shaft, carried in well-lubricated bearings, supported on two small A frames, P (see Fig. 25). On this shaft are held, by means of ebonite brushes and washers, three gunmetal discs or wheels, of which the centre one, I, is in shape like an eight-rayed star, whilst the two outer ones, A and B, are like crown wheels, each having four teeth. The three wheels are so set on the shaft that the teeth or projections of each of the two outer wheels interlock or fall in the space between the teeth of the other, whilst the radial teeth of the intermediate wheel occupy the intervals between the teeth of the two outer wheels. The developed surface of this triple wheel is shown in Fig. 26. The whole outer surface is turned true, and forms a barrel about 4 inches in diameter and $2\frac{1}{2}$ inches wide. On this barrel rest three brass gauze brushes, *b*, which are carried in well-insulated brush-holders, R, and

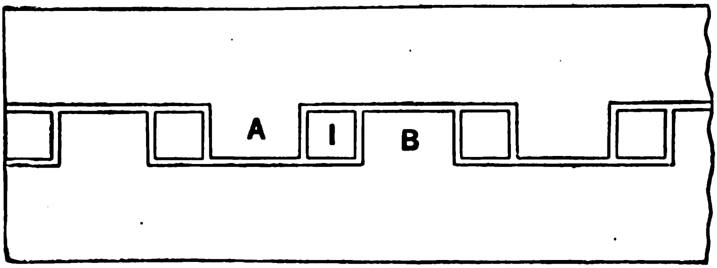


Fig. 26.

by means of three springs and levers, L, the brushes are firmly pressed against the barrel, the two outer brushes resting on the continuous portions or flanges of the two outer wheels and B, and the middle brush occupies the centre line and makes contact either with the wheel A or wheel B, or with the intermediate wheel I, according to their position. It will be seen, then, that as the commutator runs round, the middle brush is alternately brought into metallic connection with first one and then the other of the two brushes on either side. The function of the middle wheel, I, is to afford a stepping-piece to prevent any shock or jar as the middle brush passes over from one connection to the other. It also prevents the middle brush from short-circuiting the two outer brushes at any time. If, then, one terminal of the galvanometer is connected to the brush pressing against the wheel A, and one terminal of a battery is connected to the brush pressing against wheel B, and one terminal of a condenser is connected to the middle brush, the other terminals of the battery, galvanometer, and condenser being connected together, it will easily be seen that as the commutator rotates, the condenser is first charged at the battery, and then discharged through the galvanometer. It is convenient to employ a speed of 1200 and 1700 revolutions per minute. To count the rotations of the commutator, a worm, W, on the shaft drives a wheel, G, of such gear that the latter makes one revolution for every hundred revolutions of the commutator. This wheel carries a pin, which at each revolution causes a hammer, *h*, to strike a gong, E. Every hundred revolutions, therefore, of the motor or commutator the gong gives one stroke, and by means of a stop-watch it is easy to take the time of ten strokes of the gong—in other words, to ascertain the time in seconds of a thousand revolutions of the motor,

and therefore of the number of commutations per second. In the case of the motor described, 1000 revolutions take place generally in 40 seconds, which is at the rate of 1500 per minute, and therefore corresponds with 100 commutations of the condenser per second.

Various methods of making the rubbing contacts have been used, and brass gauze brushes found to be the best. Carbon brushes were tried at one time, but were not so good as the brass gauze. It is essential that the commutator surface should be kept bright and clean, and the brass gauze brushes do this themselves when adjusted to the right pressure.

Associated with this commutator, it is best to make use of a galvanometer of the movable coil type. By the aid of this instrument, given a source of constant voltage by which the motor can be driven steadily, such as a secondary battery, the measurement of small capacities becomes an exceedingly easy matter.

In the case of most movable coil galvanometers the scale deflections are by no means proportional to the current, and hence when measuring a séries of capacities it is desirable afterwards to plot a calibration curve of the galvanometer scale, from which the condenser currents can be read off directly in microamperes. This, however, is always easily accomplished. In addition, we have to measure the potential of the discharging battery. For most practical purposes this can be done by a Weston voltmeter.

Then let V represent the voltage of the battery charging the condenser or aerial, C the capacity of the condenser in microfarads, A the current in microamperes through the galvanometer, and n the number of charges per second, then—

$$A = nCV$$

or

$$C = \frac{A}{nV}$$

To determine the numerical value of the capacity we have, therefore, to standardize the galvanometer or determine the ampere value of the steady current which will make the same deflection. This can be accomplished by shunting the galvanometer with a known small resistance, placing the shunted galvanometer in series with another high resistance, and then applying to the terminals of this circuit a coil of known electromotive force. If a megohm resistance is available it is generally possible, by placing this in series with the galvanometer, to standardize the galvanometer off the same battery used to charge the condenser. In this case no voltage measurement is necessary.

If the repeated discharges of the condenser of capacity C microfarads create on the galvanometer a steady deflection θ, then if the battery has an E.M.F. V, which can be considered to remain constant during the two tests, we can apply the same battery to the terminals of the shunted galvanometer of resistance G, which is placed in series with the high resistance R, and we can alter the shunt S until the same deflection θ is obtained.

Then we have—

$$\frac{nVC}{10^6} = \frac{V}{R + \frac{GS}{G+S}} \times \frac{S}{G+S}.$$

Hence

$$C = \frac{S10^6}{nR(G+S) + nGS} \quad \ldots \ldots \quad (82)$$

This determines the capacity in terms of the resistances and the frequency of the commutator.

By the aid of the above-described apparatus the measurement of very small capacities becomes as simple as the measurement of small resistances.

Measurements must always be made by difference, and account taken of the capacity of the commutator itself and of the connecting leads. Thus, for instance, if the capacity of a Leyden jar has to be measured, the jar is connected as shown in Fig. 27, the outer surface to the common terminal of the battery and galvanometer, and the inner one to the middle brush of the commutator. The com-

mutator is then run up to speed, and the speed measured by taking the time with a stop-watch of 1000 revolutions or ten bell strokes. If the galvanometer deflection remains steady, this shows the speed is uniform. When the deflection has been measured the jar is removed, and the leads open-circuited. Another run is then taken, and the galvanometer deflection measured. The value of the current A to be inserted in the formula $C=\frac{A}{nV}$ is the difference of the currents in microamperes corresponding to these two deflections.

If the capacity being measured is that of an insulated body, such as an aerial wire or other object, then it is connected to the middle brush of the commutator, and the common terminal of the battery and galvanometer must be "earthed." The same procedure as above described must be followed to eliminate the capacity of the commutator and leads.

To employ the instrument for the measurement of dielectric constants, some form of air condenser must be provided in which the dielectric can be substituted

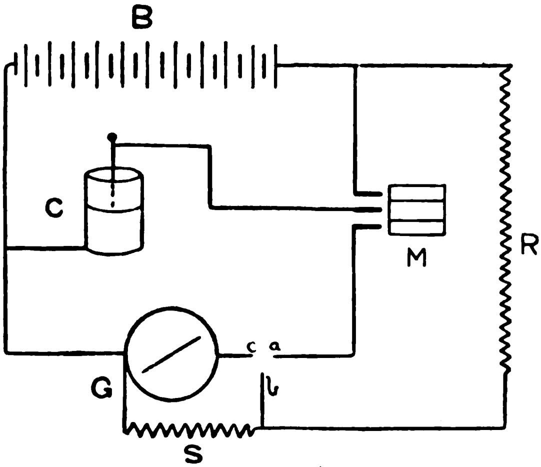


FIG. 27.--B, Battery; C, condenser; G, galvanometer; M, commutator; R, standardizing resistance; S, shunt; *a*, *b*, *c*, three-way plug switch.

for air and the capacity then measured. There are not many forms of condenser which can be used for this purpose.

If two insulated metal plates of area S square centimetres are set up in air parallel to each other at a distance *d* centimetres, we have an air condenser which has a certain capacity. Between the central portion of the plates the lines of electrostatic force spring straight across normally to the plates, and as far as this part of the capacity is concerned it can be circulated in electrostatic units by the formula usually given in the text-books, viz. :—

$$C=\frac{A}{4\pi d} \text{ (in electrostatic units)}$$

where A is some area of the plates less than that of their actual area S. The whole capacity cannot, however, be calculated by the simple rule. There is, in addition, a distribution of electric force at the edges, and beyond the edges of the plates in curved lines, and if the distance of the plates is large compared with their diameter, the capacity due to this part of the flux may amount to a large fraction of the total of the whole. Hence the above simple formula is far from giving the true capacity of a pair of parallel plates. In the same manner, the substitution of a sheet of dielectric of thickness *d* for the air between the plates

will not enable us to calculate exactly the dielectric constant. For such a sheet only occupies part of the space filled with lines of electrostatic force.

Kirchhoff has given[39] a formula for calculating exactly the capacity of a pair of parallel circular plates, each of radius r, placed at a distance d apart in the air, as follows :—

$$C=\frac{\pi r^2}{4\pi d}+\frac{r}{4\pi d}\left\{d\log_\epsilon\frac{16\pi r(d+t)}{\epsilon d^2}+t\log_\epsilon\frac{d+t}{t}\right\}+C' \quad . \quad . \quad . \quad (83)$$

where t is the thickness of the plates, C' is any part of the capacity which does not change with the distance d, and ϵ is the base of Napierian logarithms. Suppose, then, that we place between the plates a circular disc of any dielectric having a dielectric constant K, such disc being smaller than the plates, and having a radius r_1 and a thickness d_1. Let the plates be moved up to touch this disc, placed concentrically between them. Then the capacity of the system is given by the formula—

$$C_1=\frac{K\pi r_1^2}{4\pi d_1}+\frac{\pi r^2-\pi r_1^2}{4\pi d}\left\{d_1\log_\epsilon\frac{16\pi r(d_1+t)}{\epsilon d_1^2}+t\log_\epsilon\frac{d+t}{t}\right\}+C' \quad . \quad . \quad (84)$$

Hence by measurement of C_1 and the dimensions we can find K.

The assumption made is that the disc of dielectric does not disturb the distribution of the field outside itself, but only intensifies the field within itself in the ratio of K : 1. This assumption is not quite legitimate, but the method is approximately correct, and certainly far less incorrect than the assumption usually made, that the whole original capacity of the plates is merely increased in the ratio of K : 1 by inserting a plate of dielectric of the same size as the plates between them. The above method, using Kirchhoff's formula, was employed by Messrs. Pollock and Vonwiller in a measurement of the dielectric constant of plate glass.[40]

If we put $t=0$ in Kirchhoff's expression, we have the capacity of two infinitely thin circular discs at a distance d apart. It reduces them to—

$$C=\frac{\pi r^2}{4\pi d}+\frac{r}{4\pi}\log_\epsilon\frac{16\pi r}{\epsilon d}$$

or

$$C=\frac{\pi r^2}{4\pi d}\left(1+\frac{d}{\pi r}\log_\epsilon\frac{16\pi r}{\epsilon d}\right)$$

The second term in the bracket, therefore, represents that fraction by which the capacity of the real condenser exceeds that of the ideal or text-book condenser, in which the electric force is considered simply to pass normally from plate to plate. If the plates are 10 cms. in radius and 1 mm. apart, then $\frac{r}{d}=100$, and $\frac{d}{\pi r}\log_\epsilon\frac{16\pi r}{\epsilon d}$ is nearly $\frac{1}{40}$, so that the real capacity exceeds the capacity calculated from the formula $\frac{\pi r^2}{4\pi d}$ by only $2\frac{1}{2}$ per cent.

We are led, therefore, to this conclusion, that if the circular condenser plates are very large compared with their distances apart, we may calculate approximately the capacity of the condenser by the simple formula—

$$C=\frac{S}{4\pi d\times 9\times 10^5}\text{ microfarads} \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

where S is the area of one plate in square centimetres, and d is their distance apart in centimetres, $\frac{d}{S}$ being very small.

On the other hand, to abolish the irregular edge distribution, we may make use

[39] G. Kirchhoff, *Gesammelte Abhandlungen*, p. 112, "Zur Theorie des Condensators." See equation (18).

[40] See Pollock and Vonwiller, "Some Experiments on Electric Waves in Short Wire Systems and on the Specific Inductive Capacity of a Specimen of Glass," *Phil. Mag.*, 1902, vol. 3, ser. 6, p. 586.

of a *guard plate*. One of the condenser plates has in it a large aperture which is nearly filled by another insulated smaller plate. The two last plates are fixed in the same plane.

The outer margin of the smaller plate is called the guard plate. When the small plate and its guard plate are charged to one common potential, differing from the potential of the opposed larger plate, the lines of electrostatic force spring straight across between the two plates, and the capacity of the small plate with respect to the opposed one is very nearly given by formula $\frac{S}{4\pi d}$, where S is the area of the small plate and d its distance from the other. There is, however, some difficulty in applying the charge and discharge method to this arrangement, as the guard plate must be discharged at the same instant as the guarded plate, but not through the galvanometer.

In place of plates we may employ concentric cylinders. If R_1 be the inside diameter of the outer cylinder, and R_2 the outside diameter of the inner cylinder, and l the common length of both cylinders, all measured in centimetres, then the capacity in electrostatic units with air as dielectric is given by—

$$C = \frac{l}{2 \log_e \frac{R_1}{R_2}} \quad . \quad . \quad . \quad . \quad . \quad (86)$$

provided we neglect the distribution of force at the ends of the cylinders. This can only be legitimately done when their length is very great compared with the difference between R_1 and R_2.

If a substance having a dielectric of constant K is substituted for air, the capacity becomes—

$$C = \frac{Kl}{2 \log_e \frac{R_1}{R_2}} \text{ (electrostatic units)}$$

$$\text{or} \qquad C = \frac{Kl}{4144680 \,(\log_{10} R_1 - \log_{10} R_2)} \text{ microfarads} \quad . \quad . \quad . \quad (87)$$

There is, however, a distribution of electric force in curved lines at the ends of the cylinders, which in the case of short cylinders render the above formula inapplicable.

The only form of condenser in which this edge effect is absent is in the case of concentric spheres. If a solid sphere of metal of radius R_2 is supported concentrically with a hollow sphere of inner radius R_1, the dielectric being air, it is easy to show that the capacity in electrostatic units is given exactly by the expression—

$$C = \frac{R_1 R_2}{R_1 - R_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

If we substitute for air any other insulator quite filling up the space between the spheres and the capacity becomes K times as great, then K is the dielectric constant of that insulator. A form of double cone condenser was designed by the author for certain experiments on the dielectric constant of liquids or frozen liquids. It consists of two coaxial cones of metal (see Fig. 28), which can be adjusted to have any desired interval between the inside of one cone and the outside of the other. An ebonite or glass peg at the bottom holds the cones coaxially. This interspace can be filled with liquid and the capacity of the condenser so formed taken.[41]

There are several simple cases of conductors insulated in space in which the capacity can be calculated from the dimensions of the conductor. Thus if a metal sphere is hung up in infinite space, that is, all other conductors removed by a very great distance, the capacity of the sphere in electrostatic units is numerically equal to its radius in centimetres. Since 1 mfd. is equal to 900,000 electrostatic units,

[41] See Fleming and Dewar on "The Dielectric Constant of Certain Frozen Electrolytes at and above the Temperature of Liquid Air," *Proc. Roy. Soc. Lond.*, 1897, vol. 61, p. 299.

the capacity C of a sphere of radius R centimetres hung up in a medium of dielectric constant K, all other bodies being very far off, is given by the rule—

$$C = \frac{KR}{9 \times 10^5} \text{ (microfarads)}$$

On the other hand, we must not regard an ordinary sized room as representing infinite space electrically speaking. If a sphere 1 metre in diameter is hung up in a room 30 feet by 30 feet by 15 feet, the real capacity of the sphere would be about 10 per cent. greater than that given by the above rule.

Another useful case is that of a flat circular disc. The capacity of a disc of diameter d centimetres insulated in free space is $\frac{d}{\pi}$ electrostatic units, or $\frac{d}{\pi \cdot 9 \cdot 10^5}$ mfds.

A circular disc about 5 feet in diameter insulated by being hung up by a silk string in the centre of a large room has a capacity about 10 per cent. more than that given by the above formula. In measuring such very small capacities a convenient unit is the micro-microfarad (mmfds.), which is one-millionth of a microfarad. Hence a thin circular disc of which the diameter is 28·27 cms. $= 9\pi$ cms., has a capacity in free space of 10 mmfds. Hung up in a large room, it would really have about 11 mmfds. capacity.

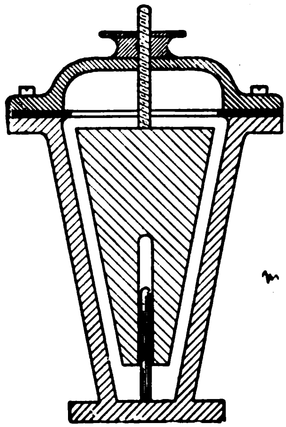


FIG. 28.—Cone Condenser.

Another important case is that of a thin long circular-sectioned wire suspended in space. Such a wire may be taken to be a limiting form of an ellipsoid of revolution. The capacity C of an ellipsoid with semi-axes, a, b, and c in infinite space, is given by the expression [42]—

$$\frac{1}{C} = \frac{1}{2}\int_0^{\infty} \frac{du}{\sqrt{(a^2+u)(b^2+u)(c^2+u)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

If we put $b = c$ and if $\frac{b}{a}$ is a small fraction compared with unity, the above integral becomes equal to—

$$C = \frac{2a}{2 \log_e \frac{2a}{b}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

If we call l the length of a wire and d its diameter, then we may say that the capacity of such a wire in free space in electrostatic units is given by—

$$C = \frac{l}{2 \log_e \frac{2l}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

The capacity, therefore, of a wire l cms. long and d cms. in diameter, insulated from the earth and considerably removed from it, is—

$$C = \frac{l}{4 \cdot 6052 \log_{10} \frac{2l}{d} \times 9 \times 10^5} \text{ microfarads} \quad . \quad . \quad . \quad (92)$$

This last formula may be put in the form—

$$C = \frac{l}{4 \cdot 1447 \log_{10} \frac{2l}{d}} \text{ micro-microfarads} \quad . \quad . \quad (93)$$

[42] See "Handbook for the Electrical Laboratory and Testing Room," J. A. Fleming, vol. ii. chap. ii. p. 114.

and gives us a very useful formula for the approximate predetermination of the capacity of a vertical wire used as an antenna for wireless telegraphy. As an illustration of the effect of the proximity of the earth we may, however, give the following figures :—

A circular metallic disc 60 inches in diameter was suspended and insulated in one room of the Pender Electric Laboratory of University College, a room about 40 feet by 50 feet by 18 feet. The calculated capacity by the formula $\frac{d}{\pi}$ is 53·44 mmfds., the measured capacity was found to be 59·95 mmfds., or 10 per cent. greater.

A wire was set up in the open air suspended and insulated from a mast. The length was 111 feet, and diameter 0·085 inch, or 0·215 cm. The calculated capacity from the ellipsoid formula is 181 mmfds. The observed capacity was 205 mmfds., or 10 per cent greater. When a number of such wires are hung up side by side the united capacity is always much less than that of the sum of each wire alone. Thus four wires, each 111 feet long and 0·215 cm. in diameter, were hung up 6 feet apart; the united capacity was found to be 583 mmfds., and not 820 or 4 × 205 mmfds. In the same way, 160 such wires suspended and insulated, the wires arranged in an inverted cone shape with angle of about 60°; the wires being 2 feet apart at the top and in contact at the bottom, were found to have a united capacity of 2685 mmfds., or only about 10 or 11 times that of one single wire of the same length and diameter. The above figures show how difficult it is to obtain any very large capacity by suspending insulated sheets or wires of metal in the open air. If we attempt to multiply the sheets or wires, they simply reduce each other's capacity, and the sum total is very far below the sum of the individual capacities.[43]

As the case of a long thin wire insulated in air is important from the point of view of wireless telegraphy, we give another method of determining the capacity by calculation which is due to Professor A. Slaby.[44]

Let a circular-sectioned cylinder of metal have a length l and a diameter $2r$. Take the centre as origin and consider any slice of the cylinder of length dx at the distance x. Then let ρ be the density of the electric charge on the surface. Hence the surface charge on the ring of width dx is $2\pi r\rho dx$.

The potential dV due to this annular charge at the origin is—

$$dV = \frac{2\pi r\rho dx}{\sqrt{r^2+x^2}}$$

and since the potential in the cylinder is everywhere the same, we obtain the potential of the cylinder by taking the integral—

$$V = 2\int_0^{\frac{l}{2}} \frac{2\pi r\rho dx}{\sqrt{r^2+x^2}}$$

$$\text{Now} \quad \int \frac{dx}{\sqrt{r^2+x^2}} = \log_e\left(x+\sqrt{r^2+x^2}\right)$$

$$\text{Hence} \quad V = 4\pi r\rho\left\{\log_e\left(\frac{l}{2}+\sqrt{r^2+\frac{l^2}{4}}\right) - \log_e r\right\}$$

But $2\pi r\rho l$ is the whole charge Q on the cylinder, and by definition the capacity $C = \frac{Q}{V}$. If, then, r is small compared with $\frac{l}{2}$, we have—

$$C = \frac{l}{2\log_e\frac{l}{r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

This is the same formula as (91).

Returning, then, to the practical measurement of capacity, it may be noticed

[43] For additional information on this point, see Fleming and Clinton, "On the Measurement of Small Capacities and Inductances," *Phil. Mag.*, May 1903, ser. 6, vol. 5, p. 493.

[44] See A. Slaby, "On Wireless Telegraphy," *Elektrotechnische Zeitschrift*, Aug. 1904; or *l'Éclairage Electrique*, Oct. 19, 1904, vol. 41, p. 179.

that the commutator above described may be used in another way for the measurement of capacity. If the commutator shown in Fig. 25 is employed not to charge and discharge the condenser through a galvanometer, but to discharge it through a short thick wire, the condenser and commutator are equivalent to a resistance because they pass through a certain quantity of electricity per second. The condenser of capacity C farads discharged n times per second is equivalent to a resistance of $\frac{1}{Cn}$ ohms. Hence it may be inserted in one arm of a Wheatstone's bridge and measured as a resistance.

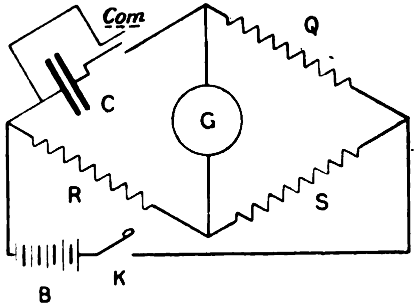


FIG. 29.—A Capacity and Commutator Balanced as a Resistance on a Wheatstone's Bridge.

Let C be the condenser (see Fig. 29), and Q, S, and R the other resistances forming the arms of the bridge. Let B be a battery and G a galvanometer. Then when Q, S, and R are varied so that no current flows through the galvanometer when the key K is depressed, the capacity C is given by the expression—

$$Cn = \frac{S\left\{1 - \frac{S^2}{(Q+S+G)(R+B+S)}\right\}}{RQ\left\{1+\frac{SB}{Q(R+B+S)}\right\}\left\{1+\frac{SG}{R(Q+S+G)}\right\}} \quad . \quad . \quad . \quad (95)$$

For the proof of the above formula the reader must be referred to the author's "Handbook for the Electrical Laboratory and Testing Room," vol. ii. p. 145, 2nd edition.

The above methods are absolute methods, that is, they determine the capacity

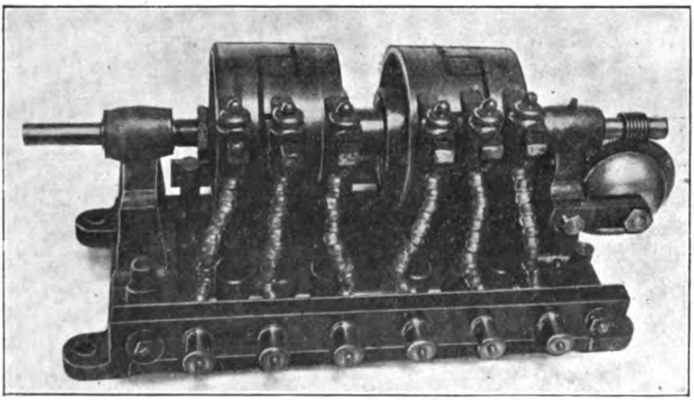


FIG. 30.—Double Commutator for the Comparison of Capacities.

in terms of a frequency and resistances. We can, however, employ a double commutator of the kind shown in Fig. 30, but having two independent commutators on the same shaft driven at the same speed by one motor. We can, therefore, commutate two condensers at the same time, and insert one condenser plus its commutator in each arm of a Wheatstone's bridge. We can thus compare directly

by a null method the unknown capacity of one condenser with the known capacity of another condenser having a different value just as if they were two resistances.[45]

In addition to these commutator methods there is another method of determining the capacity of a condenser by comparing it with a standard of mutual inductance, which is often very useful. This method is commonly called the Carey Foster method of measuring capacity, although, as a matter of fact, it was devised by Professor G. Carey Foster in 1887 as a means of determining a mutual inductance in terms of a capacity and two resistances.[46] The method is as follows :—

In one arm of a Wheatstone's bridge a condenser, C, is inserted, and in the conjugate arm is one coil of a mutual inductance, L, such as the Campbell variable standard, the connections being as shown in Fig. 31. The other arms, P and Q, are non-inductive resistances, and the battery circuit contains the other coil, N, of the mutual inductance, the resistance of which is S ohms. The battery circuit is connected as shown in Fig. 31, with one lead to one terminal of the galvanometer, G.

It is convenient, therefore, to consider the resistance of the arm Q to be included in that of the inductive coil R, and to call all the resistance shunting the galvanometer R. Let $x+y$, y, and z be the currents circulating in the meshes of this bridge when the battery key is depressed. It is convenient to consider the currents as all flowing in the same direction, so that the actual current in the galvanometer is $x=(x+y)-y$. If L, M, and N are the self and mutual inductances of the coils in the mutual inductance standard, then the kinetic energy T of the system is given by—

$$T=\tfrac{1}{2}Ly^2-Myz+\tfrac{1}{2}Nz^2 \quad . \quad . \quad . \quad (96)$$

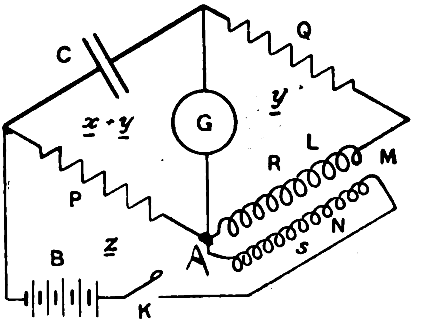


FIG. 31.—Connections of Carey Foster Bridge for Comparison of a Capacity and a Mutual Inductance.

We have to place a *minus* sign before the second term, because the coils must be so connected for this test that the E.M.F. due to mutual inductance tends to oppose that due to self-inductance in the mesh y, but to help it in the mesh z. Also the rate at which energy is dissipated in the system denoted by F is given by

$$F=P(\overline{x+y}-z)^2+G(\overline{x+y}-y)^2+Ry^2+Sz^2 \quad . \quad . \quad . \quad . \quad (97)$$

where P, R, S, and G stand for the resistances of the bridge arms and of the galvanometer.

Now Maxwell showed[47] that the electromotive force in any mesh or circuit can be expressed by an equation similar in form to Lagrange's dynamical equations, viz. for the y cycle or mesh—

$$E=\frac{d}{dt}\left(\frac{dT}{dy}\right)+\frac{1}{2}\frac{dF}{dy} \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

If, then, we substitute in equation (98) the values derived from (96) and (97), and write p for $\frac{d}{dt}$, we have the three equations for the three meshes $x+y$, y, and z as follows :—

$F = P(x+y)^2 - 2P(x+y)z + Pz^2 + G(x+y)^2 - 2G(x+y)y + Gy^2 + Ry^2 + Sz^2$

[45] This method was described by Mr. A. Campbell, see *Proc. Phys. Soc. Lond.*, vol. xxiv. p. 181, 1912; but as a matter of fact the method had been known and used in the author's laboratory many years previously.

[46] See G. Carey Foster, "Note on a Method of Determining Coefficients of Mutual Inductances," *Phil. Mag.*, vol. 23, ser. 5, p. 121, 1887.

[47] See "A Treatise on Electricity and Magnetism," J. Clerk Maxwell, vol. ii. chap. vi. 2nd edit.

For the z cycle $\quad N p z - M p y + P z - P(x+y) + S z = E$

for the y cycle $\quad L p y - M p z + G y - G(x+y) + R y = 0$

for the $x+y$ cycle $\quad P(x+y) - P z + G(x+y) - G y = -\frac{1}{C}\int (x+y)\,dt = -\frac{x+y}{Cp}$

Rearranging these terms, we have—

$$-Px - (P + Mp)y + (P + S + Np)z = E$$
$$-Gx + (R + Lp)y - Mpz = 0$$
$$\left(P + G + \frac{1}{Cp}\right)x + \left(P + \frac{1}{Cp}\right)y - Pz = 0$$

Solving for x we have—

$$x = \frac{E\left\{\left(\frac{M}{C} - RP\right) + P(Mp - Lp)\right\}}{\Delta} \quad . \quad . \quad . \quad . \quad . \quad (99)$$

where Δ is a function of P, R, S, L, M, N, and p which does not concern us.

Hence if E is steady or constant the condition for $x=0$ is thus—

$$M = CRP \text{ or } C \doteq \frac{M}{RP}$$

Accordingly, if this relation holds good, the galvanometer will show no current when the battery circuit is made or broken by the key, provided the battery E.M.F. remains constant.

If M is a known mutual inductance and R and P known resistances, we can use the method to determine the value of the capacity C by suitably selecting values for the bridge arms R and P and varying M until the galvanometer indicates no "kick" or sudden current when the key in the battery circuit is raised or lowered.

If the battery is replaced by an alternator or other means of making a variable current, then the condition for zero current in the galvanometer is not merely that M must be equal to CRP, but also that M must be equal to L. This last condition must hold good if a telephone and alternating current are used in place of a galvanometer and battery.[48]

8. Measurement of Small Capacities with High Frequency Electromotive Forces.—In the measurement of small capacities such as those of Leyden jars, antennæ, and aerial conductors generally made with high frequency electromotive forces, there are some sources of error against which the experimentalist must be on his guard. If, for instance, the capacity of a Leyden jar of the ordinary type is measured with the rotating commutator as described in § 7, at a low frequency, that is, some frequency of the order of 100, and if the capacity of the same jar is subsequently measured with a high frequency, that is to say, a frequency of the order of a million more or less, a marked difference will in general be found between these two results. The capacity of such a small condenser with high frequency electromotive force can be best measured by the aid of the author's Direct Reading Cymometer (see Chap. VI. § 15). By means of this instrument the capacity can be measured easily for different high frequencies and with different electromotive forces. The principle on which high frequency measurement of capacity can be made has already been described in § 4 of this chapter. It consists in determining, by means of the cymometer, the frequency of the oscillations set up in a circuit composed of a known inductance and the capacity to be measured. Hence by varying the inductance we can vary the frequency for the same capacity, and if the condenser under test and the inductance form an oscillatory circuit with the spark gap, we can vary the charging electromotive force by varying the length of the spark gap. If we measure in this manner the capacity of a Leyden jar for frequencies varying, say, from 1 to 2 million, and with various spark gaps, say, 1 to 4 mms., it will be found that the capacity of the Leyden jar increases with the length of the spark gap for the same frequency.

[48] See A. Gray, "Absolute Measurements in Electricity and Magnetism," vol. ii. part 2, p. 507.

The cause of this variation is the brush discharge which takes place at the edges of the tinfoil coating of the jar. When the Leyden jar oscillatory discharge is taking place an electric glow will be seen fringing the edge of the tinfoil. This really amounts to an escape of electricity from the tinfoil over the glass, and is equivalent to an increase in the capacity of the jar. This augmentation may amount to 5 or 10 per cent. of the capacity measured with a low frequency electromotive force, and is therefore by no means negligible. It can, however, be completely prevented by immersing the jar in highly insulating oil, so as to prevent glow discharge at the edges of the tinfoil. If the condenser is constructed of glass plates having tinfoil coatings put on in the usual manner, then no sensible variation in the high frequency capacity is found when the plates are immersed as described in oil when using varying values of the spark gap length, that is, of the charging electromotive force.

On the other hand, with sufficient increase in frequency of the oscillations, the capacity is found to decrease when glow discharge is arrested by immersing the condenser in oil. The author has found that in comparing the capacity of a condenser with glass dielectric at low frequency (100) and a high frequency (10^6), the difference in the capacity produced by the glow discharge at the edges of the tinfoil is far greater than the difference due to mere electrical frequency. This increase of capacity due to the glow discharge depends not merely upon the spark length employed in making the measurement, but also upon the frequency of the break of the induction coil, so that in measuring the capacity of Leyden jars by the cymometer, or any other method employing high frequency electromotive force, observers should always be careful to state the spark length, the spark frequency, and also the inductance of the circuit or the frequency of the oscillations.

As an instance of the kind of variations which may occur in such measurements, the following results are given of observations taken on the capacity of a Leyden jar of a size commonly used in wireless telegraphy. The capacity of this jar measured with the commutator at a frequency of 100 was found to be 0·001263 mfd. The capacity of the same jar was then measured with the author's cymometer for various spark lengths and inductances in series with the jar, as shown in the table below.

TABLE SHOWING THE VARIATION IN CAPACITY OF A LEYDEN JAR WITH CHARGING VOLTAGE AND FREQUENCY.

Length of spark gap.	Spark voltage.	Inductance in centimetres $=L$.	Observed oscillation constant of circuit $=\sqrt{CL}$.	Calculated capacity of jar in microfarads. $=C$.	Frequency of oscillations used $=n$.
1 mm.	4600	5,000	2·55	0·001300	$1·98 \times 10^6$
1 ,,	,,	10,000	3·65	0·001332	$1·38 \times 10^6$
1 ,,	,,	15,000	—	—	—
				mean = 0·001316	
2 mm.	8100	5,000	2·60	0·001352	$1·94 \times 10^6$
2 ,,	,,	10,000	3·69	0·001361	$1·36 \times 10^6$
2 ,,	,,	15,000	4·52	0·001362	$1·12 \times 10^6$
				mean = 0·001358	
3 mm.	11,400	5,000	2·70	0·001458	$1·85 \times 10^6$
3 ,,	,,	10,000	3·77	0·001421	$1·34 \times 10^6$
3 ,,	,,	15,000	4·60	0·001411	$1·10 \times 10^6$
				mean = 0·001430	
4 mm.	14,500	5,000	2·72	0·001480	$1·84 \times 10^6$
4 ,,	,,	10,000	3·81	0·001451	$1·32 \times 10^6$
4 ,,	,,	15,000	4·71	0·001479	$1·07 \times 10^6$
				mean = 0·001470	

The capacity of the same jar measured with low frequency $n=100$ is 0·001263 mfd.

In this case the jar was not immersed in oil, and the difference shown between the high frequency measurements are largely dependent upon the different charging voltages used and irregularities in the break of the induction coil.

9. Variation of Dielectric Constant with Temperature and Time of Charge.—Dielectric constants are much affected (i.) by the temperature of the insulator, (ii.) by the charging voltage, and (iii.) by the mode and time of its application, viz. whether steady or reversed, and if reversed, on the speed or frequency of the reversals.

Just as the bending, twisting, or strain of an imperfectly elastic or semi-viscous solid under stress depends upon the temperature, stress, and mode of application of the stress, so is it in the electrical case. The lower the temperature, the shorter the time of application of the electric force, the smaller, generally speaking, do we find the value of the dielectric constant. Observers, however, have not been always careful to define the manner in which their experiments have been conducted, and hence we find great differences between the recorded values of the dielectric constant assigned to any one substance.

For a very large number of solid insulators the dielectric constant is approximately equal to 2·6 times the density. When, however, we examine various solvents, such as water, alcohol, glycerine, nitro-benzol, etc., we find that the introduction into a chemical molecule of certain radicles or atomic groups, such as hydroxyl (HO), nitryl (NO), and ammonyl (NH_2), has the effect of creating at normal temperatures abnormally large dielectric constants. Thus the dielectric constant of pure water at ordinary temperatures is about 80, and that of ethylic alcohol is 25. Chemically speaking, water is a hydrate of hydrogen, H(HO), and alcohol is ethylic hydrate, $C_2H_5(HO)$.

The discovery was, however, made by Sir James Dewar and the author, working together, that extremely low temperatures, such as that of liquid air, had the effect of greatly reducing these abnormally large dielectric constants.

As regards temperature change, with few exceptions, we can say that decrease of temperature decreases the dielectric constant. Also that decrease in the time of charging or application of the electric force decreases dielectric constant. This is well shown by a series of observations by MM. J. Curie and P. Compan.[49] They measured the dielectric constant of three samples of crown glass in the form of sheet at temperatures between 13° C. and that of liquid air −185° C., and for various times of charging from 10 seconds to 0·05 of a second, and the results are tabulated below.

DIELECTRIC CONSTANT OF CROWN GLASS AS AFFECTED BY TEMPERATURE AND TIME OF CHARGING.

Duration of charge in seconds.	Temperature, 13° C.	Temperature, 0° C.	Temperature, −19° C.	Temperature, −75° C.	Temperature, −185° C.
10	11·25	9·47	8·44	7·09	6·49
1	9·32	8·44	7·81	7·09	6·49
0·1	8·04	7·75	7·42	7·09	6·49
0·05	7·85	7·50	7·36	7·09	6·49

In the above case the variation of the dielectric constant (K) with temperature can be expressed by a simple linear formula—

$$K = K_0 + AT$$

where K_0 is the dielectric constant at absolute zero, T is the absolute temperature,

[49] See *Comptes Rendus*, June 1902, vol. 134, p. 1295, "Sur le pourvoir Inducteur Spécifique des diélectriques aux basses Temperatures."

and A is a constant. For three samples of crown glass, Curie and Compan found—

K_o.	A.
6·03	0·00524
6·83	0·00520
6·24	0·00535

Similar results were found with ebonite, mica, and quartz.

It is evident, therefore, that the variation of dielectric constant (D.C.) with time of charging disappears at very low temperatures. An extensive series of experiments on the dielectric constants of various bodies at very low temperatures was carried out by Sir James Dewar and the author in 1896 and 1897, the results of which were published in the *Proceedings of the Royal Society of London*. The following is a list of the published papers :—

(1) "On the Dielectric Constant of Liquid Oxygen and Liquid Air," *Proc. Roy. Soc.*, vol. 60, p. 360;

(2) "Note on the Dielectric Constant of Ice and Alcohol at very Low Temperatures," *Proc. Roy. Soc.*, vol. 61, p. 2.

(3) "On the Dielectric Constants of pure Ice, Glycerine, Nitrobenzol, and Ethylene Dibromide at and above the Temperature of Liquid Air," *Proc. Roy. Soc.*, vol. 61, p. 316.

(4) "On the Dielectric Constant of Certain Frozen Electrolytes at and above the Temperature of Liquid Air," *Proc. Roy. Soc.*, vol. 61, p. 299. This paper describes the cone condenser and methods used.

(5) "Further Observations on the Dielectric Constants of Frozen Electrolytes at and above the Temperature of Liquid Air," *Proc. Roy. Soc.*, vol. 61, p. 381.

(6) "The Dielectric Constants of Certain Organic Bodies at and below the Temperature of Liquid Air," *Proc. Roy. Soc.*, vol. 61, p. 358.

(7) "On the Dielectric Constants of Metallic Oxides Dissolved or Suspended in Ice cooled to the Temperature of Liquid Air," *Proc. Roy. Soc.*, vol. 61, p. 368.

(8) "A Note on some Further Determinations of the Dielectric Constants of Organic Bodies and Electrolytes at very Low Temperatures," *Proc. Roy. Soc.*, vol. 62, p. 250.

The general results of all these observations was to show that reduction of temperature lowered the dielectric constant, in some cases in a very marked degree. Also they showed that the result of increasing the frequency when using an alternating electromotive force was to reduce the dielectric constant, in some instances in the most marked manner, but in other cases hardly at all.

In a later chapter we shall discuss the relation between dielectric constant and optical refractive index, known as Maxwell's law. According to this law the dielectric constant K should be numerically equal to the square of the refractive index, μ^2, in those cases in which the magnetic permeability is equal to that of air. If, however, we take μ to be the optical refractive index, then exceptions are far more numerous than the coincidences with the law.

The great majority of liquid and solid dielectrics at ordinary temperatures do not obey Maxwell's law, but it was shown by the investigations of the above-named authors that when cooled to very low temperatures the abnormally large values of some dielectric constants disappeared and are brought into much closer agreement with the square of the optical refractive index. The table on p. 165 shows some of the results obtained by Fleming and Dewar.

B. B. Turner determined with great care the dielectric constants of certain pure liquids, which are given in the table below and agree fairly well with those in the table [50] on next page.

Substance.	Dielectric constant K at 18° C.
Water	81·07
Nitrobenzol	36·45
Orthonitrotoluol	27·71
Ethyl chloride	10·90
Aniline	7·298
Ether	4·367
Metaxylol	2·376
Benzol	2·288

[50] See *Zeitschrift für Phys. und Chem.*, 1900, vol. 35, p. 385; also *Science Abstracts*, vol. 4, p. 503.

DIELECTRIC CONSTANTS (K) AT DIFFERENT TEMPERATURES TAKEN WITH ALTERNATING ELECTRIC FORCE HAVING A FREQUENCY OF 120.

Substance.	K at 15° C.	K at −185° C.	Square of optical refractive index.
Water	80	2·4 to 2·9	1·779 for D line
Formic acid	62	2·41	...
Glycerine	56	3·2	...
Methyl alcohol	34	3·13	...
Mononitrobenzol	32	2·6	...
Ethyl alcohol	25·8	3·11	1·831
Acetone	21·85	2·62	...
Ethyl nitrate	17·72	2·72	...
Amyl alcohol	16·0	2·14	1·951
Aniline	7·51	2·92	...
Castor oil	4·78	2·14	2·153
Ethylic ether	4·25	2·31	1·805
Olive oil	3·16	2·18	2·131
Carbon bisulphide	2·67	2·24	2·01
Petroleum oil	2·07	...	2·075
Turpentine	2·23	...	2·128
Benzol	2·38	...	2·26

The values obtained for the dielectric constants of various well-known solid insulators differ very much, but the following table gives some accepted values :—

DIELECTRIC CONSTANTS OF VARIOUS SOLID INSULATORS.

Substance.	Dielectric constant K at 15° C.	Square of optical index of refraction μ.
Flint glass (dense)	10·1	2·924
,, ,, (light)	6·57	2·375
Crown ,, (hard)	6·96	...
Calcite	7·7	2·734 A
Fluorspar	6·7	2·05
Mica	6·64	2·526
Tourmaline	6·05	2·63
Rock salt	5·85	2·36
Quartz	4·55	2·41
Sulphur	2·9 to 4·0	4·89 B
Shellac	2·7 to 3·0	...
Ebonite	2·05 to 3·15	...
India-rubber (pure brown)	2·12	...
,, (vulcanized)	2·69	...
Paraffin wax	2·0 to 2·3	...

Values given for the dielectric constant of various substances by different observers differ considerably, and as the circumstances of the measurement with respect to the time of charging and the electric force used have not been identical, we cannot consider the so-called "constant" as more than an exceedingly rough guide in the predetermination of capacity. Particularly is this the case with regard to glass. This material is of very variable composition, and its dielectric constant, according to some observers, varies very much with the time of charging. Hence, caution must be taken not to apply indiscriminately the results of low frequency dielectric measurements in high frequency work.

M. v. Hoor has carried out investigations on the effect of variation in the electric force employed on the resulting measured dielectric constant. The

electric force was measured in volts per centimetre of thickness, that is, by dividing the charging voltage by the thickness of the dielectric. The results for some dielectrics are given below[51] :—

VARIATION OF DIELECTRIC CONSTANT WITH ELECTRIC FORCE.

Substance.	Electric force in volts per centimetre.	Dielectric constant.
Paraffined paper	55·5	3·65
" "	0·528	3·68
Crown glass, No. 1 . . .	22·9	10·7
" " "	4·46	12·8
Crown glass, No. 2 . . .	27·2	6·92
" " "	1·037	7·22
Gutta-percha	41·000	3·155
"	0·491	3·26
Megohmit	5·95	5·09
"	0·286	5·31

It will be seen that there is a considerable variation in the case of glass, the dielectric constant increasing as the electric force diminishes. In connection with this, it is worth while to note that the variation of conductivity in dielectrics is in the same direction. It has been found that for most insulators the insulation resistance decreases as the applied electromotive force increases.[52]

In these respects glass has a disadvantage as a dielectric compared with ebonite, as far as its use with high frequency currents is concerned.

Another very important cause of variation in dielectric constant is the frequency of electric force. It is evident we may take the ratio of electric displacement to electric force either with a steady electric force, uniformly acting in one direction, or with a periodically reversed electric force, having any assigned frequency. In the case of some dielectrics, such as ebonite or sulphur, there is usually said to be very little difference between the dielectric constant found with low frequency alternating electric force and that under high frequency electric force. On the other hand, with glass there is said to be a very marked difference, according to the experiments of many observers, with the exception, however, of Pollock and Vonwiller, who deny that glass exhibits any very marked variation of dielectric constant with frequency. This was confirmed by Dr. J. Hopkinson and Professor E. Wilson, who say that the dielectric constant of English light flint glass is constant for low frequencies and up to a frequency $n=2\times10^6$ (see *Phil. Trans. Roy. Soc. Lond.*, 1897, vol. 189, p. 109).

The author has, however, found that both glass and ebonite give evidence of a decrease in dielectric constant with frequency, and that only liquid hydrocarbons can be considered as having a dielectric constant independent of the frequency.

It has been found, both by Professor Sir J. J. Thomson and by M. R. Blondlot, that at a frequency of 25×10^6 the dielectric constant of glass has a value as low as 2·7 or 2·8. For a low frequency of steady force, the value, as shown by the tables already given, is from 7 to 10.[53]

Again, all observers who have determined the dielectric constant of water or ice with low frequency force, say between 1 and 200 alternations of electric force per second, have found a value for the dielectric constant not far from 80.

If, however, the dielectric constant of ice is determined at −185° C. with a frequency of 120~, then its dielectric constant is found to be about 2·4 to 2·9. In

[51] See *Elektrotechnische Zeitschrift*, vol. 22, p. 716; or *Science Abstracts*, vol. v, p. 32.

[52] See A. W. Ashton, "On the Resistance of Dielectrics and on the Effect of an Alternating Electromotive Force on the Insulating Properties of India-rubber," *Phil. Mag.*, 1901, ser. 6, vol. 2, p. 501.

[53] See Prof. Sir J. J. Thomson, *Proc. Roy. Soc.*, 1889, vol. 46, p. 293, "On Specific Inductive Capacities of Dielectrics under Rapidly Alternating Electromotive Force"; also M. R. Blondlot, *Comptes Rendus*, 1891, vol. 112, p. 1058. Compare, however, with Pollock and Vonwiller, *Phil. Mag.*, 1902, ser. 6, vol. 3, p. 586.

some cases, such as ethylic alcohol, a very moderate increase in the frequency suffices to sensibly reduce the dielectric constant.

It has been pointed out by Fleming and Dewar that reduction of temperature, even when operating with low frequency alternation of electric force, has the same effect as an increase of frequency alone at constant ordinary temperature in reducing the abnormally large dielectric constants of certain bodies to a value more in accordance with Maxwell's law.

For these reasons, therefore, glass is a dielectric not very suitable for making condensers to be employed in exact scientific work with high frequency currents.

Its cheapness, however, and other good electrical and mechanical qualities, make it a very convenient substance to use for commercial work.

10. Conductivity and Energy Losses in Dielectrics.—The conductivity of, and dissipation of energy in dielectrics is a subject to which great attention has been paid of late years. Every so-called insulator or dielectric possesses in some degree true conductivity for electricity which is a cause of energy dissipation in it when it is subjected to electromotive force. When the force is alternating there is an additional dissipation of energy said to be due to *dielectric hysteresis.*

This last term has, however, been often used rather as a cloak for ignorance than with a precisely defined meaning.

In the case of a perfect dielectric used as the insulator of a condenser there should be no internal dissipation of energy by charge and discharge. If the alternating electromotive force is sinoidal, then the capacity current should be also of the same wave form and 90 degrees different in phase, the current being in advance of the electromotive force. The power factor, or cosine of the angle of phase difference, should therefore be zero. As a matter of fact, in the case of most actual condensers the power factor is not zero, and when subjected to alternating electromotive force they rise in temperature, and this points to some internal cause of energy dissipation in the dielectric.

With respect to the measurement of dielectric conductivity, it has been the custom to apply a steady unidirectional electromotive force to a condenser for a certain time, say one minute, and then to call the ratio of this applied voltage to the resulting current flowing at the end of the minute the "insulation resistance" of the dielectric. Although such a figure may have a certain commercial value for cable manufacturers it has no definite scientific meaning. The application of an E.M.F. to a dielectric results in the production of a true current of conduction through it superimposed upon a so-called displacement current. Both of these increase with time of application.

The former may be defined as a non-recoverable flow of electricity through the dielectric, and the latter as a flow which reverses after a time, if the E.M.F. is withdrawn and the condenser electrodes short-circuited. If we call movement of electricity positive when in one direction and negative when in the opposite direction across a plane in the dielectric perpendicular to the flow, then we may say that as regards the displacement current the algebraic sum of the movement is zero after a sufficiently long time, if the electromotive force is applied, removed, and the electrodes of the condenser then short-circuited. If the electromotive force is an alternating or periodic force then the phenomena become more complicated. There is an energy waste or dissipation depending upon a true conductivity of the dielectric, and there may also be an energy loss depending on the fact that there is a difference of phase between the electric force and the displacement over and above that difference depending on the true conductivity.[54]

Part or all of the true resistance loss may depend upon the presence of water in the dielectric if this substance is fibrous in structure and hygroscopic, or treated with water in process of manufacture.

[54] For a *résumé* of knowledge on the subject of energy losses in dielectrics up to 1895 the reader is referred to a Paper by P. Gasnier in *The Electrician*, vol. 36, p. 7, November 1, 1895. In Winkelmann's *Handbuch der Physik*, 2nd Ed., vol. 4, part i. p. 77, will be found a section by L. Graetz on the properties of dielectrics which, by the aid of a list of original papers, brings information down to 1902. In a Paper by Mr. E. H. Rayner on "High Voltage Tests and Energy Losses in Insulating Materials," a further list of original papers on this subject is given. See *Journal Inst. Elec. Eng.*, vol. 49, p. 3, 1912.

One of the chief additions to our knowledge of the properties of dielectrics which has been made of recent years is the proof that the conductivity of a dielectric, even such substances as glass or celluloid, which are non-hygroscopic, is or may be very much greater for alternating than for unidirectional electromotive force. Hence no measurements made with direct currents will tell us the conductivity of a dielectric for alternating currents. This distinction is very important when we are dealing with very high frequency currents.

By far the simplest mode of regarding the phenomenon in the case of simple periodic electromotive forces is to call the total current through a dielectric under the action of a given electric force the *dielectric flux*. If the electric force is sinoidal or a simple sine function of the time, then the electric flux is a similar function, but is out of phase with the force. The total flux can therefore be resolved into two components, one which is step with the force and one which is 90° in advance of it in phase.

The latter results in a current or movement of electricity which may be called the capacity current or true dielectric current, the former results in a current which may be called the conduction current. The latter, therefore, dissipates energy, but the capacity current does not.

The capacity current is measured by the product of the capacity of the condenser, and the time rate of change of the applied electromotive force or potential difference of the condenser electrodes. The conduction current is measured by the product of the conductance of the condenser and the P.D. of the electrodes.

If we call C the capacity and S the conductance of the condenser and V the potential difference of the electrodes, and if $p=2\pi$ times the frequency: then the total current through the condenser is given by $(S+jpC)V$, where j is the sign of perpendicularity and equivalent to $\sqrt{-1}$.

The quantity $S+jpC$ is called the *admittance* and its reciprocal $(S+jpC)^{-1}$ is called the *impedance* of the condenser. If σ is the conductivity per centimetre cube between opposed faces, then for a plane sheet condenser $S=A\sigma/l$ and $C=AK/4\pi l$, where A is the area of cross-section and l the thickness of the dielectric. Hence the admittance per cm. cube is $\sigma+jpK/4\pi$. The ratio S/Cp is the cotangent of the angle of phase difference of the impressed voltage and total current, and when this angle is nearly 90° the cotangent is equal to the cosine. Hence S/Cp measures the *power factor* of the condenser. The quantity σ, called the dielectric conductivity, is found to be a function of the frequency when periodic electric forces are employed.

The best method of measuring the quantities S and C for a given plate condenser, and therefore of determining σ and K for a given sample of dielectric at a given temperature and frequency, is by a bridge arrangement as devised by the author and the late Lieut. G. B. Dyke, which is a modification of a method due to M. Wien.[55]

This arrangement, called a capacity bridge, enables us to separate out and measure the true capacity and true conductivity for any condenser when employing alternating electromotive force of simple sine wave form and any frequency. This method is as follows: A Wheatstone's bridge is made up, two of the ratio arms consisting of two air condensers, C_3 and C_4, of variable capacity and negligible conductance. The two other arms are filled in, one with a similar air condenser, C_2, in series with a variable non-inductive resistance, R_2, and the other with the leaky condenser C formed with the conductive dielectric as insulator (see Fig. 32). The conductivity of this last dielectric is represented as due to a conductance S in parallel with a condenser of capacity C.

The bridge circuit contains a telephone, and the bridge must be supplied with an alternating current of pure sine form.[56] The condition which must be fulfilled

[55] See J. A. Fleming and G. B. Dyke, "On the Power Factor and Conductivity of Dielectrics when Tested with Alternating Electric Currents of Telephonic Frequency at Various Temperatures," *Journal Inst. Elec. Eng.*, London, vol. 49, p. 323, 1912. Also M. Wien, *Wied. Ann. der Physik*, vol. 44, p. 689, 1891.

[56] For instructions as to the mode of obtaining and testing such a pure sine form E.M.F. the original Paper (*loc. cit.*) must be consulted.

that there may be no sound in the telephone is that the impedances of the four arms of the bridge must be in proportion. The impedance of the leaky condenser or shunted condenser C is $(S+jpC)^{-1}$, under simple periodic currents of frequency $n=p/2\pi$. The impedance of the condenser C_2 in series with resistance R_2 is $R_2 - \frac{j}{C_2 p}$ and the impedances of the condensers C_3 and C_4 are $-j/C_3 p$ and $-j/C_4 p$ respectively. Hence the four being in proportion we have—

$$\left(R_2 - \frac{j}{pC_6}\right)(S+jpC) = C_3/C_4 \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Equating real and imaginary parts we have—

$$\frac{S}{Cp} = pC_2R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

$$\frac{C_3}{C_4} = \frac{C}{C_2} + R_2S$$

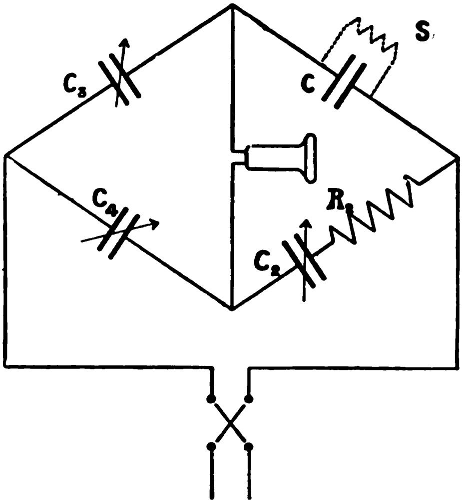


FIG. 32.—Fleming and Dyke Capacity Bridge.

The solution of these equations gives us the separate values of S and C in terms of C_2, C_3, C_4, and R_2, when we know the pulsation p.

We can therefore determine the capacity and conductance of a plate form of condenser made up with any dielectric for alternations of any frequency.

In the investigation by the author and Lieut. Dyke a very extensive series of measurements was made with all ordinary dielectrics at various temperatures, and for frequencies between 800 and 5000 covering the range of ordinary telephony. The result was to show that the conductivity of dielectrics for alternating currents is a function of the frequency.

For a number of them the conductivity (σ) per centimetre cube can be represented as a linear function of the frequency n of the form $\sigma = a+bn$, where a and b are constants.

In many cases the lines are not quite straight and an additional term has to be added, so that $\sigma = a+bn+cn^2$ more nearly represents the function. The conductivity curves in terms of frequency for glass and ebonite are shown in Figs. 33 and 34. The conductivity of dielectrics is, therefore, quite different under alternating and under continuous electromotive force. This investigation was continued for much higher frequencies by Dr. G. E. Bairsto in 1911-1914 in a long research

carried out in the Pender Electrical Laboratory, University College, London, at the author's suggestion.

The results show that in the case of most imperfect dielectrics, such as slate,

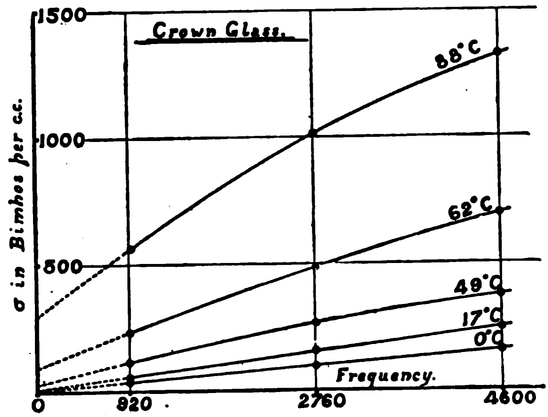


FIG. 33.—Curves showing the Variation in Conductivity of Crown Glass for Various Frequencies and Temperatures.

marble, chalk, and also in the case of gutta-percha and india-rubber, there is a certain frequency for which this alternating current's conductivity is a maximum.[57]

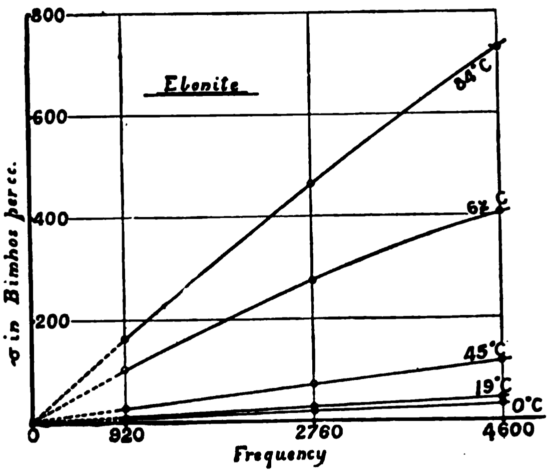


FIG. 34.—Curves showing the Variation in Conductivity of Ebonite for Various Frequencies and Temperatures.

The conductivity of these insulators is best represented in Bimhos per centimetre cube, which means in billionths of a mho, and this unit is a conductivity equal to that of a million megohms.

The alternating current conductivity of such a substance as slate for alterna-

[57] G. E. Bairsto, Thesis for D.Sc. degree, Univ. of Lond., 1914.

tions of one or two millions a second may be hundreds or thousands of times greater than its true direct current conductivity.

As an example of the maximum values obtained with high frequency currents, we may give the following figures from Dr. Bairsto's results :—

Dielectric.	Maximum conductivity in Bimhos per centimetre cube at 15° C.	Frequency at which this maximum is attained.
Dry blotting paper	28,000	600,000
Crown glass	30,000	500,000
Vulcanized india-rubber . . .	60,000	550,000
Gutta-percha	40,000	800,000
Marble	22,000	1,200,000
Slate	$2 \cdot 5 \times 10^6$	2,500,000

Hence in dealing with the problems of radiotelegraphy, in which the conductivity of such imperfect insulators is concerned, a very great error would be committed in taking as the alternating conductivity any value obtained with direct current. Moreover, it is necessary to distinguish between true dielectric conductivity and that due to the presence of moisture in fibrous dielectrics.

It has been shown by the author and Mr. Dyke by experiments with alternating currents, and by Mr. S. Evershed by experiments made with continuous currents, that a large part of the conductivity in the case of paper, and other such fibrous or porous dielectrics, is due to moisture contained in them.[58] In spite of all the labour bestowed upon the investigation of the properties of dielectrics it can hardly be said that the sources of the dissipation of energy in them are thoroughly understood. The important matter for the radiotelegraphist is that there are these sources of internal energy waste in dielectrics. Hence when a condenser is constructed with them and charged or discharged with free oscillations, a part of the energy stored up in the condenser is dissipated as heat in the dielectric.

This causes the oscillations to be more quickly damped out or die away than they would otherwise be.

It becomes important, therefore, to test condensers such as Leyden jars or condensers made with glass or ebonite plates for energy loss, and this may be stated in watts for a known condenser current by determining the equivalent resistance of the condenser.

These losses are in general too small to be determined directly by any form of Wattmeter, but the following method due to the author has been successfully applied.[59]

The condenser to be tested is joined up in series with a circuit consisting of a copper wire of known diameter and resistance wound on a square frame so that its high frequency resistance can be calculated, and its inductance measured. This circuit forms a non-radiative or feebly radiative circuit, hence if free oscillations are set up in this circuit they are damped out by resistance and condenser losses only. In circuit is also inserted a hot wire ammeter of the kind described in the following section of this chapter as suitable for measuring high frequency currents, and the circuit is interrupted also by a pair of mercury cups so that various short-wire resistances can be inserted into the circuit to vary its resistance by a known amount without sensibly altering its inductance. This condenser circuit, which will be called the secondary circuit, is placed in contiguity to another primary circuit in which highly damped oscillations are set up by means of an impact discharger. The primary circuits consist of a square circuit of one or two turns of wire of the same size as the secondary circuit, having in series with it one or two Leyden jars and an impact discharger.

[58] See S. Evershed, "The Characteristics of Insulation Resistance," *Journal Inst. Elec. Eng.*, London, vol. 52, p. 51, 1913.

[59] See J. A. Fleming and G. B. Dyke, "The Measurement of the Energy Losses in Condensers traversed by High Frequency Oscillations," *Proc. Phys. Soc. Lond.*, vol. 23, p. 117, 1911.

This discharger is described in a later chapter of this book, and is a means of producing what is called a quenched spark or a spark or condenser discharge which is suddenly interrupted, the circuit being almost immediately opened after the initial oscillation (see Chap. III. § 16). If, then, such a discharger is used in the primary circuit, the oscillations in that circuit are highly damped and practically the discharge is non-oscillatory. On the other hand, the effect on the secondary circuit is to set up in the latter its free vibrations. As there are an enormous number of sparks per second, we obtain very steady feebly damped oscillations in the secondary circuit.

Suppose, then, we measure the R.M.S. value of the current in the secondary circuit, we have four causes for the damping out of the free oscillations taking place in that circuit. These are as follows :—

1. The high frequency resistance R′ of the square circuit, which can be calculated from the dimensions of the wire and from the frequency, which last can be experimentally determined.

2. The high frequency resistance of the interpolated resistance r, which is also known.

3. The resistance of the ammeter r'.

4. The unknown source of energy loss in the condenser, which may be represented as due to a resistance ρ, which is not constant, but is a function of the condenser current. If, then, A is the R.M.S. value of this current in the secondary circuit as read on the ammeter, the total energy loss per second in that circuit is given by $A^2(R'+r_1+r'+\rho)$, and if the inductance of the circuit is L and the frequency of the oscillations is n, then the total decrement Δ of the circuit is equal to $10^9(R'+r_1+r'+\rho)/2nL$, where $\delta=10^9\rho/2nL$ is that part of the decrement contributed by the condenser.

If, then, we alter the resistance in the secondary circuit from r_1 to r_2 we have two corresponding observed values of the current, say A_1 and A_2. If an impact discharger is used in the primary circuit, there is no reaction between the secondary and primary, because the primary spark ceases almost at once, and hence the moment the secondary oscillations begin the primary circuit is open. The impact discharger communicates at every discharge the same energy to the secondary circuit, and that energy is entirely dissipated as heat, hence we must have the equation—

$$A^2(R'+r_1+r'+\rho)=a\ constant=A^2R \quad . \quad . \quad . \quad . \quad (102)$$

Accordingly, a curve whose ordinates are the total high frequency resistance R and the mean square current A^2 is an equilateral hyperbola. If, however, the resistance ρ is a function of the current, then if we slightly change the added resistance r of the circuit from one value r_1 to another r_2, and take two corresponding readings of the current, we shall have the following equations :—

$$A_1^2(R'+r_1+r'+\rho)=A_2^2(R'+r_2+r'+\rho),$$

or

$$\rho=\frac{A_2^2r_2-A_1^2r_1-(A_1^2-A_2^2)(R'+r')}{A_1^2-A_2^2} \quad . \quad . \quad . \quad . \quad (103)$$

The value of ρ is considered to belong to the mean value of A_1 and A_2.

In this manner we have been able to measure for various forms of condenser the value of the equivalent resistance ρ, and of that part of the total decrement Δ contributed by the condenser. The results show that in all cases this equivalent resistance ρ increases with the condenser current. The power W dissipated in the condenser is measured by the product of this resistance ρ and the square of the corresponding mean current A, whilst the decrement δ is the quotient of $10^9\times$ this resistance by the quantity $2nL$, where n is the frequency and L the total inductance of the circuit. In all cases the frequency of the oscillations n for each experiment was measured by a cymometer.

It is clear, from these observations, that even in the case of air condensers and oil condensers, energy losses are not entirely absent, whilst for certain forms of glass plate condenser this energy loss may be very considerable. It is best estimated in microwatts per centimetre cube of the dielectric.

The following tables give the results for a Leyden Jar and Glass Plate Condenser.

TABLE IX.

Condenser tested . . . . . . Leyden jar.
Capacity . . . . 0·00204 microfarad.
Frequency . . . . $1·29 \times 10^6$.
Nature of dielectric . . . . . . Glass.
Surface of dielectric . . . . . 634 sq. cms.
Thickness of dielectric . . . . . 0·27 cm.
Volume of dielectric . . . . . 171 cub. cms.

R.M.S. value of condenser current in amperes. A.	Effective resistance of condenser in ohms. ρ	Power dissipated in condenser in watts. W.	Semi-period decrement of condenser. $\delta/2$	Power loss in condenser in microwatts per c.c. D.	R.M.S. value of electric force in dielectric in electrostatic units. E.
3·19	0·117	1·190	0·00302	6,970	2·39
2·80	0·103	0·808	0·00266	4,730	2·10
2·55	0·096	0·624	0·00248	3,650	1·91
2·30	0·084	0·444	0·00217	2,600	1·72
1·93	0·080	0·298	0·00207	1,740	1·45

The results fit in with the formula $D = 610E^{2·76}$.

TABLE X.

Condenser tested . . . . . . Glass plate condenser in oil.
Capacity . . . . 0·00130 microfarad.
Frequency . . . . $1·68 \times 10^6$.
Nature of dielectric . . . . . . Glass.
Surface of dielectric . . . . . . 309 sq. cms.
Thickness of dielectric . . . . . 0·17 cm.
Volume of dielectric . . . . . . 52·5 cub. cms.

R.M.S. value of condenser current in amperes. A.	Effective resistance of condenser in ohms. ρ	Power dissipated in condenser in watts. W.	Semi-period decrement of condenser. $\delta/2$	Power loss in condenser in microwatts per c.c. D.	R.M.S. value of electric force in dielectric in electrostatic units. E.
3·36	0·820	9·25	0·0176	176,000	4·81
3·17	0·803	8·06	0·0173	154,000	4·54
3·02	0·785	7·15	0·0169	136,000	4·32
2·85	0·770	6·25	0·0166	119,000	4·08
2·55	0·694	4·50	0·0149	85,800	3·65
2·31	0·678	3·62	0·0146	69,000	3·30
2·16	0·676	3·16	0·0145	60,200	3·09
2·05	0·638	2·68	0·0137	51,000	2·94
1·93	0·664	2·47	0·0143	47,100	2·76
1·71	0·623	1·82	0·0134	34,700	2·45
1·17	0·550	0·754	0·0118	14,400	1·67
1·10	0·498	0·604	0·0107	11,500	1·57
1·04	0·458	0·495	0·0099	9,440	1·49
0·97	0·446	0·419	0·0096	7,990	1·39
0·85	0·370	0·266	0·0080	5,060	1·22
0·67	0·434	0·195	0·0093	3,720	0·96
0·62	0·406	0·154	0·0087	2,930	0·89
0·59	0·411	0·144	0·0088	2,740	0·84
0·55	0·430	0·129	0·0093	2,460	0·79
0·48	0·280	0·064	0·0060	1,220	0·69

The above observations agree with the formula $D=3720E^{2\cdot 42}$ well down to $E=1\cdot 4$, then not so well.

It is found that the energy losses in air and oil condensers made with certain oils are, relatively speaking, very small, but may yet amount to something of the order of several thousand microwatts per centimetre cube for electrostatic forces of the order of 2·0 electrostatic units or 600 volts per centimetre. They are also determined by the current density. In the case of condensers made with glass or ebonite dielectrics these internal energy losses may become very large for electrostatic forces attaining a value not even larger than 10, and hence contributing greatly to raise the decrement in any oscillatory circuits of which these condensers form part. Different kinds of glass seem to vary very much in this respect, and this points to the great importance of testing Leyden jars and glass plate condensers intended to be used as radiotelegraphic transmitters, especially for dielectric loss as well as for capacity. With the arrangements above described this test can be most easily and accurately carried out, and that glass may be selected which is best for the purpose. The experiments also show that for experimental work condensers made with oil as dielectric are best, whilst for large scale condensers air as a dielectric gives a small energy loss, provided the electrostatic force is not raised to a value at which brush discharges begin to produce sensible ionization of the air. If this is the case the internal losses may become considerable, even for air condensers. The moral to be drawn from the above described observations is that in the case of condensers used in radiotelegraphy the current density in the condenser should be kept as small as possible to reduce the dielectric losses.

The effect of these losses is to damp out more quickly the free oscillations set up in a circuit containing the condenser. Hence the importance of using in all cases air condensers if possible and not ebonite or glass. A similar set of measurements was made by Dr. L. W. Austin (see "Energy Losses in some Condensers used in High Frequency Circuits," *Bulletin of the Bureau of Standards*, Washington, U.S.A., vol. ix., No. 190, 1912).

Austin also adopted the method of representing the losses in a condenser as due to a hypothetical resistance with a certain current and voltage.

He substituted for the condenser under test an air condenser with inductionless resistance in series with it, and adjusted that resistance until the frequency and current in the circuit was the same in both cases, the frequency being varied by changing the value of an inductive coil of negligible resistance placed in series with the condensers. In this manner he found the following results for various condensers, having a terminal potential difference of 14,500 volts (maximum value) and a current of 7 to 8 amperes passing through them :—

Condenser.	Capacity in mfds.	Equivalent resistance in ohms.
Leyden jar, No. 1	0·00603	1·08
,, ,, No. 2	0·00605	1·19
Telefunken jar	0·00612	1·59
United wireless jar	0·00603	1·83
Moscicki jar	0·00548	0·57
Paper condenser	0·00580	2·19
Moulded micanite	0·0041	2·91
Glass plates in oil	0 0042	0·58
Fessenden compressed air, 15 atmospheres	0·00575	0·14

The above table shows that paper and micanite have very large dielectric losses. Also Austin found that Leyden jars immersed in oil show losses not much exceeding those in compressed air condensers.

In Leyden jars charged to various voltages between 10,000 and 20,000 the equivalent resistance lies between 1·0 and 1·8 ohms, and the loss increases as the square of the voltage.

Austin's results agree on the whole fairly well with those of the author.

We may then briefly consider what portion of this dielectric loss should properly be said to be due to dielectric hysteresis. The term hysteresis means "lagging behind," and is applied in the case of the magnetization of iron to the energy dissipated when a mass of iron is carried through a cycle of magnetization. The magnetization then lags behind the magnetizing force in consequence of hysteresis. The curve which delineates the magnetization in terms of the magnetizing force is a closed loop. The area of this loop is a measure of the work done per cycle in carrying the iron through the complete series of operations in which the force is applied, removed, applied negatively, and then removed again.

It is found that there is in certain dielectrics a similar effect, since the dielectric displacement or strain lags in phase behind the electric force, and we have therefore a hysteresis loop formed when we graphically delineate the effect of a cycle of operations. This lag or difference of phase between the strain and the force is properly described as dielectric hysteresis, and the energy dissipated in consequence can be described as hysteretic loss.

It is, however, necessary to distinguish carefully between energy loss due to ohmic resistance, electrolytic action, or electric discharges, and that (if any) due to dielectric hysteresis. It is a matter of great difficulty to free any insulator so completely from water or other electrolyzable material that under alternating electric force no heat is produced in it by true joulean action. It has been considered that this could be eliminated by making a measurement first with alternating electromotive force and then with continuous current at the same R.M.S. voltage, and employing a voltmeter to measure the power taken up in both cases.

The first measurement is then assumed to give the total losses, and the second the C^2R, or heating losses, and also the electrolytic losses. This method was adopted by Mr. Steinmetz,[60] and he came to the conclusion that there was a true dielectric hysteresis loss, varying as the square of the electromotive force. There are objections to this method, on the ground that the resistance of a dielectric is an ill-defined quantity, and in any case is a function of the voltage and time of application. Moreover, loss by creeping over the surface of the dielectric or brush discharges at the edges of the electrodes is not eliminated. In the same manner measurements of power factor by the wattmeter, or measurements of the angle of lag made on open-circuited cables, may give a value to the total loss due to all causes in the insulator of a cable, but they do not settle the question whether there is an energy dissipation due simply to change in the polarization or electric strain, analogous to true magnetic hysteresis in iron. In fact, just as we must distinguish between true magnetic hysteresis and eddy current loss in sheet iron, so in the case of insulators we must distinguish between that which may properly be called "dielectric hysteresis" and other sources of energy dissipation. Another mode of procedure was suggested by Ricardo Arno.[61] He placed a cylinder of an insulating material in a rotating electrostatic field, and found that it was set in rotation. Professor R. Threlfall has also conducted an extensive and well-devised series of experiments with a modification of Arno's apparatus, and carefully examined various sources of error.[62] Threlfall used his dielectrics in the form of ellipsoids of revolution, and created the rotating field by mechanically rotating a sort of air condenser with a steady uniform field. Between the plates of this condenser the ellipsoid was suspended. He carefully dried the surface of the dielectric and suspended it by a quartz fibre, and shielded the mirror and attachments from electrostatic action. Out of a very large number of experiments on ebonite, sulphur, resin, and other dielectrics, he came to the following conclusions:—

(i.) When an ellipsoid of a solid dielectric is placed in a rotating electric field, it is set in rotation even when all sources of true electric conduction are eliminated.

[60] See C. P. Steinmetz, *Electrical Engineer*, New York, March 16, 1892; or *The Electrician*, London, 1892, vol. 28, p. 602.

[61] See R. Arno, *Accademia dei Lincei*, Oct. 6, 1892, and April 30, 1893. See *The Electrician*, 1893, vol. 30, p. 516; vol. 31, p. 201; vol. 32, p. 222; vol. 33, p. 210.

[62] See R. Threlfall, "On the Conversion of Electric Energy in Dielectrics," *Physical Review*, 1897, vol. iv. p. 457; vol. v. p 21.

This indicates that the electric strain or polarization lags behind the electric force in phase. Hence, in one sense, this is a "hysteresis" effect.

(ii.) The effect is absent in liquid dielectrics.

(iii.) In solid dielectrics, if we denote the internal electric force by F, and the energy expenditure per cycle due to the true dielectric hysteresis by W, then—

$$W = aF^n$$

The exponent n is a number lying between 1·5 and 1·95 for ordinary homogeneous dielectrics, but it is not exactly 1·6.

(iv.) This hysteresis loss is very variable in passing from specimen to specimen, and an exact value cannot be assigned to any *substance* as contrasted with a *specimen*.

(v.) As regards the factor a, it is a very small number in the case of paraffin wax, but in the case of glass and ebonite may approach a value 0·03 or 0·04. Thus for a particular sample of ebonite the formula found was—

$$W = 0{\cdot}029\ F^{1{\cdot}163}$$

and for a flint-glass spheroid—

$$W = 0{\cdot}038\ F^{1{\cdot}92}$$

whilst for a paraffin ellipsoid—

$$W = 0{\cdot}0008\ F^{1{\cdot}56}$$

The constant a varies according to Threlfall greatly from sample to sample, but the index n is much more constant for samples of the same material, but varies from material to material.

This loss can be expressed as a percentage of the energy stored per unit of volume. For if F is the uniform internal electric force in one unit of volume, then the energy stored is $\frac{1}{2}$ CF² where C is the capacity per cubic unit of volume. But $C = \frac{K}{4\pi}$ for cubic unit, hence the energy stored $T = \frac{KF^2}{8\pi}$ per cubic unit.

But the energy expended in hysteresis is $W = aF^n$, therefore—

$$\frac{W}{T} = \frac{8\pi a}{K} F^{n-2} = \beta \text{ (say)}$$

Let F = 1, then—

$$\beta = \frac{8\pi a}{K}$$

Taking the case of sulphur, Threlfall gives the following figures: K = 3·162, a = 0·0139, n = 1·91. Hence β = 0·1112, or nearly 11 per cent.

For ebonite, K = 3·5, a = 0·029, n = 1·765, and β = 0·212 = 21·2 per cent. The above ratio, denoted by β, must not be confused with the ratio of the total dielectric energy dissipation to the volt-amperes or product of the condenser current and impressed voltage. This last is the power factor (P.F.) of the condenser. Taking sinoidal variation of condenser current and voltage, and assuming no energy loss except the true hysteresis, we have the volt-amperes per unit volume in equivalent electrostatic measure given by CpF^2 or by $\frac{K}{2}F^2$ per period. Hence for unit internal force the power factor (P.F.) is equal to $\frac{2W}{K}$ or to $\frac{2a}{K}$.

For the sulphur mentioned above this gives P.F. = 0·009, and for ebonite P.F. = 0·017.

Threlfall also made experiments with an apparatus designed by Ebert for producing a very high frequency revolving electric field, and placed various solid dielectrics in it. There is no need to describe the apparatus in detail, for this the reader must refer to the original paper.[63] The results, however, showed that for

[63] See Ebert, *Wied. Annalen*, 1894, vol. 53, p. 144.

a frequency as high as 10^7 dielectric hysteresis was absent. This result shows that any heating of condensers which occur with high frequency currents must be due to electric conduction, electrolysis or discharges over the surface, and not to true dielectric hysteresis.

In regard to the general question of dielectric hysteresis, Professor A. W. Porter and Dr D. K. Morris have pointed out that it is necessary to distinguish between merely viscous effects and true hysteresis.[64] If an electric force is applied to a dielectric it may take time to establish the corresponding electric strain, in which case viscosity is present. On the other hand, the strain may have the same value whether it has been arrived at by descending from a large force or rising up from a small one to the same value. In this last case true hysteresis is absent. In experiments made with a particular mica condenser, Porter and Morris found dielectric viscosity but no true dielectric hysteresis. If viscosity is present, it follows that when the impressed force varies in value but not direction the resulting flux value is a function of the frequency.

On the other hand, there is no doubt that in the majority of dielectrics the presence of moisture and conducting particles or heterogeneity of structure gives rise to true conduction currents in the mass of the dielectric, and therefore to an energy dissipation. In the case of high frequency currents, it is this surface creeping, internal discharge or conduction, that gives rise to the major part of the energy waste and dielectric heating in condensers, and not to true dielectric hysteresis.

11. The Measurement of High Frequency Electric Currents. Hot Wire Ammeters.—In dealing with electric oscillations and high frequency currents we require special forms of ammeter for making the required current measurements. In some cases instruments can be employed for comparative measurements which depend for their action upon the production of a magnetic field round the conductor through which these oscillations pass. In this case the oscillations have to pass through an insulated wire wound up into some form of coil. It is, however, difficult to graduate such instruments so as to make their scales read correctly the mean-square or effective value of oscillatory currents of various frequencies sent through the coil. This arises from the fact that with high frequency currents in close coils of insulated wire a considerable dielectric current passes from coil to coil, and is, so to speak, shunted out of the wire to a degree depending upon the frequency. Hence, as a rule, coil or electro-magnetic instruments are not so suitable as those of the straight hot-wire type for the direct measurement in amperes of a high frequency current. It is not, however, every form of hot-wire ammeter which is available for the purpose. Most hot-wire ammeters in use for measuring continuous or low frequency alternating currents are constructed on the shunt principle. The main part of the current flows through a fixed coil on metal strip, and a shunted portion passes through a by-pass wire, which is thereby heated, expanded, and provides the means of indication or measurement. This shunted circuit is, however, inadmissible in the case of measurement of high frequency currents. The ratio in which the current is divided between the shunt and working wire is a function of the frequency. Accordingly the only form of ammeter which is available and accurate for the measurement of high frequency electric currents is the hot-wire ammeter, in which the whole current passes through the working wire of the instrument.

Even then certain other precautions are necessary. The wire used must not be contained in a metal case or tube, and if it is a single wire, should not be thicker than 0·25 mm., so that its high frequency resistance is identical with its ordinary or steady resistances. Hence, if the working wire has to carry a current of several amperes, it must be made up of a sufficient number of separated or insulated fine copper or platinoid wires arranged in a bundle or strand, not closely compressed, but open. The best plan is to use thin bare wires slightly separated from each other. The stranded wire is stretched between two fixed points, and the expansion produced by the high frequency current traversing it then creates a sag, which is measured by some indicating needle.

[64] Porter and Morris, *Proc. Roy. Soc.*, 1895, vol. 57, p. 469; also *The Electrician*, April 12, 1895, vol. 34, p. 735.

A form of high frequency hot-wire ammeter devised by the author is made as follows :—

On a vertical hardwood board are fixed two metal pins, A and B, between which are stretched with equal tightness a number of fine copper or platinoid wires. They are kept taut by a metal loop drawn back by a spiral spring, O (see Fig. 35). A second fine wire is also stretched between the point C and another fixed pin, D, and to the centre of this last wire is attached a third wire, EF, the upper end of which is fastened to the short arm of a pivoted index needle. When electric oscillations are passed through the wires AB, they expand and sag, and this creates a still greater sag in the wire CD, and hence allows the short end of the index needle to rise, and the pointer end to move down over a scale. By passing various measured continuous currents through the wires AB, the scale of this instrument can be graduated directly to read amperes. Hence, if any high frequency current or steady groups of oscillations are passed through the parallel wires, the needle indicates at once their root-mean-square value. By suitably selecting the length and number of the working wires of the instrument, it can be

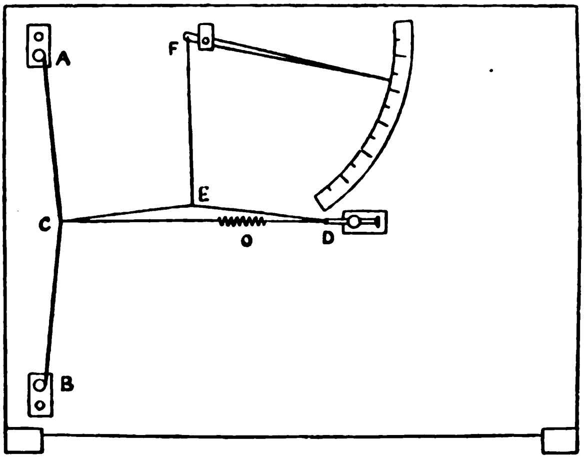


FIG. 35.—Hot-wire Ammeter for High Frequency Currents. (Fleming.)

adapted for any range of measurement. For the measurement of very small high frequency currents, such as those of about 0·01 to 0·1 ampere, the author has devised another form of single wire hot-wire instrument,[65] and a hot-wire voltmeter on the same principle had been previously but independently described by Professor Threlfall.[66]

The following form of hot-wire ammeter can be so made as to measure currents as small as 2 milliamperes, and is easily calibrated at the time of using it.

The ammeter consists of a wooden box (AB, see Fig. 36), 104 cms. in length, 8 cms. in height, and 6 cms. in width. The top of this box opens on hinges, and in the centre is fixed an achromatic convex lens, *l*, having a focal length of 10 cms. The front of the box is cut down to form a window, W, which is glazed with a sheet of thin transparent mica (see Fig. 36). In the box is fixed a square rod of well-seasoned pine, 1 metre in length and 2·5 cms. in width and breadth. To each end of this rod are fixed two small brass uprights to which terminal screws

[65] See J. A. Fleming, "On a Hot-wire Ammeter for the Measurement of Very Small Alternating Currents," *Phil. Mag.*, May 1904, ser. 6, vol. 7, p. 595; or *Proc. Phys. Soc. Lond.*, 1904, vol. 19, p. 173.

[66] See R. Threlfall, "On a New Form of Sensitive Hot-wire Voltmeter," *Proc. Phys. Soc. Lond.*, 1904, vol. 19, p. 58; also *Phil. Mag.*, ser. 6, vol. 7, p. 371, 1904.

are attached, and also small spring pieces of brass, *p*, *p*, which are pressed in by screws passing through the uprights.

To these springs at each end of the rod are attached fine wires, either of pure silver or of some high resistance alloy, such as constantan, platinoid, etc., according to the use to which the instrument is to be placed.

In the instrument constructed by the author, these wires are of platinoid, the length of the wires being 1 metre and the diameter 0·05 mm. The distance apart of these wires is about 5 mms. The extremities of these wires are soldered to the two spring pieces at the ends of the wooden rod, and the tension of these wires can be adjusted by means of the screws passing through the small uprights and pressing against the spring pieces.

To the centre of the wooden rod carrying the above-mentioned fine wires are fastened two very delicate spiral springs, *s*, which have their other ends looped over the long straight wires. These spiral springs are made of extremely fine platinoid wire, and they serve to keep the ammeter wires tight.

If one of the wires is heated by passing a current through it, it sags down slightly. The sag is indicated in the following manner:—The two wires are

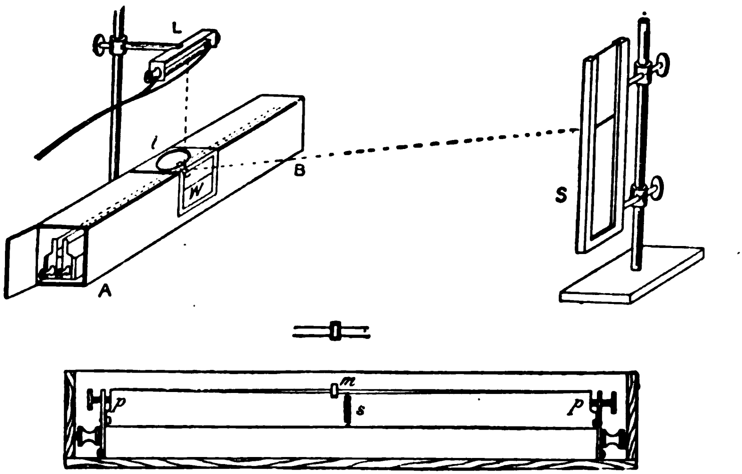


FIG. 36.—Hot-wire Milliammeter. (Fleming.)

embraced by an exceedingly small loop of paper, *m*, made from a strip of paper 2 mms. in width and about 12 or 15 mms. in length.

To this loop of paper is attached with a touch of shellac a fragment of silvered microscopic glass about 2 mms. in width and 5 mms. in length.

The tension of one of the wires is so adjusted that when no current is passing through either of them one wire sags more than the other, and this little loop of paper and its attached mirror sets itself at an angle of about 45° to the horizontal. This is attained by slightly relaxing the tension on one of the wires. Upon the lid of the containing box is carried an incandescent lamp, having a straight or horseshoe-shaped filament, and in front of the box is placed a vertical strip of ground glass, S, carried in a brass grooved frame, which can be adjusted to any height on a vertical metal rod. The height of the incandescent lamp is so adjusted that the lens forms a clear image of the filament or of one leg of the filament upon the ground glass in the form of a horizontal line of light. With a good lens this image can be made very sharp. The lens actually used was the objective of an old opera-glass. A hood of metal or asbestos placed over the lamp prevents the direct rays of the lamp falling on the ground-glass screen. The screen can be conveniently placed about a metre from the wire box.

If, then, a small current is passed through the slacker of the two measuring wires, its sag will increase and the small mirror attached to the two wires will be tilted, and the image of the filament on the ground glass will move down, but return again to its original zero, as soon as the current is removed.

As a preliminary step, both the wires must be aged by sending intermittently a small current through them for a considerable time, this current being continually interrupted.

In the instrument actually made, the platinoid wires have a resistance of about 168 ohms each ; hence, if an electromotive force of 2 volts is applied to the ends of the wires, a current of about $\frac{1}{84}$ of an ampere passes through them.

The instrument is calibrated in the following manner:—A secondary cell having a measured electromotive force, say, of about 2 volts is connected in series with one of the working wires through a resistance box of the usual plug pattern. By varying this resistance, different currents are passed through the wire, and the position of the spot of light on the screen corresponding to the different current is noted.

If the wire employed is of platinoid or of constantan, its resistance will not be altered appreciably by different small currents passed through it, and hence the resistance of the wire can be determined once for all, with a sufficient degree of approximation for practical purposes, by means of a potentiometer. When this has once been done, a few observations taken with a cell of known electromotive force and a plug resistance box used as above, enable the observer to mark off on the ground-glass strip with a pencil the position of the line of light for various known currents lying within a certain range. The strip of ground glass may then be removed and applied to a sheet of squared paper, and a curve plotted down showing the deflections in terms of the actual currents. This curve proves to be a parabola (see Fig. 37), because if we plot the logarithms of the deflections and the logarithms of the currents we have a straight line delineated, making an angle with the horizontal, the tangent of which is equal to 2. If, then, we replace the ground-glass screen in its original position and pass through the ammeter wire any current, continuous or alternating, lying within the range of the graduation, the resulting deflection of the line of light on the screen can be at once marked off on the ground glass, and from the curve of calibration obtained as above described the ampere value of this current becomes at once known.

In the instrument actually used the deflection of the line of light on the scale placed at a distance of about 80 cms. from the mirror, produced by an application of 2 volts to the wire, is about 3 cms., and 4 volts produce about 12 cms. deflection ; hence, current of about $\frac{1}{100}$ of an ampere, or 10 milliamperes, produces a deflection which can be accurately read to within 2 or 3 per cent., and a current as small as 5 milliamperes thus can be measured.

The particular class of wire with which the instrument should be strung depends on the uses to which it is to be put. If the object is to read a current of as small a value as possible, then the wire must be as fine as possible, and made of a material of high specific resistance, such as constantan.

Before the great European war which began in 1914 it was possible to obtain from Germany very fine wires drawn from different pure metals and high-resistance alloys drawn down to diameters varying between 0·05 mm. and 0·02 mm. The resistance of a constantan wire of the latter size per metre is about 1350 ohms, whilst a wire of pure silver of the larger size has a resistance of only 8 ohms per metre.

The sag of the wire used in the above-described instrument depends essentially upon its temperature, and its temperature depends upon the rate at which energy is being expended in it, per unit of its surface. Accordingly, for the measurement of the smallest currents the wires must be of high-resistance material and as small as possible in diameter, whilst for the measurement of small voltages the wire must be made of a material like silver with high conductivity.

The temperature of a fine wire traversed by a feeble current can be raised by placing the wire in a vacuum ; because then the cooling effect of air convection is removed. By including the wire in a high vacuum the sensitiveness of a hot-wire ammeter made as above described can be increased from 20 to 25 times.

The author has recently devised a form of hot-wire ammeter with the wire enclosed in a highly exhausted glass tube. The sag of the wire is measured optically from the outside.

For measuring even smaller high frequency currents, Mr. W. Duddell[67] has devised an ingenious instrument, which is, in fact, an application of Boy's micro-radiometer. A small rectangular circuit, F (Fig. 38), is suspended by a quartz fibre, S, in the field of a strong magnet, M, and the ends of this coil terminate in

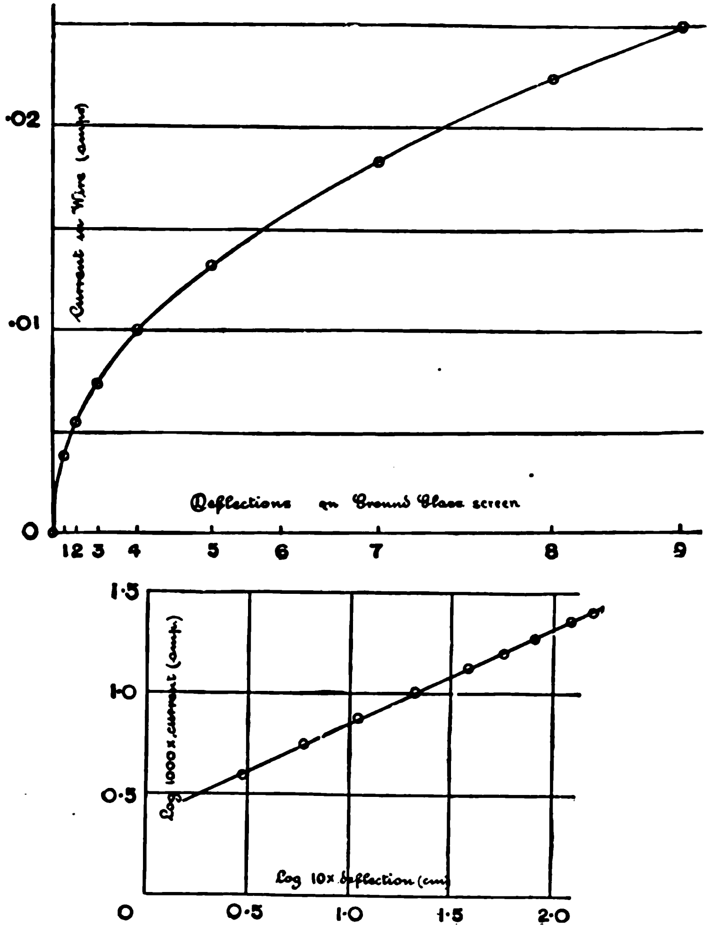


FIG. 37.—Calibration Curves of a Hot-wire Ammeter.

a bismuth-antimony thermocouple, T, one junction of which rests over and just clear of a thin strip of metallic foil or a wire, AB, through which the current to be measured is passed (see Fig. 38). The heat generated in this strip acts by convection and radiation on the thermo element, and creates in the associated circuit a current which causes it to be deflected in the magnetic field in which it is suspended. By attaching a mirror to this movable coil, Mr. Duddell has been able to measure alternating currents having a root-mean-square value of less

[67] See Mr. W. Duddell, "On Some Instruments for the Measurement of Large and Small Alternating Currents," *Proc. Phys. Soc. Lond.*, 1904, vol. 19, p. 233.

than $\frac{1}{1000}$ of an ampere. This instrument has proved of use in measuring the oscillatory currents in wireless telegraph antennæ.

For this purpose Mr. Duddell has employed thin strips of gold leaf as the heating circuit placed under the thermopile, and through this strip the electric oscillations pass and create in it heat, and enable a measurement to be made of their root-mean-square or effective value. It is easy to detect and measure the effective (R.M.S.) value of the current produced by a Bell telephone, when suitably spoken into, by the aid of this Duddell thermo-galvanometer.

Another method of employing a wire heated by electrical oscillations for the measurement of their effective or root-mean-square value has been much used in Germany. It is an application of a well-known instrument, usually called the Riess electric thermometer. It was originally invented by Sir W. Snow-Harris in 1827 (see *Phil. Trans. Roy. Soc.*, 1827). In its modern form it consists of a glass

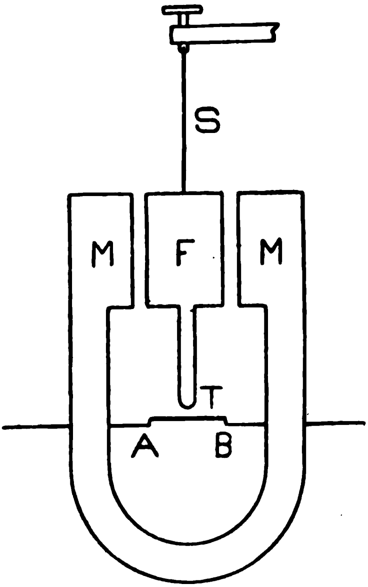


Fig. 38.—Principle of the Duddell Thermo-Galvanometer.

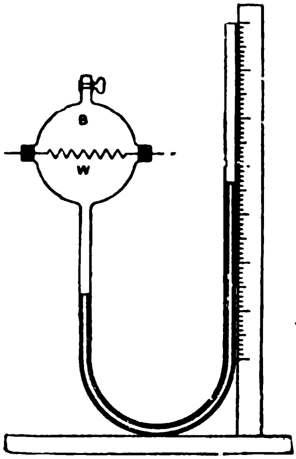


Fig. 39.—Snow-Harris or Riess Hot-wire Ammeter.

bulb (B) or tube (see Fig. 39), enclosing a fine wire (W) or stranded bundle of fine wires, with suitable electrodes. To the bulb is connected a U-tube having liquid in it, and also a lateral tube with glass stopcock is attached to the bulb for the purpose of equalizing the air pressure within and without. If an electric current is passed through the wire it heats it, and if the current remains constant a condition is soon reached in which the air in the bulb gains as much heat per second from the wire as it loses by radiation and convection. Then the pressure of the air in the bulb becomes steady, and is higher than that of the external air. Accordingly, the manometer liquid rises in one limb of the U-tube and falls in the other. A scale can be attached which shows the position of the liquid when various currents reckoned in amperes are passed through the wire. Since currents with equal R.M.S. value produce equal heat in the wire in the same time, the instrument becomes a means for measuring the R.M.S. value of electric oscillations or trains of oscillations. This instrument has to be used with some precautions to avoid errors due to expansion of the air in the bulb by heat other than that created in the wire, and is not generally so much to be trusted as a hot-wire

ammeter of the type just described, in which the heating effect of the current produces a sag in a wire strained between two fixed points.

The most generally useful type of hot-wire ammeter for the measurement of high frequency currents is one in which we determine the mean-square value of the current passing through the wire by means of a thermo junction in contact with it.

The author has designed an ammeter for this purpose made as follows. On two bracket supports (see Fig. 40) are carried a pair of brass T-pieces which are placed 5 or 7 cms. apart.

These T-pieces may be 4 or 5 cms. in length. To these are soldered a certain number of bare copper or platinoid wires, W, not larger than No. 40 S.W.G. size.

These wires must be spaced well apart. To the centre of one of these wires is attached a thermo-electric junction of copper and iron, J, or nickel and iron formed of very fine wires of these metals, not more than 0·05 mm. in diameter and not longer than 2 or 3 cms. The outer ends of these last fine wires are soldered to thick terminals, T_1, T_2. The terminals are connected to a low resistance galvanometer, a very convenient form being a Paul single-pivot galvanometer

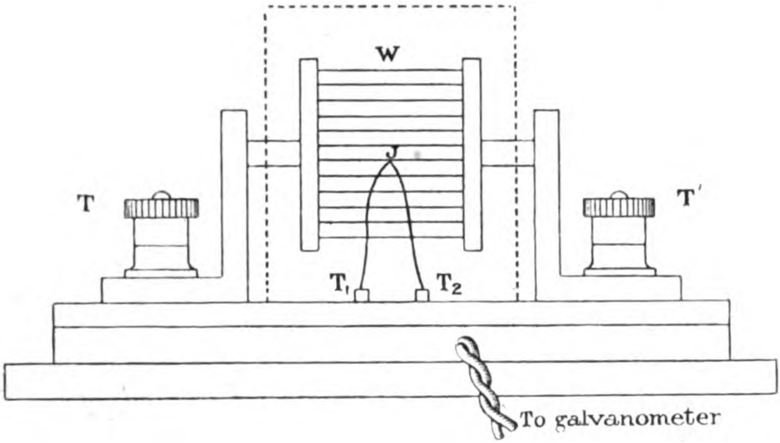


FIG. 40.—Hot-wire Thermo-electric High Frequency Ammeter. (Fleming.)

having a resistance of not more than 4 or 5 ohms. When a high frequency current is sent through the fine copper wires these are heated. A thermo-electromotive force is created at the junction in contact with one of the hot wires and deflects the needle of the galvanometer in connection with it.

The ammeter can be calibrated by sending measured continuous currents through it, and observing the corresponding deflection of the thermocouple galvanometer. A curve is then delineated, showing the currents in terms of the corresponding deflections, which is parabolic in form. If then any oscillatory current is sent through the ammeter wires, and the deflection of the galvanometer needle is observed, a reference to the curve enables us to determine the root-mean-square value of these oscillations.

Hot-wire ammeters of this kind are easily made and calibrated, and very convenient for the measurement of electric oscillations, damped or undamped.

12. The Bolometer Bridge Method of Measuring High Frequency Currents. —A third method of using a heated wire as an ammeter is the Bolometer Bridge method. When a wire is traversed by electrical oscillations and is thereby heated, its resistance increases. This increase in resistance can be measured on a Wheatstone's bridge. By a separate experiment we can find the value of the steady continuous current, which equally heats the wire. For this purpose the wire to be heated by the oscillations, called the bolometer wire, must consist of a

very fine iron or platinum wire, which must be arranged in a lozenge or diamond form (see Fig. 41). It is desirable to employ two equal circuits of the same sized wire. Let *a*, *b*, *c*, *d*, and *a′*, *b′*, *c′*, *d′*, be these two circuits, and let them be joined with two other resistances, R and S, and with a battery and galvanometer, G, so as to form a Wheatstone's bridge. To two opposite corners, *a*, *c*, of one of the diamond-shaped circuits are connected wires which lead into a circuit, in which oscillations are set up. These oscillations pass through the wires *a*, *b*, *c*, *d*, and heat them, but as the bridge connections are made to points at equal potential, *b* and *d*, there is no tendency for the oscillatory currents to stray into the bridge circuits. The balance of the bridge can thus be obtained both when the oscillations are flowing and when they are absent. The same circuit, *a*, *b*, *c*, *d*, can then be heated by a continuous current, so as to cause an equal increase in resistance, and the measurement of this equi-heating continuous current gives us the R.M.S. value of the electric oscillations.

A very sensitive bolometer bridge of the above type was employed by Professor C. Tissot, of the French Navy, in his admirable researches on resonance in antennæ.[68]

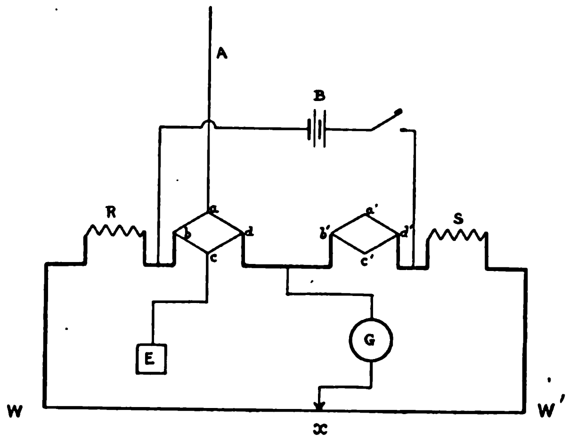


FIG. 41.—Bolometer Bridge.

He employed as the bolometer wire extremely pure platinum wire freed entirely from iridium, so that it had as low a resistance and as high a temperature coefficient as possible.

By special purification he was able to obtain platinum having a temperature coefficient of 0·0032, increasing, therefore, 0·3 per cent. in resistance per degree centigrade with rise in temperature

This platinum was drawn down into wire of extreme fineness by the Wollaston method, viz. by preparing a compound wire platinum inside with a sheath of silver outside, then drawing this down as fine as possible, and finally dissolving off the silver by nitric acid.[69] In this manner he prepared platinum not more than 0·01 mm. in diameter.

Small rectangles were then made by attaching this wire to four terminals, and these were sealed up in an exhausted glass vessel. The sides of the rectangle were about 1·5 cm. in length, and had a resistance of about 17 ohms. These

[68] See M. Camille Tissot, "Étude de la Résonance des Systèmes d'Antennes dans la télégraphic sans fils." Gauthier-Villars, Paris, 1906.

[69] For a number of useful references on this matter and information on the Wollaston process the reader is referred to the United States Patents, Nos. 767971 and 767981, granted to John Stone, dated August 16, 1904.

rectangles then were arranged as the two arms of a Wheatstone's bridge, as in Fig. 41, and one of the rectangles had its opposite corners connected respectively to the oscillatory circuit or to the antenna and earth-plate in which it was desired to measure the oscillatory current.

The sensibility of such a bolometer wire to current is greatly increased by placing the wire in a good vacuum, because the loss of heat from it by convection is greatly reduced, and therefore a given current raises its temperature higher and therefore increases by a larger percentage its resistance.

When combined with a properly arranged bridge and sensitive mirror galvanometer, such a bolometer wire is capable of indicating extremely small oscillatory currents. The bridge is first balanced so that the galvanometer needle remains at zero. The oscillations are then sent through one of the fine wire rectangles forming one arm of the bridge, it becomes heated, and the bridge balance is upset, and the needle deflects.

M. Tissot found that with one of his bolometers having a resistance of 42 ohms when used in the circuit of a receiving antenna he could measure the antenna current produced by a radiotelegraphic station 50 kilometres distant.

A bolometer bridge is therefore of great use in radiotelegraphic researches, because it enables us not merely to detect but to measure the mean-square value of the feeble oscillatory currents, whether damped or undamped, set up in a receiving antenna.

13. Electro-dynamic Current Indicators for High Frequency Currents. Fleming Alternating Current Galvanometer.—A form of alternating current galvanometer devised by the author in 1884 has of late years been found useful for the comparative measurement of high frequency currents. A copper or silver disc, R (see Fig. 42), is suspended by a very fine wire or bifilar suspension, so that it hangs within a coil, C, with the plane of the disc at 45° to the plane of the coil. If the coil is traversed by an alternating current, this creates induced currents in the disc, and it tends to set itself in a position with its plane more nearly parallel with the magnetic field of the latter.

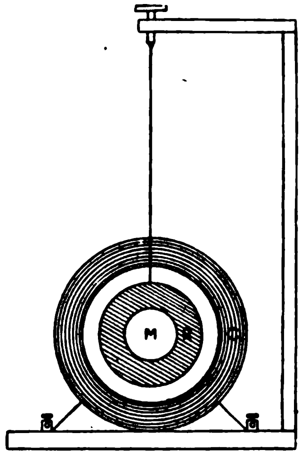


FIG. 42.—Electrodynamic Ammeter for Alternating Currents. (Fleming.)

The reason for this is because a closed conducting circuit having inductance when placed in an alternating magnetic field tends to set itself so as to decrease as much as possible the magnetic flux perforating through it. The proof of this will be found in an article on "Alternate Current Measurement," by Dr. W. E. Sumpner, in the *Proceedings of the Royal Society*, series A., vol. 80, p. 310, 1908.

The theory of this suspended disc dynamometer has been given in another form by Professor G. W. Pierce.[70] Let us assume that the disc is a ring of resistance R, and inductance L, and held in the centre of a coil of N turns, with its plane at an angle ϕ with that of the coil. Let M be the mutual inductance between the coil and ring. Let the coil be traversed by alternating currents of frequency $n=\frac{p}{2\pi}$, and let i_0 and i be the currents in the ring and coil respectively at any time, t. Then the torque F acting on the ring is $F=i_0 i\frac{dM}{d\phi}$.

The current in the coil is $i=I\sin pt$, and the E.M.F. induced in the ring is—

$$e_0=\frac{d}{dt}(-iM)=-M\frac{di}{dt}-i\frac{dM}{dt}$$

[70] See Prof. G. W. Pierce, "On Resonance in Wireless Telegraph Circuits," *Physical Review*, April 1905, vol. xx. p. 226.

For small deflections $\frac{dM}{dt}=0$, and we have—

$$e_0 = -\mathrm{MI}p \cos pt$$

The equation for the current in the ring is therefore—

$$\mathrm{L}\frac{di_0}{dt} + \mathrm{R}i_0 = -\mathrm{MI}p \cos pt$$

Hence $$i_0 = \frac{-\mathrm{MI}p(\cos pt - \theta)}{\sqrt{\mathrm{R}^2 + p^2\mathrm{L}^2}}$$

where $\theta = \tan^{-1}\frac{\mathrm{L}p}{\mathrm{R}}$.

Accordingly, we have for the torque at any instant—

$$\mathrm{F} = \frac{-\mathrm{MI}^2p \sin pt \cos(pt - \theta)}{\sqrt{\mathrm{R}^2 + p^2\mathrm{L}^2}} \frac{d\mathrm{M}}{d\mathrm{Q}}$$

The average value of this is—

$$\overline{\mathrm{F}} = \frac{1}{\mathrm{T}}\int_o^{\mathrm{T}} \mathrm{F}dt = -\frac{\mathrm{LM}p^2\mathrm{I}^2}{2(\mathrm{R}^2 + p^2\mathrm{L}^2)} \cdot \frac{d\mathrm{M}}{d\phi}$$

Maxwell has given an expression for the mutual inductance of two circles, whose planes make an angle ϕ with each other.[71] From this expression it is found that if the centres of the circles coincide, and their planes make an angle of 45°, we have $\frac{\mathrm{M}}{2} \cdot \frac{d\mathrm{M}}{d\phi} = \frac{\pi^4 r^4 r_0^4}{\mathrm{D}^6}$. Hence, if there are N turns in the coil, we have finally as the expression for the average torque acting on the ring—

$$\overline{\mathrm{F}} = \frac{\mathrm{L}p^2\mathrm{I}^2}{\mathrm{R}^2 + p^2\mathrm{L}^2} \cdot \frac{\pi^4 r^4 r_0^4}{\mathrm{D}^6}$$

where r and r_0 are the radii of the circular fixed coil and suspended ring respectively, and D is the distance from the centre of the ring to the perimeter of the fixed coil. The least value D can have is therefore r, which happens when the centres of ring and coil coincide.

The average torque on the ring, and therefore its deflecting moment, is proportional to the square of the current in the coil and proportional to the square of the frequency for the same instrument. If, therefore, the frequency is constant, and if the ring is suspended by a fine wire or quartz fibre so that the restoring torque varies as the deflection nearly, the deflection will measure the mean square value of the current passing through the coils. Professor G. W. Pierce has confirmed this conclusion experimentally.

With certain precautions, therefore, the instrument may be used to measure the mean-square value of trains of electric oscillations.

If it is desired to construct an instrument which shall act merely as an indicator of alternating current, but not of an ammeter, we may suspend within a coil a small needle of soft iron by a quartz fibre or bifilar suspension. The iron needle should be placed with its axis at 45° to the plane of the coil. When alternating currents or oscillations are passed through the coil, the needle will deflect, and its deflections may be rendered evident by attaching to it a small mirror as usual.

If high frequency oscillations are to be detected, the needle must not be a thick solid piece of iron, but must be a small bundle of extremely fine iron wires, each one of which is insulated from the rest by shellac varnish.

14. High Frequency Potential Measurements.—We are concerned in making two distinct potential measurements in connection with high frequency currents. First we may require the maximum value of the potential difference of two points on a circuit when it is traversed by electric oscillations; or, second, we may wish to know the root-mean-square value of the oscillation between the same

[71] See Maxwell's "Electricity and Magnetism," vol. ii. p. 308.

points. The measurement of the maximum potential is best made by observations on the length of spark which such potential difference will create between metallic balls of equal and known diameter. We have already given tables of the dielectric strength of air for various spark lengths and spark-ball sizes. We may for most practical purposes determine the maximum value of the potential difference between two places which exceeds 4000 volts or so by attaching to these points the terminals of a spark-ball discharger, the distance between the balls being measurable by a screw with divided head. These balls should be clean brass balls of 1 or 2 cms. in diameter, so that we may avail ourselves of the numerous observations which have been made on the sparking potential for various distances between such surfaces. The curves in Fig. 43 are plotted from the figures of observations taken by A. Heydweiller (see *Wied. Ann.*, 1893, vol. 48, p. 234), and give by inspection the

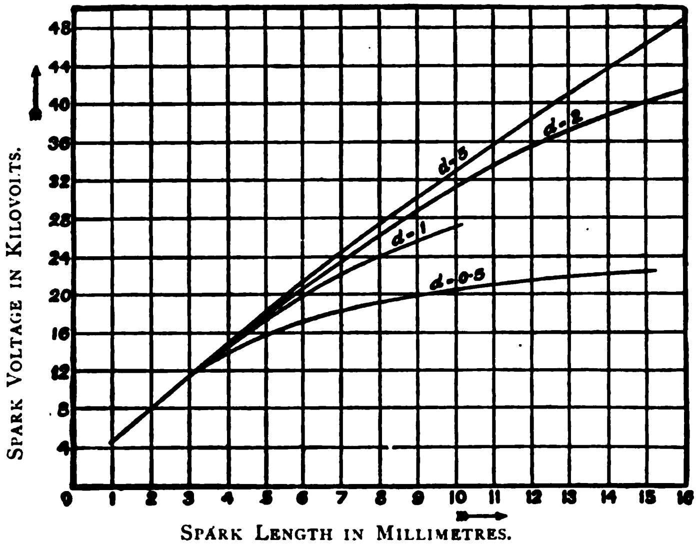


Fig. 43.—Spark Voltages for Various Spark Lengths and Spark Balls of Various Diameters, *d* cms.

voltage between spark balls varying in diameter from 0·5 cm. up to 5 cms. or 2 inches.

It will be seen that the smaller the ball the more has the curve a tendency to bend over so as to become parallel to the axis of spark length. Hence there is an advantage in the use of large spark balls for obtaining high charging potentials for condensers, since the spark voltage for a given spark length increases within a certain limit with the diameter of the balls.

The table on p. 188 gives the results of Heydweiller's observations on spark voltages between balls of various diameters and for spark lengths between 1 and 16 mms., which are graphically depicted in Fig. 43.

The table on p. 189 gives the spark voltages for various spark lengths taken between balls 2 cms. in diameter. The figures up to 1·5 cm. are taken from Heydweiller's observations, and those beyond from observations by J. Algermissen and given on the authority of Dr. J. Zenneck (see "Elektromagnetische Schwingungen und Drahtlose Telegraphie," by Dr. J. Zenneck, Stuttgart, 1905).

These tables provide the means for obtaining the spark voltage from the measurement of the spark length between metallic balls 2 cms. in diameter, but must not be applied in the case of balls much larger or smaller.

TABLE SHOWING THE MEASUREMENT OF SPARK VOLTAGES FOR VARIOUS LENGTHS OF SPARK TAKEN BETWEEN SPARK BALLS OF VARIOUS DIAMETER AT NORMAL ATMOSPHERIC PRESSURE AND TEMPERATURE BY HEYDWEILLER.

v=spark voltage.
d=spark-ball diameter in centimetres.
l=spark length in millimetres.

d=5·0 cms.		d=2·0 cms.		a=1 0 cm.		d=0·5 cm.	
l in millimetres.	v in volts.	l in millimetres.	v in volts.	l in millimetres.	v in volts.	l in millimetres.	v in volts.
5	18,360	1	4,710	1	4,800	1	4,830
6	21,600	2	8,100	2	8,370	2	8,370
7	24,540	3	11,370	3	11,370	3	11,340
8	27,330	4	14,490	4	14,550	4	13,770
9	30,090	5	17,490	5	17,310	5	15,720
10	32,850	6	20,370	6	19,920	6	17,190
11	35,580	7	23,250	7	22,050	7	18,300
12	38,310	8	26,040	8	24,090	8	19,020
13	41,010	10	31,290	9	25,590	10	20,190
14	43,680	12	35,490	10	27,000	15	22,320
15	46,230	14	38,640				
16	48,660	16	41,280				

Baille and Paschen have also made experiments on the spark voltage for different lengths of spark between metal balls of various diameters in air at

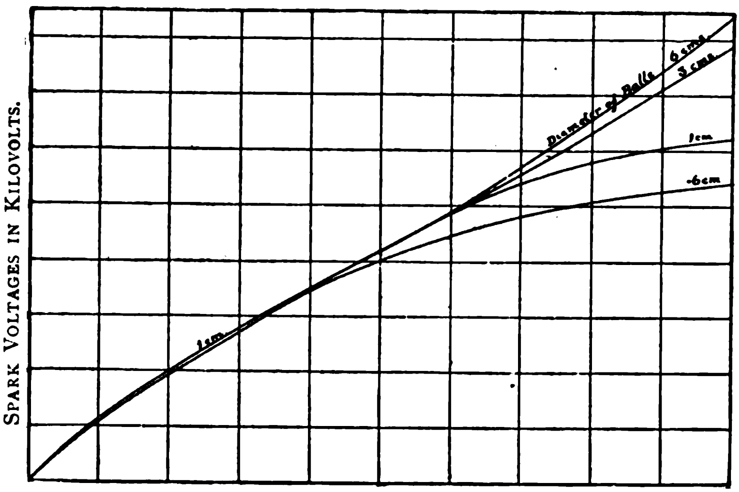


FIG. 44.—Curves showing Spark Voltages in Kilovolts for Various Spark Lengths in Millimetres. (Baille and Paschen.)

atmospheric pressure and temperature,[72] and have shown that the spark potential varies considerably with the size of the balls, and some of their results are given in the table on p. 189, and graphically in Fig. 44. From Heydweiller's observations,

[72] See Baille, *Annales de Chemie de la Physique* (5), 1882, vol. 25, p. 486.

as well as those of Baille and Paschen, it will be seen that up to a spark length of 4 mms. the variation in the diameter of spark balls between 0·5 cm. and 6 cms. makes but little variation in the spark potential, but beyond this length the spark voltage for a given spark length rises very rapidly.

TABLE SHOWING THE SPARK VOLTAGE BETWEEN BRASS BALLS, 2 CMS. IN DIAMETER, FOR VARIOUS SPARK LENGTHS.

Spark length in centimetres.	Spark voltage.	Spark length in centimetres.	Spark voltage.	Spark length in centimetres.	Spark voltage.
0·1	4,700	1·8	44,700	3·5	61,100
0·2	8,100	1·9	46,100	3·6	61,800
0·3	11,400	2·0	47,400	3·7	62,400
0·4	14,500	2·1	48,600	3·8	63,000
0·5	17,500	2·2	49,800	3·9	63,600
0·6	20,400	2·3	51,000	4·0	64,200
0·7	23,250	2·4	52,000	4·1	64,800
0·8	26,100	2·5	53,000	4·2	65,400
0·9	28,800	2·6	54,000	4·3	66,000
1·0	31,300	2·7	54,900	4·4	66,600
1·1	33,300	2·8	55,800	4·5	67,200
1·2	35,500	2·9	56,700	4·6	67,800
1·3	37,200	3·0	57,500	4·7	68,300
1·4	38,700	3·1	58,300	4·8	68,800
1·5	40,300	3·2	59,000	4·9	69,300
1·6	41,300	3·3	59,700	5·0	69,800
1·7	43,200	3·4	60,400	5·1	70,300

TABLE SHOWING THE MEASUREMENT OF SPARK VOLTAGE FOR VARIOUS SPARK LENGTHS AND SPARK-BALL DIAMETERS IN AIR AT NORMAL PRESSURE, BY BAILLE AND PASCHEN.

Spark length in centimetres.	Spark voltage for balls of diameter			
	6 cms.	3 cms.	1 cm.	0·6 cm.
0·1	4,434	4,500	4,575	4,660
0·2	7,630	7,800	8,040	8,050
0·3	10,840	10,980	11,200	11,200
0·4	13,900	14,030	14,290	13,902
0·5	16,500	16,500	16,400	15,975
0·6	19,570	19,570	19,570	17,900
0·7	22,620	22,140	21,680	19,266
0·8	26,400	25,430	23,280	20,325
0·9	29,230	28,390	24,000	21,180
1·0	33,900	31,410	24,900	21,714

It is clear on comparing all the results given for spark voltage that there are very sensible differences between the results given by various observers for the spark voltage for given lengths of spark even between balls of the same diameter.

As the voltage required to produce a spark of given length in air at ordinary pressure between metal balls is an important number, we shall collect here the results for sparks between 1 and 6 mms. in length as given by various observers.

MM. Bichat and Blondlot have measured spark voltages for various lengths of sparks in air at normal pressure taken between metal balls 1 cm. in diameter.

The observations made by T. Gray on dielectric strength of air (*loc. cit.*) between metal surfaces, which are part of spheres 70 cms. in diameter, have been

reduced to spark voltages for the stated spark lengths. Also those by T. W. Edmondson (*loc. cit.*) between balls 3 cms. in diameter, and observations made in the Physical Laboratory of University College, London, with spark balls 2·6 cms. in diameter. These spark voltages for various spark lengths from 1 to 6 mms. are set out below, and the mean of all the results is given in the last column.

Spark length in air.	Spark voltage according to observations of—				Mean results.
	Bichat and Blondlot.	T. Gray.	T. W. Edmondson.	University College.	
1 mm.	4,765	4,360	4,069	5,151	4,586
2 ,,	8,140	7,560	7,812	8,181	7,924
3 ,,	11,307	10,830	11,400	11,300	11,210
4 ,,	14,119	13,800	15,000	14,361	14,320
5 ,,	16,664	16,800	18,630	17,421	17,380
6 ,,	19,210	19,620	22,290	20,481	20,400

Heydweiller (*loc. cit.*) gives a table of collected results of spark voltages for different spark lengths between balls 0·5 cm. in diameter. He quotes from the following memoirs :—

Czermak, *Wien Ber.*, (2), 1888, vol. 97, p. 307.
Freyberg, *Wied. Ann.*, 1889, vol. 38, p. 250.
Paschen, *Wied. Ann.*, 1889, vol. 37, p. 69.
Baille, *Ann. Chim. et Phys.*, (5), 1882, vol. 25, p. 486.
Bichat and Blondlot, *Jour. de Phys.*, (2), 1886, vol. 5, p. 457.
Obermayer, *Wien Ber.*, (2), 1889, vol. 100, p. 134.
Quincke, *Wied. Ann.*, 1883, vol. 19, p. 545.

For additional information we must refer the reader to these papers, also to an important paper by Dr. A. Russell "On the Dielectric Strength of Air" (*Proc. Phys. Soc. Lond.*, November 1905), to which reference has already been made.

The results of observations by various observers who have measured the spark voltage required to produce a spark in air at normal pressure 1 mm. in length are also given below :—

Observer.	Spark voltage to produce 1 mm. spark in air at normal pressure.	Spark surfaces.
Lord Kelvin	4000 volts	Slightly curved metal plates.
Mascart	5490 ,,	Metal balls, 22 mms. diameter.
De la Rue and H. Miller	4330 ,,	Metal discs.
Bichat and Blondlot	4765 ,,	Metal balls, 10 mms. diameter.
T. Gray	4360 ,,	,, ,, 70 cms. ,,
T. W. Edmondson	4069 ,,	,, ,, 3 ,, ,,
Observations at University College, London	5151 ,,	,, ,, 2·6 ,, ,,

Mean value = 4597, say 4600 volts.

Hence we are not far wrong in accepting 4600 volts as the approximate value of the electromotive force required to produce a spark 1 mm. in length between metal balls about 1 inch in diameter in air at normal pressure and temperature.

It should be noted, however, that as the balls get hot by sparks passing, the spark voltage for a given length decreases. Hence the above figures apply only to cool balls. Moreover, ultra-violet light falling on the balls will greatly decrease the spark voltage for a given length, therefore in all such measurements the spark micrometer balls must be carefully protected from the light of other sparks or electric arcs.

The above figures enable us to tell approximately the potential to which a

condenser such as a Leyden jar is charged when it yields a spark discharge of any length between 0 and 5 cms.

The second kind of potential measurement which has to be made is that of the root-mean-square or effective value. This is accomplished by means of an electrostatic voltmeter in which the "needle" is connected to one pair or set of quadrants. Since the attraction between the fixed and movable parts varies as the square of their potential difference, the deflection of an electrostatic voltmeter connected as above mentioned measures the root-mean-square (R.M.S.) value of the potential difference of its own quadrants.

Hence if we have an oscillatory circuit containing a spark gap, and apply to the terminals of the condenser or to the spark balls an electrostatic voltmeter, we can find from the spark length the maximum voltage V, and from the voltmeter reading the mean-square voltage, and if we measure also the capacity and inductance in the circuit, we have at once the means of calculating the frequency n, and therefore the logarithmic decrement of the oscillations, as we shall show in the next chapter (see Chap. III.).

15. Measurement of Spark Frequency.—Another measurement frequently required is that of spark frequency. If an inductance coil or transformer is connected to a pair of spark balls which are short-circuited by an inductance and condenser in series with one another, then oscillatory sparks pass between the balls when the potential reaches a value corresponding to a spark length. It is impossible to calculate the power being given to the condenser circuit unless we know how many of these trains of oscillations, that is, how many oscillatory sparks take place per second. It is not possible to count these sparks, because they may come at the rate of even 50 or 100 per second. Neither can we assume that the number of sparks is equal to the number of interruptions of the primary circuit of the inductance coil or the number of alterations of the alternating circuit if a transformer is being employed. The number of sparks per second may be less or more according to circumstances. Thus, for instance, if an alternating current is being employed having a frequency of 50, it will depend upon the inductance and the capacity in the oscillating circuit, and upon the inductance inserted between the secondary circuit of the transformer and the spark balls, whether we have a number of sparks per second greater than, equal to, or less than the number of alterations per second with the alternating current supplying the transformer. The author has therefore devised the following appliances for measuring spark frequency.

A well-made wooden box perfectly light-tight and blackened in the interior, with a door on one side, is furnished with a good rapid rectilinear camera lens, L, at one side. The lens tube has the usual iris diaphragm. The box is 38 cms. high, 38 cms. wide, and 25 cms. broad. The lens tube is prolonged by another tube closed at the end, but with a very small hole, H, in the cover. In the interior of the box is a train of clockwork, W, which drives round a vertical shaft about 18 times per minute (see Fig. 45). This shaft carries a cubical block of aluminium, M, to the four sides of which are affixed carefully flatted glass plates silvered on the surface. This cubical mirror is so placed that it receives a ray passing through the small hole in the collimator tube, and gathered by the lens, and reflects it at right angles, or nearly so, so that the ray falls on a slit, S, in the side of the box, about 1 cm. wide and 7 or 8 cms. in length. Outside the box, a plate carrier, P, slides down in grooves in such fashion, that when the slide is drawn out the exposed sensitive plate glides past the slit in the box. The same clockwork that drives round the cubical mirrors lowers the photographic plate at a uniform rate so that it travels over the slit.

If, then, the pinhole in the end of the collimator is illuminated intermittently by the image of a spark thrown on it, then the ray passing through the lens and reflected from the revolving mirror is brought to a focus on the photographic plate and sweeps across it, imprinting an image on the plate at intervals depending on the frequency of the spark. Four times in each revolution of the block M a train of images sweeps over the gradually falling photographic plate, and when this is developed we find it covered with rows of black spots, each of which denotes the occurrence of a spark (see Fig. 46 (*b*)). It is clear that the number of sparks per second bears a definite relation to the speed of revolution of the mirror and to the

angle subtended by the slit at the mirror, and to the speed at which the photographic plate is lowered past the slit in the camera.

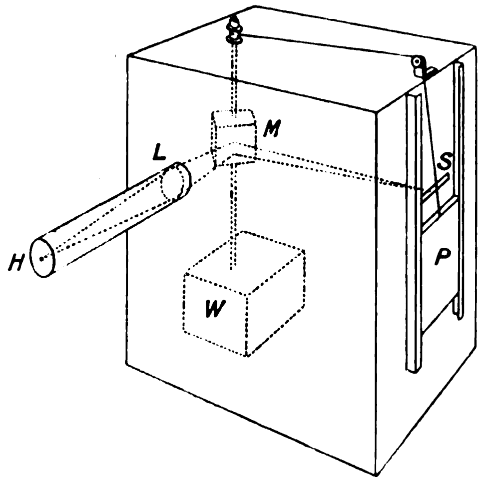


FIG. 45.—Photographic Spark Counter. (Fleming.)

It is convenient to have a mark on the photographic plate carrier, and a scale at the side to determine the time T taken by the plate to move down, say, 10 cms.

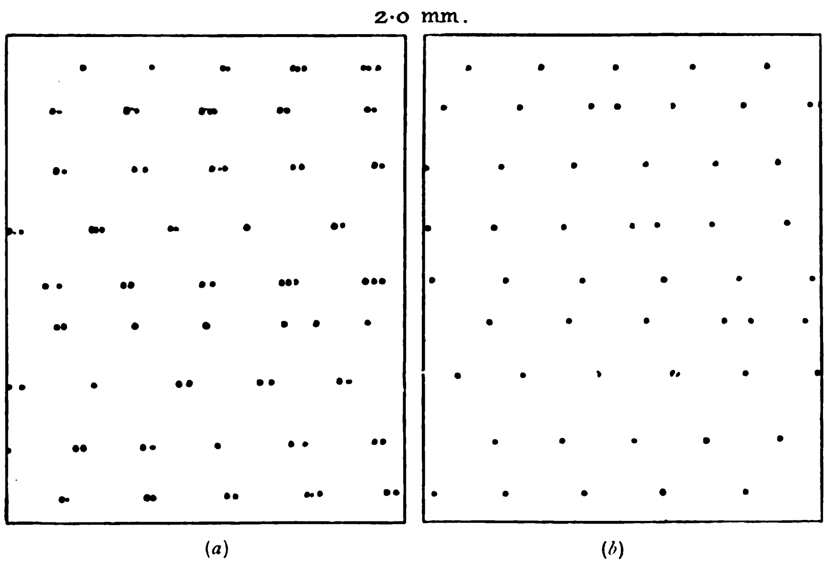


FIG. 46.—Records of Photographic Spark Counter.

Let l be the length of this slit, which in the camera designed by the author is 7·9 cms., and let θ be the angle in degrees which this slit subtends at the mirror

surface. Then $\frac{\theta}{360}$ is the fraction of one complete revolution which the reflected ray turns through in sweeping over a slit of length l.

If the photographic plate descends a distance d cms. per revolution of the mirror block, and takes a time T seconds to descend a distance D cms., then the time taken by the mirror to turn through one revolution is $\frac{Td}{D}$ and to turn through 1° is $\frac{Td}{360D}$, and to turn through an angle θ is $\frac{Td\theta}{360D}$. Hence half this time, or $\frac{1}{2}\frac{Td\theta}{360D}$, is the time taken for the ray to sweep over the slit of length l. Hence the time interval t corresponding to a length of 1 cm. on the photographic plate is $t=\frac{Td\theta}{lD720}$ seconds $=$ CT. If, then, there are N spark images on the plate in M rows, the average number per row is $\frac{N}{M}$, and the average space interval in cms. between images is $\frac{WM}{N}$, where W is the width of the plate in cms. Therefore $\frac{WMCT}{N}$ is the average time interval in seconds between the sparks, or $\frac{N}{MWCT}$ is the spark frequency.

In the spark counter constructed by the author, the constant C is equal to 0·00102, or very nearly $\frac{1}{1000}$, and the spark frequency n is given by the formula—

$$n=\frac{\text{Total number of spark images on plate}}{\text{number of rows of images}\times 0{\cdot}00102\ \text{T}\times 7{\cdot}79}$$

$$=\frac{1000}{8\text{T}}\times\frac{\text{Number of spark images on plate}}{\text{number of rows of images}}$$

The formula is checked by the following device. On the shaft of an electric motor is placed a sheet tin disc 2 feet in diameter having four holes an inch in diameter near the edge at quadrantal positions. Behind the disc is placed a small arc lamp, so that the light shines through these holes. When the motor is set in revolution with a speed of 3000 R.P.M., and its speed carefully determined with a tachometer, we have flashes of light emitted by the arc through the holes at the rate of about 200 per second. If we treat these flashes as if they were sparks and photograph them with the spark counter, we then find, on developing the plate, a number of black dots, which are the images of the collimator hole intermittently illuminated by the flashes coming at a known rate per second. On applying the formula given above to calculate the number of flashes from the number of images, we find an agreement with the actual number within 1 per cent. Such a control plate gives us, therefore, the means of confirming the accuracy of the formula and testing the photographic counter.

When such a spark counter is used to photograph the oscillatory spark at the spark balls of a radiotelegraphic transmitter, we find that the results are extraordinarily different according to the nature of the potential generator used, whether induction coil or transformer—also on the nature of the interrupter if a coil is used, and especially upon the length of the spark gap, and whether it has an air blast applied to it or not.

16. The High Frequency Resistance of Multiple-Stranded Insulated Wire.—Reference has already been made in § 1, p. 109, to an effect of increased resistance which may be found in stranded conductors, even if constructed of very fine insulated wires bunched or twisted together. The causes of the increased resistance which solid wires offer to high frequency currents has already been fully explained in § 1 of this chapter, and also that below a certain diameter of wire this increase is negligible. Hence it has generally been assumed that if a stranded conductor is made up of very fine insulated wires, say No. 40 S.W.G., twisted together, such compound conductor will have the same resistance for high frequency as for steady currents. This assumption is not quite correct. Each fine wire of

the strand when traversed by an alternating current has an external alternating magnetic field, and this field moving to and fro through the surrounding wires creates in them eddy currents which dissipate energy, and is therefore equivalent to an increase in resistance.

Hence, although such stranding may prevent the non-uniformity of current distribution across the cross-section of the conductor, and so eliminate one cause of augmented resistance, it brings into play another cause which may under some conditions make the resistance of the stranded conductor greater than that of a solid equi-sectioned conductor.

This matter has recently been investigated by Dr. G. W. O. Howe in a paper in *The Proceedings of the Royal Society* (Vol. 93, A, Sept. 1917, p. 468), and he has given some formulæ applicable in certain cases. If N insulated copper wires, each of diameter d, are bunched together to form a straight-stranded circular-sectioned cable of diameter D, then for alternating currents of frequency n such a conductor will have a resistance R_n which is given by—

$$R_n = R_o\left(1 + \frac{4 \cdot 2 N^2 n^2 d^6}{10^6 D^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

where R_o is the steady current resistance. If such a multiple-stranded wire of square overall section is wound into a spiral, the turns of the cable being close together and having S turns per centimetre, then Howe shows (*loc. cit.*) that the resistance of the spiral R_n is given by—

$$R_n = R_o\left(1 + \frac{28 N^2 n^2 S^2 d^6}{10^6}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

where R is the resistance to steady currents.

If the turns of the square-sectioned multiple conductor do not touch each other, but have each a width t centimetres, then Howe shows that the resistance to alternating currents of frequency n will be—

$$R_n = R_o\left(1 + \frac{35 \cdot 6}{10^6} N n^2 d^4 a S^2 t^2\right) \quad . \quad . \quad . \quad . \quad . \quad (106)$$

where a is the ratio Nd^2/D^2.

Hence it must not be assumed that in all cases mere stranding of fine insulated wire together is sufficient to produce a conductor which will have the same resistance for high frequency currents as for steady currents. The stranded conductor may even have a higher resistance than the solid conductor of equal cross-section.

Howe has calculated tables for various sizes of fine wire and various diameters of cable made up with them, showing the high frequency resistance for various frequencies, for which we must refer the reader to the original paper.

CHAPTER III

DAMPING AND RESONANCE

1. The Logarithmic Decrement of Electric Oscillations and the Damping. —We have seen that when electric oscillations are excited in a circuit having resistance, inductance, and capacity, by permitting a sudden discharge to take place across a spark gap in it, we have produced in the circuit high frequency alternating currents which continually decay in amplitude, thus constituting a train of *damped oscillations.*

This decay may arise from several causes, acting singly or together, but is essentially dependent upon some action which dissipates the initial energy imparted to the condenser.

We shall consider first the simplest case.

Let the circuit be a closed inductive circuit of constant resistance, the capacity in it consisting of a condenser, the plates of which are very near together. Also let the capacity and inductance have such values that the periodic time of a free oscillation in the circuit is large, compared with the time taken by an electric impulse to travel round the circuit. Since this velocity is the same as the velocity of light, the above condition is fulfilled when the length of the circuit does not exceed a few metres, and the natural periodic time is something of the order of a millionth of a second. Under the above circumstances the method of investigation applied in Chap. I. § 5 to the discharge of a condenser is valid. This is the case for most oscillatory circuits likely to be employed in practice. Then let C be the capacity in the circuit, L the inductance, and R′ the high frequency resistance of the circuit employed.

The inductance and capacity can be measured or calculated, and the frequency is then very approximately determined by the expression—

$$n=\frac{1}{2\pi\sqrt{CL}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation C and L must both be measured in consistent units—that is, in farads and henrys, or both in electro-magnetic units. If C is measured in microfarads and L in centimetres, then, as already shown—

$$n=\frac{5\times10^{6}}{\sqrt{CL}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is convenient to measure C and L in these last-named units in practice.

The quantity $\sqrt{CL}$ is then called the *oscillation constant* of the circuit, and varies inversely as the frequency, for—

$$\sqrt{CL}=\frac{5\times10^{6}}{n}$$

Thus, if the oscillation constant has a value, say, 10, this means that the product of the numbers representing the capacity reckoned in microfarads and the inductance in centimetres is 100.

If we denote the current in the circuit at any instant by i, then it has been shown in Chap. I. § 2 that we may express i as a function of the time by the equation—

$$i=\mathrm{I}\epsilon^{-at}\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where a stands for $\frac{R'}{2L}$ and I is some constant.

If in this equation we put successively, as on page 3—

$$t=\frac{\phi}{\pi}\frac{T}{2},\ t=\frac{\phi}{\pi}\frac{T}{2}+\frac{T}{2},\ t=\frac{\phi}{\pi}\frac{T}{2}+T, \text{ etc.},$$

we obtain the values of the successive maximum currents, I_1, I_2, I_3, etc., in opposite directions, where $\phi=\tan^{-1}p/\alpha$.

If we take the ratios of these successive maxima, we have—

$$\frac{I_1}{I_2}=\frac{I_2}{I_3}=\frac{I_3}{I_4}, \text{ etc.}=\epsilon^{\frac{\alpha T}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The quantity α is called the *damping factor*, and $\alpha T=\alpha/n=R'/2nL$ is denoted by δ, and is called the *logarithmic decrement of the oscillations*.

Accordingly— $$\frac{I_1}{I_3}=\frac{I_3}{I_5}=\frac{I_5}{I_7}, \text{ etc.}=\epsilon^{\delta}$$

or $$\delta=\log_\epsilon\frac{I_1}{I_3}=\log_\epsilon\frac{I_3}{I_5}, \text{ etc.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The damping factor α is a quantity the *dimensions* of which are those of the reciprocal of a *time*, whilst the logarithmic decrement is a mere numeric.

The quantity $\epsilon^{-\delta}$ is called the *damping* of the oscillations, and is the ratio of one maximum to the one preceding it in the same direction.

If the circuit contains no spark gap, and is a nearly closed or non-radiative circuit, the logarithmic decrement is a constant determined by the capacity, inductance, and resistance of the circuit.

The successive maximum values of the currents, or potential differences of any two points, decrease in accordance with an exponential law, and the logarithmic decrement can be calculated at once, when we know the inductance, resistance, and capacity in the circuit.

For since $\delta=\frac{R'}{2nL}$, and since $n=\frac{1}{2\pi\sqrt{CL}}$, we have the value of the logarithmic decrement for the closed non-radiative circuit given by the expression—

$$\delta=\pi R'\frac{\sqrt{C}}{\sqrt{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The value of the high frequency resistance R′ can be calculated from the ordinary or ohmic resistance by the formulæ already given (see Chap. II. § 1), when we know the frequency, provided the wire is approximately straight or bent into a curve of large radius.

The cases, however, in which we can apply the above expression (6) are not numerous, since most nearly closed oscillatory circuits for which we wish to know the decrement contain a spark gap, the resistance of which is not constant, or else are wound in spirals, and in addition we have in the case of all open circuits a loss of energy due to electro-magnetic radiation, producing a damping of the oscillations far exceeding that due to true resistance alone.

Furthermore, it must be noticed that when two circuits are coupled together inductively the establishment of an electric oscillation in one circuit creates an induced oscillation in the other. Hence, even if the secondary circuit is a closed non-radiative circuit without spark gap, its logarithmic decrement is increased by the mere fact of the proximity of the primary circuit, on which the secondary circuit exercises a reciprocal inductive action.

2. The Mean-square and Root-mean-square Value of a Train of Oscillations.—Let i denote the current in a circuit at any time, t, after the commencement of a train of oscillations in it having a frequency n. Let δ stand for the logarithmic decrement. Then, assuming that δ is a small quantity in comparison with 2π, and therefore the angle ϕ very nearly 90°, it follows from equation (4) of Chap. I. that the current can be expressed as a function of the time in the form—

$$i=I_1\epsilon^{\delta/4}\epsilon^{-\alpha t}\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this expression I_1 stands for the first maximum value of the oscillations which occurs at a time $t=\frac{T}{4}$ reckoned from initial zero.

If we square equation (7) we obtain—

$$i^2 = I_1^2\epsilon^{\delta/2}\epsilon^{-2\alpha t}\sin^2 pt \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which gives us an expression for the square of the instantaneous value of the current. Suppose that an electric oscillation is passed through a very thin wire or electrolytic conductor, of which the high frequency resistance is the same as its steady or ohmic resistance, R, then the rate at which heat is generated at any instant is given by the expression—

$$Ri^2 = RI_1^2\epsilon^{\delta/2}\epsilon^{-2\alpha t}\sin^2 pt \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

We cannot easily measure the instantaneous rate of evolution of heat, but if we allow a series of trains of oscillations at the rate of N per second to pass through the conductor, there will be a certain steady rate of production of heat, which is measured by the mean value of the quantity Ri^2, taken at numerous equidistant intervals during the second.

It is important, therefore, to determine an expression for the mean value of the square of the currents forming a train of electric oscillations.

The mean value of the square of the currents taken at equidistant intervals of time during an oscillation is called the *mean-square value* of the current (M.S. value).

The *mean-square* (M.S.) value of the oscillations when multiplied by the effective resistance of the circuit gives us the average rate of production of heat in the circuit or the dissipation of energy in it.

If there are N of these trains of oscillations per second, then, since each oscillation is completed in a small fraction of a second, we can say that the mean value of the square of i during one second of time is given by the integral—

$$J^2 = N\int_0^\infty i^2 dt \text{ [1]}$$

Hence, to obtain this mean-square current we have to find the value of the integral—

$$\int_0^\infty \epsilon^{-2\alpha t}\sin^2 pt \, . \, dt = \int_0^\infty \epsilon^{-2\alpha t}\left(\frac{1-\cos 2pt}{2}\right)dt$$

or of $$\int_0^\infty \tfrac{1}{2}\epsilon^{-2\alpha t}dt - \int_0^\infty \tfrac{1}{2}\epsilon^{-2\alpha t}\cos 2pt \, . \, dt$$

A reference to any treatise on the integral calculus will show that—

$$\int \epsilon^{nx}\cos mx \, . \, dx = \epsilon^{nx}\left(\frac{n\cos mx + m\sin mx}{m^2+n^2}\right)$$

Hence we have—

$$\int \epsilon^{-2\alpha t}\cos 2pt \, . \, dt = \epsilon^{-2\alpha t}\left(\frac{2p\sin 2pt - 2\alpha\cos 2pt}{4(\alpha^2+p^2)}\right)$$

If, therefore, we denote by J the *root-mean-square value* [2] (R.M.S. value) of the N groups of oscillations per second, so that J is defined by the integral—

$$J = \sqrt{N\int_0^\infty i^2\,dt}$$

and if i denotes the current at any time, t, such that—

$$i = I_1\epsilon^{\delta/4}\epsilon^{-\alpha t}\sin pt$$

we have—

$$J^2 = NI_1^2\epsilon^{\delta/2}\int_0^\infty \epsilon^{-2\alpha t}\sin^2 pt \, . \, dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

[1] By some writers the mean-square value of the current, or the value of the integral J^2, is called the *integral value* of the oscillations or of the oscillation train.

[2] The expression *root-mean-square value* (R.M.S. value) is an abbreviation for the long expression, "the value of the square root of the mean of the squares of the currents or electromotive forces taken at numerous equidistant intervals of time throughout a single period, or of the time of a train of oscillations."

Substituting in (10) the proper value of the definite integral as given above, we have—

$$J^2 = N I_1{}^2 \epsilon^{\delta/2} \frac{p^2}{4a(a^2+p^2)}$$

Now $p = 2\pi n$ and $a = n\delta$, hence—

$$\frac{p^2}{a^2+p^2} = \frac{1}{1+\left(\frac{\delta}{2\pi}\right)^2}$$

$$\text{and} \quad J^2 = \frac{N I_1{}^2 \epsilon^{\delta/2}}{4n\delta} \cdot \frac{1}{1+\left(\frac{\delta}{2\pi}\right)^2}$$

In all practical cases $\frac{\delta}{2\pi}$ is a quantity not greater than 0·1, and often as small as 0·01. Hence, in those cases in which $\frac{\delta^2}{4\pi^2}$ is small compared with unity, we can say that the mean-square value of the oscillations having N trains per second is given by—

$$J^2 = \frac{N I_1{}^2 \epsilon^{\delta/2}}{4n\delta} = \frac{N I_1{}^2 \epsilon^{\delta/2}}{4a}$$

If the oscillations are not strongly damped, which is the case for a nearly closed oscillatory circuit when the high frequency resistance R′ is small compared with $4nL$ for that circuit, then the quantity $\epsilon^{\delta/2}$ is nearly unity, and the value of the mean-square current J^2 for M trains of oscillations per second is—

$$J^2 = \frac{N I_1{}^2}{4a}$$

$$\text{Hence} \quad J = \sqrt{\frac{N}{4a}} \cdot I_1 \qquad \text{(11)}$$

The root-mean-square current J is therefore proportional to the first maximum value of the oscillations, and to a factor which depends upon the number of trains of oscillations per second and the damping factor of each train.

It has been already shown (see Chap. I. § 5, equation 17) that if a condenser of capacity C is charged to a potential V, and then discharged through an inductive circuit, the current i at any time, t, reckoned from the instant when the discharge commences, is given by the expression—

$$i = CpV\epsilon^{-at} \sin pt \qquad \text{(12)}$$

Comparing together equations (10) and (12), we see that—

$$CpV = I_1 \epsilon^{\delta/4} \qquad \text{(13)}$$

Hence, substituting in equation (11), we have—

$$J^2 = \frac{NC^2p^2V^2}{4n\delta}$$

If the capacity C and inductance L are measured in absolute measure, then $p^2 = \frac{1}{CL}$, and therefore—

$$J^2 = \frac{2\pi NV^2C\sqrt{C}}{4\delta\sqrt{L}} \qquad \text{(14)}$$

from which we have—

$$\delta = \frac{2\pi NV^2C\sqrt{C}}{4J^2\sqrt{L}} \qquad \text{(15)}$$

If the above equation (15) is compared with (6), it will be seen that they differ only in that the quantity R′ in the latter is replaced by the quantity $\frac{NV^2C}{2J^2}$ in the

former. The energy stored in the condenser at each charge is measured by $\frac{1}{2}CV^2$, and if there are N discharges per second, which all expend themselves in heating the circuit, having high frequency resistance R′, the R.M.S. value of the current being J, we must obviously have the equation $R'J^2=\frac{1}{2}NCV^2$. Hence (15) can be reduced from 6 merely by the application of the principle of conservation of energy.

3. Determination of the Number of Oscillations by the Aid of the Decrement.—A knowledge of the value of the logarithmic decrement of the oscillations taking place in any circuit enables us to calculate the number of oscillations of current or potential which take place before their maximum value is reduced in any assigned ratio and the oscillations practically extinguished. If we consider the decrement to be constant, and I_1 to denote the first maximum oscillation and I_m the mth, *in the same direction*, then—

$$\frac{I_1}{I_m}=\epsilon^{(m-1)\delta} \qquad (16)$$

$$\text{Hence } \log_\epsilon \frac{I_1}{I_m}=(m-1)\delta \qquad (17)$$

and from this last equation (17) we can determine m when we have selected any desired ratio for $\frac{I_1}{I_m}$. Thus, let us suppose that the oscillations are to be considered as extinguished for all practical purposes when $\frac{I_1}{I_m}=100$, that is, when the last is only 1 per cent. of the first. We have then—

$$m=\frac{4{\cdot}605+\delta}{\delta} \qquad (18)$$

Thus if $\delta=0{\cdot}015$ we have $m=308$.

This gives us the number of *complete oscillations* which elapse before the maximum value of the oscillation is reduced to 1 per cent. of its initial or first maximum value.

4. Practical Determination of the Logarithmic Decrement of Electric Oscillations.—It will be seen from the previous sections that the practical determination of the logarithmic decrement for any oscillatory circuit and for certain types of circuit is an important matter, since the physical effects which arise in many cases depend in large degree upon the duration of the train of oscillations or number of complete oscillations in a group or wave train.

The majority of oscillating circuits for which we desire to predetermine the logarithmic decrements have a spark gap in them.

It has been shown by Dr. J. Zenneck[3] that when an oscillating circuit contains a spark gap the simple exponential law of decay no longer holds good, and the logarithm of the ratio of successive maxima is no longer constant. The decadence of the maxima is then approximately represented by a straight line and not by a logarithmic curve. This is equivalent to a continual increase in the logarithmic decrement as the oscillations decrease. Hence if we assign such a value of the logarithmic decrement as to fit the slope of the straight line at each point, this value increases with time as the amplitude dies away. This arises from the fact that the resistance of the spark increases as the amplitude of the oscillations decays.

Each successive oscillation is a little smaller than it would be if they diminished strictly according to a logarithmic law. Accordingly we can no longer predetermine the logarithmic decrement by calculation, but must arrive at it by experiment.

Experimentalists have, however, generally assumed that the decrement is constant, and that the decay of oscillations follows an exponential law.

The important matter is to determine the mean value of the decrement

[3] See J. Zenneck, *Ann. der Physik*, March 1904, vol. 13, p. 822; or *Science Abstracts*, July 1904, vol. 7, A.

Generally speaking, the greater part of the whole resistance of such oscillatory circuits as are used in wireless telegraphy is located in the spark gap, and an erroneous assumption is made if the resistance of the spark is taken as constant throughout the duration of a train of oscillations.

If, however, as a first approximation, we agree to take the spark resistance as constant, then several methods present themselves by means of which we may determine experimentally the logarithmic decrement for non-radiative circuits containing a spark gap.

1st. We may find the ratio between two successive maximum currents or oscillations in the same direction. This will give us the value of ϵ^{δ}, from which δ can be determined.

2nd. We may find the ratio between the square of the first maximum potential difference or current and the mean of the squares of all the potential differences or currents during a train. This, as shown below, will give us the value of $4n\delta$.

3rd. We may find the resistance of the spark independently of the rest of the circuit, and then if L is the high frequency inductance, and R′ is the high frequency resistance of the metallic part of the circuit, and r the resistance of the spark, we may determine the value of δ corresponding to r from the equation—

$$\delta = \frac{R' + r}{2n L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where n is the corresponding frequency of the oscillations.

The first-named method of determining the logarithmic decrement was suggested by Sir Ernest Rutherford in an important paper.[4] The process of measurement is as follows:—Consider two similar coils of wire, A and B, placed in series in a discharge circuit. Let these coils be wound in opposite directions. Let two similar steel needles magnetized to saturation be placed in coils A and B, their north poles facing in the same direction. If then a train of electric oscillations is created in these coils by discharging a condenser through them, it will be found that the reduction of magnetic moment of the needles is not the same in both cases. Let I_1, I_3, I_5, etc., be the maximum oscillations of the discharge in the one direction, and I_2, I_4, I_6, etc., be the maximum oscillations in the opposite direction. Then suppose that the half-oscillation I_1 is in such a direction as to increase the magnetization of the needle in the solenoid A, the needle in the coil A, being already saturated, will have no magnetic effect produced upon it by this first oscillation. The second oscillation, I_2, in the opposite direction, however, demagnetizes the surface skin, and the third oscillation, I_3, tends to remove the effect of the second, and so forth.

In the solenoid B, the first half-oscillation, I_1, is in such a direction that it demagnetizes the needle, and the second, I_2, tends to remagnetize it in its original direction, and so on.

Since the maximum value of the first oscillation, I_1, is greater than that of the second oscillation, I_2, and the third greater than the fourth, and so on, the needle in the coil B will be more demagnetized than that in A. If, however, we increase the number of turns per centimetre on the coil A, until the magnetic effects on the two needles are exactly the same, then assuming that the values of the currents decrease in geometrical progression, the maximum value of the magnetic force due to oscillation I_2 acting on the needle in the coil is equal to the maximum value due to the oscillation I_1 on the needle in coil B. Since the sum of a geometrical progression is proportional to its first term and to a function of its common ratio, it follows that the sum of $I_1 - I_2 + I_3 - I_4 +$ etc. is to the sum of $I_2 - I_3 + I_4 - I_5 +$ etc. in the ratio of I_1 to I_2, and hence the resultant effect of the entire train of oscillations in demagnetizing the steel needles is in each case proportional to the magnitude of the first effective demagnetizing oscillations. The statements above made follow, therefore, as a consequence of this fact.

Let N_1 and N_2 be the number of turns per centimetre of length which must be

[4] See Sir E. Rutherford, "On a Magnetic Detector of Electric Waves and some of its Applications," *Phil. Trans. Roy. Soc. Lond.*, A, 1897, vol. 189, p. 1.

put on the two coils A and B respectively to make their demagnetizing effects equal.

Then $$4\pi N_1 I_1 = 4\pi N_2 I_2$$

Therefore $$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Hence we have by definition—

$$\frac{\delta}{2} = \log_e \frac{I_1}{I_2} = \log_e \frac{N_2}{N_1}$$

and the logarithmic decrement is determined from the logarithm of the ratio of the turns per centimetre of length of the two coils. The *damping* $\epsilon^{-\delta/2}$ is then equal to the ratio $\frac{N_1}{N_2}$.

Instead of employing two separate solenoids or coils, a more convenient plan was adopted by Sir E. Rutherford. A narrow piece of sheet zinc, ABC (see Fig. 1), was bent into an almost complete circle, 7 cms. in diameter. This was fixed upon a block of ebonite. At the centre of the circle a thin glass tube, OM, was placed, which serves as the axis of a metal arm, LM, which pressed against the circumference of a circle and could be moved round it. The magnet consisted of about 30 very fine steel wires 0·003 inch in diameter, made into a compound magnet 1 cm. long. The wires were insulated from each other by shellac varnish, and the small needle was fixed inside a thin glass tube which

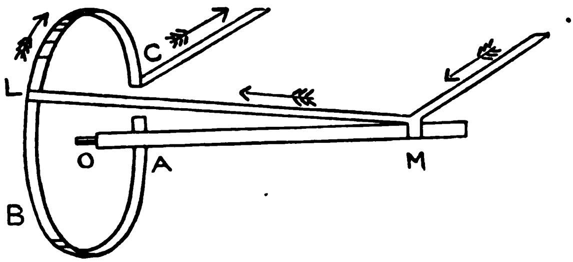


FIG. 1.—Rutherford's Method of Measuring the Decrement of Electric Oscillations.

could be easily slipped in and out of the central glass tube OM. Round the circumference of the zinc circuit was placed a divided scale, and the whole arrangement was placed in position before a small mirror magnetometer. The magnetized needle was then magnetized to saturation and placed in position in the centre of the circular strip, and its action on the magnetometer was compensated by another magnet. An oscillatory discharge in the right direction was then passed round the circular circuit and a deflection of the magnetometer was observed, due to the partial demagnetization of the detector needle. The detector needle was then removed and magnetized again to saturation.

Since the magnetic field H at the centre of a circle due to an arc of length l is given by the expression—

$$H = \frac{lI}{a_2}$$

where a is the radius of the circle and I the current in it, we see that the magnetic force acting on the magnetic needle is proportional to the length of the arc traversed by the discharge.

A series of observations showed that the deflection of the magnetometer was approximately proportional to the magnetic force acting on the needle, provided the magnetic force was well below the value required to completely demagnetize

the steel. To determine the damping of the oscillations, a discharge was passed in such a direction as to partly demagnetize the needle and the deflection of the magnetometer noted. The magnetic detector was then removed, magnetized to saturation and replaced, and a discharge passed in the opposite direction, and by various trial experiments the length of the effective arc of the circular circuit through which the discharge passed was adjusted in each case until the deflection was the same. When this was the case the ratio of the lengths of these arcs was proportional to the maximum values of the first and second oscillations. The damping $\epsilon^{-\delta/2}$ is then equal to the ratio of the lengths of the arcs of the circular circuit employed in the two experiments.

In this way the rate of decay of oscillations in many circuits was examined when the circuit contained a very short spark gap and the discharge circuit consisted of copper wires. It was found that the damping in this case was hardly appreciable. If, on the other hand, the copper wires were replaced by iron wires, then the damping rose to a large value and it also increased as the length of the spark gap increased, although not in the same proportion. The reason for this increase in the damping when iron wire is used is the absorption of energy due to the magnetization losses. The oscillatory current is practically confined to the surface of the conductor, but it does penetrate sufficiently into the iron to effect a certain degree of magnetization, and involves, therefore, some amount of hysteretic energy loss. If the iron wire is electroplated with copper or is galvanized, then it is found that, as regards damping, the wire becomes practically identical with a solid copper or solid zinc wire. The following table of observations by Sir E. Rutherford (*loc. cit.*) gives the results of experiments made with spark gaps of various lengths in a copper wire circuit, and shows the corresponding damping $\epsilon^{-\delta/2}$ and the total resistance R of the spark gap and circuit calculated from the formula—

$$R = 2n L\delta$$

EXPERIMENTS ON THE DAMPING OF ELECTRIC OSCILLATIONS

Discharge circuit rectangular and made of copper wire, 184 cms. by 90 cms.; inductance of circuit, L = 7400 cms.; capacity, C = 2000 electrostatic units; frequency = 1·25 million per second.

Length of spark gap in millimetres.	The damping, or ratio of two first oscillations. $\epsilon^{-\delta/2} = I_2/I_1$	Total resistance of spark gap and circuit in ohms.	Resistance of spark in ohms.
0·6	0·98	...	...
1·2	0·97	1·1	0·7
2·4	0·93	2·6	2·2
3·7	0·90	3·7	3·3
4·9	0·79	8·4	8·0
6·1	0·70	12·4	12·0

In these experiments the calculated high frequency resistance of the wires of the discharge circuit was 0·4 ohm, so that the excess over and above this value is the resistance of the spark.

Professor Rutherford found, as others have done, that the damping of the oscillations increased very rapidly with the length of the spark. It was also found that the damping depended on the capacity of the condenser used when the inductance and spark length were kept constant. For instance, with a spark gap having a length of 3·2 mms., and with different capacities, as in the table below, he found that the damping decreased with the increase of the capacity. It may be noted that to reduce capacity measured in electrostatic units to the equivalent in microfarads it is requisite to divide by 9×10^5.

Capacity in electrostatic units.	Damping.	Spark resistance in ohms.
1000	0·94	2·2
2000	0·90	2·6
4000	0·81	3·8

Similar experiments have also been made by the above-named method by Miss H. Brooks.[5] In this case the circuit in which the oscillations were set up consisted of a copper wire rectangle, the wire having a diameter of 0·7 mm., and the sides of the rectangle lengths of 145 and 125 cms. respectively. Hence the high frequency inductance was about 10^4 cms. The condenser used in the first experiments was a Leyden jar having a capacity of 0·00277 mfd. In other experiments another jar of less capacity was employed. The frequency of the oscillations was therefore close to 10^6. The quantity $2n\mathrm{L}$ had a value 2×10^{10} C.G.S. units nearly, or 20 ohms, and the logarithmic decrement δ, assumed constant, was therefore equal to one-twentieth part of the total resistance of the spark and circuit reckoned in ohms, since $\delta=\dfrac{\mathrm{R}'+r}{2n\mathrm{L}}$.

The values $\epsilon^{-\delta/2}$ and resulting values of δ for various spark lengths, found, as above described, by Professor Rutherford's method, are given in the table below.

CAPACITY OF CONDENSER = 0·00277 mfd.

Spark length. S.	Damping. $\epsilon^{-\delta/2}$.	Logarithmic semi-decrement. $\delta/2$.
1 mm.	0·905	0·100
3 ,,	0·880	0·128
5 ,,	0·885	0·122
7 ,,	0·865	0·145
9 ,,	0·860	0·152
11 ,,	0·845	0·168
13 ,,	0·845	0·168

CAPACITY OF CONDENSER = 0·000805 mfd.

Spark length. S.	Damping. $\epsilon^{-\delta/2}$.	Logarithmic semi-decrement. $\delta/2$.
2 mm.	0·875	0·133
3 ,,	0·840	0·175
5 ,,	0·765	0·269
7 ,,	0·680	0·387
9 ,,	0·590	0·529
11 ,,	0·515	0·663

The above numerical values of the damping $\epsilon^{-\delta/2}$ plot out into straight lines when the corresponding values of the spark length S are taken as abscissæ.

The earliest definite measurements of the logarithmic decrement of electric oscillations by the second method above mentioned were made by V. Bjerknes.

[5] See Miss H. Brooks on "The Damping of Oscillations in the Discharge of a Leyden Jar," *Phil. Mag.*, vol. 2, ser. 6, p. 92.

It depends upon a comparison between the maximum value of the potential difference of the spark balls in an oscillating circuit, and the root-mean-square value of the potential as measured by an idiostatic electrometer.[6] If an electrometer of the Kelvin type is constructed, having a suspended paddle-shaped "needle," and a pair of quadrants placed on opposite sides and against opposite ends of the "needle," we have an instrument which is sensitive to high frequency alternating potentials, and measures their root-mean-square value. If, then, the two quadrants are connected to the ends of a circuit in which oscillations are taking place, the quadrants are alternately positively and negatively charged; but as the induced charges on the needle change places with the change of charge on the quadrants, the needle deflects constantly in the same direction. This deflection, if small, is proportional to the square root of the mean of the squares of the potential difference of the quadrants.

Since the train of oscillations always lasts far less than one second, we may say that this root-mean-square value of the potential difference is proportional to the square root of the integral $\int_0^\infty v^2 dt$, where v is the instantaneous potential difference.

We have seen (Chap. I. § 5) that when a condenser is discharged across a spark gap through a low resistance, the potential difference v of the plates at any time, t, can be expressed by an equation of the form—

$$v = V_0 \epsilon^{-at} \cos pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where V_0 is the initial potential difference of the plates.

From the above equation we have—

$$\int_0^\infty v^2 dt = V_0^2 \int_0^\infty \epsilon^{-2at} \cos^2 pt = \frac{V_0^2}{4a} = U^2 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Hence U is the root-mean-square value of the potential difference of the condenser plates. But $a = n\delta$, therefore—

$$\delta = \frac{1}{4n} \cdot \frac{V_0^2}{U^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and δ becomes known from the values of V_0, U, and n.

Since we cannot obtain a steady deflection of the electrometer by one single spark or train of oscillations, it is necessary to permit a series of discharges at a uniform rate of N per second to take place, and the value of δ is then given by the expression—

$$\delta = \frac{N}{4n} \cdot \frac{V_0^2}{U^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The value of V_0 can be obtained from the spark length by the aid of the tables of spark potentials already given (see Chap. II. § 14), for the value of the potential difference of the plates of the condenser when the discharge begins is equal to the spark potential. The value of U is obtained by connecting the calibrated electrometer across the terminals of the condenser. The number of sparks per second, and also the frequency of the oscillations, must be obtained, the latter being calculated from the inductance and capacity in the circuit. In this manner we arrive at the value of the logarithmic decrement of the oscillations.

Some of Bjerknes' observations were made with an open oscillatory circuit of the Hertzian type, and for such an oscillator he found that the decrement per half-period depended on the length of the spark gap, but had values varying from 0·13 to 0·20, as the spark length increased from 1 to 5 mms. The damping in this case does not arise simply from resistance, but is mainly due to the radiation of energy in the form of electric waves, and this matter is further considered in § 8 of this chapter, and also in Chap. V. Whilst for an *open* or radiative circuit of the Hertzian type the decrement per half-period may amount to 0·2 or over, for a nearly closed circuit, such as a Hertzian resonator, Bjerknes found that the decrement might be as low as 0·001.

[6] See V. Bjerknes, "Ueber die Dämpfung schneller Electrischer Schwingungen," *Wied. Annalen der Physik*, 1891, vol. 44, p. 74.

5. Determination of the Mean Logarithmic Decrement in Oscillatory Circuits—Drude's Researches.—A very extensive research was conducted by Professor P. Drude on the influence of spark length and other factors on the logarithmic decrement of condenser circuits containing a spark gap. These results have a very practical bearing upon wireless telegraphy.[7]

The method he adopted was one which in principle originated with V. Bjerknes. If a primary oscillatory circuit containing a spark gap and condenser has oscillations set up in it, and if this circuit acts upon a closed secondary circuit containing a condenser but no spark gap, we have induced oscillations set up in this latter circuit. If the secondary circuit has such a form that its inductance can either be calculated or measured, and if the capacity in it is also known, as already shown, we can calculate its natural time period and deduce its high frequency resistance, and therefore logarithmic decrement, provided that the condenser in this secondary circuit is of such form that no energy is dissipated by radiation or in any other way except by resistance. If we insert in this circuit a thin wire at some point, we can, by means of a thermo-element or other means, determine the *integral* or *mean-square value* of the secondary current.

If this secondary circuit has such a form that we can vary its inductance, and therefore natural time period, we can find the corresponding values of the integral or mean-square value of the secondary current. For some particular inductance of the circuit this integral value of the secondary current will have a maximum value.

When this takes place the secondary circuit is said to be *in resonance* with the primary circuit (see § 6 of this chapter). V. Bjerknes and P. Drude[8] have shown that we can then determine the sum of the logarithmic decrements of the primary and secondary circuits from the observed values of the maximum integral current, and of any other value of the integral current not differing greatly from this critical one, when we know also the percentage deviation of the time period of the secondary circuit in the last case from that which sets up resonance.

Let i_2 denote the secondary current at any instant, and J^2 the integral current, so that—

$$J^2 = \int_0^\infty i_2^2 dt$$

Also let T_2 denote the periodic time of the secondary circuit. Then let J_m^2 and T_m denote the values of these quantities when the secondary circuit is so adjusted as regards inductance that J^2 has its maximum value. Again, let—

$$T_2 = T_m(1 + \eta) \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

Since $T_2 = \frac{1}{n_2}$ and $T_m = \frac{1}{n_1}$, where n_1 and n_2 are the frequencies of the two circuits, we have—

$$\eta = \frac{n_1 - n_2}{n_2} = \frac{n_1}{n_2} - 1$$

Let δ_1 and δ_2 denote the logarithmic decrements of the primary and secondary circuits.

Drude, following Bjerknes, then shows that the sum of the decrements of the two circuits is given by the equation—

$$\delta_1 + \delta_2 = 2\pi\eta\sqrt{\frac{J^2}{J_m^2 - J^2}}\,{}^{9} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

[7] See P. Drude, "Die Dampfung von Kondensatorkreisen mit Funkenstrecke," *Ann. der Physik*, **1904**, vol. 15, part 4, p. 709.

[8] See V. Bjerknes, *Ann. der Physik*, **1895**, vol. 55, p. 121, "Ueber Elektrische Resonanz." See P. Drude, *Ann. der Physik*, **1904**, vol. **13**, p. 512, "Ueber induktive Erregung zweier Elektrischer Schwingungskreise mit anwendung auf Perioden und Dämpfungsmesung, Tesla transformatoren, und drahtlose Telegraphie."

[9] See P. Drude, *Ann. der Physik*, **1904**, vol. 15, p. 716, to which we must refer the reader for the rather long proof of the above formula, which is derived from another equation (84) in an article by P. Drude in *Ann. der Physik*, vol. 13, p. 527. A proof of this formula is given in § 14 of this chapter (see equation 145).

Near resonance when n_2 is nearly equal to n_1 the quantity η becomes identical with $1-\frac{n_2}{n_1}$.

We have translated Drude's formula into our notation, but in referring to the original paper the reader should note that whilst his logarithmic decrement is defined as due to a complete period as in the above formula, he takes J, and not J^2, to represent the integral current. We shall indicate in a later section of this chapter (see § 14) the method by which the above formula is obtained.

Drude's experiments were conducted with a secondary circuit which had the form of a rectangle, the sides being made partly of metal rods and partly of tubes, so that by sliding them in and out of each other the length of the rectangle, and therefore its inductance, could be varied and calculated. A condenser of known capacity was inserted in this circuit, and also a short piece of fine wire, to which a thermo-junction was attached. This last was connected to a galvanometer. Drude first proved that when a single spark discharge was made in the primary circuit the induced secondary oscillation heated the fine wire, and therefore the thermo-junction, and produced a "throw" or ballistic deflection of the galvanometer coil proportional to the integral effect of the secondary current or oscillation.

He then showed that if l_m was the length of side of the secondary rectangle corresponding to resonance or to J_m, and if dl was any small variation of this length, the quantity η was equal to $\frac{1}{2}\frac{dl}{2l_m}$, and hence the above formula (25) transforms into—

$$\delta_1+\delta=\pi\,.\,\frac{dl}{l_m}\sqrt{\frac{J^2}{J_m^2-J^2}} \qquad (26)$$

His experimental procedure was then to take a number of observations of the integral current J^2 corresponding to various values of the side of the secondary rectangle, and to plot a curve called a *resonance curve*, in which ordinates represented the "throw" of the galvanometer, and abscissæ the length of the side of the rectangle forming the secondary circuit. For the fuller explanation of the nature of a resonance curve, the reader must refer to § 14 of this chapter.

Drude then calculated for any given length of side the inductance, and hence the decrement δ_2, of the secondary circuit.

The observations were then reduced as follows: Corresponding to each particular length of side of the secondary circuit there is a certain "throw," s, of the galvanometer when a single primary spark is taken which measures the secondary integral current. If s_1 and s_2 represent two throws corresponding to two lengths of side, l_1 and l_2, one greater and one less than the value s_m corresponding to resonance by equal amounts, we can say that—

$$dl=\frac{l_1-l_2}{2}\text{ and } l_m=\frac{l_1+l_2}{2}$$

and
$$J^2=\frac{s_1+s_2}{2}\text{ and } J_m^2=s_m$$

Therefore we have—

$$\delta_1+\delta_2=\pi\,.\,\frac{l_1-l_2}{l_1+l_2}\sqrt{\frac{(s_1+s_2)\div 2}{s_m-(s_1+s_2)/2}} \qquad (27)$$

Taking a number of values of l and s from the resonance curve Drude deduced the values $\delta_2=0{\cdot}0083$, $\delta_1=0{\cdot}08$.

The above values are the semi-period decrements. They show, therefore, that the primary circuit is much more damped than the secondary circuit.

By a large number of observations made with various spark lengths and with spark balls of various materials, Drude arrived at the following conclusions :—

1. For every condenser circuit with a spark gap there is a certain length of spark for which there is a minimum damping.

2. For zinc spark balls and small sparks this critical spark length lies between 1 and 2 mms., and the logarithmic decrement between 0·05 and 0·08.

3. To obtain small damping, it is necessary to employ a condenser absolutely free from brush discharges or dielectric hysteresis, and this can only be done by constructing the condenser of metal plates placed in petroleum oil.

4. Zinc spark balls give the smallest damping, and preserve their active effect in producing an oscillatory spark longest. Cleaning the surfaces increases the spark activity and reduces the decrement.

5. The integral effect in the secondary circuit increases at first with increasing spark length and then diminishes again.

6. The resistance of the spark depends very much upon the capacity and inductance in the oscillating circuit. Hence we cannot speak of a spark of given length having a definite resistance. With a large capacity and small inductance the spark resistance for given length is less than with small capacity and large inductance.

7. The effects of increased air pressure, and of light falling on the spark balls, and of the material of the spark balls on the logarithmic decrement, were carefully investigated. Drude says that spark balls made of zinc are superior to those made of brass in their active spark-producing qualities.

8. The effect of brush discharges on the edges of condenser plates, and of dielectric hysteresis when glass was employed as a dielectric, showed itself in increased total logarithmic decrement, and therefore in a decreased number of oscillations per train.

From the point of view of practical wireless telegraphy by electric waves, the important deduction to be made from these experiments is the advantage of employing short sparks. In those cases in which high charging potentials are required, it is better to gain this by using a number of short sparks in series rather than one long one. This can be done by placing a number of insulated metal plates in series with very small spaces between them, and connecting the two terminal balls to the oscillating circuit (see Chap. VIII. § 16).

6. **The Resistance of an Oscillatory Spark.**—Since the resistance of the spark in a condenser circuit traversed by electric oscillations is an important factor in determining the decay of the oscillations, considerable attention has been given to experimental methods for determining directly the resistance of an oscillatory electric spark and its variation with quantity, frequency, and spark length.

The factors which can be varied are—

(i.) The length of the spark.

(ii.) The quantity of electricity which passes initially, as measured by the spark potential and the capacity of the condenser discharging.

(iii.) The oscillation frequency determined by capacity and inductance of the circuit.

(iv.) The group frequency, or number of oscillatory sparks per second.

(v.) The material and form of the discharging surfaces.

(vi.) The pressure and nature of the gas in which the spark takes place.

A full examination of the effect of all these factors has not yet been made. In many cases the conditions of experiment have not been stated accurately, and between most of the experiments on spark resistance so far conducted a considerable difference of conditions has existed, so that comparisons are difficult. Some observers have endeavoured to measure the equivalent ohmic resistance of a single oscillatory spark. Since, however, in wireless telegraphy and Hertzian wave work generally we nearly always employ a continuous series of oscillatory sparks, the investigations made with isolated sparks are not of predominant interest.

A knowledge of the logarithmic decrement as obtained from the ratio of the first two oscillations gives us a lower limit to the resistance of the spark, provided we know the high frequency resistance of the rest of the circuit. Thus, in the experiments on damping made by Miss Brooks by Rutherford's method already

described, the inductance of the metallic part of the discharge circuit was 10^4 cms. The capacity employed was 0·00277 mfd., and hence the frequency was nearly 10^6. The quantity $2\pi L$ was therefore nearly 2×10^{10} cms. per second, or 20 ohms. The high frequency resistance of the wire of the discharge circuit was found to be 0·6 ohm as calculated by Lord Rayleigh's formula. Hence if r is the spark resistance in ohms and δ the logarithmic decrement, and if we assume the spark resistance constant during the train of oscillations, we have—

$$\delta = \frac{0{\cdot}6 + r}{20}$$

Taking the values of δ given in the first table on p. 203 for spark lengths of 1, 3, 5, 11 mms. respectively, we calculate the corresponding spark resistance as follows:—

Spark length. S.	Logarithmic decrement per semi-period. $\delta/2$.	Spark resistance r.
1 mm.	0·100	3·4 ohms.
3 ,,	0·123	4·52 ,,
5 ,,	0·122	4·28 ,,
11 ,,	0·168	6·12 ,,

It must be noted that these observations refer to the resistance of single sparks or discharges, and not to the resistance produced when a large number of discharges per second are made across the spark gap. There is evidence that in this last case the resistance of the train of sparks is very much less, for the same spark length, than the values given in the above table. Accordingly, these results are valid only for the circumstances of the experiment.

Miss Brooks found that the pressure of the air round the spark exercised a very marked effect on the damping, reduction of pressure within certain limits reducing the damping.

An interesting deduction is made in her paper from known facts as to the electric charge carried on gaseous ions, viz. that the expenditure of energy in the manufacture of the ions just necessary to carry the discharge across the gap does not account for the whole of the damping, but it is evident that a vastly larger number of ions are created by the discharge than is necessary to carry the discharge across. It is to the recombinations of these ions that the heat and light of the spark are probably due.

Experiments were also made on the effect of variation of the capacity of the condenser. It was found that when this capacity was greater than about 0·001 mfd. the damping was practically independent of the capacity, but that for very small capacities the damping increased rapidly with decrease of capacity. Hence the damping reaches a steady state when the discharge current exceeds a certain very moderate value.

Adolf Slaby also made some interesting observations on the resistance of an oscillating spark.[10] His method consisted in forming an oscillating circuit containing in series the ordinary spark gap S_1 of fixed length, a spark gap, S_2, of variable length, a condenser, C, and inductance, L, and a variable resistance, R, in the form of a graphite rod, as well as a sensitive hot-wire ammeter, A (see Fig. 2). The spark gap of variable length was shunted by an electrolyte resistance, U, consisting of a tube containing a solution of sulphate of copper having a resistance of 410 ohms. This permitted the condenser to be charged, but did not sensibly shunt the disruptive discharge. The variable spark gap was then altered in length from zero, and, corresponding to various lengths, readings of the ammeter were taken. This gave the current (R.M.S. value) in terms of the spark

[10] See *Elektrotechnische Zeitschrift*, Oct. 27, 1904; also *The Electrician*, Nov. 11, 1904, vol. 54, p. 150.

length. The spark gap S_2 was then made zero, the graphite resistance R varied, and readings of the ammeter again taken. This gave the current in terms of the graphite resistance. These two sets of observations were plotted as curves with current as ordinates, and for equal ordinates they showed the resistance of the spark gap expressed in ohms. In repeating these experiments, the author has found it to be an advantage to employ a long inductance coil of low resistance instead of the tube of sulphate of copper, and in place of the graphite rod to use a long column of 10 per cent. dilute sulphuric acid of variable length.

The final results indicated that the resistance of the spark gap rises parabolically with spark length for small lengths, but afterwards increases linearly. Slaby found that with increase in the capacity of the oscillating circuit the resistance of the spark per millimetre of length decreased.

The following results, taken from the figures given in Professor Slaby's paper, show the resistance of the spark for various spark lengths when the capacity in the circuit had a value of 360 electrostatic units, or 0·0004 mfd. :—

Spark length.	Spark resistance.
1 mm.	0·25 ohms.
2 ,,	0·90 ,,
3 ,,	2·30 ,,
4 ,,	5·0 ,,

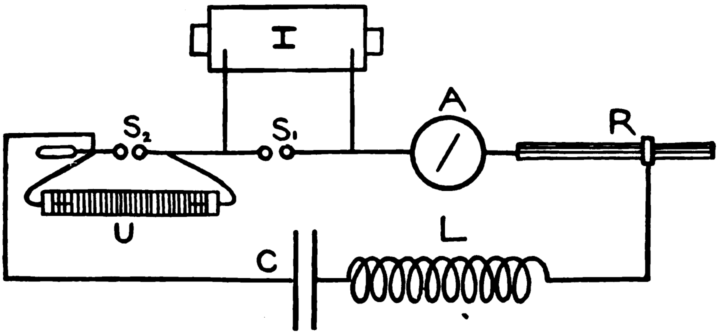


Fig. 2.—Slaby's Method of Measuring the Resistance of Oscillatory Electric Sparks.

The spark resistance for given lengths depends greatly upon the capacity used, that is, upon the quantity of electricity discharged across the gaps. This is shown by the curves in Fig. 3, taken from Professor Slaby's observations. There is also good evidence that the spark resistance varies with the number of discharges per second when these are numerous. Again, if the conductance of the spark is plotted in terms of the frequency, it is found that as the period increases the conductance diminishes, at first linearly and afterwards more rapidly.

For all periods, with the same spark voltage, small spark lengths have more conductivity per unit of length than long ones. Hence the advantage of using a number of small spark gaps in series, instead of one long one, in certain cases where small damping is required, is very great.

Professor Slaby found that, in the case of an ordinary wireless telegraph aerial wire, the damping due to the resistance of the wire itself is negligible. For a 120-foot copper wire antenna 3 mms. in diameter it amounts at most to about 0·8 per cent. Hence we may say that in the case of the oscillating circuits used in Hertzian wave telegraphy the damping of the oscillations is almost entirely due to the resistance of the spark, and to the radiation of energy from the aerial wire.

To obtain, therefore, a wave train not rapidly damped, the resistance of the spark and of the rest of the circuit must be very small, or the supply of energy must be very large if radiation is taking place.

Another method of measuring the spark resistance in the case of a non-

radiative circuit has been employed by several observers. It depends on the fact that if a condenser is discharging with oscillations through nearly closed circuit partly metallic but containing a spark gap, and if the condenser itself does not in any way dissipate energy internally by hysteresis or brush discharges, then the rate at which the energy is given out by the condenser must be equal to the sum of the rates at which it is being dissipated in the metallic part of the circuit, and in the spark.

If we call J the root-mean-square value of the instantaneous discharge current i when reckoned in amperes, so that

$$J^2 = N\int_0^\infty i^2 dt$$

where N is the number of discharges per second, and if R′ is the high frequency resistance of the circuit; then J^2R' is the rate at which energy is dissipated in the metallic part of the circuit. Again, if r is the resistance of the spark, J^2r is the rate at which energy is dissipated in the spark. We have then to find experi-

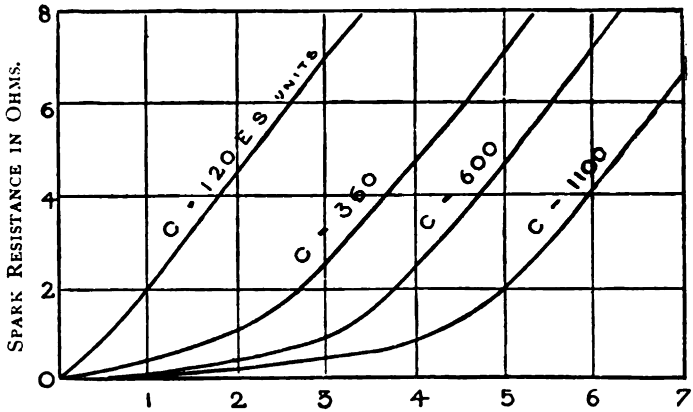


Fig. 3.—Curves showing Spark Resistances for Various Spark Lengths and Capacities (Slaby). The numbers on the curves denote the capacity corresponding to them reckoned in electrostatic units.

mentally the value of the root-mean-square discharge current for the discharges per second.

This may be done as follows:—

We employ a hot-wire ammeter suitable for measuring currents of 1 to 10 amperes or upwards, the hot wire of which consists of a number of fine copper wires placed in parallel. If we refer to the formula for the high frequency resistance of round copper given in Chap. II., viz.—

$$R' = R\left\{\pi d\sqrt{\frac{n}{\rho}} + \tfrac{1}{4}\right\}$$

we shall see that if $n = 10^6$ we have—

$$\frac{R'}{R} = 40d \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Hence, for this frequency, when d is as small as 0·25 mm. there is no sensible increase in resistance. Accordingly, a copper wire of No. 40 S.W.G. size has not an appreciably greater resistance for currents of a frequency of 10^6 than for steady currents. If, therefore, we make an ammeter as in Fig. 40, Chap. II., and calibrate it with continuous currents, we shall be able to read off on it the root-mean-square

value of a high frequency current passing through it. Suppose, then, that we place such an ammeter in a circuit consisting of a round-sectioned copper wire bent in the form of a rectangle, and complete this circuit by a condenser and spark gap. The condenser must be of such a type that there is no internal dissipation of energy in it, and is best made of metal plates placed in highly insulating oil. We then take a series of discharges at the rate of N per second, the sparks passing at regular intervals. It will be found that the ammeter gives a steady deflection and indicates a current, say, of A amperes. If we calculate from the inductance, capacity, and ohmic resistance of the circuit, the high frequency resistance R′, the quantity A^2R' gives us the rate at which energy is being dissipated in the metallic part of the circuit.

If we know the spark potential V corresponding to the spark length, then the quantity $\frac{1}{2}NCV^2$ gives us the rate at which energy is derived from the condenser. Hence the quantity $\frac{1}{2}NCV^2 - A^2R'$ must be the rate at which energy is being expended in the spark, and therefore the resistance of the spark r must be given by the expression—

$$r = \frac{\frac{1}{2}NCV^2 - A^2R'}{A^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

In the above expression, however, we assume that the resistance of the spark is that of the spark which is due to the condenser discharge alone. The actual spark which happens is, however, an admixture of two sparks, or rather of a spark and an arc. At the moment when the dielectric between the spark balls breaks down, not only does the condenser begin to discharge with oscillations, and thus form the true oscillatory spark, but the induction coil or transformer or other source of charging voltage produces a discharge across the gap which is of the nature of an electric arc. The greater this arc the less will be the resistance between the discharge balls. Hence it is not quite easy to define what is meant by spark resistance, and the discrepancy between the results of various observations on spark resistances may to some extent be due to the fact that arc and spark resistance are mixed up together in different proportions. We can determine to what extent there is a true electric arc effect mixed up with the true spark discharge as follows: If C is the capacity of the condenser in microfarads, and V the potential in volts corresponding to the spark length, and N the number of charges per second, then the charging current in amperes flowing into the condenser should be $\frac{NCV}{10^6}$ since this is the quantity per second delivered to the condenser. If we then insert a hot-wire ammeter in between the spark balls and the induction coil or transformer, and find a greater value than that given by the above expression for the current flowing out of the source of supply, we know that the difference between the observed and calculated current must be passing across the spark gap as an electric arc.

The chief difficulty, however, in applying this last-mentioned method of determining spark resistance is in the correct measurement of the spark frequency and spark voltage. The spark frequency can be found by means of the author's spark counter (see Chap. II. § 15). It is by no means correct to assume that there is only one discharge spark for each break of the circuit of the induction coil or alternation of the transformer, whichever is used to create the discharges. If the spark gap is short there may be several oscillatory discharges per break of the induction coil or per alternation of the transformer. On the other hand, if the spark gap is long and the capacity large there may be a lesser number of oscillatory sparks than alternations of the charging potential. The second difficulty consists in correctly estimating the spark voltage, that is, the potential to which the condenser is charged. When the spark balls become hot the spark voltage for a given spark length is decreased, and the only way to determine the voltage is to place in parallel with the actual spark balls used another pair consisting of brass balls of a known diameter, say, 2 cms., and ascertain to what distance these last balls must be approached in order that discharge may just begin to take place between them, and from that distance to determine the corresponding spark voltage by the tables given in Chap. II. § 14. Even when the spark frequency

and spark voltage are correctly estimated, it is found that great discrepancies exist between the spark resistances obtained for sparks of known lengths. This is unquestionably due to the different degrees in which true electric arc discharge is mixed up with spark discharge.

Professor Slaby also investigated the effect of change of size and material of the spark balls upon the spark resistance. He found that for short sparks (less than 4 mms. in length) an increase in diameter of the spark balls was accompanied by an increase of spark resistance, but that for longer sparks the difference was imperceptible. As regards the effect of material, he tried balls of 1 cm. in diameter made of brass, copper, lead, aluminium, magnesium, cadmium, zinc, tin, iron, steel, silver, gold, and platinum, and spark lengths from 0·5 to 3 mms., using apparently a very small capacity. His results showed that for the 0·5-mm. spark cadmium and tin and silver balls gave spark resistance about half that of the other metals, which were about equal, but for the 3-mm. sparks iron and steel balls gave spark resistance about 30 per cent. greater than that of the remaining metals.

The following table gives the chief results :—

Spark length in millimetres.	Spark resistance in ohms between 10-mm. diameter balls made of—									
	Brass.	Pb.	Cu.	Al.	Mg.	Cd.	Zn.	Sn.	Fe.	Ag.
0·5	0·9	0·9	1·3	1·3	1·3	0·5	1·0	0·5	0·9	0·6
1·0	2·4	1·8	2·8	2·8	2·8	1·5	2·2	1·2	2·2	1·5
1·5	4·0	3·3	4·4	4·6	5·5	3·0	3·5	2·5	4·5	2·5
2·0	5·9	5·5	6·4	7·1	9·5	5·2	5·6	4·6	7·7	3·8
2·5	8·9	9·3	9·3	10·6	14·6	8·4	8·4	8·2	11·8	5·8
3·0	12·8	14·6	12·6	15·5	...	12·4	12·2	13·3	16·4	8·9

The results show that a far less spark resistance for a 3-mm. spark voltage could be obtained by using four tin balls placed 1 mm. apart, so as to obtain three 1-mm. sparks in series between tin surfaces, than by using a single spark of 3 mms. in length between brass and iron balls.

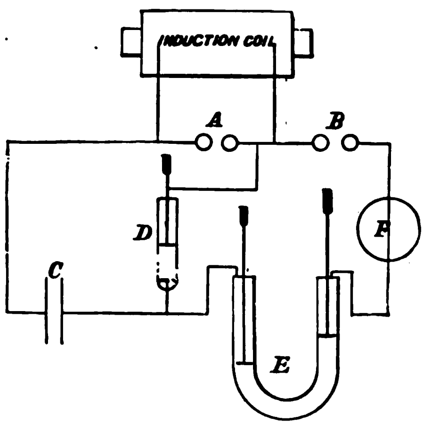


FIG. 4.—Arrangement of Apparatus used in Measurement of Spark Resistances at University College, London. *E*, U-tube of dilute sulphuric acid; *D*, high resistance of copper sulphate solution; *F*, hot-wire ammeter; *B*, adjustable spark balls; *C*, condenser.

The author repeated these experiments of Professor Slaby, but found it convenient to substitute for the graphite resistance R in Fig. 2 a U-tube, E, full of dilute sulphuric acid of known strength and resistivity (see Fig. 4). A thermometer immersed in this liquid gives its temperature, and a greater or less length of the column of fluid can be inserted in the oscillatory circuit by moving up or down metallic piston electrodes in the arms of the U-tube. In place of the tube D of sulphate of copper an inductance spiral may be used. Using this apparatus and spark balls (B) of iron, zinc, or brass 1·25 inch in diameter, the author determined the spark resistance for various lengths of spark and for a capacity of 1070 mmfds. as shown in the curves in Fig. 5. These curves show that with increasing spark length the resistance of the spark between iron balls increases rapidly when compared with that between brass or zinc balls.

It should be noticed that the method adopted by Professor Slaby and by the author of measuring spark resistance involves a constant current in the discharge circuit, and may therefore be said to give the spark resistance for various spark lengths corresponding to a certain constant current. In radiotelegraphy, by the spark method, we generally find that the length of the spark itself determines the quantity of electricity which passes at each discharge, and hence the resonance method of measuring spark resistance has been preferably employed, for then the oscillatory current is not constrained to have any constant value, but is allowed to take the value determined by the capacity and inductance of the oscillatory circuit. This method consists in determining first a resonance curve for the oscillatory circuit in the manner described in the last section of this chapter. If, then, we employ for the metallic part of the oscillatory circuit which contains the spark gap, such a circuit that we can calculate its high frequency resistance R′, and if we call the resistance of the spark r, then if L is the inductance, and n the frequency, and

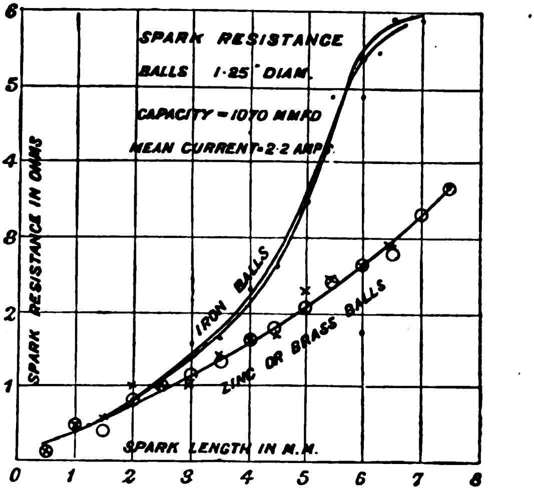


FIG. 5.—Spark Resistances for Various Spark Lengths between Iron, Brass, and Zinc Balls. (Fleming.)

δ_1 the decrement of the spark circuit, we have seen that $\delta = \frac{R'+r}{2nL}$, and hence we can calculate r when we know the other quantities.

The author carried out, in conjunction with Mr. Richardson, a series of experiments on the decrement and spark resistance of oscillatory circuits, both when the spark was subjected to an air blast and when it was not blown upon. The circuit consisted of a rectangle of copper wire 0·162 cm. in diameter, the sides of the rectangle being respectively 34·17 cms. and 142·1 cms. The inductance was 5012 cms. and the high frequency resistance 0·31 ohm for currents of a frequency of $1{\cdot}25 \times 10^6$. The capacity in series with this rectangle was an oil condenser having a capacity of 0·002645 mfd. The spark balls were brass balls 3 cms. in diameter. The spark resistance was measured by the Bjerknes resonance method for various spark lengths with and without an air blast on the spark balls. The decrements per semi-period and the spark resistances are given in the table on p. 214. The frequency employed was in all cases near to $1{\cdot}25 \times 10^6$.

Experiment shows that the spark resistance tends to fall slightly with increasing current in the oscillation circuit, and that the air blast conduces to render this current more uniform.

7. Magnetic Damping.—If we employ as the oscillatory circuit a wire made of magnetic material, then, in addition to the damping or decay of the oscillations

caused by the resistance of the wire and that due to the spark (if any) in the circuit, there is an additional damping due to the work absorbed in producing the magnetic changes in the circuit. Bjerknes found for equal-sized resonators made of wire 0·5 mm. thick a logarithmic decrement of 0·017 if the metal was copper, but 0·13 if it was iron or nickel.[11] The fact that such magnetic damping occurs is proof that magnetic metals retain their magnetizable qualities, and therefore hysteretic energy dissipating power, even when the magnetizing force is being reversed millions of times per second.

SPARK RESISTANCES AND LOGARITHMIC DECREMENTS FOR SHORT HIGH FREQUENCY SPARKS.

Spark length in mm.	Air blast on spark gap.		No air blast on spark gap.	
	Log. dec. per half period of circuit.	Spark resistance in ohms.	Log. dec. per half period of circuit.	Spark resistance in ohms.
1	0·0467	0·86	0·0423	0·75
2	0·0443	0·80	0·0447	0·81
3	0·0397	0·68	0·0383	0·65

Bjerknes showed by experiment that an iron or nickel wire, when used as an oscillatory circuit, has sensibly greater damping than a copper one, also that the deposition of the thinnest film of electro-deposited copper on the iron wire sufficed to annul this extra damping. This also was confirmed by similar experiments made by Professor Rutherford and by Miss Brooks.[12] This fact alone affords proof that electric oscillations are confined to the surface skin of the wire.

We have already seen (see Chap. II. p. 140) that this concentration of the current at the surface is more marked with magnetic conductors than in the case of non-magnetic materials.

At one time it was considered doubtful whether exceedingly rapid alternations of magnetic force could magnetize iron, and therefore give up energy to it in consequence of magnetic hysteresis.[13] There are many facts, however, which show that the penetration of the high frequency current into the iron conductor, though small, is sufficient to bring about surface magnetization, and therefore hysteretic loss. Suppose we set up a pair of condensers or Leyden jars and connect their outer coatings by a thick copper wire, in which a couple of loops of two or three turns are formed, and their inner coatings to the secondary terminals of an induction coil (see Fig. 6).

If in one of the loops formed in this circuit we introduce a glass bulb, B (see Fig. 6), containing air or any other gas highly rarefied, we find that at each discharge of the coil a bright ring of light is formed in the bulb. This is an induced discharge in the rarefied gas acting as a secondary circuit. The discharge may be so adjusted that the introduction of any object into the other loop in the condenser circuit which absorbs the energy of the oscillation quenches the glow in the gas.

Professor Sir J. J. Thomson has shown that it is possible to arrange the experiment so that the introduction of a cylinder of copper, or bundle of copper wire, into the second coil of the primary circuit does not much affect the luminous discharge in the gas, but the introduction of a similar-sized cylinder of iron or equal bundle of iron wire, W (see Fig. 6), immediately destroys it. This, Professor

[11] See V. Bjerknes, *Wied. Ann. der Physik*, 1892, vol. **47**, p. 69, and vol. 48, p. 592, 1893. The above numbers are half of those given by Bjerknes to adjust them to our definition of the log. dec.
[12] See also Miss H. Brooks, *Phil. Mag.*, ser. 6, vol. **2**, p. 92.
[13] See Hertz, *Ann. der Physik und Chemie*, 1888, vol. 34, p. 558.

Thomson points out, must be a consequence of the energy absorption involved in magnetizing the iron, so that although its electrical conductivity is much less than copper, yet owing to the fact that its permeability is much higher than unity, its damping effect on the electrical oscillations is on the whole greater.[14]

Accordingly, we are led to the conclusion that even at these high frequencies the iron is magnetized by the action of electrical oscillations, and possesses a permeability which is probably as high as 300 or 400.

Direct photographic proof of the magnetizability of iron by oscillatory discharges has been obtained by Dr. E. W. Marchant, and the two photographs of oscillatory sparks shown in Figs. 25 and 26 of Chap. I. illustrate this fact well.[15] The first photograph is that of the spark taken when a condenser of 0·06 mfd. was discharged through a coil having an inductance of about 5 microhenrys, the potential of the discharge being 13,500 volts. The coil contained in this case no iron core. The second photograph shows the spark when a core of 550 fine iron wire, No. 28, was inserted into the paper tube on which the wire was wound.

These photographs show that the effect of the iron is to increase the time period or to slow down the oscillations, and in addition, owing to the increase in

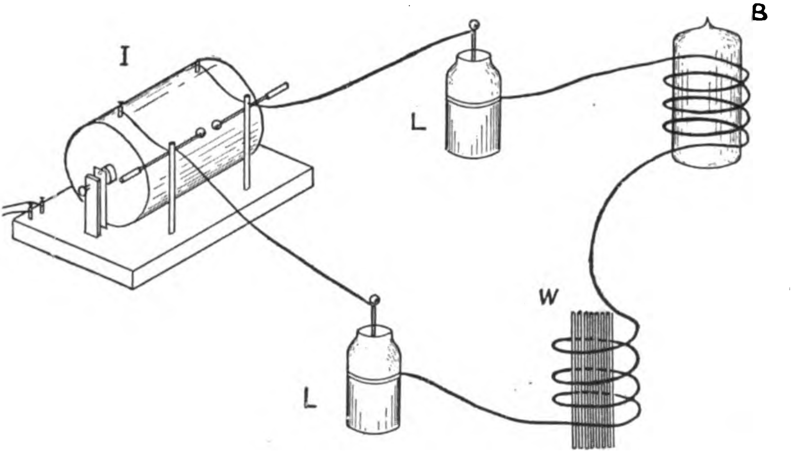


FIG. 6.—I, Induction Coil; L, L, Leyden Jars; B, Vacuum Bulb; W, Iron Core introduced into Coil.

the permeability of the iron as the discharge current dies away, we see that the interval between successive oscillations increases—in other words, the oscillations are no longer isochronous.

Again, it has been shown by Professor J. J. Trowbridge that electric oscillations on iron wires are damped out more quickly than on copper wires, and that there is an energy absorption in the case of iron greater than can be accounted for by its electrical resistance.[16]

An excellent investigation by Mr. C. E. St. John has confirmed the above results.[17] By creating stationary electric waves on wires, Mr. St. John has shown that the inductance of iron wires is greater than that of similar-sized copper wires when made into circuits of the same form, and conveying electric oscillations of a frequency of about 56 millions by 3·4 to 4·3 per cent., and he has confirmed the

[14] See "Researches in Electricity and Magnetism," p. 323; also see J. J. Thomson, *Phil. Mag.*, Dec. 1891, p. 460.

[15] Taken from a letter by Dr. Marchant to *Nature*, Aug. 30, 1900.

[16] See Prof. J. Trowbridge, "The Damping of Electric Oscillations on Iron Wires," *Phil. Mag.*, Dec. 1891, ser. 5, vol. 32, p. 504.

[17] See Mr. C. E. St. John, "Wave Lengths of Electricity on Iron Wires," *Phil. Mag.*, Nov. 1894, ser. 5, vol. 38, p. 425.

result that in the case of iron wire there is a more rapid damping out of the oscillations.

The experiments show that the permeability of the iron even at this high frequency is on an average still as high as 385.

The result of an extensive research was published in 1913 by Professor E. W. Marchant (see *Proc. Roy. Soc. Lond.*, vol. 88A, p. 254, 1913), in which he gave details of many measurements made to determine the relation between magnetizing force and permeability in the case of iron or nickel, when the magnetizing force was produced by condenser discharges. He confirmed the accuracy of the Kelvin formula (see Chap. I. § 5) when using an air condenser and air core inductance. When an iron wire core was inserted in the inductance spiral the oscillations were found to be more quickly damped out, the damping being mostly due to eddy current loss in the iron; also the time for each half oscillation increases with the duration of the discharge. This phenomenon is a consequence of the increase in the permeability of the iron as the current, and consequently the magnetizing force, gradually decreases. The variation of effective permeability as found from the oscillograms agrees within limits of experimental error with that found for the same iron with steady magnetizing forces. The same effects were found in the case of nickel. The frequencies used in these experiments varied from about 20,000 to 60,000.

In discussing the various forms of detectors for electric waves, we shall have to notice some which depend for their action upon the fact that electric oscillations can alter the magnetic hysteresis of iron, as well as increase its effective magnetic permeability.

The practical deduction to be made from the above facts is that the rate at which electric oscillations decay on an iron wire is much greater than that at which they would decay if a non-magnetic wire of the same size is used; in other words, the logarithmic decrement is greater. This is in some small degree due to a difference in electric resistance, but chiefly to the magnetic permeability of the iron. Therefore the moral is, that iron wires must not be used for constructing oscillatory circuits in which it is desired that the oscillations shall be as little damped as possible. Hence iron must not be used for wireless telegraph aerials. Nevertheless, well-galvanized iron wire can be used, since it has been shown that a very thin layer of zinc placed on iron is sufficient to confine the electric oscillations to the zinc, and prevent them from penetrating to the iron beneath and giving up their energy to it.

8. Damping Due to Radiation and other Causes.—It will have been evident, from the facts considered in the two previous sections, that any source of dissipation of energy in the oscillatory circuit shows itself by causing damping or decay in the oscillations. Hence not only does ohmic resistance of the circuit or spark gap and magnetic hysteresis (if any) in the wire circuit create damping, but dielectric hysteresis (if any), or true dielectric conduction, or brush discharges over the dielectric surface of the condenser used, are also possible additional causes. Also, if energy is being sent off from the oscillatory circuit in the form of electric waves or radiation, this also creates very considerable damping. In a later chapter we shall study more particularly the forms of circuit which can thus radiate. Meanwhile we may say that if the distance between the two surfaces which form the condenser is small compared with the linear dimensions of the smaller of the two plates, then the circuit containing this condenser is called *a closed oscillation circuit.* If, however, the distance is large compared with the linear dimension of the smaller of the two surfaces, the circuit is called *an open oscillation circuit.*

Typical instances of closed or feebly radiative and open or strongly radiative circuits are shown in Fig. 7.

Again, if we couple a nearly closed oscillatory circuit consisting of a condenser, C, spark gap, S, and inductance coil, L, with another suitably tuned open circuit, M (see Fig. 8), the open circuit can have electric oscillations created in it inductively by the other, and these oscillations can in turn create a disturbance called an electro-magnetic wave in the surrounding æther. Hence energy is, so to speak, sucked out of the closed circuit by the radiating circuit, and considerable

damping ensues. The closed circuit alone cannot radiate if the condenser plates are close together, but it can radiate if coupled with an open one.

If oscillations are created in a nearly closed circuit by connecting the spark balls to the secondary terminals of an induction coil, then experiment shows that these oscillations are very persistent, the logarithmic decrement is small, and the damping almost wholly due to the resistance of the spark. Several dozen oscillations may take place before the electrical disturbance dies away. On the other hand, if we set up oscillations in an open circuit, the decay of the oscillations is much more rapid, and is almost entirely due to the fact that they impart their energy to the surrounding æther, and create electric waves by a process discussed more in detail in a subsequent chapter. There are, therefore, very few oscillations, a dozen at most, before the electrical motion has practically ceased. There is, therefore, a very great difference between these two forms of circuit. The closed circuit is called a persistent oscillator, and the open one a good radiator.

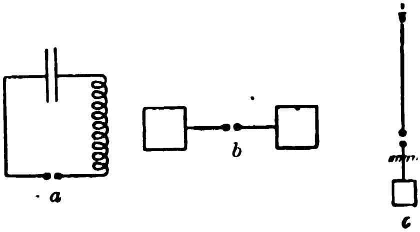


FIG. 7.—*a*, Closed Oscillation Circuit; *b*, *c*, Open Oscillation Circuits.

As we can always avoid using iron or magnetic wires for oscillatory circuits, and also condensers in which internal dielectric energy losses occur, we need not concern ourselves further with the increase in damping which arises from hysteresis losses, whether magnetic or dielectric. On the other hand, we are unable to arrest the decay of oscillations due to resistance or radiation. The total logarithmic decrement in any oscillatory circuit is therefore made up of two parts—

(i.) Damping due to resistance, and (ii.) damping due to radiation.

The relative numerical value of these two decrements, or parts of the total decrement, depends upon the nature of the oscillatory circuit.

In the case of an open circuit oscillator, such as a Hertzian oscillator, consisting of two rods placed in line with their ends nearly touching, and furnished with spark balls, or in the case of a Marconi aerial, consisting of a pair of spark balls, one connected to the earth and the other to a vertical insulated wire, the radiation decrement very greatly exceeds the resistance decrement.

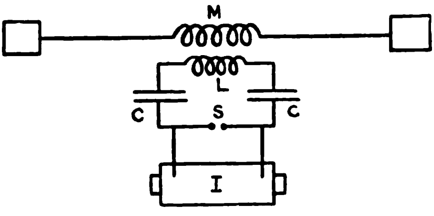


FIG. 8.—Inductive Coupling of a Closed and Open Oscillation Circuit.

In the case of a certain circular Hertz resonator, consisting of a nearly closed metallic circuit interrupted by an air condenser, S. Lagergren found that the radiation decrement was 0·14, whilst the resistance decrement was only 0·0128, the total decrement being 0·153.[18]

M. Planck, however, found that the radiation decrement for an open or radiative circuit of the Hertzian type was as high as 0·3, whilst the resistance decrement was only 0·09, the total decrement being 0·39.[19]

Bjerknes has shown that for certain equal oscillators made respectively of copper and platinum, in the case of the copper 75 per cent. of the energy lost is due to radiation, and 25 per cent. is dissipated by resistance; whereas for the platinum 37·5 per cent. was lost by radiation and 62·5 per cent. by resistance.

The predetermination of the radiation logarithmic decrement can only be achieved in a few cases by reason of the difficulty of the calculations.

[18] See S. Lagergren, "Ueber die Dämpfung Electrischer Resonatoren," *Wied. Ann. der Physik*, 1890, vol. 64, p. 290.
[19] See Max Planck, *Wied. Ann.*, 1897, vol. 60, p. 599.

Taking the case of a linear oscillator consisting of two metal spheres at the extremities of two metal rods, provided at their inner ends with small spark balls (see Fig. 9), Hertz calculated the energy stored up and the energy lost per oscillation to be as follows [20]:—

Let l be the length of the oscillator, measured from ball to ball, let λ be the wave length of the radiation emitted, and Q the charge on either half of the oscillator just before the spark discharge begins. Then Hertz shows that the energy lost by radiation from this oscillator per half period is given by the expression—

$$\frac{8\pi^4 Q^2 l^2}{3\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

For the proof of the above formula we must refer the reader to § 9, Chap. V., of this treatise.[21] Also in § 10, Chap. V., it is shown as a consequence that if C is the capacity of one part of the oscillator with respect to the other, and δ is the radiation logarithmic decrement as defined—

$$\delta = \frac{16\pi^4 l^2 C}{3\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

We may apply this to a case given by Hertz himself (see "Electric Waves," p. 150).

The oscillator consisted of two metal rods, each 5 mms. in diameter and 50 cms. in length. To the ends of these rods were attached metal spheres, 30 cms. in diameter (see Fig. 9). The rods were placed in line, with a spark gap of 7·5 mms. between the small knobs terminating the metal rods.

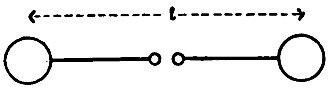


Fig. 9.—Dumb-bell Oscillator. (Hertz.)

We have first to calculate the capacity of one-half of the oscillator with respect to the other. The capacity of a sphere in electrostatic units is numerically equal to its radius in centimetres.

The capacity of one sphere with respect to the other is, however, only half of the above value, because the two spheres may be considered to be in series with each other. Hence the capacity with which we are concerned is equal to 7·5 cms. Inserting this value for C in the expression (31), and the values $\pi^4 = 97{\cdot}4$, $l = 100$, and $\lambda = 480$ (as given by Hertz), we find for the value of the radiation decrement δ—

$$\delta = \frac{16 \times 97{\cdot}4 \times (100)^2 \times 7{\cdot}5}{3 \times (480)^3} = 0{\cdot}17 \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To obtain the total decrement, we have to add to this the resistance decrement. Since the resistance is almost entirely due to the oscillatory spark, that of the rod being negligible, it will be sufficient to calculate it by the formula $\frac{r}{2nL}$, where r is the resistance of the spark, n the frequency, and L the inductance of the rods.

The high frequency inductance of the straight rod, 100 cms. in length ($=l$) and 5 mms. in diameter ($=d$), can be approximately calculated by the formula—

$$L = 2l\left(\log_e \frac{4l}{d} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Hence L = 1134 cms. The frequency is therefore nearly 50×10^6.

The value of $2nL$ is therefore 102×10^9, or nearly 100 ohms.

We have already seen that the resistance of a 7-mm. oscillatory spark may be rather over 5 ohms, and hence the logarithmic decrement due to resistance would then be about $\frac{5}{100}$, or 0·05. Hence, for the oscillator in question, when in operation we have a radiation decrement equal to 0·34, and a resistance decrement equal to

[20] See Hertz, "Electric Waves," English translation by D. E. Jones, p. 150.

[21] See also "Æther and Matter," Adams Prize essay, by Sir Joseph Larmor, M.P., F.R.S., p. 225, where the same formula is deduced by a different method.

0·05, and a total logarithmic decrement of 0·4 nearly. Hence the loss of energy by radiation per oscillation is more than 10 times as great as that due to the resistance of the spark.

V. Bjerknes, in an important paper,[22] has shown, by the method already explained, that the total damping of a Hertzian oscillator is very large, and he obtained experimentally for a certain Hertzian oscillator total logarithmic decrements with various spark lengths, as follows :—

Spark length.	Total logarithmic decrement.
1 mm.	0·28
2 ,,	0·30
3 ,,	0·32
4 ,,	0·34
5 ,,	0·40

The gradual increase in the value of δ is without doubt due to the steady increase in the spark resistance with spark length, which increases the part of the decrement due to resistance. The agreement between the calculated value of the total decrement and that obtained experimentally by Bjerknes for a 5-mm. spark is very close.

In another case, Bjerknes measured the logarithmic decrement of a Hertz radiator consisting of two metal rods, each 5 mms. in diameter and 50 cms. in length, having attached at the ends circular discs of metal 30 cms. in diameter. The opposite ends terminated in spark balls, and the rods were placed in line with each other.

The capacity in free space of a circular disc of diameter D cms. in electrostatic units is $\frac{D}{\pi}$. Hence in this case the capacity of each disc in space was nearly 10 cms. The capacity of one-half of the oscillator with respect to the other is therefore 5 cms., or a little more, on account of the capacity of the rod.

Bjerknes found that the wave length of the wave radiated from the oscillator was 431·2 cms. Hence, substituting in the formula for δ the values $C=5$, $\lambda=4·312$, $l=100$, we have—

$$\delta=\frac{16\times 97·4\times(100)^2\times 5}{3\times(431·2)^3}=0·16$$

The resistance decrement, for the Hertz oscillator previously mentioned, has a value of about 0·05. Hence the total decrement should be 0·37.

Bjerknes found experimentally, for this oscillator, a total logarithmic decrement of 0·4, which agrees fairly well with the above calculated value.[23]

An important case, which, however, can only be treated approximately, is that of the Marconi aerial wire in its original form. As we shall see in a subsequent chapter, Marconi made telegraphy without wires by means of electric waves possible by his invention of the earthed antenna or linear radiator.

A vertical insulated wire has a spark ball at the lower end which is placed in apposition to another spark ball connected to the earth. The two balls are connected to the secondary terminals of an induction coil. When the coil is in action the aerial wire is charged and discharged alternately with oscillations across the spark gap. It is well known that these oscillations are strongly damped. We can obtain a fair approximation to the logarithmic decrement and to the damping, as follows :—

If a wire is set up vertically to the surface of the earth, the earth being a fairly good conductor, and if the wire receives an electrical charge, it forms an *electrical image* of itself in the conductivity earth ; the electric force in surrounding space being due to the joint action of the charge in the wire and that on an oppositely

[22] See V. Bjerknes, "Über die Dämpfung Schneller Electrischer Schwingungen," *Wied. Ann. der Physik*, 1891, vol. 44, p. 74.

[23] See V. Bjerknes, *Bihang till K. Svenska Vet. Akad. Handlungen*, 1893, 20 Afd., I. nr. 5, II. p. 6, "Ueber Electrische Resonanz"; see also M. Planck, *Wied. Ann. der Physik*, 1897, vol. 60, p. 595.

charged inverted duplicate of itself below the surface called its electrical image. The effect is closely analogous to the optical effect produced if a luminous rod is set up vertically on a mirror. The illumination at any point in space will be due to the rod and to its optical image in the mirror.

A straight aerial wire may be regarded as an extreme case of a prolate ellipsoid. Hence if the wire has a height h and a diameter d we may consider that the wire and its electrical image can be replaced by an ellipsoid of revolution with a semi-major axis h and a semi-minor axis equal to $d/2$.

We have already seen in Chap. II. § 7 (90) that the capacity of such an ellipsoid of revolution is given by the expression[24]—

$$C' = \frac{h}{\log_e \frac{4h}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Hence the capacity of the wire itself may be considered as approximately equal to that of a semi-ellipse or to half of the above value. This capacity is distributed all along the wire, but it is clear that we can replace this distributed capacity by a located or concentrated capacity at the summit of a wire of negligible capacity.

It can be shown that if an ellipsoid of revolution is divided by equidistant parallel planes taken perpendicularly to its axis of revolution, each of the zones into which the surface is divided has the same electrical capacity *in situ*. Hence if the vertical wire is not too near the earth we may assume that its capacity per unit of length is the same all the way up it. As a matter of fact, in actual aerial wires the bottom portions have larger capacity per unit of length than the upper ones, by reason of their greater proximity to the earth.

We have in the next place to calculate the equivalent located capacity of such a linear oscillator.

In discussing the case of the Hertzian oscillator above, we have assumed that the electrical capacity was limited to the capacity of the two spheres placed at the outer ends of the linear oscillator or wire interrupted in the centre by a spark gap.

In the case of the single-wire antenna, we have capacity distributed all along it, and we must calculate what must be the capacity which, concentrated at the top of the aerial, would, when charged with the potential found at the summit, give a total electric charge equal to that actually resident on the wire. We shall show in the next chapter (see Chap. IV. § 7) that when the fundamental oscillations are excited on such a wire the maximum potential increases all the way up the wire from the earthed end to the top in accordance with a simple sine law. This fact has been experimentally confirmed. Hence if V denotes the maximum potential of an element of the wire at any distance, x, from the earth, and if V_h is the potential at the top of the aerial wire of height h, then the expression—

$$V = V_h \sin\left(\frac{\pi}{2} \cdot \frac{x}{h}\right)$$

gives us a value for V which complies with the terminal conditions. Let c be the capacity of the wire per unit of length, and hence the whole capacity of the wire C is given by—

$$C = ch = \frac{h}{2 \log_e \frac{4h}{d}}$$

The maximum charge of electricity dQ on any element of length dx of the wire is then—

$$dQ = cV dx = \frac{V_h \sin\left(\frac{\pi}{2} \cdot \frac{x}{h}\right) dx}{2 \log_e \frac{4h}{d}}$$

[24] J. A. Fleming and W. C. Clinton, "On the Measurement of Small Capacities and Inductances," *Phil. Mag.*, ser. 6, vol. 5, p. 492; see also Chap. II. § 7.

To obtain the whole charge of the wire, we have to integrate the above expression between the limits h and 0. Hence we have—

$$Q = \frac{V_h}{2\log_\epsilon \frac{4h}{d}} \int_0^h \sin\left(\frac{\pi}{2} \cdot \frac{x}{h}\right) dx = \frac{V_h}{2\log_\epsilon \frac{4h}{d}} \cdot \frac{2}{\pi} h = \frac{2}{\pi} C V_h$$

Hence the distributed capacity C could be replaced by a capacity $\frac{2}{\pi}C$ located at the top, and this if charged to the potential of the top of the aerial would give a charge equal to that actually distributed along the aerial. The quantity $\frac{2h}{\pi}CV_h$ is called the *electric moment* of the antenna.

The antenna with its distributed capacity may therefore be replaced by an imaginary antenna having a capacity $\frac{2}{\pi}C$ located at the top, at a height h above the earth. In each case there will be an electrical image of opposite sign in the earth. The capacity of the antenna with respect to its image in the earth is the same as that of the equivalent located capacity placed at the summit of a wire without capacity, with respect to its image in the earth. This, as in the case of the Hertzian oscillator consisting of a pair of spheres at the outer ends of a rod, is therefore equal to half the capacity of the antenna above ground, and is therefore given by the expression—

$$C = \frac{1}{2}\,\frac{2}{\pi}\,\frac{h}{2\log_\epsilon \frac{4h}{d}} = \frac{h}{2\pi \log_\epsilon \frac{4h}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Hence in the general expression for the radiation decrement (31) we have to substitute the above value for C. Again, as we shall show in Chap. IV., the wave length of the wave radiated from a simple rod oscillator earthed at the lower end is approximately five times the height of the aerial wire. Hence in (31) we have to substitute $5h$ for λ or $125h^3$ for λ^3. Also since in (31) we used the letter l for the whole length of the oscillator we have to substitute $2h$ for l. Making these substitutions the formula (31), viz.—

$$\frac{\delta}{2} = \frac{16\pi^4 l^2 C}{6\lambda^3}$$

it becomes

$$\frac{\delta}{2} = \frac{64\pi^3}{1500 \log_\epsilon \frac{4h}{d}}$$

But since $\pi^3 = 31{\cdot}006$, the above expression is very nearly equivalent to—

$$\frac{\delta}{2} = \frac{4}{3}\,\frac{1}{\log_\epsilon \frac{4h}{d}} = \frac{2{\cdot}67}{\log_\epsilon \frac{4h}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35a)$$

An expression in close agreement with the above was given by M. Abraham (see *Annalen der Physik*, vol. 66, p. 435, 1898) differing only from (35a) in that the constant is 2·44 instead of 2·67 (see Chap. V. § 12 (71)).

Also a formula has been obtained by L. Cohen by a different method, which is quite identical with (35a), in a paper on *Electromagnetic Radiation* published in the *Journal of the Franklin Institute*, U.S.A., for April 1914.

To sum up, we may say that for any ordinary form of Hertzian oscillator, including a Marconi vertical wire aerial radiator, the logarithmic decrement per half-period due to radiation has a value not far from 0·1 or 0·2, whilst the logarithmic decrement per half-period due to the resistance of the spark is very considerably less, say about 0·01 or 0·02.

This means that the oscillations are practically extinguished in about ten complete oscillations or less. For since $\epsilon^{20 \times 0{\cdot}2} = \epsilon^4 = 54{\cdot}6$, a logarithmic decrement of 0·2 implies that the tenth *complete* oscillation has a value which is only 2 per

cent. of the first oscillation, and is therefore practically negligible. To facilitate the calculation of the decay of oscillations for given decrements, we append a table of the powers of ϵ for various fractional and integer exponents. In Fig. 10 are

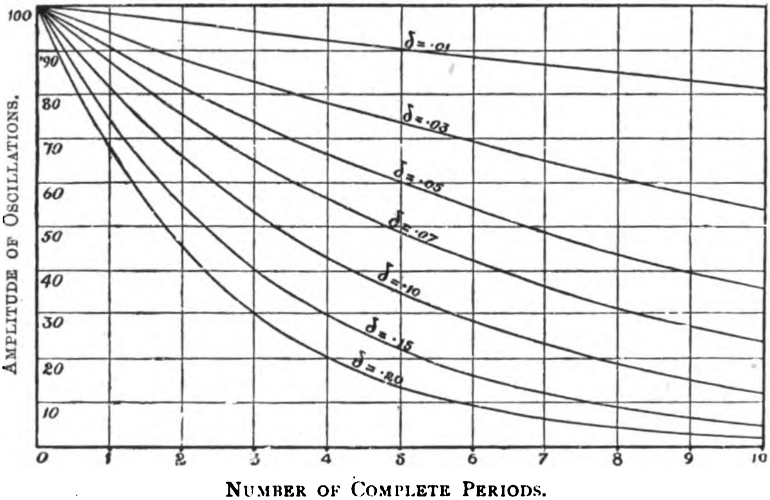


FIG. 10.—Curves showing the Amplitude of the Oscillatory Potential or Current at the end of each Complete Period for Various Values of the Decrement δ per semi-period.

shown a series of curves which are the plotting of the equation $y = \epsilon^{-\delta}$ for different values of δ marked on the curves.

VALUES OF ϵ^{δ}.

$\epsilon = 2{\cdot}71828$.

δ	ϵ^{δ}.	δ	ϵ^{δ}.	δ	ϵ^{δ}.	δ	ϵ^{δ}.
0·00	1·000	0·10	1·105	1·00	2·72	5·50	244·6
0·01	1·010	0·20	1·220	1·50	4·48	6·00	403·4
0·02	1·020	0·30	1·35	2·00	7·39	6·50	763·6
0·03	1·030	0·40	1·49	2·50	12·18	7·00	1096·0
0·04	1·041	0·50	1·63	3·00	20·10	7·50	1808·0
0·05	1·052	0·60	1·82	3·50	33·12	8·00	2981·0
0·06	1·062	0·70	2·02	4·00	54·6	8·50	...
0·07	1·072	0·80	2·22	4·50	88·0	9·00	...
0·08	1·083	0·90	2·46	5·00	148·4	10·00	...
0·09	1·094						

If the amplitude of the first oscillation is taken as unity, the ordinates of any curve show the successive amplitudes at the end of each period, corresponding to decrements per half-period marked on the curve.

An expression for the ratio between the radiation decrement δ_r and the resistance decrement δ_s has been established by Max Planck,[25] who arrives at the formula—

[25] See Max Planck, "Ueber Electrische Schwingungen welche durch Resonanz erregt und durch Strahlung gedampft werden," *Wied. Ann. der Physik*, 1897, vol. 60, p. 577.

$$\frac{\delta_r}{\delta_s}=\frac{8\pi^2}{3}\left(\frac{l}{\lambda}\right)^2\frac{3\times10^{10}}{r\times10^9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where r is the spark resistance in ohms, and l and λ are the length and wave length of the oscillator. Since λ is from 4 to 5 times l, and r may be 5 to 10 ohms generally, we have—

$$\frac{\delta_r}{\delta_s}=26{\cdot}32 r\frac{l^2}{\lambda^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

which may have a value from 5 to 16 or so, according as we take $r=5$ or 10 and $\frac{\lambda^2}{l^2}=16$ or 25.

A formula very similar to that given by Planck can be deduced for the ratio of the energy expended in radiation and that expended in the spark.

Consider the first half-period $\left(\frac{T}{2}\right)$ of an oscillation in which the maximum current is I_1, and therefore the root-mean-square value $\frac{I_1}{\sqrt{2}}$. We have, for the value of the energy expressed in electrostatic units (E_s) expended in the spark (of which the resistance is r ohms) in the first half-period, the expression—

$$E_s=\frac{r\times10^9\times I_1^2\times T}{9\times10^{20}\times2\times2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The numeric 9×10^{20} in the denominator is the factor for converting resistance measured in electromagnetic units to resistance measured in electrostatic units.

In one half-period the energy expended in radiation, also expressed in electrostatic units (E_r), is given by Hertz's formula—

$$E_r=\frac{8\pi^4Q^2l^2}{3\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

As we have already shown (see p. 198, equation (13)) that—

$$C^2p^2V^2=Q^2p^2=I_1^2\epsilon^{\delta/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Substituting the above value Q^2 in Hertz's expression, we have—

$$E=\frac{2\pi^2l^2I_1^2}{3n^2\lambda^3}\epsilon^{\delta/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Hence, dividing equation (41) by (38), and remembering that for electric radiation through space $n\lambda=3\times10^{10}$, we obtain—

$$\frac{E_r}{E_s}=\frac{8\pi^2}{3}\epsilon^{\delta/2}\left(\frac{l}{\lambda}\right)^2\frac{3\times10^{10}}{r\times10^9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Hence the ratio is independent of the amplitude, and is the same for each half-oscillation, and therefore for the whole train.

The formula (42) differs from Planck's formula (36) only by the factor $\epsilon^{\delta/2}$, and this is nearly unity if δ is small. This factor, however, is not unity if δ has a value such as 0·4, for then $\epsilon^{\delta/2}$ is near to 1·2. It is easy to show that if the decrement δ has such a small value that $\epsilon^{\delta/2}$ is unity, then we must have—

$$\frac{E_r}{E_s}=\frac{\delta_r}{\delta_s}$$

where δ_r is the radiation decrement and δ_s the resistance decrement. Taking the expression for δ_r derived from Hertz's formula for the radiation per half-period (see Chap. V. § 10), and expressing the capacity C in *farads*, we have—

$$\delta_r=\frac{16\pi^4l^2Cu^2}{3\lambda^3 . 10^9}$$

where $u=3\times10^{10}$.

Also, since the resistance decrement is given by—

$$\delta_s = \frac{r}{2n\mathrm{L}}$$

where r is the spark resistance in ohms, and L is the circuit inductance in henrys, we have by division, remembering that $4\pi^2\mathrm{CL}n^2 = 1$—

$$\frac{\delta_r}{\delta_s} = \frac{8\pi^2 l^2 n}{3r\lambda^2 \cdot 10^9}$$

But the above formula is Planck's (36), and differs only from (42) by the absence of the factor $\epsilon^{\delta/2}$. Hence generally—

$$\frac{\mathrm{E}_r}{\mathrm{E}_s} = \frac{\delta_r}{\delta_s}\epsilon^{\delta/2} = \frac{80\pi^2}{r}\epsilon^{\delta/2}\left(\frac{l}{\lambda}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Fig. 11.—Rowland's Syntonic Pendulums.

where δ is the total decrement.

We may apply this last formula (43) to calculate the *radiative efficiency* of a Marconi aerial radiator having the form of a simple wire of length l and a total decrement $\delta = 0{\cdot}4$, which would be the case if the spark had a length, say, of 5 mms., and therefore a resistance of about 5 ohms. Under these conditions $\epsilon^{\delta/2} = 1{\cdot}22$ nearly and $r = 5$. Then, since the wave lengths λ of the radiated wave would be rather more than four times the length l of the radiator, we have approximately—

$$\frac{\mathrm{E}_r}{\mathrm{E}_s} = 12{\cdot}2$$

and we may say that the energy radiated is 12 times that dissipated in the spark, or the efficiency of radiation is nearly 92·5 per cent.

9. Free and Forced Oscillations. Resonance.—In all departments of physics in which we are concerned with vibrating bodies or systems of any kind, we find ourselves confronted with a phenomenon which is generally described by the term *resonance*. This term was originally coined in connection with certain effects noticed in acoustics, but its real origin being dynamical, it has been generalized and extended.

In its simplest form it can be exemplified by an experiment due to Professor H. A. Rowland.[26] Let a wooden lath (see Fig. 11) be provided at the bottom with a weight, and let it be suspended at the top so as to be capable of vibrating like a pendulum in one plane. It is then said to have one degree of freedom. At a point just below the point of suspension let a steel pin be placed through the rod, so as to project out at right angles to the rod and the plane of oscillation. When the rod vibrates, this pin makes small excursions to and fro. Provide a number of strings with bullets at the bottom and a loop formed in the string at the other end, by which to hang these simple pendulums on the pin of the master pendulum. Let these strings be of such length that one of the pendulums is equal in length to the master, one is one-third the length, one is a quarter, and one is an odd length, no exact fraction. If then the master pendulum is set in vibration, and any of the simple pendulums be successively hung on the pin, these last will be set in sympathetic vibration if its natural time period T, expressed by $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length of the string and g is the acceleration of gravity, is equal to that of the master pendulum to some exact submultiple of it. Otherwise the simple pendulum will not be set in motion by the other.

The time period for small swings of the master pendulum is given by the expression—

$$T = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{K}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

[26] See H. A. Rowland, "Collected Physical Papers," p. 29.

where I is the moment of inertia of the mass, and K is the quotient of the torque required to produce a small angular displacement, θ, by the angle θ. The proof of the above formula is simple. If we neglect all sources of energy dissipation such as friction, we may say that the restoring torque $K\theta$ is proportional to the product of the moment of inertia round the axis of rotation and to the angular acceleration. Accordingly—

$$-I\frac{d^2\theta}{dt^2}=K\theta \qquad \qquad (45)$$

The left-hand quantity has the minus sign because the displacement is assumed to decrease with the time. Hence the equation of motion is—

$$I\frac{d^2\theta}{dt^2}+K\theta=0 \qquad \qquad (46)$$

A particular solution of the above equation is—

$$\theta=A \sin \beta t \qquad \qquad (47)$$

Since $\sin \beta t=\sin (\beta t+2\pi)=\sin \beta\left(t+\frac{2\pi}{\beta}\right)$, it follows that $\frac{2\pi}{\beta}$ is equal to the periodic time of the motion, because after the lapse of a time T the displacement repeats itself. Hence—

$$\frac{2\pi}{\beta}=T, \text{ or } \frac{2\pi}{T}=\beta \qquad \qquad (48)$$

By differentiating (47) and substituting it in the original equation (46), we find that $\beta=\sqrt{\frac{K}{I}}$. Hence we have—

$$T=2\pi\sqrt{\frac{I}{K}}$$

If, then, exceedingly small impulses act on the system, at intervals exactly equal to its free periodic time, each one of these impulses acts to increase the effect of the last, and very large oscillations may be accumulated by extremely small individual impulses.

This fact can be illustrated by a number of simple instances. Stretch a string somewhat loosely between two fixed supports, and attach to it two simple pendulums. Set one of these in vibration in a plane perpendicular to the vertical plane which contains the stretched string. It will communicate small impulses to the loose support, and through it to the other pendulum, which will thereby be set in motion (see Fig. 12). Since, however, action and reaction are equal and opposite, the first pendulum is brought gradually to rest as it communicates its motion to the second. Then the second conveys back the energy to the first, and so the pendulums continue to set each other in motion and transfer the energy of motion from one to the other.

The general dynamical principle that any system capable of being set in vibration can have large oscillations created in it by infinitely small impulses coming at intervals equal to its own free period of vibration has extensive application.

It is not only applicable to cases of mechanical motion, but to electrical systems of conductors possessing capacity and inductance disturbed by electromotive force. If there be any case in which a system has potential energy when disturbed, and is subject to such constraints that its potential energy is increased by a displacement, it will, if left to itself, tend to go back to the condition of minimum potential energy, and in so doing will overshoot the mark. The acquired kinetic energy is then returned to the potential form, and a vibrational condition is set up in which energy is continually transformed from potential to kinetic and *vice versâ* at each transformation, some of the kinetic being dissipated as heat.

We have already seen that an inductance, L, in series with a capacity, C, constitutes an electrical system having one degree of freedom. An electromotive force acting on it causes an increase in the potential energy, and if the system is then

abandoned to itself it will execute electrical oscillations, the time period T of which is given by the formula—

$$T=2\pi\sqrt{CL} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

If, then, small electromotive forces act on the system at regular intervals they will increase continually this potential energy, provided that their time period agrees very exactly with that of the circuit. A very little difference, however, is sufficient to prevent the cumulative effect.

In dealing with this part of the subject we shall see that we meet continually with the product $\sqrt{CL}$, viz. the product of the square root of the capacity of a condenser and the inductance of a coil placed in series with it. It is convenient to call this product the *oscillation constant* of the circuit.

Again, in considering the separate parts, we find the phenomena are determined by the quantities Lp or $2\pi nL$ and $\frac{1}{Cp}$ or $\frac{1}{2\pi nC}$, where n is the frequency. The quantity Lp is now called the *reactance of* the inductive circuit, and the author has employed the term *captance* to signify the quantity $\frac{1}{Cp}$.

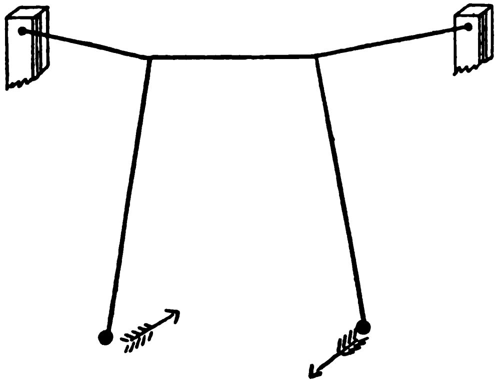


FIG. 12.—A Pair of Coupled Pendulums.

The quantity $p=2\pi n$, or the number of oscillations in 2π seconds, is conveniently called the *oscillation number.*

The reactance and captance are quantities of the dimensions of resistance, and may be measured in ohms. Hence, if there be a circuit consisting of a condenser and inductance in series, which is submitted to simple periodic or sinoidal electromotive force, the current in the circuit creates two electromotive forces, one of which opposes and the other helps change of current. If I is the maximum value of the current, then LpI is the maximum value of the counter-electromotive force due to reactance or inductance, and $\frac{I}{Cp}$ is the maximum value of the adjuvant electromotive force due to captance or capacity. The *vector equation* connecting current I and impressed electromotive force E (maximum values being understood) is—

$$E=RI+j\left(LpI-\frac{I}{Cp}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

where j stands for the sign of perpendicularity, or that the vector $\left(LpI-\frac{I}{Cp}\right)$ is at right angles to that denoted by RI. Accordingly, the impressed electromotive force must have components which have a vector sum equal to that of the several electromotive forces acting against or with it. Hence, by the ordinary

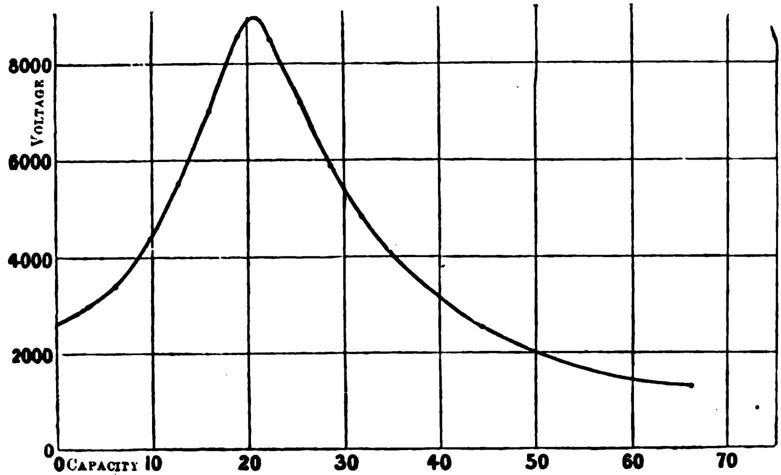


FIG. 13.—Variation of Terminal Voltage with Capacity in the Case of a Circuit having Capacity and Inductance when acted upon by a Periodic Electromotive Force.

rules for obtaining the size of vectors expressed by complex quantities, we have[27]—

$$(I)=\frac{(E)}{\sqrt{R^2+\left(Lp-\frac{1}{Cp}\right)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

where (E) and (I) denote the mere numerical values of E and I. Accordingly, if we keep E, n, and R constant, and vary L and C, the current I will have a maximum value when $Lp=\frac{1}{Cp}$, or when the reactance is equal to the captance, or when—

$$LCp^2=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

The above is the condition for resonance in a single circuit.

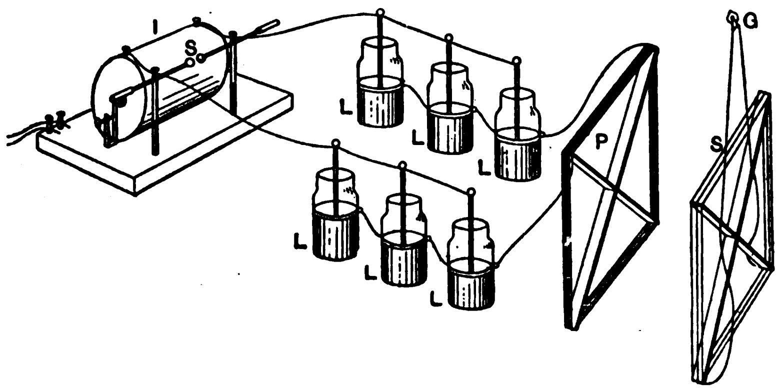


FIG. 14.—Production of Electric Oscillations in a Secondary Circuit assisted by Resonance. I, induction coil; L, L, Leyden jars; P, primary circuit; S, secondary circuit; G., incandescent lamp.

[27] For a short explanation of the method of dealing with alternating current problems by means of these complex or vector expressions, the reader is referred to the next section of this chapter.

If we attempt to test the above formula by placing a condenser of variable capacity across the terminals of an alternator, we are met with the difficulty that change in the capacity alters the phase difference of the current and electromotive force of the alternator, and therefore affects its excitation.[28]

In this case the result found is a mixed effect. Nevertheless, the measurement of the current shows that as the capacity or inductance is varied, the current tends to a maximum value, which it reaches when the condition is fulfilled. Under these conditions, the inductive circuit in series with a capacity acts as if it were perfectly non-inductive, and the current has the value it would have if a non-inductive resistance equal to the resistance of the inductive circuit was substituted for the capacity and inductance employed.

Hence, if we plot out the current flowing in the circuit under constant sinoidal electromotive force, or the electromotive force corresponding to constant current, when the capacity or inductance is varied, we have a curve such as that shown in Fig. 13, which rises sharply to a maximum value, which it reaches when the inductance, capacity, and frequency are so related that $LCp^2 = 1$. When we are employing high frequency electromotive forces, very striking effects can be produced with quite small inductances and capacities placed in series with each other, and the circuits so formed are remarkably responsive to exceedingly small periodic electromotive forces which agree in period with the natural time period of the circuit so formed.

To obtain these cumulative or resonance effects, it is necessary, however, to employ circuits with small damping, or which are persistent oscillators. We can illustrate the chief facts as follows :—

Let two circuits, P and S (see Fig. 14), be formed, each of 8 or 10 turns of insulated wire wound round square frames, the side of each frame being 1 metre in length. Let one circuit, P, have a Leyden jar or jars and spark gap associated with it, so as to form an oscillatory circuit. Let the other circuit, S, be placed at some little distance, and its ends joined by a small incandescent lamp. Then if oscillations be produced in the first circuit P by discharges of an induction coil, I, and if the second circuit be placed parallel to it, and at no great distance, oscillations will be induced in this second circuit, and these, if the circuits are near enough, will cause the small glow lamp to be illuminated.

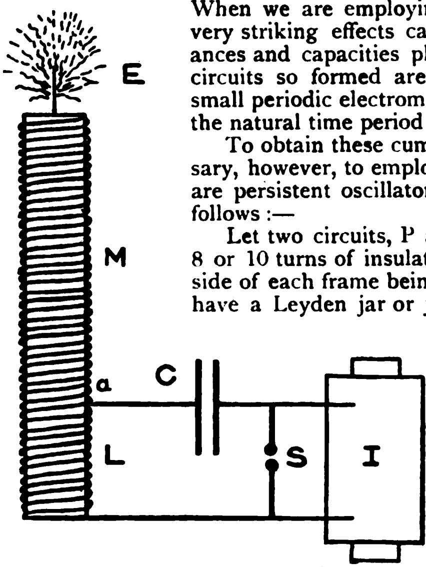


Fig. 15.—Resonance Helix.

In this case we have what are called forced oscillations produced in the secondary circuit. If, however, we cut the secondary circuit S and introduce a condenser formed of a Leyden jar or jars, we can arrange such a capacity that the secondary circuit has the same oscillation constant as the primary. That is, for each circuit, P and S, the quantity $\sqrt{CL}$, where C is the capacity and L the inductance, has the same value.

When this is done we find that the inductive effect of the primary circuit on the secondary circuit is greatly increased, and that we can put the secondary circuit much further off and yet light up the incandescent lamp in it to the same brilliancy. This increased effect is due to resonance. By making the oscillation constant of the primary and secondary circuits the same, we have "tuned," as it is called, the two circuits to each other, and the inductive effects are vastly enhanced. We can in a similar manner exhibit the effects of resonance in connection with open circuits. Let a spiral of bare copper wire, ML (see Fig. 15), be wound in turns not touching each other round an ebonite or wooden frame

[28] See J. A. Fleming, "The Alternate Current Transformer," vol. ii. p. 394, where some of these mixed resonance effects in the case of alternators and transformers working on cables having capacity are discussed.

or cylinder. An oscillatory circuit is then formed of a part, L, of this helix, and a condenser, C, and spark gap, S, excited by an induction coil, I, as usual. The point of contact *a* with the section of the spiral circuit which lies towards the middle of the helix must be capable of being shifted. The helix is then divided into two unequal parts, one part, L, is being employed as the inductance in an oscillatory circuit, and the other, M, is a free or open circuit in contact with this oscillatory circuit. If we set up oscillations, and shift the point of contact *a* so as to lengthen or shorten the free part of the helix, we shall find such a position that a powerful electric brush, E, starts from the free end of the helix, showing that strong electric oscillations are being set up in it. This arrangement is much used for creating high frequency electric brush discharges as used in medical work. It is then known as an Oudin Resonator (see Fig. 16).

The above-described phenomena are called resonance effects, and two electric circuits so coupled together that oscillations in one act by induction to create oscillations in the other, constitute an *oscillation transformer*. We have, however, in this preliminary description, for the sake of simplicity, avoided reference to the reaction which one circuit exercises on the other. We cannot define more precisely what we mean by saying that two circuits are in resonance with each other, or tuned together or syntonized, until we have examined a little more in detail what really takes place in such cases.

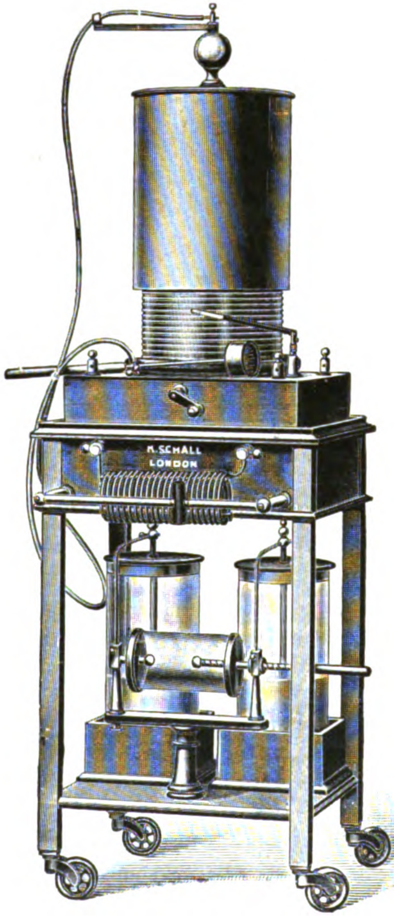


FIG. 16.—Oudin Resonator for Creating High Frequency Electric Brush Discharges.

The laws governing the action of oscillation transformers when very high frequency currents are employed differ greatly from those which hold good in the case of low frequency alternating current transformers. For example, if we desire to make a step-up transformer for raising potential when employing low frequency alternating currents, we should construct one in which the two coils had a very different number of turns, and a low electromotive force applied to the terminals of the coil of the smaller number of terms would be raised in value, so that the terminal potential difference of the two coils would be almost in the ratio of the number of their turns. In the case of high frequency oscillations, the ratio of transformation of potential is not in the proportion of the number of turns of the two circuits.

Before, however, we can discuss the theory of oscillation transformers, it is necessary to explain briefly the simplest analytical method of dealing with problems in alternating currents.

10. The Representation of Alternating Currents by Complex Quantities.—The study of alternating current phenomena, and therefore also of electric oscilla-

tions, is assisted by the adoption of simple mathematical methods for representing the quantities with which we are concerned. The usual method of procedure is to express the instantaneous value of a periodic current or electromotive force as a function of the maximum value during the phase, and of the time expressed as a fraction of the complete periodic time. In actual practice we are chiefly concerned, however, with the maximum value, or with the root-mean-square value (R.M.S. value) of the periodic quantity during the period, and we can simplify the analytical treatment if we can avoid introducing the symbol for time.

The R.M.S. value of a periodic current or electromotive force is defined as follows :—

Let i be the value, say, of the current at any time, t, reckoned from the beginning of the period, and let T be the periodic time, then the R.M.S. value, J, is given by the expression—

$$\mathrm{J}=\sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

Hence, if the quantity i varies in a simple harmonic manner, so that—

$$i=I\sin pt,$$

where I is the maximum value, then—

$$\mathrm{J}=\frac{I}{\sqrt{2}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

We can always, therefore, determine J from I when the equation to the curve is given.

All we need, therefore, in discussing problems connected with simple periodic currents is to represent in some manner the phase or direction and maximum magnitude of the current or electromotive force.

This is most conveniently done by means of *complex quantities*.

If a denotes any line or vector of given length drawn horizontally and to the right, then with the usual convention $-a$ will denote an equal horizontal line to the left. We may consider, therefore, that the vector $+a$ is converted into the $-a$ by turning it through two right angles, or by operating on it by -1. Hence the symbol for turning it through one right angle must be such that when twice repeated on itself it turns $+1$ into -1. This operation is denoted by $\sqrt{-1}=j$. Hence ja denotes a line drawn perpendicularly to a. The quantity j is therefore an algebraic *sign of perpendicularity*. It follows that $j^2=-1$ or $j=\sqrt{-1}$, and j has the same analytical signification as $\sqrt{-1}$, viz. when applied as a multiplier or operator to a vector it turns it through a right angle.

Hence any line may be represented as the vector sum of two lines, consisting of a horizontal of a units in length and a vertical component of b units in length. The proper representation of it is, therefore, $\pm a \pm jb$. The length or *size* of this line is $\sqrt{a^2+b^2}$.

Quantities of the form $a+jb$ are called *complex quantities*, and $\sqrt{a^2+b^2}$ is called the *modulus* of $a+jb$. The ratio $\frac{b}{a}$ is called the *slope* of the vector.

Hence lines or vectors may be drawn from any point to represent the maximum values of simple periodic quantities. The elements or steps a or b will represent the instantaneous values of these quantities, and their moduli will represent their actual measured maximum values, and if divided by $\sqrt{2}$ their R.M.S. values.

These complex quantities have certain properties, the chief of which may here be briefly mentioned. We shall take a single capital letter to represent a vector as a vector, and the same letter in brackets to represent its *size*.

Thus $\mathrm{E}=a+jb$ represents a *vector*, and $(\mathrm{E})=\sqrt{a^2+b^2}$ represents its *size*.

The reader should note and verify the following rules for dealing with these complex quantities and their moduli or size :—

(i.) Multiplication by j turns a vector through a right angle in a counter-clockwise or positive direction of rotation.

If $a+jb$ is any vector, then $j(a+jb)=-b+ja$ is a vector of the same size at right angles to $a+jb$.

(ii.) Multiplication by $-j$ turns a vector through a right angle in the clockwise direction.

If $a+jb$ is any vector, then $-j(a+jb)=(b-ja)$ is a vector of the same size at right angles negatively.

(iii.) If we denote the slope of the vector by θ, then $\frac{b}{a}=\tan\theta$, and if we denote the size of the vector by (A), then $a=(\mathrm{A})\cos\theta$ and $b=(\mathrm{A})\sin\theta$. Therefore $\mathrm{A}=(\mathrm{A})(\cos\theta+j\sin\theta)$.

The quantity $(\cos\theta+j\sin\theta)$ is called a *rotator*, for if applied to any vector it rotates it through an angle θ without changing its size.

Thus we can easily show, by multiplication and collection, that the size of the vector X, where $\mathrm{X}=(a+jb)(\cos\theta+j\sin\theta)$, is $\sqrt{a^2+b^2}$.

It is the same as the size of the vector $a+jb=\mathrm{A}$.

The vector X, however, is turned through an angle θ in the positive direction, beyond the vector A. If we insert in the operator $(\cos\theta+j\sin\theta)$ the exponential values of sine and cosine, viz.—

$$\sin\theta=\frac{\epsilon^{j\theta}-\epsilon^{-j\theta}}{2j} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

$$\text{and} \quad \cos\theta=\frac{\epsilon^{j\theta}+\epsilon^{-j\theta}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

$$\text{we have} \quad \cos\theta+j\sin\theta=\epsilon^{j\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Hence $\epsilon^{j\theta}$ and $\epsilon^{-j\theta}$ are also rotating operators, causing rotation of vectors through an angle θ in the positive or negative direction when applied to them.

If in place of θ we write pt, where t signifies time and $p=\frac{2\pi}{T}$, T being the periodic time, we see that $\mathrm{A}(\cos pt+j\sin pt)=\mathrm{A}\epsilon^{jpt}$ signifies a vector of length (A) *continually rotating* round one extremity with an angular velocity p.

Additional important properties of complex quantities are as follows :—

(i.) If two complexes are multiplied together, the modulus or size of their product is the product of their separate moduli or sizes. Thus if $a+jb$ and $c+jd$ are two vectors, their sizes are $\sqrt{a^2+b^2}$ and $\sqrt{c^2+d^2}$. Also $(a+jb)(c+jd)$ is another vector, and its size is $\sqrt{a^2+b^2}\,.\,\sqrt{c^2+d^2}$.

This is easily proved by multiplication and collection of terms.

(ii.) The same rule may be extended to quotients, powers, and roots of complex quantities. Accordingly, any such compound complex quantity as $\frac{a+jb}{c+jd}\sqrt{e+jf}$ may be written out in the canonical form $\mathrm{A}+j\mathrm{B}$, and its size $\sqrt{\mathrm{A}^2+\mathrm{B}^2}$ determined.

We need not, however, take the trouble to make this calculation, because the size of the above vector can be written down at once by the above rule, for it is equal to—

$$\frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}\sqrt{\sqrt{e^2+f^2}}$$

Since a complex quantity represents a vector or line, it is obvious that if two complex quantities are equal, their horizontal and vertical steps, or real and unreal parts, must be respectively equal. Thus, if—

$$a+jb=c+jd$$

we must have $a=c$ and $b=d$.

A process continually required is that of separating a complex quantity into its real and unreal parts. Thus, if we have the complex equation—

$$\frac{a+jb}{c+jd}(e+jf)=i+jk$$

we can separate out the steps as follows: Multiply numerator and denominator by $c-jd$; we then have—

$$i+jk=\frac{aec-bfc+afd+ebd}{c^2+d^2}+j\frac{acf+ebc+aed-bfd}{c^2+d^2}$$

Hence $$i=\frac{aec-bfc+afd+ebd}{c^2+d^2}$$

and $$k=\frac{acf+ebc+aed-bfd}{c^2+d^2}$$

The above rules will afford the reader most of the information necessary to follow the application of complex quantities to the representation of simple periodic quantities.

This method consists in representing the maximum value of a simple harmonic electromotive force or current by a vector denoted by such a complex as $a+jb$. Then we fix its position in space because the slope of the vector is such that $\frac{b}{a}=\tan\theta$, and its length or size by $\sqrt{a^2+b^2}$. An expression such as $A\epsilon^{jpt}$ represents then a rotating vector and its real part, viz. $A\cos pt$, represents its instantaneous value or projection on a certain axis, and A represents the magnitude or size of its maximum value.

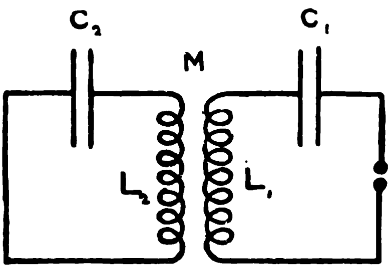


FIG. 17.—Two Coupled Oscillation Circuits.

In connection with simple periodic quantities, a theorem of great utility is as follows: If $A\sin pt$ represents any simple harmonic quantity, and $B\cos pt$ represents another of different amplitude but the same frequency, then $A\sin pt+B\cos pt$ also represents a simple periodic quantity of amplitude $\sqrt{A^2+B^2}$, but differing in phase from the first, viz. $A\sin pt$ by an angle ϕ, such that $\tan\phi=\frac{B}{A}$. Hence, $A\sin pt+B\cos pt=\sqrt{A^2+B^2}\sin(pt+\phi)$.

To prove the theorem, divide both sides by $\sqrt{A^2+B^2}$, then since $\sin\phi=\frac{B}{\sqrt{A^2+B^2}}$ and $\cos\phi=\frac{A}{\sqrt{A^2+B^2}}$, because $\tan\phi=\frac{B}{A}$, the identity is evident.

11. Theory of Coupled Oscillation Circuits having Capacity and Inductance in Series.—Let us consider two circuits, each having inductance, L, capacity, C, and resistance, R, and specify these quantities for the two circuits respectively by the suffixes 1 and 2. We shall follow first the lines of investigation in an interesting paper by A. Oberbeck.[29] Let these circuits be placed in inductive connection with each other by making the inductance in each circuit one of the coils of an oscillation transformer (see Fig. 17). Let these two circuits have a mutual inductance, M.

Let oscillations be set up in one circuit. It is required to find the resulting currents in the two circuits and potential differences of the condenser plates, due to the mutual reaction of the circuits.

Let i_1 and i_2 be the currents at any instant, and v_1 and v_2 the potential differences of the condenser plates. Then we have the fundamental equations—

$$L_1\frac{di_1}{dt}+M\frac{di_2}{dt}+R_1i_1-v=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

$$L_2\frac{di_2}{dt}+M\frac{di_1}{dt}+R_2i_2-v_2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

[29] See A. Oberbeck, "Ueber den Verlauf der Electrischen Schwingungen bei den Tesla'schen Versuchen," *Wied. Ann. der Physik.*, 1895, vol. 55, p. 623. See also G. W. Pierce, "On Experiments on Resonance in Wireless Telegraph Circuits," *Physical Review*, vol. 24, February 1902, p. 152.

If we differentiate each equation with respect to time, and remember that—

$$i_1 = -C_1\frac{dv_1}{dt},\ i_2 = -C_2\frac{dv^2}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

we arrive at the equations—

$$C_1L_1\frac{d^2i_1}{dt^2} + C_1R_1\frac{di_1}{dt} + C_1M\frac{d^2i_2}{dt^2} + i_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (60)$$

$$C_2L_2\frac{d^2i_2}{dt^2} + C_2R_2\frac{di_2}{dt} + C_2M\frac{d^2i_1}{dt^2} + i_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (61)$$

By differentiating the last equations twice, and eliminating between the results and the originals, we can separate the variables and obtain the equation—

$$C_1C_2(L_1I_2 - M^2)\frac{d^4i_1}{dt^4} + C_1C_2(R_1L_2 + R_2L_1)\frac{d^3i_1}{dt^3}$$

$$+ (C_2L_2 + C_1L_1 + C_1C_2R_1R_2)\frac{d^2i_1}{dt^2} + (C_1R_1 + C_2R_2)\frac{di_1}{dt} + i_1 = 0 \quad . \quad . \quad (62)$$

and a similar one in i_2.

In nearly all cases with which we are concerned in radiotelegraphy, the resistance of the oscillatory circuits is small compared with their reactance, and hence we shall not commit sensible error by neglecting the terms involving R_1 and R_2 in the above equations.

If also we write k for $\frac{M^2}{L_1L_2}$ where k is called the *coefficient of coupling*, and assume that the currents vary in a simple harmonic manner, we may assume that both i_1 and i_2 are quantities which vary as the real part of ϵ^{jpt} where $j = \sqrt{-1}$.

$$\text{Hence} \qquad \frac{d^2i_1}{dt^2} = -p^2i_1 \text{ and } \frac{d^4i_1}{dt^4} = p^4i_1$$

Making these substitutions, the equation (62) reduces to—

$$C_1L_1C_2L_2(1 - k^2)p^4 - (C_1L_1 + C_2L_2)p^2 + 1 = 0. \quad . \quad . \quad . \quad . \quad (63)$$

Now the natural time period T_1 of one circuit taken alone is such that $T_1^2 = 4\pi^2C_1L_1$, and that of the other, T_2, is given by $T_2^2 = 4\pi^2C_2L_2$, and the quantity p in equation (63) denotes 2π divided by the frequency or frequencies of the circuits when coupled together. If we write $\frac{4\pi^2}{T^2}$ for p^2 in the equation (62), and substitute for C_1L_1 and C_2L_2 their equivalents as above, we obtain the following biquadratic in T, viz.—

$$T^4 - (T_1^2 + T_2^2)T^2 + T_1^2T_2^2(1 - k^2) = 0 \quad . \quad . \quad . \quad . \quad . \quad (64)$$

The solution of which is—

$$T^2 = \frac{T_1^2 + T_2^2 \pm \sqrt{(T_1^2 - T_2^2)^2 + 4k^2T_1^2T_2^2}}{2} \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Hence it is clear there are two values of T corresponding to the two roots of the equation, and this signifies that when the oscillatory circuits are coupled together so as to act inductively on each other, then oscillations are set up in each circuit of two periods differing from each other, and from the natural free periods T_1 and T_2 of each circuit taken separately. Let us call the time periods of these two oscillations T′ and T″, then we have—

$$T' = \sqrt{\frac{T_1^2 + T_2^2 + \sqrt{(T_1^2 - T_2^2)^2 + 4k^2T_1^2T_2^2}}{2}} \quad . \quad . \quad . \quad . \quad (66)$$

$$T'' = \sqrt{\frac{T_1^2 + T_2^2 - \sqrt{(T_1^2 - T_2^2)^2 + 4k^2T_1^2T_2^2}}{2}} \quad . \quad . \quad . \quad . \quad (67)$$

Several cases of interest then present themselves.

I. Let $T_1 = T_2$, that is, let two circuits be supposed to have the same periodic time when separated far apart from each other. This is the case of *isochronism*,

or, as it is usually called, of *resonance*. If, then, in (66) and (67) we put $T_1 = T_2 = T$, we have—

$$T' = T\sqrt{1+k} \qquad (68)$$

$$T'' = T\sqrt{1-k} \qquad (69)$$

or if we consider frequencies n' and n'' and n instead of time periods, we have—

$$n' = \frac{n}{\sqrt{1+k}} \qquad (70)$$

$$n'' = \frac{n}{\sqrt{1-k}} \qquad (71)$$

from which we have—

$$k = \frac{n''^2 - n'^2}{n''^2 + n'^2} \qquad (72)$$

II. If the circuits are in resonance, that is, if $T_1 = T_2 = T$, and if the coefficient k is zero or extremely small, then we have $T' = T'' = T$. In other words, there are oscillations of only one frequency set up in each circuit. This is the case if the circuits are far apart or *feebly coupled*.

III. If the circuits are in resonance, but closely coupled or near together, so that the coefficient $k = 1$, we have $T' = \sqrt{T_1^2 + T_2^2}$ and $T'' = 0$. Hence, in this case also there are oscillations of only one periodicity, which is the square root of the sum of the squares of the periodic times of the two circuits when far apart.

We have in the next place to consider the transformation ratio of an oscillation transformer connecting two circuits having inductance and capacity. We shall assume that the resistances of the circuits, and therefore the damping, to be negligible, and we can then write the equations (57) and (58) when the values of i_1 and i_2 given in (59) are substituted, in the form—

$$C_1L_1\frac{d^2v_1}{dt^2} + C_2M\frac{d^2v_2}{dt^2} + v_1 = 0 \qquad (73)$$

$$C_2L_2\frac{d^2v_2}{dt^2} + C_1M\frac{d^2v_1}{dt^2} + v_2 = 0 \qquad (74)$$

By differentiating the above equations twice with regard to t and eliminating between the resulting and original equations, we arrive at two other equations, viz.—

$$C_1C_2(L_1L_2 - M^2)\frac{d^4v_1}{dt^4} + (C_1L_1 + C_2L_2)\frac{d^2v_1}{dt^2} + v_1 = 0 \qquad (75)$$

and a similar equation in v_2.

These equations have particular solutions of the form—

$$v_1 = A_1 \cos p_1t + B_1 \cos p_2t \qquad (76)$$

$$v_2 = A_2 \cos p_1t + B_2 \cos p_2t \qquad (77)$$

This may be proved by differentiating (76) and (77) and substituting in the original equations (73), (74), and (75), which they will be found to satisfy.

Since v_1 is a simple periodic quantity it may be represented by the real part of $\epsilon^{jpt} = \cos pt + j \sin pt$, and then we have—

$$\frac{d^4v_1}{dt^4} = p^4v_1 \quad \text{and} \quad \frac{d^2v_1}{dt^2} = -p^2v_1$$

Hence, substituting these values in (75), we obtain a biquadratic in p, viz.—

$$p^4 - \frac{C_1L_1 + C_2L_2}{C_1C_2(L_1L_2 - M^2)}p^2 + \frac{1}{C_1C_2(L_1L_2 - M^2)} = 0 \qquad (78)$$

If the roots of this are p_1^2 and p_2^2, we have then—

$$p_1^2 + p_2^2 = \frac{C_1L_1 + C_2L_2}{C_1C_2(L_1L_2 - M^2)} \qquad (79)$$

$$p_1^2p_2^2 = \frac{1}{C_1C_2(L_1L_2 - M^2)} \qquad (80)$$

and hence

$$p_1^2 - p_2^2 = \frac{\sqrt{(C_1L_1 - C_2L_2)^2 + 4C_1C_2M^2}}{C_1C_2(L_1L_2 - M^2)} \qquad (81)$$

From (76) and (77) find the values of $\frac{d^2v_1}{dt^2}$ and $\frac{d^2v_2}{dt^2}$ and substitute in the original equations (73) and (74), and we find that—

$$\left.\begin{aligned} a_1 = \frac{A_1}{A_2} &= \frac{p_1^2MC_2}{1-p_1^2L_1C_1} \\ a_2 = \frac{B_1}{B_2} &= \frac{p_2^2MC_2}{1-p_2^2L_1C_1} \end{aligned}\right\} \qquad (82)$$

Let V_1 be the maximum potential difference of the plates of the primary condenser, viz. the discharge potential. Hence, when $t=0$, $v_1=V_1$, and $v_2=0$. Let V_2 be the maximum potential difference of the plates of the secondary condenser. Then we have at the instant $t=0$—

$$A_1+B_1=V_1 \qquad (83)$$

$$A_2+B_2=0 \qquad (84)$$

For shortness, put $\frac{A_2}{A_1}=a_1$ and $\frac{B_2}{B_1}=a_2$. Then it is obvious that—

$$\left.\begin{aligned} A_1 &= \frac{V_1a_2}{a_2-a_1} & B_1 &= -\frac{V_1a_1}{a_2-a_1} \\ A_2 &= \frac{V_1a_1a_2}{a_2-a_1} & B_2 &= -\frac{V_1a_1a_2}{a_2-a_1} \end{aligned}\right\} \qquad (85)$$

The solutions of (73) and (74) are then—

$$v_1 = \frac{V_1}{a_2-a_1}(a_2 \cos p_1t - a_1 \cos p_2t) \qquad (86)$$

$$v_2 = \frac{V_1a_1a_2}{a_2-a_1}(\cos p_1t - \cos p_2t) \qquad (87)$$

and

$$V_2 = V_1\frac{2a_1a_2}{a_2-a_1} \qquad (88)$$

In this last expression insert the proper values of a_1 and a_2 from (82), and we have—

$$V_2 = -V_1\frac{2MC_1}{\sqrt{(L_1C_1-L_2C_2)^2+4M^2C_1C_2}} \qquad (89)$$

Accordingly, when the circuits are syntonized, that is, when $C_1L_1=C_2L_2$, we have—

$$V_2 = -V_1\frac{\sqrt{C_1}}{\sqrt{C_2}}$$

Hence the transformation ratio in this case depends only on the relative capacity of the condensers in the primary and secondary circuits.

We have then for the potential difference v_2 of the terminals of the secondary condenser at any instant the expression—

$$v_2 = V_1\frac{\sqrt{C_1}}{\sqrt{C_2}}(\cos p_1t - \cos p_2t) \qquad (90)$$

and for the secondary current i_2 the equation—

$$i_2 = V_1\sqrt{C_1C_2}(p_2 \sin p_2t - p_1 \sin p_1t)$$

From (79) and (81) it can be shown that when the circuits are syntonized so that $C_1L_1=C_2L_2=CL=\frac{1}{p^2}$, where $\frac{2\pi}{p}$ is the natural time period of each circuit, we have—

$$p_1 = \frac{p}{\sqrt{1+k}} \text{ and } p_2 = \frac{p}{\sqrt{1-k}} \qquad (91)$$

where

$$k = \frac{M}{\sqrt{L_1L_2}}$$

Accordingly, the secondary current i_2 is then the sum of two currents of different frequency and amplitude, for—

$$i_2 = V_1\sqrt{C_1C_2}\frac{p}{\sqrt{1-k}}\sin\frac{p}{\sqrt{1-k}}t - V_1\sqrt{C_1C_2}\frac{p}{\sqrt{1+k}}\sin\frac{p}{\sqrt{1+k}}t$$

The oscillation of greatest frequency has also the greatest amplitude.

If the circuits are not syntonized, then—

$$V_2 = -V_1\frac{2kC_1\sqrt{L_1L_2}}{\sqrt{(C_1L_1 - C_2L_2)^2 + 4k^2C_1L_1C_2L_2}} \quad . \quad . \quad . \quad . \quad (92)$$

And if $k=1$ or $M = \sqrt{L_1L_2}$, the above becomes—

$$\frac{V_1}{V_2} = -\frac{\sqrt{L_1}}{\sqrt{L_2}}\left(\frac{C_1L_1 + C_2L_2}{2C_1L_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

The second factor on the right-hand side is generally not far from unity in value and then if N_1 and N_2 are the numbers of turns on the primary and secondary circuits of the oscillation transformer, we have approximately—

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

We see, therefore, that in the case of the oscillation transformer, with its two circuits loosely coupled and tuned in resonance, the damping being negligible, the ratio of transformation is determined solely by the capacities in the two circuits; whereas when the circuits are not tuned, but closely coupled, the ratio is determined by the relative number of turns on the two circuits.

The discussion of the more general case in which the damping of the two circuits is not negligible leads to greater analytical difficulties, and is dealt with in the subsequent sections.

Accordingly, the design of an oscillation transformer to transform high frequency currents is based on very different facts to the design of low frequency transformers. In the latter case the transformer changes terminal voltage almost in the ratio of the numbers of turns on the two circuits. In the former case, if the circuits are of equal time period separately, the change ratio depends solely on the capacities in the two circuits.

By making the primary capacity sufficiently large compared with the secondary capacity, we can increase in the same proportion the terminal voltages of the two condensers. In this case there are two periods of oscillation in the coupled circuit, the mean of the squares of the two periodic times being equal to the square of the common time period. The two may, however, become equal when the coupling is sufficiently loose.

The other case of a pair of closely coupled circuits, with different time periods, presents us with an instance of forced oscillations. The single resultant forced time period has a square equal to the sum of the squares of the time periods of the two circuits separately.

Following the investigation of Oberbeck (*loc. cit.*), we may give a numerical example which will illustrate the foregoing.

Let there be two coupled circuits having capacity, inductance, and mutual inductance.

Let $L_1 = 1000$ cms., $L_2 = 25,000$ cms., $M = 3000$ cms.; also let $C_1 = 0{\cdot}001$ mfd. $= 10^{-18}$ electromagnetic units, and let $C_2 = 0{\cdot}00004$ mfd. $= 0{\cdot}04 \times 10^{-18}$ electromagnetic units.

Then $L_1C_1 = L_2C_2 = 10^{-15}$

Therefore $T_1 = T_2 = 2\pi\sqrt{L_1C_1} = T$

or $T^2 = \frac{4\pi^2 10}{10^{16}}$

Also $\theta^2 = 4\pi^2 M\sqrt{C_1C_2} = \frac{4\pi^2 6}{10^{16}}$

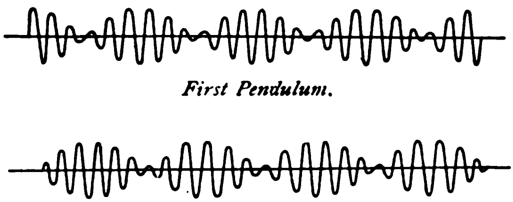
Accordingly $T' = 2\pi\frac{4}{10^8}$ and $T'' = 2\pi\frac{2}{10^8}$

Hence the two frequencies n_1 and n_2 are $n_1 = \frac{10^8}{25 \cdot 132}$ and $n_2 = \frac{10^8}{12 \cdot 566}$, whilst the common frequency $= n_0 = \frac{10^8}{19 \cdot 844}$, this last being the frequency which would exist in each circuit if they were separate and far apart.

It is important that the reader should fully understand the reason for the appearance of these two oscillations of different frequencies in syntonic coupled isochronous circuits. An assistance may be obtained by referring again to the experiment with the two equal pendulums hung on a loose string, to which reference was made in § 9 of this chapter. (See Fig. 12.)

Consider, then, the case of these two equal pendulums. Each has the same time period when vibrating alone. If, however, they are hung side by side on a loose string, we have seen that when one pendulum is set in motion it imparts its motion to the other little by little, and in so doing brings itself to rest. Then the second pendulum in turn gives back its motion to the first, and so on. It is clear that the energy is transferred from one to the other, or backwards and forwards between the two pendulums.

Let us suppose that a band of paper moves underneath the pendulums in a direction at right angles to the plane in which each is vibrating, and let each of



First Pendulum.

Second Pendulum.

FIG. 18.—Oscillations of a Pair of Coupled Pendulums.

these pendulums trace on the paper a wavy curve representing its alternately increasing and diminishing vibrations. We should have upon the paper two curves traced represented by the curves in Fig. 18. Each curve would be a curve having a two-fold periodicity, viz. a certain frequency of oscillation and a much smaller frequency of variation in maximum amplitude. Also the instant of maximum amplitude in one curve would coincide with the instant of zero maximum in the other. It is clear that the ordinate of such a curve can be represented by the expression

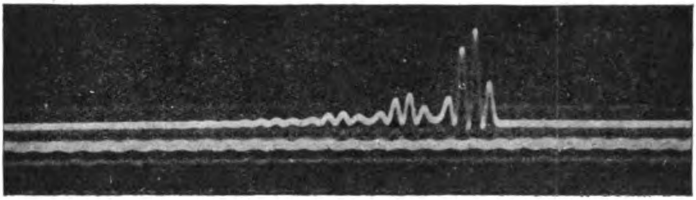
$$y = \mathrm{A} \cos qt \sin pt$$

for this represents a simple harmonic motion having a frequency $n = p/2\pi$, but having a maximum amplitude A cos qt varying with a frequency $\mathrm{N} = q/2\pi$. But now the expression A cos qt sin pt is equal to

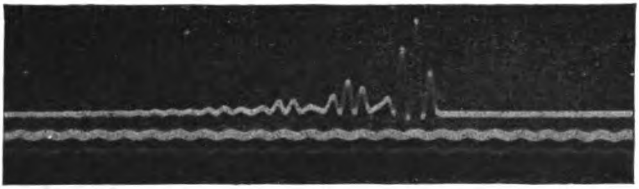
$$\tfrac{1}{2}\mathrm{A} \sin (p+q)t + \tfrac{1}{2}\mathrm{A} \sin (p-q)t$$

Hence each periodically varying curve as shown in Fig. 18 can be resolved into the sum of two undamped simple harmonic curves, one having a frequency greater than n, and the other having a frequency less than n. The frequency of the variation of the maximum ordinate is $\frac{q}{2\pi}$. A similar phenomenon is met with in acoustics. If two organ pipes slightly out of tune with each other are sounded together the resultant sound waxes and wanes in a manner which produces an effect called the *beats* of the sound. It is obvious that the graphical representation of the loudness of the resultant sound would be a curve as in Fig. 18, and this resolves itself into the sum of two simple harmonic vibrations of different

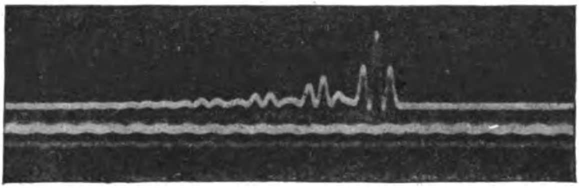
frequencies. The same thing happens in the case of two coupled oscillatory circuits. The oscillations in one circuit create by induction oscillations in the



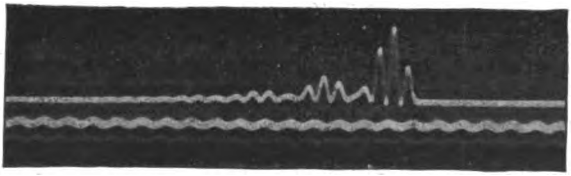
$C_1 = 9{\cdot}55$ mfds. $C_2 = 0{\cdot}000875$ mfd.
$L_1 = 4{\cdot}62$ millihenrys. $L_2 = 70{\cdot}15$ henrys.
$k = 0{\cdot}385$.



$C_1 = 11{\cdot}87$ mfds. $C_2 = 0{\cdot}001063$ mfd.
L_1, L_2, and k, same as above.



$C_1 = 9{\cdot}55$ mfds. $C_2 = 0{\cdot}001063$ mfd.
L_1, L_2, and k, same as above.



$C_1 = 11{\cdot}87$ mfds. $C_2 = 0{\cdot}000875$ mfd.
L_1, L_2, and k, same as above.

FIG. 19.—Oscillograms of Oscillations in Coupled Circuits. (Taylor-Jones.)

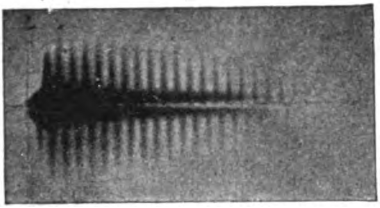
other. Also these latter induce back-oscillations in the primary. Therefore the current amplitude in each circuit undergoes a periodic variation of maximum value, and can therefore be resolved into the sum of two oscillations of different frequency.

This has been well shown by experiments made by Dr. E. Taylor-Jones in investigations made specially for the purpose of recording the oscillations in a coupled circuit photographically.[30] He employed an electrostatic oscillograph of his own design as a means of delineating the oscillations.[31]

He used two circuits with the following constants. The secondary circuit was the secondary coil of an induction coil, having an inductance of $70{\cdot}15 \times 10^9$ cms. or 70·15 henrys, and its resistance was 14,022 ohms. The primary circuit consisted of 1200 turns of No. 14 copper wire wound on a glass tube. Its resistance was 1·0378 ohms, and inductance 4,619,000 cms. or 4·619 millihenrys. The coefficient of coupling of these circuits was such that $k = 0{\cdot}1483$ or $k = 0{\cdot}385$. Hence the circuits were somewhat closely coupled. The capacity in the primary circuit was of the order of 10 mfds., and that in the secondary about 0·001 mfd.

The frequencies n_1 and n_2 were calculated from Oberbeck's formula as given in equation (65.)

These frequencies were of the order of 500 to 1000 per second. The resultant oscillation or potential difference of the secondary condenser was then recorded



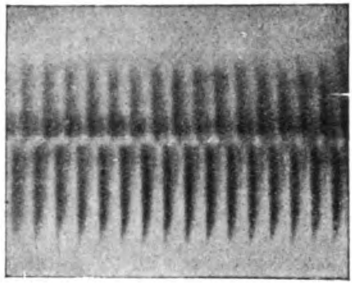


FIG. 20.

(*a*) Oscillogram of Damped Oscillations in a Single or Uncoupled Circuit. (Dieselhorst.)

(*b*) Oscillogram of Undamped Oscillations in a Single or Uncoupled Circuit. (Dieselhorst.)

photographically, and is shown in the series of curves given in Fig. 19. These show very admirably the existence of the "beats" in the damped oscillation due to the coexistence of oscillations of two frequencies in the resultant oscillation. The same was proved to be the case when persistent oscillations were employed produced by a Duddell musical arc instead of the damped oscillations due to a condenser discharge.

A confirmation of the above autographic delineation of the resultant oscillation in a coupled circuit is given by the experiments of Dr. Dieselhorst with the oscillograph vacuum tube described in Chap. I. § 6. Making use of this, Dr. Dieselhorst photographed the damped oscillations of a condenser discharge, and obtained the photograph shown in Fig. 20 (*a*), in which the decadent amplitude of the black shaded lines represents the gradual damping out of the oscillations. When, however, this condenser circuit was coupled electromagnetically to a radiating and tuned antenna, so that it became one member of a pair of coupled circuits, the oscillograph photograph was as shown in Fig. 21, in which the black lines representing the oscillations are separated into bunches, and these interspaces correspond to the beats, and show that there must have been two superimposed oscillations.[32]

A photograph taken by the same means with an uncoupled or single oscillatory circuit in which persistent oscillations exist is shown in Fig. 20 (*b*).

A most complete experimental investigation of this subject has been made by

[30] See E. Taylor-Jones, "Electrical Oscillations in Coupled Circuits," *Phil. Mag.*, January 1909.

[31] "A Short-Period Electrometer," *Phil. Mag.*, August 1907.

[32] See *Electrical Engineering*, April 23, 1908, p. 625.

Professor George W. Pierce.[33] He arranged two coupled circuits, each having capacity and inductance, and set up in one oscillations by means of a spark gap and induction coil. The periodicity of the oscillations set up in the two circuits was then measured by making them act inductively upon a tertiary circuit very loosely coupled with the primary or secondary circuit. This tertiary circuit could have its inductance and capacity varied, and the condition in which the current induced in it had its maximum value was indicated by the deflection of a high frequency dynamometer or modified form of Fleming alternating current galvanometer.[34]

By calibrating this third circuit so that, in any condition of adjustment, the product of its inductance L and capacity C is known, we know its natural frequency n, because $n=\frac{1}{2\pi\sqrt{CL}}$.

This circuit, therefore, becomes a means of measuring the frequency of the oscillations set up in the primary or secondary circuit by tuning the tertiary circuit

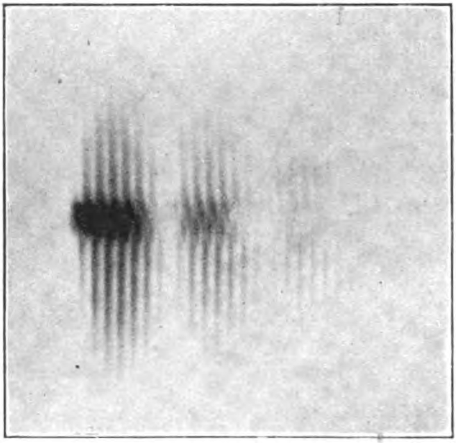


FIG. 21.—Oscillogram of Damped Oscillations in a Pair of Coupled Circuits. (Hans Boas.)

first to one frequency and then to the other, and judging of this agreement by the fact that the tertiary current reaches a maximum or sub-maximum corresponding to that setting or to resonance between it and the other circuit for that frequency.

It will be shown in a later chapter that when oscillations are set up in a circuit some of the energy is radiated in the form of electromagnetic waves, and the wavelength λ of these waves is numerically equal to the product of their time period T and the wave velocity u all measured in consistent units.

Hence in the expressions already given (66) and (67) for the time periods T_1 and T_2 of the two oscillations in coupled circuits, we may substitute λ_1 and λ_2 for the two wave-lengths, and arrive at the expressions—

$$\lambda_1'=\sqrt{\frac{\lambda_1^2+\lambda_2^2+\sqrt{(\lambda_1^2-\lambda_2^2)^2+4k^2\lambda_1^2\lambda_2^2}}{2}} \quad . \quad . \quad . \quad . \quad . \quad (95)$$

$$\lambda_2'=\sqrt{\frac{\lambda_1^2+\lambda_2^2-\sqrt{(\lambda_1^2-\lambda_2^2)^2+4k^2\lambda_1^2\lambda_2^2}}{2}} \quad . \quad . \quad . \quad . \quad . \quad (96)$$

[33] See Prof. G. W. Pierce, "Experiments on Resonance in Wireless Telegraph Circuits," Part V. The *Physical Review*, vol. xxiv. p. 166, February 1907.

[34] See Chap. II. § 13.

Professor Pierce measured carefully the wave-lengths of the waves set up in the two coupled circuits which he used, and compared the results with the above formulæ and found a very good agreement. The natural wave-length λ_2 of the

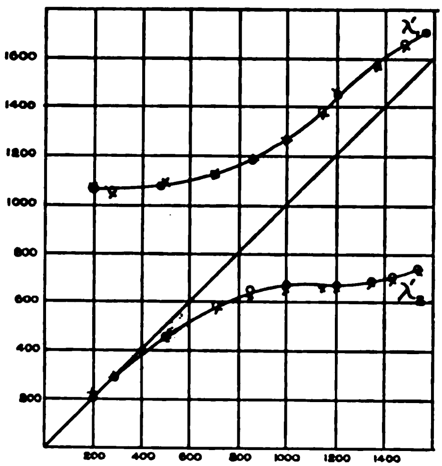


FIG. 22.

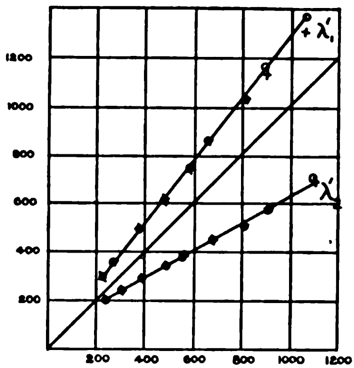


FIG. 23.

secondary circuit was kept constant and equal to 1060 metres, whilst the primary circuit was varied so as to alter its natural wave-length λ_1 from 210 to 1560 metres. The observed and calculated values of λ_1' and λ_2' are set out in the following Table, and delineated in the curves in Fig. 22.

Calculated and observed wave-lengths radiated by a coupled oscillator. Circuits not syntonized. Primary capacity = 0·00432 mfd. Secondary capacity = 0·00482. Primary inductance, varied as below. Secondary inductance = 0·066 millihenry. Primary wave-length, varied as below. Secondary wave-length $\lambda_2 = 1060$ metres.

Pr. inductance.	Pr. wave-length.	Wave-lengths calculated.		Wave-lengths observed.	
Millihenrys.	Metres.	λ_1'	λ_2'	λ_1'	λ_2'
0·1585	1560	1740	727	...	710
0·139	1460	1670	712	1650	685
0·118	1350	1567	686	1570	665
0·10	1230	1462	680	1480	660
0·082	1130	1390	660	1370	660
0·065	1000	1273	685	1280	660
0·0482	870	1185	680	1185	630
0·0315	700	1127	595	1125	565
0·0172	510	1080	467	1090	460

The agreement between observation and theory is fairly close.

Another series of observations was taken when the two coupled circuits had the same periodic time or wave-length when separated. This was the case of resonance.

In this we have—
$$(\lambda_1')^2 = \lambda_1^2(1+k) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$
$$(\lambda_2')^2 = \lambda_2^2(1-k) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

The results of observation are recorded in the Table on p. 242, and delineated in Fig. 23.

16

The convergence of the lines in Fig. 23, the ordinates of which represent the various values of λ_1' and λ_2', as the coupling becomes looser, is very striking and in accordance with theory.

Calculated and observed wave-lengths radiated by a coupled oscillator with circuits syntonized to a common wave-length λ which was varied.

Natural free wave-length of either circuit.	Wave-lengths radiated from the coupled circuits.			
	Observed.		Calculated.	
λ	λ_1'	λ_2'	λ_1'	λ_2'
1060	1290	655	1335	680
900	1095	555	1150	555
810	1025	503	1032	507
690	860	450	870	440
570	700	380	714	374
487	600	330	609	322
395	480	280	485	278
290	345	224	352	210
252	275	204	294	174

12. **The Damping in Coupled Circuits.**—Oberbeck shows (*loc. cit.*) how to calculate the damping in each of the two coupled circuits forming an oscillation transformer when the resistances are not negligible. On referring to equation (62) in § 11, we see the value of p is given by an equation of the fourth degree of the form—

$$p^4+fp^3+gp^2+hp^2+k=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

The roots of this equation are—

$$(-\alpha+j\beta),\quad(-\alpha-j\beta),\quad(-\gamma+j\eta),\quad(-\gamma-j\eta)$$

Let α be small compared with β, and γ small compared with η, as is always the case in practice. Then Oberbeck proves that—

$$\alpha=\frac{f\beta^2-h}{2(2\beta^2-g)},\qquad\gamma=\frac{f\eta^2-h}{2(2\eta^2-g)}\quad . \quad . \quad . \quad . \quad (100)$$

Let the coefficient of coupling $=\frac{M}{\sqrt{L_1L_2}}$ be denoted by k, and let us consider the case in which $L_1C_1=L_2C_2$. Then if R_1 and R_2 are the resistance of the two circuits, Oberbeck shows that—

$$\alpha=\frac{\frac{R_1}{L_1}+\frac{R_2}{L_2}}{4(1+k)},\qquad\gamma=\frac{\frac{R_1}{L_1}+\frac{R_2}{L_2}}{4(1-k)}\quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

Hence the two oscillations resulting in the coupled circuits of equal separate period are differently damped. One is more damped and the other less damped than the mean of the damping in the two separate circuits.

If we write a_1 for $\frac{R_1}{2L_1}$ and a_2 for $\frac{R_2}{2L_2}$, then we have—

$$(1+k)\alpha=\tfrac{1}{2}(a_1+a_2)$$
$$(1-k)\gamma=\tfrac{1}{2}(a_1+a_2)\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

We see, therefore, that if $k=0$, $\alpha=\frac{1}{2}(a_1+a_2)$, but if k is not zero then we have $\alpha<\gamma$.

13. **General Theory of Resonance.**—When two circuits having inductance,

resistance, and capacity are inductively connected together, we are then presented with a unique case to consider if their natural time periods of oscillation when separate are the same. Oscillations in one circuit then create a strong response in the other coupled circuit. In practice we find that this *syntony* or agreement between the time periods of the two circuits must be very exact if the phenomenon of resonance is to take place. Hence any treatment of the subject would be incomplete which did not include an examination of the manner in which a departure from equality in the free time periods of the two circuits affects the result. We shall first consider the case of a secondary circuit which has an induced electromotive force created in it by a *sustained or continuous* simple periodic current in an adjacent primary circuit. Let C be the capacity in the secondary circuit, L the inductance, and R the resistance. Let the current in the circuit at any time, t, be denoted by i, and the potential difference of the terminals of the condenser by v. Let the damping factor $\frac{R}{2L}$ be denoted by α, and the logarithmic decrement by δ.

The condenser circuit has a natural time period of oscillation n_2, which is determined by the equation—

$$n_2=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

Hence, if $p=2\pi n_2$ and $\alpha=\frac{R}{2L}$, we have—

$$\frac{1}{LC}=p^2+\alpha^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

The differential equation for the current in the condenser circuit is—

$$L\frac{di}{dt}+Ri+v=e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

where $e=E\sin qt$ is the simple harmonic electromotive force acting in the secondary circuit due to the action of the current in primary circuit having a frequency n_1 such that $2\pi n_1=q$.

Again, $i=C\frac{dv}{dt}$, and therefore, by substitution in (104), we have—

$$CL\frac{d^2v}{dt^2}+CR\frac{dv}{dt}+v=E\sin qt \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

To solve this last equation, differentiate it twice with respect to t, and eliminate $\sin qt$ with the aid of the original equation. We have then—

$$\frac{d^4v}{dt^4}+2\alpha\frac{d^3v}{dt^3}+(p^2+\alpha^2+q^2)\frac{d^2v}{dt^2}+2q^2\alpha\frac{dv}{dt}+q^2(p^2+\alpha^2)v=0$$

The auxiliary biquadratic of the above (see Boole's "Differential Equations," p. 194) is $(m^2+q^2)(m^2+2\alpha m+p^2+\alpha^2)=0$, and the roots of this equation are—

$$m=\pm\sqrt{-1}\,q$$
$$m=-\alpha\pm\sqrt{-1}\,p$$

Hence the solution of (105) is—

$$v=P\sin qt+Q\cos qt+A\epsilon^{-\alpha t}\sin pt+B\epsilon^{-\alpha t}\cos pt$$

or $$v=V\sin(qt-\phi)+V'\epsilon^{-\alpha t}\sin(pt-\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

where P, Q, A, and B are some constants such that $V=\sqrt{P^2+Q^2}$ and $V'=\sqrt{A^2+B^2}$.

This last solution indicates that the current in the secondary circuit consists of two superimposed oscillations.

(i.) A *forced oscillation* of amplitude V, which is undamped and has a frequency n_1 identical with that or the applied electromotive force.

(ii.) A *free natural oscillation*, having an initial maximum value V′, which is damped, and therefore dies out before long, leaving only the forced oscillation to persist.

If we differentiate (106) twice, and substitute the results in the original equation (105), we can neglect those terms which have a factor ϵ^{-at}, as they die away after a short time, and we are then left with the equation—

$$\frac{E}{CL}\sin qt = V(p^2 - q^2 + a^2)\sin(qt - \phi) + V2qa\cos(qt - \phi)$$

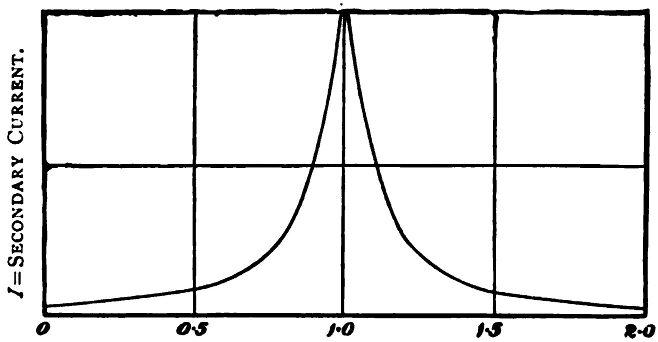
or $$\frac{E}{CL}\sin qt = V\sqrt{(p^2 - q^2 + a^2)^2 + (2qa)^2}\,\sin(qt - \phi - \psi)$$

Hence $$V = \frac{p^2 + a^2}{\sqrt{[(q^2 - p^2) - a^2]^2 + [2qa]^2}}\,.\,E \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

and since the maximum value of the condenser current $I = CVq$, we have an expression for the maximum value of the condenser current, viz.—

$$I = \frac{q}{L\sqrt{[(q^2 - p^2) - a^2]^2 + [2qa]^2}}\,.\,E \quad . \quad . \quad . \quad . \quad . \quad (108)$$

Suppose that a is small, so that a^2 is negligible in comparison with p^2. In this ase the secondary circuit is said to be feebly damped. Then, bearing in mind



$\frac{n_2}{n_1}$ = Ratio of Frequencies.

Fig. 24.—A Resonance Curve.

that $\frac{p}{q} = \frac{n_2}{n_1} = x$, we may write the equation (108) for the current in the condenser circuit in the form—

$$I = \frac{E}{L\sqrt{q^2(1 - x^2)^2 + (4a^2)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

where $q = 2\pi n_1$, $2a = \frac{R}{L}$, and $x = \frac{n_2}{n_1}$.

Let us examine the manner in which the current I varies as the ratio $\frac{n_2}{n_1}$ of the natural frequencies of the driving and driven circuits approximates to unity.

Let $x = \frac{n_2}{n_1} = 0$ then $I = \frac{E}{L\sqrt{q^2 + (2a)^2}}$

when $x = \frac{n_2}{n_1} = 1$ then $I = \frac{E}{2aL} = \frac{E}{R}$

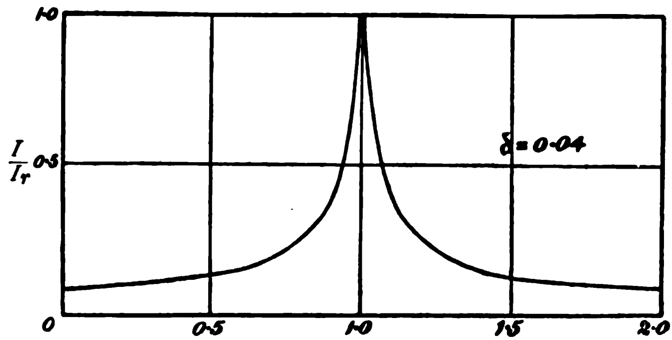
and if $x = \frac{n_2}{n_1} = \infty$ then $I = 0$

We see, therefore, that the expression for the current in the secondary circuit, considered as a function of the ratio of the natural frequencies of the two circuits,

has a maximum value when $n_1 = n_2$. If we delineate the expression for I in the form of a curve, the abscissæ of which represents to scale $\frac{n_2}{n_1}$ and the ordinates the corresponding values of I, then we have a curve as shown in Fig. 24, which is called a *resonance curve.* This curve runs up into a peak very sharply, because the value of I depends on the difference of the squares of two quantities which are approaching each other in value. The current corresponding to equality in the frequency of the two circuits is called the *resonance current.* We shall denote it by I_r.

It is obvious that the ratio of the current corresponding to any particular value of $\frac{n_2}{n_1}$ not far from unity, to the current which exists when $\frac{n_2}{n_1}$ is unity, is given by the equation—

$$y = \frac{I}{I_r} = \frac{2aq}{\sqrt{(q^2 - p^2)^2 + (2qa)^2}} = \frac{2a}{q\sqrt{\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^2 + \left[\frac{2a}{q}\right]^2}} \quad . \quad . \quad (110)$$



$\frac{n_2}{n_1}$ = RATIO OF FREQUENCIES.

FIG. 25.—A Resonance Curve.

It is most convenient to plot the ratio $\frac{I}{I_r} = y$ as ordinates to abscissæ representing $\frac{n_2}{n_1} = x$ (see Fig. 25).

A resonance curve so plotted enables us to determine the logarithmic decrement of the oscillation circuit with great ease. For if δ is the logarithmic decrement of the circuit, then $n_2\delta = a$ and $2\pi n_1 = q$. Hence—

$$\frac{2a}{q} = \frac{\delta}{\pi} \cdot \frac{n_2}{n_1}$$

Therefore
$$\frac{I}{I_r} = \frac{\frac{\delta}{\pi} \cdot \frac{n_2}{n_1}}{\sqrt{\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^2 + \left(\frac{\delta}{\pi} \cdot \frac{n_2}{n_1}\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

or if $\frac{n_2}{n_1} = x$, then when x is near unity, $1 + x$ is nearly 2, and we can transform (111) into—

$$\frac{I}{I_r} = \frac{1}{\sqrt{1 + \frac{(1 - x)^2}{\left(\frac{\delta}{2\pi}\right)^2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

or $$\delta = 2\pi(1-x)\sqrt{\frac{I^2}{I_r^2 - I^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

The practical use of this last expression for the decrement is considerable. Owing to the difficulty of measuring spark resistance, and the fact that the high frequency resistance of a circuit can only be predetermined in a few cases, we are seldom able to obtain the resistance decrement of a circuit by direct calculation.

We can, however, proceed experimentally as follows: Insert in the secondary circuit a hot-wire ammeter so as to measure the value of the root-mean-square current. Since for the same circuit this R.M.S. value J is directly proportional to the maximum value I of the currents during each train, it follows that—

$$\frac{J^2}{J_r^2 - J^2} = \frac{I^2}{I_r^2 - I^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

where the suffix r indicates the value of the current at its maximum, due to exact resonance.

If, then, we can measure or calculate from the capacity and inductance in the

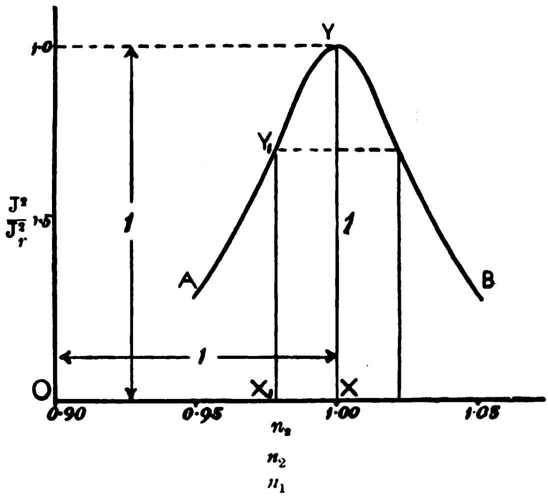


FIG. 26.—Determination of the Decrement of Electrical Oscillations by the aid of a Resonance Curve.

primary and secondary circuits the frequencies n_1 and n_2 for the various values of the secondary current J, we can plot a resonance curve of J in terms of the ratio $\frac{n_2}{n_1}$ as follows:—

Set off on some horizontal line a distance, OX (see Fig. 26), to represent unity, and on this line mark off various values of the ratio $\frac{n_2}{n_1}$ as abscissæ. Corresponding to these, set up ordinates representing the values of J^2, and taking the maximum ordinate XY to have a value, unity, on some scale, we obtain a curve, AYB, the ordinates of which represent the ratio of the square of the secondary current J^2, to the square of the maximum current, J_r^2; and the corresponding abscissæ the ratio of the natural frequencies of the two circuits.

Then, if X_1Y_1 is some value of $\frac{J^2}{J_r^2}$ corresponding to a value of $\frac{n_1}{n_2}$ *not far from unity*, we have $XX_1 = 1 - \frac{n_2}{n_1}$ and

$$\frac{X_1Y_1}{\sqrt{(XY)^2-(X_1Y_1)^2}}=\sqrt{\frac{J^2}{J_r^2-J^2}}$$

Hence from (113) $\delta=2\pi\,(XX_1)\dfrac{X_1Y_1}{\sqrt{(XY)^2-(X_1Y_1)^2}}$ . . . (115)

The reader must, however, notice that this method of obtaining the decrement δ from the resonance curve is based on two assumptions—

(i.) The distance XX_1 must be small compared with OX, so that $1-\frac{n_2}{n_1}$ is a small quantity compared with unity.

(ii.) The method is only valid when the decrement δ is small compared with 2π, so that $n\delta=\alpha$ is small compared with $2n\pi=p$, as above assumed.

Hence the method only applies to the determination of the decrement of a feebly damped oscillatory circuit, or to one in which the decrement is not greater, say, than 0·1.

If the damping is not small, then we cannot neglect δ in comparison with 2π, and in plotting the resonance curve for potential and current we have to employ the complete equations (107) and (108). We then find that the resonance curves for potential and current plotted to the same ordinates $\frac{n_2}{n_1}$ are no longer identical or symmetrical, and moreover that the maximum ordinate of the curve does not coincide with abscissa $\frac{n_2}{n_1}=1$. This leads to the conclusion that we have to distinguish between *isochronism* in two circuits and *resonance*, and that whilst these are identical for feebly damped currents, they are not so for strongly damped currents. The two diagrams in Figs. 27 and 28 for the resonance curves of potential and current for two circuits having decrements per half-period respectively of 0·1 and 0·4, show this distinction. These diagrams are taken, by kind permission, from the treatise by Professor J. Zenneck on "Electrical Oscillations and Wireless Telegraphy," p. 573.

14. Resonance between Two Coupled Circuits both having Damping.—The case of two inductively coupled oscillation circuits both having damping presents somewhat greater analytical difficulties in its discussion. It has been handled with ability by several writers. We shall first follow in outline the method employed by V. Bjerknes in dealing with this problem.[35] We assume that there are two circuits, both having capacity, inductance, and resistance, which are inductively connected. Let us suppose that oscillations are excited in one circuit by means of a spark gap as usual, and that these set up other oscillations in the adjacent secondary circuit. The problem is to predetermine the secondary current and the decrements and their relation to the constants of the two circuits. Let suffixes 1 and 2 refer to the primary and secondary circuits, let C, L, and R denote the capacity, inductance, and resistance of the circuits, α the damping factor, and δ the decrement, and p 2π times the frequency n. We have first to construct the differential equation expressing the instantaneous terminal potential difference of the condenser in the secondary circuit. Let v_2 be this potential at any time t. Then as in the previous section, we have as the equation of potential difference between the terminal of the secondary circuit condenser the expression—

$$\frac{d^2v_2}{dt^2}+\frac{R_2}{L_2}\cdot\frac{dv_2}{dt}+\frac{1}{C_2L_2}v_2=\frac{E}{C_2L_2}\epsilon^{-\alpha_1 t}\cos p_1 t \quad . \quad . \quad . \quad (116)$$

We assume, for the sake of avoiding purely analytical difficulties, that the impressed electromotive force in the secondary circuit has its maximum value E when $t=0$, and that at this instant the oscillations in the secondary circuit begin, so that $v_2=0$, and $\frac{dv_2}{dt}=0$ when $t=0$. Writing $2\alpha_2$ for $\frac{R_2}{L_2}$, and $p_2^2+\alpha_2^2$ for $\frac{1}{C_2L_2}$

[35] See V. Bjerknes, *Wied. Ann.*, 1895, vol. 55, p. 121; also *Ibid.*, 1891, vol. 44, p. 74.

as before, we have, as the expression for the potential difference of the terminals of the condenser in the secondary circuit, the equation—

$$\frac{d^2v_2}{dt^2}+2a_2\frac{dv_2}{dt}+(p_2^2+a_2^2)v_2=\frac{E}{L_2C_2}\epsilon^{-a_1t}\cos p_1t \quad . \quad . \quad . \quad (117)$$

The above expression indicates that the motion of electricity in the secondary

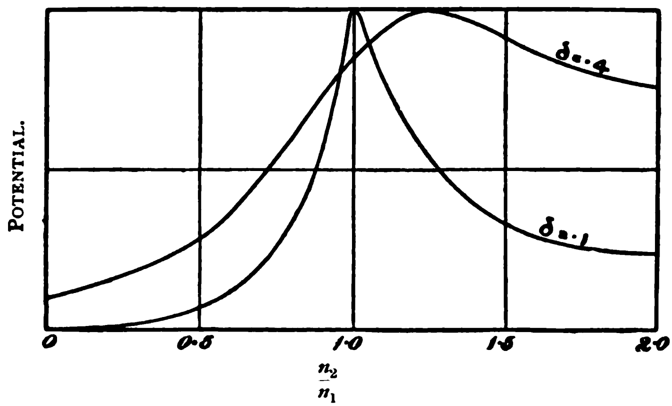


FIG. 27.—Resonance Curves Plotted in Terms of Potential for Strongly Coupled Circuits.

circuit is due to a damped inducing oscillation in the primary circuit with period $\frac{2\pi}{p_1}$ and damping factor a_1.

To solve (117), differentiate all through twice with respect to time; multiply the original (117) by $(p_1^2+a_1^2)$, the first differential by $2a_1$; and add the results to

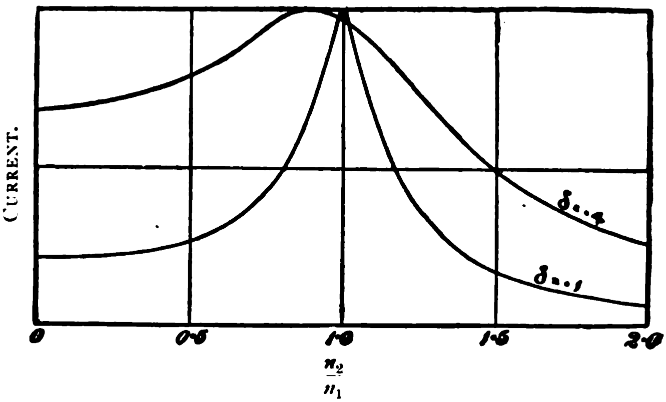


FIG. 28.—Resonance Curves Plotted in Terms of Current for Strongly Coupled Circuits.

the second differential equation. This eliminates the term $\epsilon^{-a_1t}\cos p_1t$, and gives us a differential equation of the 4th order, viz.—

$$\frac{d^4v_2}{dt^4}+2(a_1+a_2)\frac{d^3v_2}{dt^3}+\left[(p_1^2+a_1^2)+(p_2^2+a_2^2)+4a_1a_2\right]\frac{d^2v_2}{dt^2}$$
$$+\left[2a_2(p_1^2+a_1^2)+2a_1(p_2^2+a_2^2)\right]\frac{dv_2}{dt}+\left[(p_2^2+a_2^2)(p_1^2+a_1^2)\right]v_2=0 \quad . \quad (118)$$

Replacing the differential coefficients by m^4, m^3, m^2, and m respectively, we have as the auxiliary equation a biquadratic in m.

The solution of (117) is found by taking the roots of the auxiliary biquadratic having the same coefficients term for term. These roots are easily seen to be—

$$-a_1 \pm \sqrt{-1}p_1 \text{ and } -a_2 \pm \sqrt{-1}p_2$$

Hence the solution of (117) is in the form—

$$v_2 = V_1\epsilon^{-a_1t}\sin(p_1t+\theta_1) + V_2\epsilon^{-a_1t}\sin(p_2t+\theta_2) \quad . \quad . \quad . \quad (119)$$

This indicates that there are *two superimposed oscillations created in the secondary circuit.*

(i.) A *forced oscillation* of maximum amplitude V_1, having the same frequency and damping as the primary current.

(ii.) A *free oscillation* of maximum amplitude V_2, having the natural frequency and damping of the secondary circuit.

To find the values of the amplitudes V_1 and V_2, and the phase angles θ_1 and θ_2, we proceed as follows :—

Differentiate the solution (119) for v_2, and substitute the values of v_2 and $\dfrac{dv_2}{dt}$ found from (119) in the original equation (117). We obtain the expression—

$$\frac{E}{L_2C_2}\epsilon^{-a_1t}\cos p_1t = V_1[p_2^2 - p_1^2 + (a_2-a_1)^2][\epsilon^{-a_1t}\sin(p_1t+\theta_1)$$
$$+ V_1[2p_1(a_2-a_1)]\epsilon^{-a_1t}\cos(p_1t+\theta_1) \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Bearing in mind that $\dfrac{1}{L_2C_2} = p_2^2 + a_2^2$, and that $\sqrt{A^2+B^2}\cos pt = A\cos(pt+\theta) + B\sin(pt+\theta)$, provided that $\tan\theta = \dfrac{B}{A}$, it follows at once that—

$$V_1 = \frac{p_2^2 + a_2^2}{\sqrt{[p_2^2 - p_1^2 + (a_2-a_1)^2]^2 + 4p_1^2(a_2-a_1)^2}} \cdot E \quad . \quad . \quad . \quad . \quad (121)$$

and $$\tan\theta_1 = \frac{p_2^2 - p_1^2 + (a_2-a_1)^2}{2p_1(a_2-a_1)}$$

The above expressions give us the maximum amplitude and phase of the *forced* oscillation in the secondary circuit.

To find the same constant for the free oscillation, we must take equation (119), and put $t=0$ and $v=0$; and also differentiate (119), and put $t=0$ and $\dfrac{dv}{dt}=0$. We then have—

$$\left.\begin{aligned} V_1\sin\theta_1 + V_2\sin\theta_2 &= 0 \\ -V_1a_1\sin\theta_1 + V_1p_1\cos\theta_1 - V_2a_2\sin\theta_2 + V_2p_2\cos\theta_2 &= 0 \\ \text{or} \quad V_2p_2\sin\theta_2 &= -V_1p_2\sin\theta_1 \\ \text{and} \quad V_2p_2\cos\theta_2 &= -V_1[(a_2-a_1)\sin\theta_1 + p_1\cos\theta_1] \end{aligned}\right\} \quad . \quad . \quad (122)$$

Squaring and adding, we obtain—

$$V_2^2p_2^2 = (a_2-a_1)^2V_1^2\sin^2\theta_1 + p_2^2V_1^2\sin^2\theta_1 + p_1^2V_1^2\cos^2\theta_1 + 2p_1\overline{a_2-a_1}V_1^2\sin\theta_1\cos\theta_1 \quad (123)$$

and having regard to the value of $\tan\theta_1$ given in (121), we find that (123) reduces to—

$$V_2^2p_2^2 = V_1^2[p_2^2 + (a_2-a_1)^2] \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

Hence it follows that—

$$\frac{V_2}{V_1} = \frac{\sqrt{p_2^2+(a_2-a_1)^2}}{p_2}$$

and that $$V_2 = \frac{(p_2^2+a_2^2)\sqrt{p_2^2+(a_2-a_1)^2}}{p^2\sqrt{[p_2^2-p_1^2+(a_2-a_1)^2]^2+4p_1^2(a_2-a_1)^2}} \cdot E \quad . \quad . \quad . \quad . \quad (125)$$

and $$\tan\theta_2 = \frac{p^2}{a_2-a_1} \cdot \frac{p_2^2-p_1^2+(a_2-a_1)^2}{p_2^2+p_1^2+(a_2-a_2)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

The above expressions enable us to define the secondary current precisely.

Each potential oscillation, forced and free, acts to produce its own current in the secondary circuit, and if we call I_1 and I_2 the maximum values of the forced and free secondary currents, we have these related to the potential maxima as follows:—

$$I_1 = C_2 V_1 p_1$$
$$I_2 = C_2 V_2 p_2$$

Accordingly, we have the following expressions for the amplitude of the currents:—

$$\left.\begin{array}{c}\text{Forced current}\\ \text{maximum}\\ \text{amplitude}\end{array}\right\} = I_1 = \frac{p_1 E}{L_2\sqrt{[p_2^2 - p_1^2 + (a_2 - a_1)^2]^2 + 4p_1^2(a_2 - a_1)^2}} \quad . \quad (127)$$

$$\left.\begin{array}{c}\text{Free current}\\ \text{maximum}\\ \text{amplitude}\end{array}\right\} = I_2 = \frac{\sqrt{p_2^2 + (a_2 - a_1)^2}.E}{L_2\sqrt{[p_2^2 - p_1^2 + (a_2 - a_1)^2]^2 + 4p_1^2(a_2 - a_1)^2}} \quad . \quad (128)$$

Hence the actual current in the secondary circuit is the resultant of two damped oscillations, differing in phase and frequency. The further discussion of the problem is facilitated by adopting a procedure due to V. Bjerknes.[36]

He assumes that we may consider the secondary current as a single current of variable amplitude expressed as a function of the time, of the form—

$$i = C_2\left(\frac{p_1 + p_2}{2}\right) M \cos(mt + m') \quad . \quad . \quad . \quad . \quad . \quad (129)$$

where M is a function of the time and of the damping factors and other circuit constants.

$$\text{Let } m = \frac{p_1 + p_2}{2}, \quad n = \frac{p_1 - p_2}{2}, \quad \mu = \frac{a_1 + a_2}{2}, \quad \nu = \frac{a_1 - a_2}{2}.$$

Let the original differential equation for the potential difference of the terminals of the secondary condenser be—

$$\frac{d^2v}{dt_2} + 2a_2\frac{dv}{dt} + (p_2^2 + a_2^2)v = \frac{E}{L_2C_2}\epsilon^{-a_1 t}\sin(p_1 t + \phi) \quad . \quad . \quad . \quad (130)$$

Bjerknes shows that the solution of the above equation can be given in the form—

$$v = M\sin(mt + m')$$

where—

$$M^2 = \frac{E^2}{16L_2^2C_2^2m^2(n^2 + \nu^2)}\left(P_1 + 2.\frac{1 + \cos 2\phi}{m}P_2 + 2.\frac{\sin 2\phi}{m}P_3\right) \quad . \quad . \quad (131)$$

and
$$\left.\begin{array}{l}P_1 = \epsilon^{-2\mu t}(\epsilon^{-2\nu t} + \epsilon^{2\nu t} - 2\cos nt\\ P_2 = \epsilon^{-2\mu t}(n\epsilon^{2\nu t} - n\cos 2nt - \nu\sin 2nt\\ P_3 = \epsilon^{-2\mu t}(\nu\epsilon^{2\nu t} - \nu\cos 2nt + n\sin 2nt\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (132)$$

Bjerknes then discusses various cases, and delineates curves showing the variation of M with time.

1st case. Let the primary and secondary circuits have the same periodic time and damping, viz. $p_1 = p_2$ and $a_1 = a_2$. Then we have—

$$M = \pm\frac{E}{2L_2C_2m}.t.\epsilon^{-\mu t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

The graph of this equation is shown in Fig. 29 (A). In this case the amplitude of the oscillations first increases and then slowly falls away again.

2nd case. Let the two circuits have equal periodic times but unequal damping. Then—

$$M = \pm\frac{E}{4L_2C_2m\nu}\epsilon^{-\mu t}(\epsilon^{-\nu t} - \epsilon^{-\nu t}) \quad . \quad . \quad . \quad . \quad . \quad (134)$$

[36] See V. Bjerknes, "On Electrical Resonance," *Wied. Ann.*, 1895, vol. 55, p. 121.

The graph of this equation is given in Fig. 29 (B) for logarithmic decrements $\delta_1 = 0{\cdot}4$, $\delta_2 = 0{\cdot}04$.

3rd case. Let the damping of the two circuits be the same, but the frequencies different. Then we have—

$$M = \pm \frac{E}{2L_2C_2mn} \epsilon^{-\mu t} \sin nt \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

The graph of this equation is shown in Fig. 29 (D), and it presents us with that periodic waxing and waning which is known in acoustics as the phenomenon of *beats*.

4th case. Let the damping and frequency of the two circuits be different, then the value of M is given in (131), and the graph will vary according to the relative values of the constants, but two cases are shown in Fig. 29 (E and F). Bjerknes

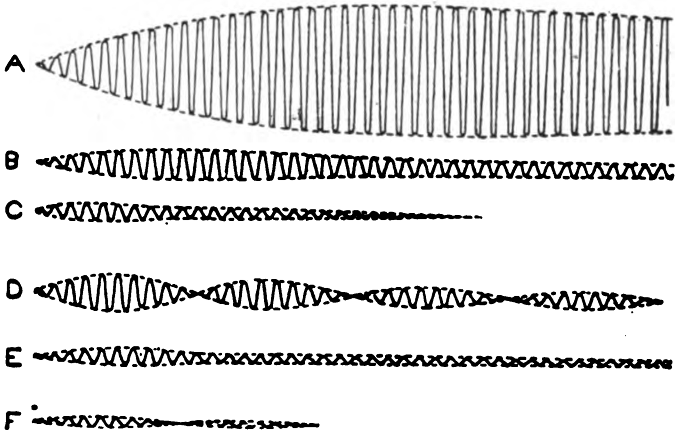


FIG. 29.—Bjerknes' Curves representing Various Types of Possible Secondary Oscillations.

then passes on to show how the *integral value* or *mean-square value* of the resultant secondary current can be calculated.

If we denote the mean-square value of the potential difference of the terminals of the secondary circuit condenser by U, and the corresponding value of the current by J, then U is defined by the equation—

$$U^2 = \int_0^\infty v^2 dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

since, owing to the frequency of the oscillations, a time of 1 second may be considered to be *infinite* as far as the decay of oscillations is concerned. Then, since $v = M \sin (mt + m')$—

$$v^2 = \frac{M^2}{2} - \frac{M^2}{2} \cos 2\,(mt + m') \quad . \quad . \quad . \quad . \quad . \quad (137)$$

In taking the integral, that part due to the cosine term of the above equation is zero, and hence—

$$U^2 = \tfrac{1}{2} \int_0^\infty M^2 dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

Also the mean-square value of the current is given by—

$$J^2 = C_2^2 m^2 U^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

where C_2 is the capacity of the secondary condenser. Hence—

$$J^2 = \tfrac{1}{2} C_2^2 m^2 \int_0^\infty M^2 dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (140)$$

We must refer the reader to Bjerknes' paper (*loc. cit.*) for the steps of the reasoning by which he finally deduces an important equation for the mean-square value of the secondary current, which in our notation is—

$$J^2 = \frac{E^2}{16 L_2^2} \cdot \frac{a_1 + a_2}{a_1 a_2} \cdot \frac{1}{(p_1 - p_2)^2 + (a_1 + a_2)^2} \quad . \quad . \quad . \quad . \quad . \quad (141)$$

Since—

$$i = C_2 \frac{dv}{dt}$$

we have—

$$J^2 = \int_0^\infty i^2 dt = C_2^2 \int_0^\infty \left(\frac{dv}{dt}\right)^2 dt$$

If we substitute the values of V_1 and V_2 given in (120) and (125) in the equation (119), and then differentiate with regard to t, square and integrate with regard to t, and neglect powers of $(p_1 - p_2)$ and $(a_1 - a_2)$, we reach, after some troublesome reductions, the above equation. (See also "La Télégraphie sans fil," by MM. J. Boulanger and G. Ferrié, 7th ed., p. 165.)

This equation gives us the value of the current which would be read on a hot-wire ammeter of suitable type inserted in the secondary current.

It shows us that J increases as p_1 and p_2 or the frequencies of the two circuits become more nearly equal. Let us denote by J_r the value of the secondary current when $p_1 = p_2$, and call J_r the *resonance current*; then—

$$J_r^2 = \frac{E^2}{16 L_2^2} \cdot \frac{1}{a_1 a_2 (a_1 + a_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

Accordingly, the ratio of J^2 to J_r^2 is given by—

$$\frac{J^2}{J_r^2} = \frac{(a_1 + a_2)^2}{(p_1 - p_2)^2 + (a_1 + a_2)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (143)$$

Hence

$$\frac{J_r^2 - J^2}{J^2} = \frac{(p_1 - p_2)^2}{(a_1 + a_2)^2}$$

or

$$(a_1 + a_2) = (p_1 - p_2) \frac{J}{\sqrt{J_r^2 - J^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

If δ_1 and δ_2 are the logarithmic decrements of the two circuits, then $a_1 = n_1 \delta_1$, and $a_2 = n_2 \delta_2$. Hence if we insert these values of a_1 and a_2 in (144), and assume that *the frequencies of the two circuits n_1 and n_2 are nearly the same*, we can write (144) in the form—

$$\delta_1 + \delta_2 = 2\pi \left(1 - \frac{n_2}{n_1}\right) \frac{J}{\sqrt{J_r^2 - J^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

This useful equation gives us the means of determining the sum of the decrements of the two circuits when we have a resonance curve plotted showing the variation of $\frac{J^2}{J_r^2}$ with $\frac{n_2}{n_1}$. Thus, suppose we insert a suitable hot-wire ammeter in the secondary circuit and vary the inductance of that circuit so as to change its natural time period n_2, and if we know n_1 we can plot a curve, as in Fig. 26, called a resonance curve, in which the ordinates represent the values of $\frac{J^2}{J_r^2}$ and the abscissæ denote the fraction $\frac{n_2}{n_1}$. This curve has a maximum ordinate equal to unity, and a corresponding abscissa also equal to unity. Draw any other

ordinate *near to* the maximum and let x denote $1-\frac{n_2}{n_1}$ and y denote $\frac{J^2}{J_r^2}$. Then from (145) we have—

$$\delta_1+\delta_2=2\pi x\sqrt{\frac{y}{1-y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

and the measurement of x and y enables us to find $\delta_1+\delta_2$. If, then, we can calculate one decrement from other data, we have the second decrement from this last equation.

This is the equation which was applied to determine the decrement of an oscillatory circuit having a spark gap in it in the researches of P. Drude, to which reference has been already made in § 5.

It is sometimes convenient to employ equation (146) in the form—

$$y=\frac{J^2}{J_r^2}=\frac{1}{1+\frac{4\pi^2x^2}{(\delta_1+\delta_2)^2}}$$

This equation holds good only when $x=1-\frac{n_2}{n_1}$ is small, and when δ_1 and δ_2 are also small compared with 2π.

In the paper by Bjerknes, above mentioned (*Wied. Ann.*, vol. 55, 1895, p. 145), he proceeds to show how we can transform the equation (143) for the ratio of the mean-square values of the secondary current J^2 to the resonance current J_r^2 into another form. Let ω stand for the sum of the decrements $\delta_1+\delta_2$ per semi-period, or for the mean value of the decrements of the primary and secondary circuits, and let T_1 be the time period of the oscillator or primary circuit, and T_2 that of the secondary circuit or resonator, then Bjerknes proves that—

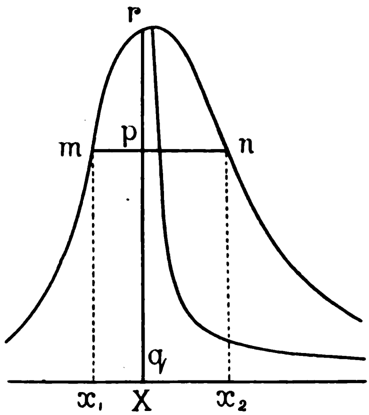


FIG. 30.

$$\frac{J^2}{J_r^2}=\frac{\omega^2T_2^2+\pi^2S(T_1-T_2)}{\omega^2T_2^2+\pi^2(T_1-T_2)^2}$$

where S is a certain parameter, the meaning of which will appear presently.

If we put y for J^2, Y for J_r^2, X for T_2, and x for T_1, the equation (141) transforms into—

$$\frac{y}{Y}=\frac{\omega^2X^2+\pi^2S(x-X)}{\omega^2X^2+\pi^2(x-X)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

The above equation is the equation of the resonance curve. It can easily be thrown into the form—

$$x^2y-Axy-Bx+Cy+D=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

which is the expression for a curve of the third degree.

If the expression (147) is rearranged in terms of $(x-X)$ and $(y-Y)$, it can be put into the form—

$$\pi^2y(x-X)^2-\pi^2SY(x-X)+\omega^2X^2(y-Y)=0 \quad . \quad . \quad . \quad . \quad (149)$$

The point $x=X$ and $y=Y$ is then a point on the curve. Bjerknes calls this the *point of isochronism*. It is a point near to the maximum value, but not coincident with it.

If we put $y=Y$ in (147), it gives us $(x-X)=S$, which shows that S represents the length of a chord through the point of isochronism.

If the curve represented by (147) or (148) is delineated (see Fig. 30), and if we

draw a chord across it parallel to the axis of x, and call x_1 and x_2 the abscissæ of these intersections, we have $(x_1 - X)$ and $(x_2 - X)$ as roots of the equation (149), and therefore by the theory of equations we have—

$$(x_1 - X) + (x_2 - X) = \frac{Y}{y}$$

$$(x_1 - X)(x_2 - X) = \frac{\omega^2 X^2 (y - Y)}{\pi^2 y}$$

The quantity—

$$\tfrac{1}{2}[(x_1 - X) + (x_2 - X)]$$

or $\frac{x_1 + x_2}{2} - X$ represents the distance between the middle point of this chord and the ordinate of the point of isochronism. If we call this distance z, then we have—

$$yz = \frac{SY}{2} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

This shows that *the locus of the middle points of all the chords of the resonance curve drawn parallel to the axis of x* is an *equilateral hyperbola.*

The asymptotes of this hyperbola are the axis of x and the ordinate of the point of isochronism.

If, then, we draw the resonance curve and bisect all the chords, we can describe the hyperbola and find its asymptote, and hence the point $x = X$, $y = Y$.

If we draw any chord mn near the top (see Fig. 30), it cuts this asymptote, and is cut by it into two sections, and these four lengths are—

$$\left.\begin{aligned} mp &= x_1 - X = a \\ np &= x_2 - X = b \\ qp &= \quad y \quad = c \\ rp &= Y - y = d \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

If we then insert the values of $x_1 - X$, $x_2 - X$, y and $Y - y$ given in (151) in equation (149), we arrive at the equation—

$$\omega^2 = \frac{abc}{d}\frac{\pi^2}{X^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (152)$$

$$\text{or} \quad \omega = \frac{2\pi}{X}\sqrt{\frac{abc}{d}} = \delta_1 + \delta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (153)$$

Hence the sum of the decrements of the oscillator and resonator, or of the two coupled circuits, can be obtained from the resonance curve by drawing the equilateral hyperbola, which is the locus of the middle points of all its chords, and finding its vertical asymptote.

We shall apply this theorem, in a later chapter, to the determination of the numerical values of the decrement in certain cases.

A very masterly discussion of the problem of the inductive transformation of electric oscillations has also been given by Professor P. Drude in a well-known memoir.[37] He discusses the theory of an oscillation transformer or Tesla coil, consisting of a secondary circuit wound on a cylinder of length h and diameter d, in one or more layers of wire, and embraced by a primary circuit of one or a few turns of wire, the primary circuit being a circle having its centre on the axes of the secondary circuit.

He takes L_{11} to denote the self-inductance of the primary, and L_{22} that of the secondary, and L_{12} to denote the mutual inductance of the secondary on the primary, and L_{21} that of the primary circuit on the secondary. In the case of two simple linear circuits with currents equal in all parts of the circuit, we should have $L_{12} = L_{21} = M$, or the mutual inductance of the two circuits. In the case of

[37] P. Drude, "Über induktiv Erregung zweier Elektrische Schwingungskreise mit Anwendung auf Perioden und Dampfungsmessung Tesla transformatoren und Drahtlose Telegraphie," *Ann. der Physik*, 1904, vol. 13, p. 512.

such an oscillation transformer as is here discussed, L_{21} is always greater than L_{12} in the ratio—

$$L_{21} : L_{12} = 1 : \sin \pi\frac{a}{h}$$

where a is some quantity less than the length h of the secondary spool.

The reason for this difference is that the total flux of induction produced by the actual current of unit strength in the centre coil of the secondary circuit is less than that which would be produced by a current having the same value, viz. unit strength in all parts of the secondary coil, because the actual current in the secondary coil is greatest in the centre of the wire and zero at the terminals or open ends. Drude then defines the coefficient of coupling k by the expression—

$$k^2 = \frac{L_{12} \cdot L_{21}}{L_{11} \cdot L_{22}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (154)$$

If, then, v_1 is the potential difference of the primary condenser terminals at any instant, and v_2 that of the secondary terminals at the same instant, Drude establishes two equations which in our notation are as follows :—

$$L_{11}C_1\frac{d^2v_1}{dt^2} - L_{12}C_2\frac{d^2v_2}{dt^2} + R_1C_1\frac{dv_1}{dt} + v_1 = 0 \quad . \quad . \quad . \quad . \quad (155)$$

$$L_{22}C_2\frac{d^2v_2}{dt^2} - L_{21}C_1\frac{d^2v_1}{dt} + R_2C_2\frac{dv_2}{dt} + v_2 = 0 \quad . \quad . \quad . \quad . \quad (156)$$

and obtains solutions for these in the form—

$$v_1 = A\epsilon^{x_1t} + A_2\epsilon^{x_2t} + A_3\epsilon^{x_3t} + A_4\epsilon^{x_4t} \quad . \quad . \quad . \quad . \quad . \quad (157)$$

$$v_2 = B\epsilon^{x_1t} + B_2\epsilon^{x_2t} + B_3\epsilon^{x_3t} + B_4\epsilon^{x_4t} \quad . \quad . \quad . \quad . \quad . \quad (158)$$

and finally, by a long course of reasoning, he proves that if the circuits are adjusted to resonance, so that $L_{11}C_1 = L_{22}C_2$, we have the value of the secondary terminal potential difference given by the expression—

$$v_2 = \frac{\rho}{2}V_1\sqrt{\frac{C_1}{C_2} \cdot \frac{L_{21}}{L_{12}} \cdot \frac{k^2}{k^2 - \left(\frac{\delta_1 - \delta_2}{2\pi}\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad (159)$$

Where V_1 is the maximum value of the primary condenser potential difference, and ρ is a function of the sum $(\delta_1 + \delta_2)$ of the two logarithmic decrements, δ_1 and δ_2 of the primary and secondary circuits when separate, and also of the coefficient of coupling, k. The function expressing ρ is of the form—

$$\rho = \epsilon^{At}\cos Pt - \epsilon^{Bt}\cos Qt \quad . \quad . \quad . \quad . \quad . \quad . \quad (160)$$

where A, B, P, and Q are functions of $\delta_1 + \delta_2$, k and the frequencies of the two circuits. Hence v_2 has its maximum value corresponding to the maximum value of ρ. Drude gives a series of curves, reproduced in Fig. 31, which delineate the form of the function expressing ρ in terms of k for certain values of $\delta_1 + \delta_2$ between 0·15 and 1·00. It is seen that these curves all have a maximum ordinate corresponding to a coefficient of coupling k near to 0·6, and also that for the value $k = 1$ the value of ρ is for all curves 0·5. If δ_1 and δ_2 are both zero, then ρ has the value unity for all values of k. Hence, if we denote the maximum value of ρ by $\bar{\rho}$ for any value of $\delta_1 + \delta_2$, we can express the maximum value of the secondary terminal potential difference V_2 by the equation—

$$V_2 = \frac{\bar{\rho}}{2}V_1\sqrt{\frac{C_1}{C_2} \cdot \frac{L_{21}}{L_{12}} \cdot \frac{k^2}{k^2 - \left(\frac{\delta_1 + \delta_2}{2\pi}\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad (161)$$

If, then, $\delta_1 + \delta_2 = 0$, we have $\bar{\rho} = 1$ and—

$$V_2 = \frac{V_1}{2}\sqrt{\frac{C_1}{C_2}} \cdot \sqrt{\frac{L_{21}}{L_{12}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (162)$$

This expression for the ratio of $\frac{V_2}{V_1}$ for the undamped oscillations becomes identical with that given by Oberbeck if $L_{21} = L_{12}$. From the curves given in Fig. 31 we can calculate the ratio of the maximum secondary terminal potential difference V_2 to the primary condenser terminal potential difference for any assumed value of k and for values $\delta_1 + \delta_2$ corresponding to the curves given.

Thus, suppose the decrements of the circuits are such that $\frac{1}{2}(\delta_1 + \delta_2) = 0{\cdot}15$, and that the coupling is such that $k = 0{\cdot}6$. We see then from the curves that $\rho = 0{\cdot}87$, and since $\left(\frac{\delta_1 + \delta_2}{2\pi}\right)^2 = \frac{1}{441}$, we can say that the secondary terminal potential

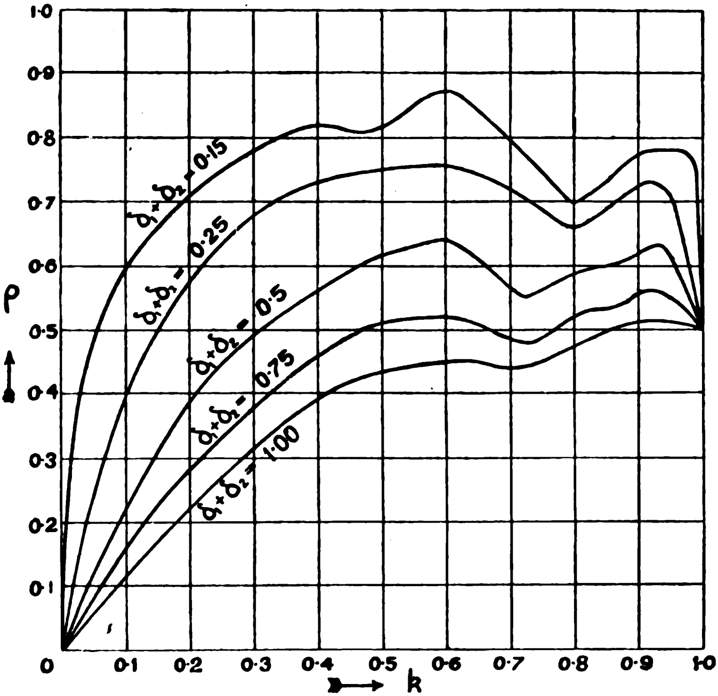


FIG. 31.—Drude's Curves.

Note.—In the above diagram δ_1 and δ_2 are the semi-period decrements.

difference is 87 per cent. of that which it would be if the circuits were undamped, and the same primary charging voltage employed.

In the course of his analysis Drude establishes an equation for the mean-square value of the secondary current J^2, which, when expressed in our notation, is as follows :—

$$J^2 = \frac{V_1^2}{16} \cdot \frac{L_{21}^2}{L_{11}^2 \cdot L_{22}^2} \cdot \frac{a_1 + a_2}{a_1 a_2} \cdot \frac{1}{(p_1 - p_2)^2 + (a_1 + a_2)^2} \quad . \quad . \quad . \quad (163)$$

where a_1 and a_2 are the damping factors of the two circuits. This equation had already been obtained by Bjerknes (see § 14 of this chapter). If we bear in mind that the maximum value of the current in the primary circuit $I_1 = \frac{V_1}{L_{11} p_1}$, and that the maximum value of the electromotive force created in the secondary circuit by

the primary current is $L_{21}I_1p_1$, we see that $\frac{V_1L_{21}}{L_{11}}$ is the same quantity as that denoted by E in the expression of Bjerknes (see equation 142 of this chapter), and that the expressions therefore given for J^2 by Drude (163) and Bjerknes (141) agree with one another.

It follows, therefore, that the maximum value of the mean-square secondary current in an oscillation transformer is given by the expression—

$$J^2_{max}=\frac{V_1^2}{16}\cdot\frac{L_{21}^2}{L_{11}^2\cdot L_{22}^2}\cdot\frac{1}{\alpha_1\alpha_2(\alpha_1+\alpha_2)} \quad . \quad . \quad . \quad . \quad (164)$$

or if we put for α_1 and α_2 their values in terms of the decrements and common frequency n, we have $\alpha_1=n\delta_1$ and $\alpha_2=n\delta_2$. Hence—

$$J^2_{max}=\frac{V_1^2}{16}\cdot\frac{L_{21}^2}{L_{11}^2\cdot L_{22}^2\cdot n^3}\cdot\frac{1}{\delta_1\delta_2(\delta_1+\delta_2)} \quad . \quad . \quad . \quad (165)$$

If the coupling is such that $L_{12}\cdot L_{21}=L_{21}^2=k^2L_{11}L_{22}$, we have, since $C_1L_{11}=C_2L_{22}=\frac{1}{4\pi^2n^2}$:

$$J^2_{max}=V_1^2C_1C_2\cdot\frac{n\pi^4k^2}{\delta_1\delta_2(\delta_1+\delta_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (166)$$

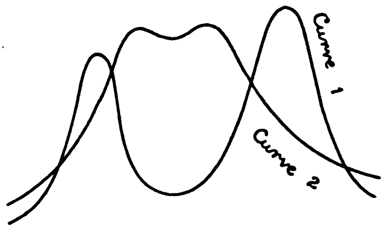


FIG. 32.—Double-Humped Resonance Curves for Coupled Circuits.

The above formula is very convenient for calculation, and shows us, amongst other things, the importance of securing a small decrement for the primary or condenser circuit if the mean-square value of the secondary current is to be large. We shall find this formula of use to us in calculating the current in the antenna of a wireless telegraph transmitter plant. It should be noted that both Drude and Bjerknes assume only *one* train of oscillations per second in their formulæ. Hence if there are N trains per second the values given for J^2 must be multiplied by N.

If we take the resonance curve in the above manner for a pair of coupled circuits in resonance, then, as proved above, we shall have oscillations of two frequencies set up in each circuit, and therefore find two frequencies, corresponding to which there will be maximum currents in the tertiary or cymometer circuit. Hence the resonance curve, when described, will be found to be a curve with double hump, as in Fig. 32. If these humps are fully separated, which implies that the frequencies of the two oscillations lie far apart, then we may apply the above-mentioned method of determining the decrement to each hump separately. This is the case when the coupling is very close.

If, however, the coupling is loose, then the humps are not widely separated, as in Curve 1, Fig. 32, but tend to run into one hump with a depression on its summit, as in Curve 2, Fig. 32. It is then impossible to apply the above method of determining the decrement to each hump separately. C. Fischer has, however, described a method, due to J. Zenneck, by which the two nearly superimposed

waves can be examined separately.[38] If I. and II. in Fig. 33 represent the coupled oscillatory circuits and M the measuring circuit, then loops 1 and 2 are formed in the circuits I. and II., which are coupled inductively with two loops 3 and 4 placed in the measuring circuit. If loops 1 and 2 are placed so far apart that they do not sensibly act on each other, and if 4 is coupled with 2, then there will be two waves set up in the measuring circuit. If the measuring circuit is brought into a condition of resonance with one wave by altering its capacity, then on bringing the loop 3 near to loop 1 a minimum reading of the ammeter in the measuring circuit will be found, so that any approach or removal of 3 to or from 1 increases the current. If 3 is kept in this position the resonance curve obtained by continuously varying the capacity and inductance of M shows only one maximum, which is that due to the wave of one frequency. By reversing the connections of loop 1 and slightly shifting 3 the resonance curve of the other wave can be drawn. The principle which lies at the base of this method is that the oscillatory circuits I. and II. are each the seat of a current, which may be considered to be the sum of two damped oscillations differing in amplitude,

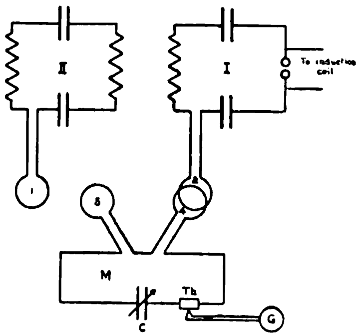


FIG. 33.

damping, and frequency. Thus the solution of the differential equation (118) of the fourth order is of the form—

$$i_1 = A_1\epsilon^{-\delta t}\sin pt + B_1\epsilon^{-\gamma t}\sin qt \quad . \quad . \quad . \quad . \quad . \quad (167)$$

and for the secondary circuit we have—

$$i_2 = A_2\epsilon^{-\delta t}\sin pt + B_2\epsilon^{-\gamma t}\sin qt \quad . \quad . \quad . \quad . \quad . \quad (168)$$

and hence for the resultant magnetic field in the neighbourhood of these circuits we have—

$$M = (a_1A_1 + a_2A_2)\epsilon^{-\delta t}\sin pt + (a_1B_1 + a_2B_2)\epsilon^{-\gamma t}\sin qt \quad . \quad . \quad (169)$$

where a_1 and a_2 are some constants depending on the locality. Hence if we couple the circuits I. and II. independently to the measuring circuit, but in different degrees, and bear in mind that the current in circuit I. is also exactly opposite in phase to that in II., we can partly neutralize the effect of one of the oscillations in II. in the measuring circuit by the action of the same oscillation in I., and thus leave the measuring circuit only influenced by one of the oscillations existing in the circuit II. C. Fischer gives in his paper (*loc. cit.*) examples of very loosely, moderately loosely, and tightly coupled circuits, and delineates the resultant resonance curves, and determines by this means the decrements of the two oscillations. He gives the following values for certain oscillation circuits of the decrements of the two oscillations for loose, fairly loose, and tight coupling determined in this manner :—

[38] See C. Fischer, "A Method of Examining Separately the Two Waves on Coupled Oscillators," *Ann. der Physik* vol. 19, p. 182, 1906, or *Science Abstracts*, vol. ix. A, 1906, *abs.* No. 573.

Coupling.	Decrement δ.	Decrement γ.
Loose = 3 per cent.	0·08 (0·07)	0·09 (0·056)
Moderate = 7 per cent.	0·057 (0·063)	0·068 (0·075)
Tight = 39 per cent.	0·076 (0·08)	0·09 (0·09)

The values put in brackets denote the decrements which were obtained by applying the Bjerknes method directly to each hump as if it were a separate resonance curve. The differences show that for values of the coupling below, say, 10 per cent., the two methods give different values. In the same manner there is a concordance between the values of the coefficient of coupling obtained from

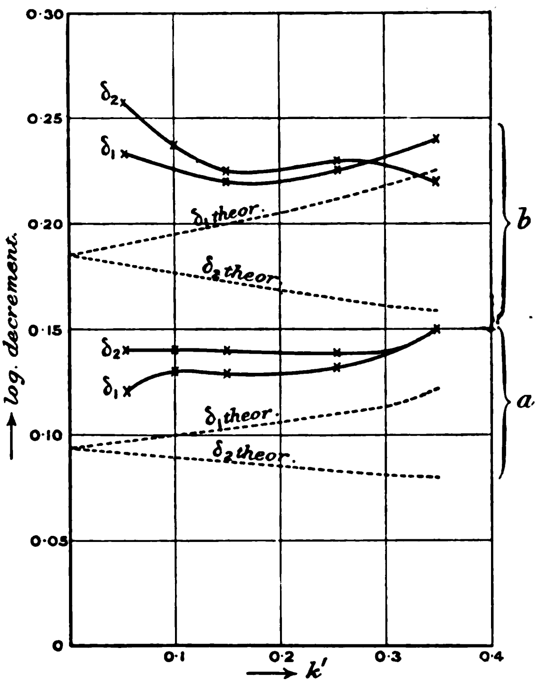


FIG. 34.—Curves representing the Results of C. Fischer's Comparisons of the Observed and Calculated Decrements for Coupled Circuits.

observations with the two humps if these are wide apart, but not if they are close together.

The theory of the damping of the two oscillations in coupled circuits has also been examined by C. Fischer,[39] and the results of experiments by the above method compared with the predictions of the theory given by P. Drude.[40] Fischer found that the frequencies of the two oscillations n_1 and n_2 existing in two coupled circuits, each of natural free frequency N, are given by the equations—

$$\frac{N^2}{n_1^2} = 1 - k, \qquad \frac{N^2}{n_2^2} = 1 + k \quad . \quad . \quad . \quad . \quad . \quad . \qquad (170)$$

and hence that—

$$\frac{1}{n_1^2} + \frac{1}{n_2^2} = \frac{2}{N^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (171)$$

[39] See C. Fischer, "On Coupled Condenser Circuits," *Ann. der Physik*, vol. 22, p. 265, 1907, or *Science Abstracts*, vol. x. A, 1907, *abs.* 702.

[40] See P. Drude, *Ann. der Physik*, vol. 13, p. 528, 1904.

Fischer confirmed the above relation both for large and small damping coefficients. Experiment showed, however, that Drude's equations for the decrements δ_1 and δ_2 of these two oscillations, viz.—

$$\delta_1 = \frac{D_1 + D_2}{2} \frac{n_1}{N} \qquad \delta_2 = \frac{D_1 + D_2}{2} \frac{n_2}{N}$$

where D_1 and D_2 are the decrements of the oscillations in the two coupled circuits when separate, is not even true qualitatively, as the oscillation of greatest frequency has not *always* or even generally the largest decrement. Also the actual decrements found are larger than those predicted by the theory. This has also been noticed by M. Wien.[41]

The results of C. Fischer's observations on this matter are delineated in Figs. 34 and 35, where the dotted lines represent the decrements of a pair of coupled

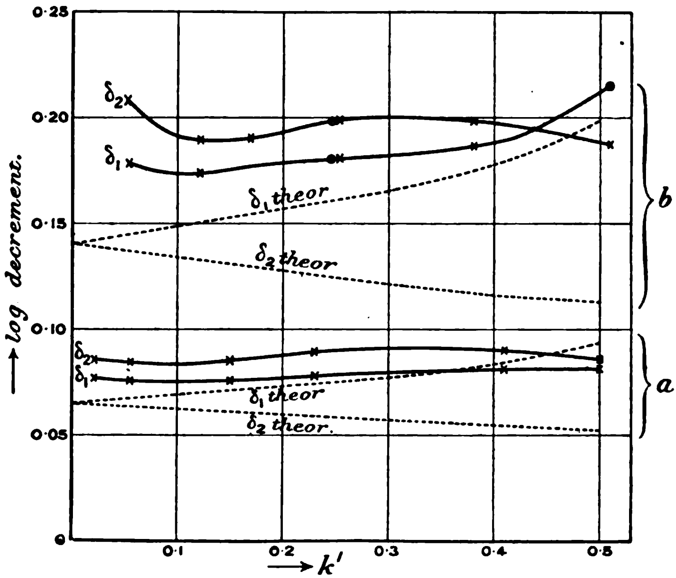


FIG. 35.—Curves representing the Results of C. Fischer's Comparisons of the Observed and Calculated Decrements in Coupled Circuits.

circuits calculated by the formulæ of Drude, and the firm lines the observed decrements for various couplings k'. These curves show the discrepancy between the observed and calculated decrements.

15. Impact Excitation.—In the application of coupled oscillatory circuits in radiotelegraphic apparatus there is a disadvantage in the employment of circuits coupled so closely as to produce a marked reaction between them, principally because the resonance curve is then not sharply peaked as in Fig. 25, but flat topped or double humped as in Fig. 32. We have already mentioned in Chap. I. § 15 the investigations of M. Wien which led to a method of creating in a secondary circuit feebly damped oscillations of a single frequency by the impulsive action of a strongly damped or quenched oscillation in the primary circuit.

We have also explained in § 11 of this chapter the manner in which the reaction of the two circuits produces an effect equivalent to *beats* in both primary and secondary oscillations when the primary oscillations are allowed to continue.

[41] See M. Wien, "The Intensity of the Two Waves in Coupled Transmitters," *Science Abstracts*, vol. ix. A, 1906, *abs.* 2076.

If then we set up in the primary unquenched oscillations, we have in both circuits trains of damped oscillations undergoing periodic variations in amplitude as shown in the two upper curves in Fig. 36. If, however, the primary spark or oscillation is quenched then in the primary circuit we have a few very highly damped oscillations or perhaps a nearly dead-beat discharge. On the other hand, the resulting oscillations set up in the secondary circuit are feebly damped. Also owing to the primary circuit becoming open almost at once by the damping out of the spark there is no reaction between the circuits, and the oscillations set up in the secondary are merely its free natural oscillations. The secondary circuit receives, as it were, an electrical impulse or blow, and vibrates freely, and the oscillations are as represented by the two lower curves in Fig. 36.

Such a discharger is called a quenched spark discharger, and the method is called impact excitation of the secondary.

The particular forms of dischargers used to secure this impact excitation are described in Chap. VII. on Radiotelegraphic Apparatus in Part III. of this book.

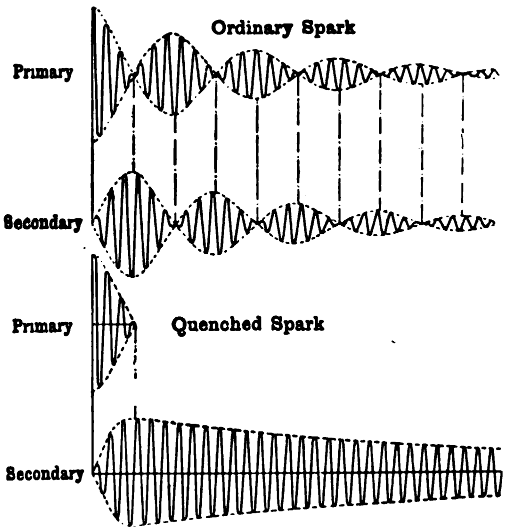


FIG. 36.—Diagram showing the Electrical Beats produced in the Primary and Secondary Circuits when a sustained Primary Spark is used, and the Single Period Oscillations in the Secondary Circuit when a Quenched Spark is employed.

If the spark is not immediately quenched, then in the period of time during which it lasts there is a reaction between the circuits, with the resulting production of a complex oscillation. These facts are well brought out by taking a resonance curve for the secondary circuit with various degrees of coupling (k) both with impact and ordinary spent discharges.[42]

The curves in Figs. 37 and 38 show a series of resonance curves for the two cases taken by the author and Lieut. Dyke with a secondary circuit having a small decrement, viz. 0·01 per half-period, and with degrees of coupling (k) of the circuits varying from 3 to 33 per cent.

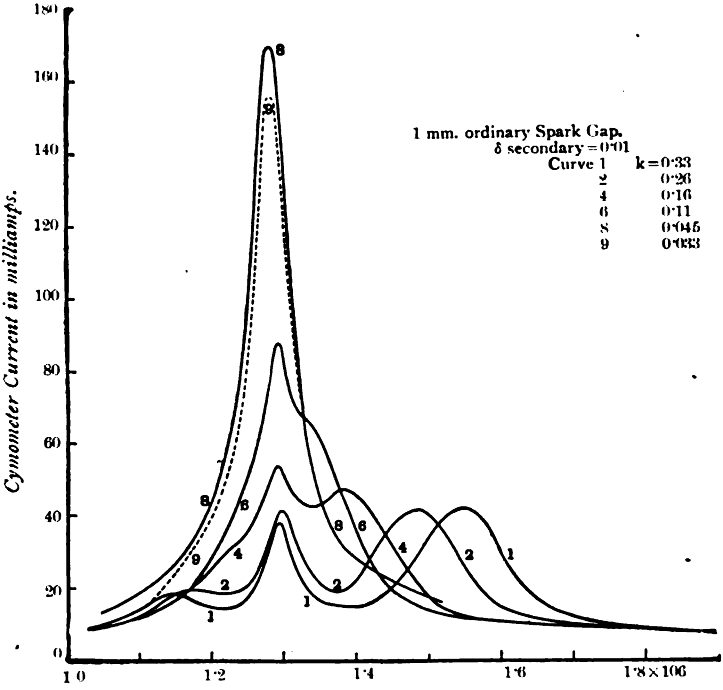
It will be seen that in the case of the impact discharger the coupling can be as close as 30 per cent. before any evidence appears of a double hump of the resonance curve, showing that there are then oscillations of two periods in the secondary circuit. On the other hand, in the case of the ordinary spark balls,

[42] See J. A. Fleming and G. B. Dyke, "Some Resonance Curves taken with Impact and Spark-Ball Dischargers," *Proc. Phys. Soc. Lond.*, vol. 23, Feb. 1911, p. 136,

the coupling can hardly be closer than 10 per cent. without producing a reaction between the two circuits sufficient to flatten the resonance curve.

Hence when we desire to remove the troublesome complications introduced by the reaction of the two coupled circuits and obtain pure trains of oscillations of a single frequency in the secondary we must employ impact excitation. The theory of this impact discharger is best treated as follows[43] :—

Let the two oscillatory circuits be supposed to have in them condensers of capacity C_1 and C_2 which are leaky and have conductances S_1 and S_2 in their dielectrics. Also let the inductive circuits have resistances R_1 and R_2 and inductances L_1 and L_2. Let free oscillations be excited in the primary,



[*By permission of the Proprietors of " The Electrician."*

FIG. 37.—Resonance Curves taken with Ordinary Spark Discharger, and various Couplings of Primary and Secondary Circuits.

and let the circuits react on each other, the mutual inductance being M. Then if these oscillations are damped they can be represented by the real part of $\epsilon^{j(p+ja)t} = \epsilon^{jPt}$, where $p = 2\pi$ times the frequency and a is the damping coefficient $= R/2L + S/2C$. Hence, when the circuits are in oscillation and left to themselves, the currents I_1 and I_2 are determined by the two equations—

$$\left.\begin{array}{l} \left\{(R_1 + jP_1L_1) + \dfrac{1}{S_1 + jP_1C_1}\right\} I_1 + jMP_2I_2 = 0 \\ jMP_1I_1 + \left\{(R_2 + jP_2L_2) + \dfrac{1}{S_2 + jP_2C_2}\right\} I_2 = 0 \end{array}\right\} \quad . \quad . \quad . \quad (172)$$

[43] See "Some Oscillograms of Condenser Discharges and a Simple Theory of Coupled Oscillatory Circuits," J. A. Fleming, *Proc. Phys. Soc. Lond.*, vol. 25, June 1913, p. 217.

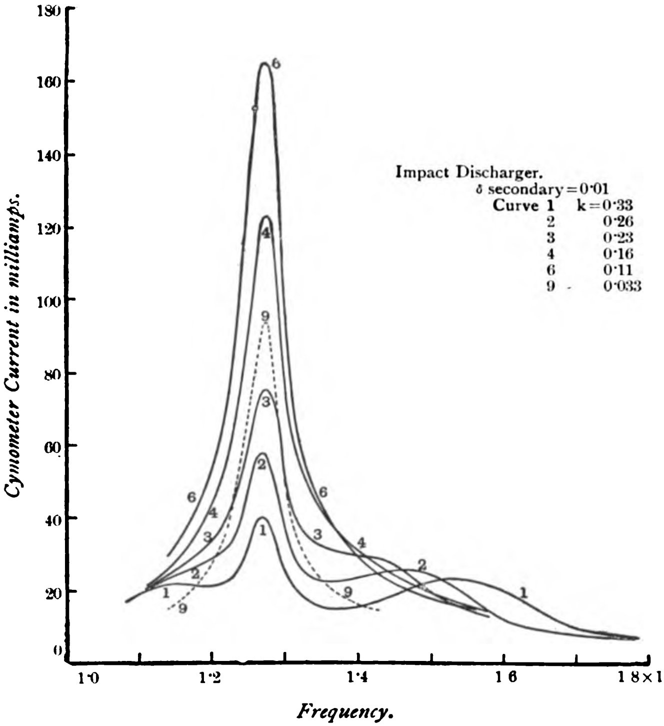
Writing Tim_1 for the function $R_1 + jP_1L_1 + (S_1 + jP_1C_1)^{-1}$ and Tim_2 for the same function for the secondary circuit, these being respectively the total impedances for damped oscillations, we have the equations—

$$Tim_1 I_1 + jMP_2I_2 = 0, \quad jMP_1I_1 + Tim_2I_2 = 0 \quad . \quad . \quad . \quad . \quad (173)$$

Eliminating I_1 and I_2, the determinant—

$$\begin{vmatrix} Tim_1, & jMP_2 \\ jMP_1, & Tim_2 \end{vmatrix} = 0. \quad . \quad . \quad . \quad . \quad . \quad (174)$$

or $Tim_1\ Tim_2 + M^2P_1P_2 = 0$ gives us an equation determining the frequency of the oscillations set up in the circuits.



[*By permission of the Proprietors of "The Electrician."*

FIG. 38.—Resonance Curves taken with Impact Discharger or Quenched Sparks, and various Couplings of Primary and Secondary Circuits.

Suppose we limit ourselves to the case of non-leaky condensers and consider the resistance of the secondary circuit to be small. Then $S_1 = S_2 = 0$, and also if $R_2 = 0$, we have $P_2 = p = 2\pi n$.

Let us denote the mutual inductance by $M = k\sqrt{L_1L_2}$, where k is the coefficient of coupling; then the determinant (174) reduces to—

$$(1 - P_1^2C_1L_1 + jP_1C_1R_1)(1 - P_2^2C_2L_2 + jP_2C_2R_2) = M^2P_1^2P_2^2C_1C_2 \quad . \quad . \quad (175)$$

or, since $P_2 = p$ and $P_1 = p + ja_1$ and $M^2 = k^2L_1L_2$, we have—

$$(1 - p^2C_2L_2)[1 - (p^2 - a_1^2)C_1L_1 - j2a_1pC_1L_1 + jpC_1R_1 - a_1C_1R_1]$$
$$= k^2(p^2 - a_1^2)p^2C_1L_1C_2L_2 + jk^2 2a_1p^3C_1L_1C_2L_2 \quad . \quad . \quad . \quad (175)$$

Equating real parts of the above equation and remembering that $a_1 = R_1/2L_1$, we find at once on substitution that

$$1 - p^2(C_1L_1 + C_2L_2) + p^4(1 - k^2)C_1L_1C_2L_2 = a_1^2(C_1L_1 - k^2p^2C_1L_1C_2L_2 - p^2C_1L_1C_2L_2) \quad (176)$$

If we assume the circuits are tuned, or that $C_1L_1 = C_2L_2 = CL$, then equation (176) reduces to—

$$(p^2CL(1 - k) - 1)(p^2CL(1 + k) - 1) - a_1^2CL(1 - (1 + k^2)p^2CL) = 0 \quad . \quad . \quad (177)$$

Suppose, then, that a_1 is very small, or that the damping in the primary is negligible, the equation (177) is seen to be satisfied by—

$$p = \frac{1}{\sqrt{CL(1 - k)}} \text{ and by } p = \frac{1}{\sqrt{CL(1 + k)}}$$

and there are, therefore, oscillations of two frequencies given by the above well-known formulæ. If, however, a_1 is very large, or the primary current rapidly quenched by increasing the resistance R_1 to infinity, then the coupling becomes zero when the primary circuit is open, and we have only one oscillation, which is the free oscillation of the secondary circuit. For intermediate values of a there will be three frequencies present. The resonance curve will then exhibit three humps as shown in the lower curves in Fig. 37.

PART II.—ELECTRIC WAVES

CHAPTER IV

STATIONARY ELECTRIC WAVES ON WIRES

1. The Propagation of Electric Potential and Current along a Conductor of Infinite Length.—Let us consider the case of a conductor infinitely long, consisting of a wire embedded in an insulator. Let the resistance, inductance, and capacity per unit of length of this wire be denoted by R, L, and C. Let the conductor of the insulator per unit of length of the wire be denoted by S.

Then if a periodic electromotive force is applied to some point in this circuit a current will be created in it.

Let the point of application of the electromotive force be taken as origin, and measure any distance x from it along the circuit. Consider an element of the conductor whose length is $\delta x = (x+\delta x) - x$ situated at this distance x from the origin.

Also at the point whose abscissa is x let the current in the conductor be denoted by i and the potential by v.

At the distance $x+\delta x$ the current will be $i - \frac{di}{dx}\delta x$, and the potential $v - \frac{dv}{dx}\delta x$.

The resistance, inductance, capacity, and dielectric conductance of the length δx of the conductor are $R\delta x$, $L\delta x$, $C\delta x$, $S\delta x$, respectively.

Hence the equations connecting v and i are obviously—

$$L\frac{di}{dt} + Ri = -\frac{dv}{dx} \quad . \quad . \quad . \quad (1) \qquad C\frac{dv}{dt} + Sv = -\frac{di}{dx} \quad . \quad . \quad . \quad . \quad (2)$$

If both i and v vary in a simple harmonic manner, so that they are proportional to the real part or horizontal step of ϵ^{jpt}, and if I and V are the maximum values of the current and potential during the period, then whatever lines be taken to represent I and V, the maximum values of $\frac{di}{dt}$ and $\frac{dv}{dx}$ will be represented by lines equal to pI and pV respectively, drawn at right angles to I and V, where p, as usual, denotes $2\pi n$.

Hence, if we consider only maximum values of the periodic functions and represent the vectors denoting them by complex quantities, we can write the equations (1) and (2) as vector equations, as follows :—

$$-\frac{dV}{dx} = jpLI + RI \qquad\qquad -\frac{dI}{dx} = jpCV + SV$$

$$\text{or} \quad -\frac{dV}{dx} = (R + jpL)I \quad . \quad . \quad (3) \qquad -\frac{dI}{dx} = (S + jpC)V \quad . \quad . \quad . \quad (4)$$

Separating the variables in (3) and (4) by differentiation, we have—

$$\frac{d^2V}{dx^2} = (R + jpL)(S + jpC)V \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\frac{d^2I}{dx^2} = (R + jpL)(S + jpC)I \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or, writing P for $\sqrt{R + jpL} \, . \, \sqrt{S + jpC}$, we obtain—

$$\frac{d^2V}{dx^2} = P^2V \quad . \quad (7) \qquad \frac{d^2I}{dx^2} = P^2I \, . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The solutions of the above equations (7) and (8) are—

$$V = a\epsilon^{+Px} + b\epsilon^{-Px} \qquad (9)$$

$$I = -\frac{P}{R+jpL}(a\epsilon^{+Px} - b\epsilon^{-Px}) \qquad (10)$$

where a and b are constants of integration.

The quantity P is a complex quantity, and can be represented in the typical form $\alpha + j\beta$.

$$\text{Hence} \quad \sqrt{R+jpL}\,.\,\sqrt{S+jpC} = \alpha + j\beta \qquad (11)$$

$$\text{and therefore} \quad \alpha^2 + \beta^2 = \sqrt{R^2+p^2L^2}\,.\,\sqrt{S^2+p^2C^2} \qquad (12)$$

$$\text{also} \quad \alpha^2 - \beta^2 = RS - p^2LC$$

Accordingly—

$$2\alpha^2 = \sqrt{(R^2+p^2L^2)(S^2+p^2C^2)} + (RS - p^2LC) \qquad (13)$$

$$2\beta^2 = \sqrt{(R^2+p^2L^2)(S^2+p^2C^2)} - (RS - p^2LC) \qquad (14)$$

The quantity α is called the *attenuation constant* of the cable, and β is the *wave-length constant.* They can be calculated when we know the *primary constants*, R, L, C, S, and n.

Next let us suppose the cable to be of infinite length in one direction, and that the impressed electromotive force is placed at the origin or accessible end. Let it be of simple harmonic form and maximum value E.

Then obviously there will be a gradual decrease in the magnitude of the maximum potential and current along the cable, since these quantities must be zero at the infinitely distant end.

Under these conditions, we then note that when $x=0$, $V=E$, and when $x=\infty$, $V=0$. It follows, therefore, that $a=0$ and $b=E$. Hence the solutions (9) and (10) when applied to the above case become transformed into—

$$V = E\epsilon^{-Px} \quad (15) \qquad I = E\epsilon^{-Px}\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}} \quad (16)$$

The quantity $\sqrt{R+jpL}/\sqrt{S+jpC}$ is called the *line characteristic* or *line impedance*, and is denoted by the symbol Z_0.

These equations give us the vector values of the potential and current at any point in the cable at a distance x from the origin at which a simple periodic electromotive force of maximum value E is applied.

$$\text{Since } P = \alpha + j\beta \text{ and } \epsilon^{-j\beta x} = \cos\beta x - j\sin\beta x \qquad (17)$$

we can write (15) and (16) in the form—

$$V = E\epsilon^{-\alpha x}(\cos\beta x - j\sin\beta x) \qquad (18)$$

$$I = E\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}}\epsilon^{-\alpha x}(\cos\beta x - j\sin\beta x) \qquad (19)$$

The reader should note that the above expressions are complex quantities, and represent V and I considered as vectors.

If we require the mere magnitude or size of V and I, that is, the numerical values of the maximum potential and current at the point x in the cable, we have to put these equations (18) and (19) into the form $A+jB$, and then find the value of the modulus $\sqrt{A^2+B^2}$ which expresses the magnitude in a scalar sense.

The reader should also notice the physical signification of the equation (18). It denotes that the maximum value V of the potential at any point in the cable is less than the maximum value of the impressed electromotive force E in the ratio $1 : \epsilon^{-\alpha x}$, and also that V is shifted backwards in phase relatively to E by an angle βx.

The factor $\epsilon^{-\alpha x}$ is called the *attenuation factor*, and the factor $(\cos\beta x - j\sin\beta x)$ is the *phase factor* for the distance x.

When we know α and β we can always graphically delineate the attenuation and phase difference. Hence as we proceed along the cable the maximum potential varies from point to point in accordance with the law of a damped oscillation. These facts may be presented graphically as follows :—

Take a line OX (see Fig. 1) to indicate the cable, and set up a perpendicular

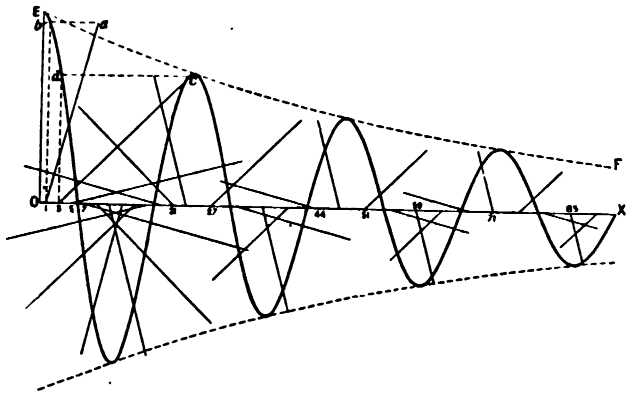


FIG. 1.—Delineation of a Curve representing the Variation of Maximum Potential along an Infinite Cable having a Simple Periodic Electromotive Force applied at one End, O.

OE to represent in magnitude and direction the maximum value of the electromotive force at the generator end. Then at equidistant points draw other lines decreasing in length in geometrical progression, and each shifted backwards or forwards in direction relatively to the preceding line by an equal angle. If we

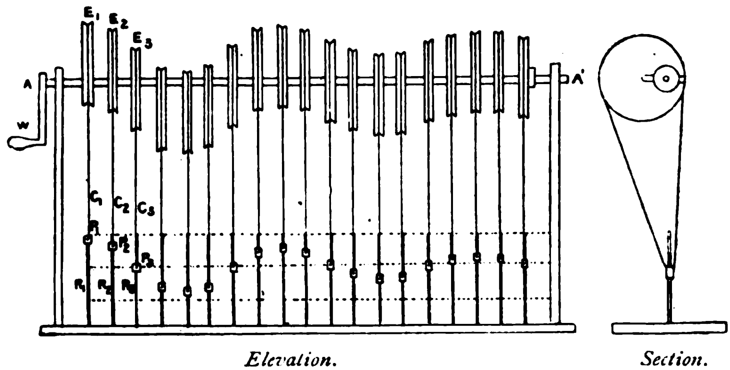


FIG. 2.—A Model illustrating the Propagation of an Alternating Current along a Cable of Infinite Length having a Simple Periodic Electromotive Force applied at one End.

suppose these lines to revolve with equal angular velocities round their ends as centres situated at equidistant intervals on the line OX ; then their projections at the same instant on vertical lines drawn through their centres will represent at that instant the actual voltage at these points in the cable. The periodic change with time and distance may be represented by a working model made in the following manner : On a long steel axle, AA, are fastened a number of eccentric pulleys, E_1, E_2, E_3, etc. (see Fig. 2). The eccentricities of these wheels decrease in

geometric progression, and each eccentric is set in phase backward behind its preceding neighbour by an equal angle. These wheels are embraced by endless cords, C_1, C_2, C_3, etc., of equal length attached to balls or blocks of metal, P_1, P_2, P_3, etc., sliding on vertical rods, R_1, R_2, R_3, etc., placed below each eccentric wheel.

When the axle carrying all the eccentricities is revolved by a handle, W, the blocks P_1, P_2, P_3, etc., will rise and fall with a nearly simple harmonic motion, and at any instant all the blocks will be situated on a sinuous curve of continually decreasing amplitude. As the eccentric axle revolves the motion of the balls will depict the progression of a wave potential along a cable having capacity, inductance, resistance, and leakance.

The equations (18) and (19) contain within them the explanation of the limitations of telephony, but we are not here concerned to discuss them generally. On this part of the subject the reader may be referred to a special treatise by the author on "The Propagation of Electric Currents in Telephone and Telegraph Conductors" (Constable & Co., London).

Since we are limiting our discussion to the effects of high frequency currents, we can reduce the complexity of the above expressions to a considerable degree. In cases where p is large the term pL in equations (13) and (14) is always much greater numerically than R, and likewise the numerical value of pC is greater than that of S. Accordingly, if we neglect R and S in comparison with pL and pC, the equations (13) and (14) for α and β reduce to—

$$2\alpha^2 = RS \quad . \quad . \quad (20) \qquad 2\beta^2 = 2p^2LC - RS \quad . \quad . \quad . \quad (21)$$

Therefore $$\beta^2 - \alpha^2 = p^2LC \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

In all cases likely to be met with in practice, RS is very small compared with p^2LC. Hence for high frequency oscillations it is sufficient to take—

$$\alpha = \sqrt{\tfrac{1}{2}RS} \quad . \quad (23) \qquad \beta = p\sqrt{LC} \quad . \quad . \quad . \quad . \quad (24)$$

Since $\cos(\beta x + 2\pi) = \cos \beta x$, it follows that $\cos \beta x = \cos \beta\left(x + \frac{2\pi}{\beta}\right)$, and therefore after moving along the conductor a distance $\frac{2\pi}{\beta}$ the current and potential again repeat themselves in value, or the wave-length of both the current and potential curves is equal to $\frac{2\pi}{\beta}$. Hence, since in all cases of wave motion the wave velocity W is connected with the frequency n and the wave-length λ by the equation $W = n\lambda$. (25) and since $\lambda = \frac{2\pi}{\beta}$ and $\beta = p\sqrt{CL}$, it follows that $W = \frac{1}{\sqrt{CL}}$. (26) or the wave velocity is inversely as the oscillation constant of the cable per unit of length.

In those cases in which the insulation of the surrounding dielectric is so high that $S=0$, and if pL is large compared with R, the vector equations (18) and (19) for the potential and current at any point in the wire at a distance x from the origin reduce to—

$$V = E\left(\cos\frac{2\pi}{\lambda}x - j\sin\frac{2\pi}{\lambda}x\right) \quad . \quad . \quad (27) \qquad I = E\frac{\sqrt{C}}{\sqrt{L}}\left(\cos\frac{2\pi}{\lambda}x - j\sin\frac{2\pi}{\lambda}x\right) \quad . \quad (28)$$

We see, therefore, that in such a case the current in the wire at any point is determined solely by the capacity and inductance per unit of length of the wire, and, moreover, that on account of the shift of phase, the current is not even in the same direction at the same time at all points in the conductor.

At two places not very far apart electricity may be flowing in opposite directions at the same moment. Also, owing to the periodic character of the expressions, the same values of V and I repeat themselves cyclically as x continually increases.

The expressions (27) and (28) are vector expressions in the form $a + jb$. To obtain the numerical values for the potential and current at any point in the cable, we have to find the size of these vectors, viz. the value of $\sqrt{a^2+b^2}$, and to obtain the actual potential at any moment we have to take the real part or horizontal step of the vector, viz. a.

Accordingly, the potential v at any distance along the cable x from the origin is given by the equation—

$$v = E\cos\frac{2\pi}{\lambda}x \quad . \quad (29) \quad \text{and similarly the current } i \text{ by } i = E\frac{\sqrt{C}}{\sqrt{L}}\cos\frac{2\pi}{\lambda}x \quad . \quad (30)$$

As we proceed along the cable, therefore, the current and potential are distributed at any moment in accordance with the ordinates of a simple sine curve. These waves of potential and current move along the cable from the generating end with a speed equal to $\frac{1}{\sqrt{CL}}$.

2. Stationary Electric Waves on Wires of Finite Length.—We have next to consider the changes made in the above expressions for the potential and current in the linear conductor when it is of finite length.

Consider first a wire infinitely extended in both directions. At two places separated by a distance $2l$ let two simple harmonic electromotive forces of opposite sign and of maximum value $+E$ and $-E$, that is, differing in phase by 180°, be applied. Then in the space between one of these sources and the point halfway between the two sources it is clear that the current must be distributed exactly as is the case in a finite wire of length l with one simple harmonic electromotive force of maximum value E placed at one end (see Fig. 3).

For it is clear that in a terminated or finite cable the current must always be zero at the end opposite to that at which the electromotive force is applied. Also in the case of the two opposite electromotive forces applied at a distance $2l$ in the infinite cable it is obvious that the current at the midpoint must always be zero.

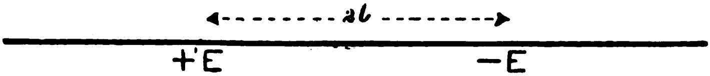


Fig. 3.—Two Sources of Alternating Electromotive Force in Opposite Phases placed in an Infinite Cable.

Again, we may cut away all that part of the infinite cable to the right or the left beyond the points of application of the electromotive forces without affecting the distribution of current in the length left behind. Hence in a piece of cable of finite length l having an electromotive force E applied at one end the distribution of current must be the same, point for point, as it is in that part of an infinite cable which constitutes the half of the intercept between the points of application of the two opposite electromotive forces $+E$ and $-E$ separated by a distance $2l$.

We have seen that in an infinite cable the current I_1 at a distance x from the source E, is given by the equation—

$$I_1 = E\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}}\epsilon^{-Px}$$

Hence, at a distance $2l-x$ from a source $-E$, the current I_2 must be—

$$I_2 = -E\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}}\epsilon^{-P(2l-x)}$$

Now consider the infinite cable with the two sources $+E$ and $-E$ at a distance $2l$. At a point lying to the right of $+E$ and at a distance x the current I due to both sources must be the algebraic sum of those due to both separately, or must be expressed by—

$$I = E\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}}\left\{\epsilon^{-Px} - \epsilon^{-P(2l-x)}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

This, therefore, must be the expression for the current in a finite conductor of length l having an electromotive force E applied at one end, the equation (31) giving us the current at a point at a distance x from the source of electromotive force E.

To obtain the potential V we must refer to the equation (4), § 1, and note that I and V are connected by the relation—

$$\frac{dI}{dx} = -(S + jpC)V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Hence, differentiating (31) and recollecting that—

$$P = \sqrt{S + jpC} \,.\, \sqrt{R + jpL} \quad . \quad (33) \qquad \text{we have—} \quad V = E\{\epsilon^{-Px} + \epsilon^{-P(2l-x)}\} \quad . \quad . \quad (34)$$

Therefore we see that the current in the finite cable of length l with harmonic electromotive force E applied at one end is obtained by taking the *difference* of two currents, one due to a source +E at the origin, and the other to an *electrical image* of this source (viz. +E) placed in imagination as much beyond the far end of the cable as the real source is from it.

Also the potential at any point is obtained by taking the *sum* of the potentials separately of the real source and an image of the source reflected in the far end of the cable.

If we consider that S may be neglected in comparison with pC, and also R in comparison with pL, as we may do, when dealing with electrical oscillations in ordinary wires, then we have the two following vector expressions for the potential V and current I at any distance x from one end of a finite wire of length l, a simple periodic electromotive force E being applied at the origin, viz.—

$$V = E\{\epsilon^{-Px} + \epsilon^{-P(2l-x)}\} \quad . \quad . \quad (35) \qquad I = E\frac{\sqrt{C}}{\sqrt{L}}\{\epsilon^{-Px} - \epsilon^{-P(2l-x)}\} \quad . \quad . \quad (36)$$

But under the above conditions, when S and R are negligible compared respectively with pC and pL, we have seen that $\alpha = \sqrt{\tfrac{1}{2}RS}$ and $\beta = p\sqrt{LC}$. If $S=0$, then $\alpha = 0$, and the attenuation is zero. This takes place when the conductivity of the dielectric is zero. Under these conditions we have $P = j\beta = jp\sqrt{CL}$, and the equations (35) and (36) may be written in the form—

$$V = E\{\epsilon^{-j\beta x} + \epsilon^{-j\beta(2l-x)}\}. \quad . \quad (37) \qquad I = E\frac{\sqrt{C}}{\sqrt{L}}\{\epsilon^{-j\beta x} - \epsilon^{-j\beta(2l-x)}\} \quad . \quad (38)$$

or—

$$V = E[\{\cos\beta x + \cos\beta(2l-x)\} - j\{\sin\beta x + \sin\beta(2l+x)\}] \quad . \quad . \quad (39)$$

$$I = E\frac{\sqrt{C}}{\sqrt{L}}[\{\cos\beta x - \cos\beta(2l-x)\} - j\{\sin\beta x - \sin\beta(2l-x)\}] \quad . \quad . \quad (40)$$

The above are vector expressions of the form $A + jB$. To obtain the scalar values or size, we must form the expressions equivalent to $\sqrt{A^2 + B^2}$, and we then have—

$$(V) = (E)\sqrt{2 + 2\cos 2\beta(l-x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

$$(I) = (E)\frac{\sqrt{C}}{\sqrt{L}}\sqrt{2 - 2\cos 2\beta(l-x)}\,. \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where (V), (E), and (I) stand for the scalar values of the vectors V, E, and I respectively.

In the above equations, if we put $x = l$, we have—

$$(V) = 2(E) \qquad\qquad (I) = 0$$

which shows that at the free end of the wire the potential rises to twice the value at the generator end, whilst the current, of course, is zero.

Bearing in mind that under the conditions assumed $\beta = p\sqrt{CL}$, and also the velocity of propagation of the wave is $W = 1/\sqrt{CL}$, and that $W = n\lambda$ where λ is the wave-length, we have as a consequence $\beta = 2\pi/\lambda$. Also let the length l of the conductor be some multiple of λ, so that $l = m\lambda$.

Then substituting these values in equations (41) and (42) and squaring, we have—

$$(V)^2=(E)^2\left(2+2\cos\frac{4\pi}{\lambda}(l-x)\right) . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (43)$$

$$(I)^2=(E)^2\frac{C}{L}\left(2-2\cos\frac{4\pi}{\lambda}(l-x)\right) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (44)$$

These equations give us the numerical value of the maximum potential and current during the phase at any point in the cable.

We will apply them to certain instances. Let the length of the cable be one-quarter of a wave-length, then when $x=l=\frac{\lambda}{4}$ we have $(V)=2(E)$ and $(I)=0$. Also

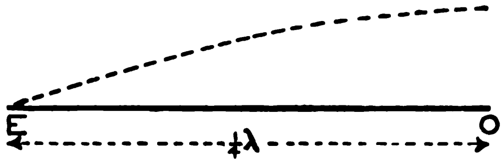


Fig. 4.—Distribution of Potential along a Finite Cable having a Simple Periodic E.M.F. placed at E (*Fundamental Oscillation*).

when $x=0$ we have $(V)=0$ and $I=2(E)\frac{\sqrt{C}}{\sqrt{L}}$. Accordingly, in this case there is a steady increase of potential and decrease of current all the way from the origin to the open or free end of the cable. The distribution of potential may be represented by the ordinates of the dotted line in Fig. 4, where the thick black line represents the cable, E being the end at which the electromotive force is applied, and O the free or insulated end of the cable.

Again, suppose we take the length of the cable equal to $\frac{3\lambda}{4}$. Then at the distances $x=0$, $x=\frac{\lambda}{4}$, $x=\frac{\lambda}{2}$, $x=\frac{3\lambda}{4}$, we have $(V)=0$, $(V)=2(E)$, $(V)=0$, $(V)=2(E)$.

Fig. 5.—Distribution of Potential along a Finite Cable having a Simple Periodic E.M.F. placed at E (*First Harmonic Oscillation*).

There are, therefore, loops and nodes of potential, and similarly loops and nodes of current. The current, however, is a maximum at those points at which the potential is zero, and *vice versâ*.

The distribution of potential may be represented by the ordinates of the dotted line in Fig. 5.

In the same manner, if we take $l=\frac{5}{4}\lambda$ and examine the distribution of potential, we always find it to be a maximum at the free end O, whilst at that point the current is zero. Also the potential is a maximum at a distance $\frac{\lambda}{4}$, from the free end, and there are loops and nodes of potential separated by distances $\frac{\lambda}{4}$, as shown by the ordinates of the dotted line in Fig. 6.

If, then, the wire or conductor has a length which is any exact multiple of one-quarter of a wave-length, so that $l = M\frac{\lambda}{4}$, where M is any integer number, then it is easy to show that there will be $\frac{M+1}{2}$ loops of potential and the same number of nodes, including those at the beginning and end of the wire. Thus if $M=1$ there is one loop at the free end and one node at the generator end; if $M=3$ there are two loops and two nodes, and so on.

It is clear, therefore, that if a conductor has a length equal to some exact integer multiple of the quarter wave-length of any harmonic electric oscillation, and if a simple periodic or sinoidal electromotive force having the corresponding frequency is applied at one end, we have *stationary electric waves* of potential and current set up on the wire, that is, a distribution of potential and current varying from point to point along the wire in accordance with the ordinates of a sine curve.

We may, if we please, consider that this is due to the interference of waves reflected at the open end of the wire with those which are travelling up the wire with a velocity $\frac{1}{\sqrt{CL}}$ from the source.

There is a perfect analogy between this electrical phenomenon and the stationary aerial waves produced in stopped organ pipes, the stopped end corresponding to the insulated end of the wire. Electric potential corresponds, then, to air pressure, and electric current to velocity of the air particles.

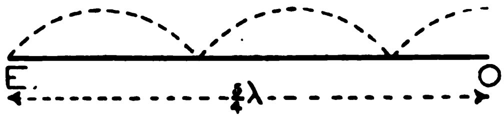


FIG. 6.—Distribution of Potential along a Finite Cable, OE, having a Simple Periodic E.M.F. placed at E (*Second Harmonic Oscillation*).

We may refer the reader to any good treatise on acoustics for a full description of the mode of production of these stationary air waves in open or closed pipes, and a knowledge of these acoustic effects is of assistance in comprehending the corresponding electrical phenomena. Otherwise we may compare the electric vibrations set up on wires or helices with the stationary waves produced in stretched cords when put in transverse vibration.

3. **Effect of Damping upon the Stationary Waves or Wires.**—If we do not neglect the damping or attenuation of the waves propagated along the finite wire, the expressions for the current and potential at any point become a little more complicated, but are easily obtained. Referring to equations (34) and (31) for the potential and current at any point in the insulated wire, we have—

$$V = E\{\epsilon^{-Px} + \epsilon^{-P(2l-x)}\} \qquad (45)$$

$$I = E\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}}\{\epsilon^{-Px} - \epsilon^{-P(2l-x)}\} \qquad (46)$$

If we put $P = \alpha + j\beta$, we then have—

$$V = E[\{\epsilon^{-\alpha x}\cos\beta x + \epsilon^{-\alpha(2l-x)}\cos\beta(2l-x)\} - j\{\epsilon^{-\alpha x}\sin\beta x + \epsilon^{-\alpha(2l-x)}\sin\beta(2l-x)\}]$$

This is a vector equation in the form $V = A + jB$.

If we scalarize or find the size of the vector, we have—

$$(V) = \sqrt{A^2+B^2}, \text{ or by substitution—}$$

$$(V) = (E)\sqrt{\epsilon^{-2\alpha x} + \epsilon^{-2\alpha(2l-x)} + 2\epsilon^{-2\alpha l}\cos 2\beta(l-x)} \qquad (47)$$

The above equation may be written—

$$(V) = (E)\epsilon^{-\alpha x}\sqrt{1 + \epsilon^{-4\alpha(l-x)} + 2\epsilon^{-2\alpha(l-x)}\cos 2\beta(l-x)} \qquad (48)$$

If $x=l$, we have $(V)=2(E)\epsilon^{-al}$.

In the same way, if we take the expression for I in (46) and write it out, we have—

$$I=E\frac{\sqrt{S+jpC}}{\sqrt{R+jpL}}[\{\epsilon^{-ax}\cos\beta x-\epsilon^{-a(2l-x)}\cos\beta(2l-x)\}$$
$$-j\{\epsilon^{-ax}\sin\beta x-\epsilon^{-a(2l-x)}\sin\beta(2l-x)\}]$$

This is a vector expression of the form $\frac{\sqrt{a+jb}}{\sqrt{c+jd}}(e+jf)$, and hence, by the rule given on p. 231 for obtaining the size of such a vector, we have—

$$(I)=(E)\left(\frac{S^2+p^2C^2}{R^2+p^2L^2}\right)^{\frac{1}{4}}\sqrt{\epsilon^{-2ax}+\epsilon^{-2a(2l-x)}-2\epsilon^{-2al}\cos 2\beta(l-x)} \quad . \quad . \quad (49)$$

If $x=l$, we have $(I)=0$.

We may also write equations (48) and (49) in the form—

$$(V)^2=(E)^2\epsilon^{-2ax}\{1+\epsilon^{-4a(l-x)}+2\epsilon^{-2a(l-x)}\cos 2\beta(l-x)\} \quad . \quad . \quad . \quad . \quad (50)$$

$$(I)^2=(E)^2\frac{\sqrt{S^2+p^2C^2}}{\sqrt{R^2+p^2L^2}}\epsilon^{-2ax}\{1+\epsilon^{-4a(l-x)}-2\epsilon^{-2a(l-x)}\cos 2\beta(l-x)\} \quad . \quad . \quad (51)$$

In each of these expressions for the potential and current in the conductor at any point x, the quantity in the bracket consists of an exponential part which varies steadily with x, and a periodic or cosine term which varies periodically. This last term is more pronounced in its effect in proportion as β is large and a small. Hence, if we make the resistance of the cable per unit of length small and the inductance large, also if we increase the capacity and reduce the leakage per unit of length as much as possible, we shall get more marked loops and nodes than if the inductance is small.

An important consequence follows from this. We can by coiling the wire into a spiral of a single layer of wire in closely adjacent turns increase the inductance per unit length of the spiral. The spiral wire acts like a linear conductor of abnormally large inductance, and hence the spiralization promotes the formation of marked loops and nodes.

Accordingly, the effect of large wire resistance or large insulation conductance per unit of length of the conductor is to damp out all evidence of loops and nodes or stationary waves on the wires. On the other hand, the effect of a large inductance and capacity per unit of length of the conductor is to render more evident the phenomena of stationary electric waves.

4. Experimental Production of Stationary Electric Waves upon Spiral Wires.—The above theoretical investigation can be tested and beautifully illustrated by means of experiments carried out with insulated wire helices on which stationary electric waves may be formed.

If we wind on a non-conducting rod a helix of insulated wire in one single layer of closely adjacent turns, we have a conductor which may be regarded as a cylindrical conductor having a certain capacity, inductance, resistance, and insulation per unit of length.

The length of the conductor is the length of the spiral, not the length of the wire forming it, and by capacity and inductance per unit length of the spiral is meant the whole capacity or inductance as it stands divided by the length of the spiral.

We have already seen that the inductance of a spiral of this kind can be nearly predetermined, if the ratio of length to diameter is large, by the formula—

$$L=\frac{\pi^2D^2N^2}{l}$$

where l is the length, D the diameter, and N the total number of turns on the spiral. Hence the inductance per unit of length is equal to $(\pi DN')^2$, where N′ is the number of turns per unit of length of the spiral. Since the length of wire wound on one unit of length of the spiral is $\pi DN'$, we have the rule that the

inductance of such a spiral per unit of length is numerically equal to the square of the length in centimetres of the wire wound on per unit of length of the helix. We can therefore make it large by employing a fine wire. Again, the capacity of such a helix is not much different from that of a metallic cylinder of the same external dimensions, and therefore not much affected by the size of the wire used. We can therefore increase the inductance per unit of length (L) without increasing the capacity per unit of length. Also, we can keep down the resistance per unit of length by using a wire of high conductivity, and we can make the insulation high by covering it with silk and winding the wire on an ebonite tube. By these means we can make a conductor of linear form, for which the constant $a=\sqrt{\frac{RS}{2}}$ is small compared with $\beta=\sqrt{CL}$, and therefore, as above explained, the nodes and loops will be sharply marked when stationary electric waves are formed upon it.

For this purpose a helix of insulated wire of the following dimensions is convenient:—

On an ebonite rod or thick tube 215 cms. long and 4·75 cms. in diameter and circular section is wound a helix of silk-covered copper wire, consisting of 5465 closely adjacent turns in one layer.

This helix of wire is 210 cms. in length, and each turn has a mean diameter of 4·78 cms. Hence the total inductance is $32{\cdot}07\times10^{6}$ cms., and the inductance per centimetre of length of the helix is $1{\cdot}527\times10^{5}$ cms. If this helix is placed in a horizontal position at a height of 50 cms. or so above a table supported on insulating stands, we can measure its capacity with respect to the earth, and for the helix above described it is found to be 45 micro-mfds.[1] Hence the capacity per unit of length C is $\frac{45}{210}$ micro-mfd.

An electric wave, therefore, travels along this spiral with a velocity of $\frac{1}{\sqrt{CL}}$, which in this case is $174{\cdot}8\times10^{6}$ cms. per second.

This velocity is about $\frac{1}{180}$th part of the velocity of light.

Hertz has described an experiment in which he established stationary electric waves on a spiral wire and compared the internodal distances with those which would be formed if the wire were stretched out straight, and he found that in the former case the velocity of the wave was much less than that of light.[2]

H. C. Pocklington has treated the matter theoretically, and he also shows that the velocity of the electric wave along a spiral is less than its velocity along the same wire stretched out straight. From the theory given above it is clear that this is due to the greatly increased inductance per unit of length of the spiral as compared with the simple linear wire.[3]

We can then proceed to set up stationary electric oscillations on the above-described spiral wire as follows: A condenser of variable capacity and a variable inductance are joined in series with each other and with a spark gap. For this purpose a condenser made as follows is convenient. Rectangular pieces of good sheet ebonite $20\times22{\cdot}5$ cms. and 3 mms. in thickness are coated on both sides with tinfoil, the area of each tinfoil sheet being $15\times17{\cdot}5$ cms. Twenty-four of these plates are prepared and grouped in six sections, each of four plates. The tinfoil sheets have tinfoil lugs attached to them, and in each set of four plates the tinfoils are so joined up as to make a condenser of nearly 0·001 mfd. capacity. The whole set of six condensers then has a capacity of 0·006 mfd., and they can be joined partly in series and partly in parallel. The six bundles of four ebonite plates are bound with silk and immersed in an ebonite box filled with vaseline oil free from water.

In a condenser so made by the author there were slight differences in capacity between the six condensers, but when all were joined in parallel the measured

[1] See J. A. Fleming, "On the Propagation of Electric Waves along Spiral Wires," *Phil. Mag.*, Oct. 1904, ser. 6, vol. 8, p. 433.

[2] See "Electric Waves," H. Hertz, English translation by D. L. Jones, pp. 158, 159.

[3] See H. C. Pocklington, "Electric Oscillations in Wires," *Proc. Camb. Phil. Soc.*, Oct. 25, 1897, vol. ix. p. 324.

capacity was 0·005835 mfd., and when the six were joined in two groups of three condensers, each in series, the two sets being in parallel, it gave a condenser of 0·001461 mfd. These capacities can be accurately determined by means of the revolving switch described in Chap. II. p. 149. The variable inductance may conveniently take the form of a helix of thick copper wire with movable contact, as described in Chap. II. p. 134.

The spark gap should consist of a pair of zinc balls adjustable as to distance. They should be enclosed in a wooden box to reduce noise, and it is an advantage to cause a blast of air to impinge on the spark gap.

The spark balls S, condenser C, or condensers C_1 and C_2, and variable inductance L, are then joined up with the long insulated helix H, as shown diagrammatically in Fig. 7. The secondary terminals of an induction coil I are connected to the spark balls, and one spark ball, namely, that next to the inductance coil L, is connected to the earth E, that is, to a gas or water pipe, by a wire. On starting, the induction coil oscillations are set up in the condenser circuit, the frequency n of which is given by the formula—

$$n=\frac{5\times 10^6}{\sqrt{CL}}$$

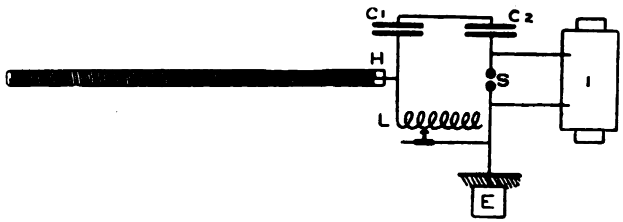


Fig. 7.—Arrangement for Producing Stationary Electric Waves on a Long Helix, H, H, by the Oscillations in a Condenser Circuit possessing Inductance and a Spark Gap.

The velocity W with which the wave travels up the helix is given by—

$$W=\frac{1}{\sqrt{C_1L_1}}=n\lambda$$

where C_1 and L_1 are the capacity and inductance of the helix per unit of length.

Also for the production of stationary waves we must have the wave-length λ of the stationary wave on the helix so adjusted that—

$$m\frac{\lambda}{4}=l$$

where m is unity or some odd integer number, and l is the length of the helix.

Combining these equations, we have—

$$W=\frac{1}{\sqrt{C_1L_1}}=\frac{5\times 10^6}{\sqrt{CL}}\cdot\frac{4l}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

or

$$\sqrt{CL}=\frac{20l\times 10^6}{m}\sqrt{C_1L_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

If, therefore, we adjust the *oscillation constant* $\sqrt{CL}$ of the condenser circuit to be equal to $20l\times 10^6\times\sqrt{C_1L_1}$, divided respectively by 1, 3, 5, 7, we then shall find that when the oscillations are established in the condenser circuit, resonant stationary oscillations are set up on the helix.

These will show themselves by making strong electric brush discharges into the air at the insulated end of the helix, and at the loops or antinodes, and in a dark room the helix will be seen to be surrounded by a glow of light, which is brightest at the antinodes of potential. It can, however, be best detected and the

position of the nodes fixed by holding a vacuum tube of the spectrum type filled with rarefied neon near the tube.

Neon is one of the rare atmospheric gases discovered by Sir William Ramsay, and Sir James Dewar has shown that it can be extracted from it by absorbing the oxygen, nitrogen, and other commoner constituents of air by means of coco-nut charcoal cooled with liquid hydrogen or liquid air. The author found some time ago that a vacuum tube of the spectrum type with a very small bore, not more than 1 mm. in the straight part of the tube (see Fig. 8), when filled with rarefied neon, formed an excellent and most sensitive means of detecting a high frequency electric field. The tube then glows with a bright red-orange light, which is visible in broad daylight. If such a tube cannot be obtained, then one of the same form, made with uranium glass and filled with rarefied carbonic dioxide gas, will answer the purpose fairly well.

To locate the loops and nodes, the vacuum tube must be held over the helix and perpendicular to it, and at varying distances from it, and moved along parallel to itself. It will then be found that in some positions it glows brightly, whilst in others it does not, and a very slight movement on either side of the last positions will make the tube illuminate again. These non-glowing positions are just over the nodes of potential on the helix. If a boxwood scale divided into centimetres and millimetres is placed below the helix and at about 10 cms. from it, it is possible to read off the distance from the end of the helix at which these antinodes and nodes of potential exist, as shown by the positions at which the neon or other vacuum tube glows or does not glow brightly. We can then adjust the inductance in the condenser circuit and the capacity of the latter, until we so arrange matters that we have a good electric brush at the end of the helix farthest from the condenser, and either no node or else 1, 2, 3, 4, etc., nodes of potential, indicating

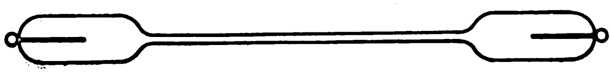


FIG. 8.—Neon Vacuum Tube.

that the helix has established on it either its fundamental or its 1st, 2nd, 3rd, etc., harmonic oscillation, as shown by the existence of a $\frac{1}{4}$ of a stationary wave or $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, etc., stationary waves.

In a research of this character the author found that, with a helix as above described, the nodes and antinodes of potential were distributed as shown by the ordinates of the dotted lines in Fig. 9. The numbers given underneath the diagram OE representing the helix show the internodal distances in centimetres, and the distance of the first potential node from the open end O of the helix. Two things are at once noticeable.

(1) The internodal distances are not equal, but increase towards the end of the helix which is attached to the condenser. This seems to show that the velocity of the wave is not the same at all parts of the helix, but is rather greater near the condenser end E. This may be due to the free ends of the helix having a slightly smaller inductance per unit of length than the middle portions.

(2) The distance from the open end of the helix O to the first node of potential N_1 is always less than half the distance N_1N_2, between the two succeeding nodes, or any pair of succeeding nodes. In fact, the distance ON_1 multiplied by 2·5 is always nearly equal to N_1N_2.

The velocity of the wave along the helix can be ascertained by measuring the wave length of the stationary wave on the helix, which is equal to twice the distance N_1N_2 between the first and second nodes, and also ascertaining the frequency of the oscillations.

The frequency can be ascertained from the condenser capacity and the inductance in the main oscillation circuit. In experiments by the author[4] the real value of the inductance used corresponding to certain scale readings of

[4] See J. A. Fleming, "On the Propagation of Electric Waves along Spiral Wires," *Phil. Mag.*, Oct. 1904, ser. 6, vol. 8, p. 417.

the variable spiral inductance employed was carefully ascertained by comparing it with the inductance of certain squares of copper wire of known size In this way a series of observations was made, noting the capacity C in the condenser

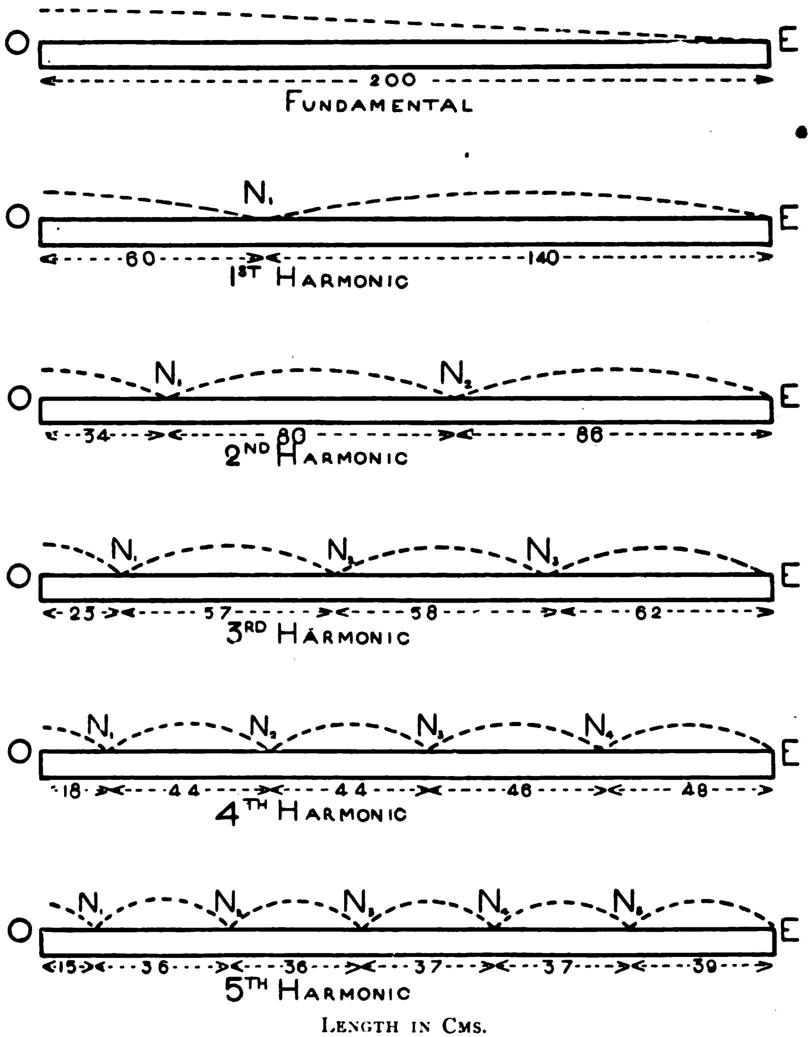


FIG. 9.—Diagrams representing the Stationary Potential Waves set up on a Long Helix, OE, by a High Frequency E.M.F. applied at one end, E. The Distance of the Dotted Lines from the Helix represents the Potential Amplitude at that Point of the Helix.

circuit, the inductance L, the calculated frequency n, the observed stationary wave-length λ on the helix, and from these data the velocity of the wave (W) was calculated. The results are shown in the table on p. 278.

It will be seen that for the first three harmonics the wave velocity is nearly 172×10^6 cms. per second, and this agrees very well with the velocity 174×10^6 cms.

per second calculated from the measured inductance and capacity of the spiral per unit of length. There is, however, a falling off in the value of W for higher harmonics. This may be due to inaccuracy in measuring the small values of the inductance L or else to some cause not yet ascertained.

Oscillation.	Capacity in mfds. in condenser circuit, C.	Inductance to cms. in condenser circuit, L.	Calculated frequency, n.	Observed wave-length, λ.	Calculated wave velocity, $W = n\lambda$.
Fundamental .	0·005835	110,000	$0{\cdot}197 \times 10^6$	(871)	(172×10^6)
1st harmonic .	0·002887	25,000	0·588 ,,	292	172 ,,
2nd ,, .	0·001461	18,000	0·977 ,,	175	172 ,,
3rd ,, .	0·001464	9,000	1·379 ,,	124	171 ,,
4th ,, .	0·001461	6,000	1·70 ,,	95	163 ,,
5th ,, .	0·001461	5,000	1·9 ,,	80	152 ,,

Two points call for notice. If we employ the velocity, viz. 172×10^6 cms. per second, obtained from observation made on the wave-lengths of the first, second, and third harmonics to calculate back the wave-length of the fundamental oscillations, we find this last to be 871 cms. The length of the helix was 210 cms. Hence it is clear that the fundamental wave-length is rather more than four times the length of the helix.

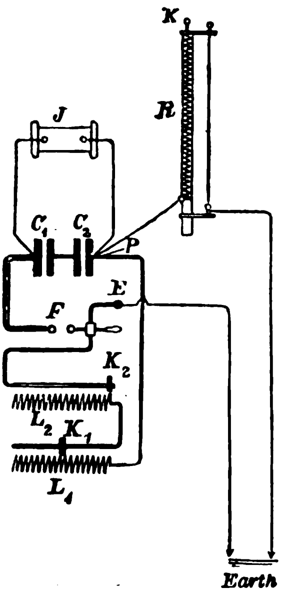


Fig. 10.—Arrangement of Seibt's Apparatus for exhibiting Stationary Potential Waves on a Vertical Helix. J, induction coil; C_1, C_2, condensers; F, spark balls; L_1, L_2, adjustable inductances; R, helix.

In the next place the distance from the open end of the helix to the first node of potential is always decidedly less than one quarter of the corresponding wave-length, that is, it is less than half the distance between the two succeeding nodes of potential. In fact, the wave-length is more nearly equal to *five times* the distance from the open end to the first node.

This indicates that the simple theory above given is not sufficient to fit the facts. A more complete theory of the production of stationary electric waves on open circuits has been given by Professor H. M. Macdonald.[5] In this investigation he shows that if the fundamental electrical oscillation is set up on a perfectly straight insulated wire, it consists in an oscillation such that the centre of the wire is a node of potential and the two extremities are antinodes, but the wave-length of the oscillation set up is shown to be not simply twice the length of the wire, as the simple theory given in § 4 of this chapter would indicate, but 2·53 times the length of the wire. Hence this indicates that if a wire has a high frequency electromotive force applied at one end, and the length of the wire is so adjusted that there is an antinode of potential at the other end, the wire vibrating in its fundamental oscillation, then the length of the wave must be nearly five times that of the wire. It is impossible to verify this with the fundamental oscillation of a single wire, because we cannot make a sufficiently sharp measurement of the position of the node, but in the case where the wire has a higher

[5] Adams Prize Essay, by H. M. Macdonald, "Electric Waves," Cambridge University Press, 1902, see p. 111.

harmonic oscillation produced upon it, as in the experiments of the author above described with the spiral, we can ascertain the length of the wave by measuring the distance between the first and second node, and then ascertain also the distance from the first node to the open end of the wire, and we find, as shown in the diagram in Fig. 9, that the distance from the end of the wire to the first node is to the distance between the first and second nodes very nearly in the ratio of the numbers 2 to 5.

It will be seen, by referring to Fig. 9, delineating the above described experiments with stationary waves on spiral wires, that for the second harmonic

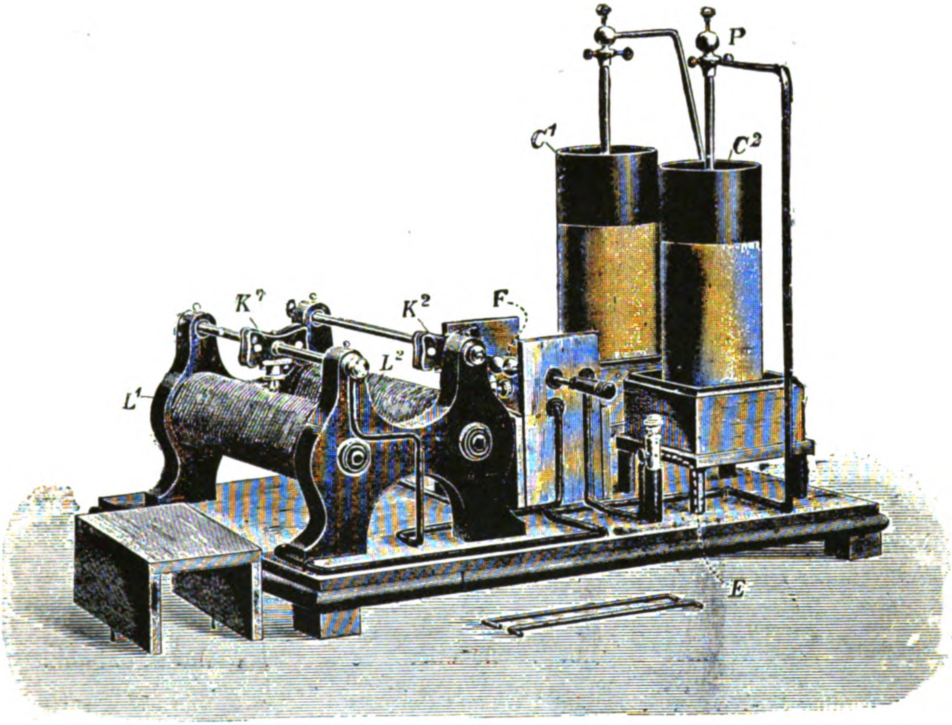


FIG. 11.—Perspective View of Condenser Circuit of Seibt's Apparatus, as made by F. Ernecke of Berlin. C^1, C^2, Leyden jars; F, spark balls in a box; L^1, L^2, variable inductance coil.

the ratio of ON_1 to N_1N_2 is 34 to 80, which is exactly as 2 to 5, and for the third harmonic the ratio of ON_1 to N_1N_2 is as 23 to 57, which is also very nearly as 2 to 5, and the same with the fourth and fifth harmonics.

These observations, therefore, with the spiral so far confirm Macdonald's theory.

Another method of exhibiting these nodes and loops on a spiral wire was devised by G. Seibt.[6]

Seibt's method is to place a spiral of insulated wire wound on a wooden rod in a vertical position, and to stretch alongside of it and parallel to it, a few centimetres away, a very fine bare wire which is connected to the earth. The spiral is connected at bottom end to an oscillating circuit consisting of a couple

[6] See "Elektrische Drahtwellen," *Electrotechnische Zeitschrift*, April 10, 17, 24, May 1, 8, 1892, vol. xxiii.

of Leyden jars, a spark gap, and a variable inductance, and oscillations are excited in this as usual by an induction coil. The apparatus is as represented in Figs. 10 and 11.

When oscillations are set up in the Leyden jar circuit, and the inductance varied so as to give the frequency of these oscillations the proper value for exciting either the fundamental or the higher harmonic oscillations in the spiral, then an electric brush discharge takes place between the surface of the spiral and the parallel earth wire (see Fig. 12).

If the spiral is vibrating in its fundamental manner, then its glow is very brilliant at the top, and drops away to nothing down at the bottom; but if it is vibrating in its higher harmonics, then the glow is distributed in patches, the brightest points marking the position of the antinodes of potential. This experiment forms a very beautiful one, but it can only be seen in a perfectly dark room. Moreover, the position of the nodes and antinodes cannot be fixed with great accuracy, but it serves to render visible, in a sense, these stationary waves. Again, if a brass rod terminating in a knob is taken in the hand and the knob held near the top of the spiral when vibrating in its fundamental manner, very long thin sparks can be drawn from it, and a strong electric brush proceeds from the end of the helix. If the knob at the end of the rod is carried down the spiral, it will be found that the spark drawn becomes shorter but more brilliant as it is taken lower down.

This indicates the gradual decrease in the potential amplitude as we pass from the open or insulating end of the spiral to the condenser end, and also it indicates the gradual increase of the current, the current flowing into the helix being a maximum at the lower end of the spiral and the potential amplitude a maximum at the upper end.

It will be seen, therefore, that to excite the fundamental oscillation in the spiral we must apply to the bottom end a high frequency electromotive force, the frequency of which is such that the wave produced by it travels a distance rather more than four times the length of the spiral during the time of one period. We have seen that the velocity with which the wave travels along the spiral is measured by $\frac{1}{\sqrt{C_1L_1}}$, where C_1 is the capacity of the spiral per unit of length and L_1 is its inductance, hence the frequency n required to produce the fundamental oscillation is given by the equation—

$$n=\frac{1}{\lambda\sqrt{C_1L_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

In the above expression λ is the wave-length of the fundamental oscillation, which, according to the simple theory, is four times the length of the spiral, and, according to Macdonald's theory, five times the length of the spiral; but, as far as the experiments of the author go, is, in fact, nearly 4·15 times the length of the spiral. Hence we may say that the frequency n_0 required to produce the fundamental oscillation on a spiral of length l is given by the equation—

$$n_0=\frac{1}{4{\cdot}15l\sqrt{C_1L_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

The frequency required to produce the first harmonic, or the oscillation having one node about one-third of the way from the open end of the spiral, is $3n_0$. If we call this n_1 we have for the frequency required to produce the first harmonic the expression—

$$n_1=\frac{1}{12{\cdot}45l\sqrt{C_1L_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

In the same way the frequency n_2 required to produce the second harmonic is five times, the third harmonic seven times, that required to produce the fundamental oscillation, and, generally speaking, the frequency required to produce the mth harmonic is $(2m+1)$ times the frequency of the fundamental. Therefore we have,

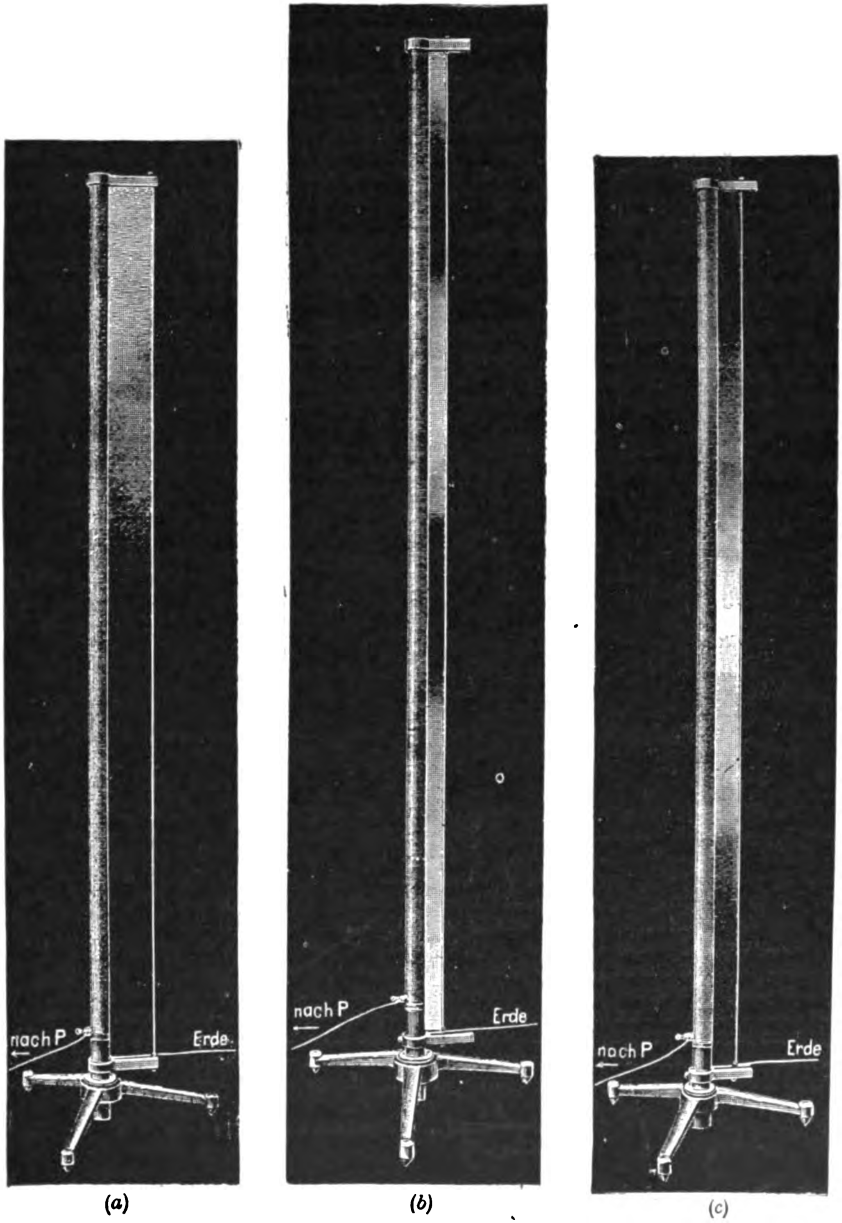


FIG. 12.—Electric Glow Discharge between a Vertical Earth Wire and a Seibt Helix when connected to a Condenser Circuit yielding High Frequency Oscillations. (*a*) Helix exhibiting fundamental oscillation (upper end insulated); (*b*) helix exhibiting second harmonic oscillation (upper end insulated); (*c*) helix exhibiting fundamental oscillation (upper end earthed).

for the frequency n_m required to produce the mth harmonic on the spiral or state of electrical vibration with m modes of potential, the expression—

$$n_m = \frac{1}{4 \cdot 15(2m+1)l\sqrt{C_1 L_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

The author has found that winding the spiral on a wooden rod is a mistake. Ordinary wood, even if dry, has considerable conductivity for high frequency currents, and therefore tends to give the spiral a greater capacity as the frequency increases. This creates a disagreement between the observed facts and the deductions from theory.

The helix must be wound on either an ebonite or a glass rod. In a subsequent chapter (Chap. VI.) we shall see how such a helix may be used as a cymometer for measuring the length of the electric waves radiated from an aerial wire as used in wireless telegraphy.

5. Direct, Inductive, and Electrostatic Coupling.—In establishing stationary electric waves upon wires, a high frequency electromotive force must be created in the wire at some point. This may be done in one of three ways, which are respectively called the *direct*, *magnetic* or *inductive*, and *electrostatic* or *dielectric coupling*.

The direct coupling consists in connecting a wire on which it is desired to establish the stationary waves directly to some point on an oscillating circuit which

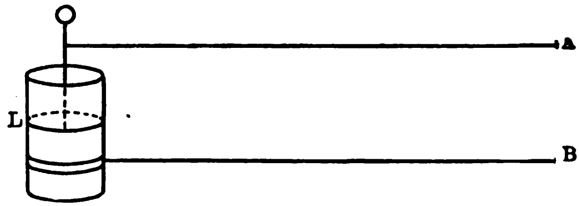


FIG. 13.—L, Leyden jar having inner and outer coatings, connected to resonant wires, A, B, of proper length. (Lodge.)

is rising and falling rapidly in potential; or we may connect two similar wires to two points on an oscillating circuit which vary oppositely in potential at the same moment. The simplest illustration of this is to connect to the inner and outer coating of a Leyden jar two long wires which are extended horizontally. When the jar is discharged (see Fig. 13) oscillations are set up and the coatings of the jar rise and fall rapidly in potential in opposite senses. Hence the wires have periodic electromotive impulses applied to their ends. Just as when a rope fixed at one end is jerked at the other and a hump or wave runs along it, so in the case of the electric wires a wave of potential travels along the wire and is reflected at the insulated end and runs back. The velocity with which the wave runs along the wire if straight is the velocity of light. We have seen (see Chap. II., equation 94) that the electrostatic capacity of a long circular-sectioned wire of diameter d and length l in free space is nearly given by the expression—

$$C = \frac{l}{2 \log_\epsilon \frac{2l}{d}} \text{ (in electrostatic units)} \quad . \quad . \quad . \quad . \quad . \quad (58)$$

In electromagnetic measure it is obtained by dividing the above expression by the numerical factor 9×10^{20}. Hence—

$$C = \frac{l}{9 \times 10^{20} \times 2 \log_\epsilon \frac{2l}{d}} \text{ (in electromagnetic units)} \quad . \quad . \quad . \quad . \quad (59)$$

The number 9×10^{20} is the square of the electromagnetic velocity u, identical with the velocity of light.

The inductance L of such a wire in electromagnetic units is given by—

$$L=2l\left(\log_\epsilon\frac{4l}{d}-1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

We may write the above equation in the form—

$$L=2l\left(\log_\epsilon\frac{2l}{d}+\log_\epsilon 2-1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

$$\text{or} \quad L=2l\left(\log_\epsilon\frac{2l}{d}-0\cdot3\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

If $\log_\epsilon\frac{2l}{d}$ is large compared with 0·3, we can say that for this wire—

$$L=2l\log_\epsilon\frac{2l}{d}$$

$$\text{and hence} \quad \sqrt{CL}=\frac{l}{3\times10^{10}}$$

$$\text{or} \quad \frac{1}{\sqrt{\frac{C}{l}\cdot\frac{L}{l}}}=3\times10^{10}=u$$

The left-hand side of the above equation is the reciprocal of the square root of the product of the capacity and inductance of the wire per unit of length, and this, we have seen, is the expression for the velocity with which the electric wave runs along the wire. The symbol u stands for the number 3×10^{10}, or the velocity of light in centimetres per second. Therefore the wave runs along the straight wire with the velocity of light. It is reflected at the open end, and if the frequency is adjusted so that the time taken by the wave to travel nearly four times the length of the wire is equal to one complete period of the electromotive force, then stationary waves are produced on the wire and a greatly exalted potential amplitude occurs at the open end.

If the frequency is 3, 5, 7, etc., times this fundamental frequency, then higher harmonic oscillations with nodes and antinodes of potential are formed on the wire.

One of the first investigators to notice and measure these stationary waves on wires so produced by direct coupling with the coatings of a Leyden jar was Sir Oliver Lodge.[7]

Let a condenser of capacity C be discharged through a low resistance circuit of inductance L, and let two long wires proceed parallel to each other and insulated in space from each other, one end of each being connected to one coating of the condenser. The capacity C_1 of the two wires in electrostatic units with respect to each other can be shown to be given by the equation[8]—

$$C_1=\frac{l}{4\log_\epsilon\frac{2D}{d}} \text{ (in electrostatic units)} \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where l is the length of each of the wires, D their distance apart, and d the diameter of each wire assumed to be of circular section.

Hence the capacity in electromagnetic units is—

$$C_1=\frac{l}{u^2 4\log_\epsilon\frac{2D}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

The high frequency inductance L_1 of these two wires, each of length l, and at a distance D cms. apart in air, has been shown (see Chap. II. § 3 (55)) to be given by the expression—

$$L_1=4l\left(\log_\epsilon\frac{2D}{d}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

[7] See O. J. Lodge, "On the Theory of Lightning Conductors," *Phil. Mag.*, August 1888, ser. 5, vol. 26, p. 217; also *The Electrician*, August 10, 1888, vol. xxi. p. 435.

[8] See "Handbook for the Electrical Laboratory and Testing Room," J. A. Fleming, vol. ii. p. 121.

Hence, multiplying (64) and (65), we have—

$$C_1 L_1 = \frac{l^2}{u^2}, \text{ or } u = \frac{1}{\sqrt{\frac{C_1}{l} \cdot \frac{L_1}{l}}} = 3 \times 10^{10}$$

The expression $\frac{1}{\sqrt{\frac{C_1}{l} \cdot \frac{L_1}{l}}}$ is the velocity of the wave along the wires, and is therefore equal to the velocity of light.

If, then, the capacity of the wires with respect to each other is small compared with that of the condenser, the discharge of the latter applies to the ends of the wires a periodic potential difference with a frequency $n = \frac{1}{2\pi\sqrt{CL}}$. In order that the stationary oscillations may be set up on the wire we must have for the fundamental oscillation such a length l for each wire that $u = 4ln = 3 \times 10^{10}$, and therefore—

$$l = \frac{\pi}{2} \times 3 \times 10^{10} \times \sqrt{CL}$$

In the above equation C is the capacity of the condenser in electromagnetic measure. If we express the condenser capacity in microfarads, and the inductance of the circuit through which it is discharging in centimetres, then we have the following very approximate formula :—

$$l = 1500\sqrt{C_{\text{mfds.}} \times L_{\text{cms.}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Thus, for instance, let a small Leyden jar having a capacity of about $\frac{1}{700}$ mfd. be discharged through a loop of thick copper wire about 4 mms. in thickness and 120 cms. long. The inductance of this circuit would be about 700 cms., and the corresponding length l of the resonant wire would be 15 ms., or nearly 50 feet. Such a length of wire, if attached to the jar inner coating, would have the fundamental oscillation set up on it, and at the far end we should have an antinode of potential and a strong brush discharge.

Hence to exhibit nodes and loops of potential on a straight wire we need higher frequency, and therefore smaller capacity and inductance in the discharge circuit.

To establish the first harmonic oscillation with one node at about one-third the length of the wire from the open end, we must have a frequency three times that required for the fundamental, that is, the product CL must be nine times as great. Accordingly, if C is made four times greater, L must be made 2¼ times greater than would be the case to excite the fundamental.

These higher harmonic oscillations are, however, better called into existence by using an arrangement due to Hertz, and modified by others, such as Sarasin and de la Rive and Lecher.

6. Creation of Stationary Electric Waves on Straight Wires.—A convenient method of establishing stationary electric waves on wires is one which Continental writers generally attribute to Lecher, and call the Lecher arrangement. As a matter of fact, it originated with Lodge and Hertz, whilst Sarasin and de la Rive gave it an improved form.

Hertz devised the form of oscillator we shall describe more in detail in the next chapter, which consists of two metal plates having rods attached, these rods being terminated in spark balls. The rods and plates are placed in one line, with the balls near together. They then constitute a condenser, with air as dielectric, which discharges across the gap when the potential difference of the plates, created by attaching the spark balls to the secondary terminals of an induction coil, exceeds a certain value determined by the width of the spark gap. This discharge sets up oscillatory currents in the rods and rapid oscillations of potential in the plates. If then two other plates are placed close to the plates of the oscillator, these first named having long parallel wires attached to them (see Fig. 14) with their ends insulated, we have the so-called Lecher arrangement.

Hertz used in some experiments only one extra plate and wire,[9] but Sarasin and de la Rive made the arrangement symmetrical by employing two plates and two parallel wires, whilst R. Blondlot showed that oscillations could be set up in a long wire circuit by coupling it electromagnetically with a circuit containing a condenser in which oscillations were created by a discharge across a spark gap.[10]

The double plate and parallel wire arrangement was also described by E. Lecher.[11] In this case the wires are said to be coupled electrostatically to the Hertz oscillator. When the secondary terminals of an induction coil are connected to the spark balls of the oscillator, and vibrations excited, the plates at the end of the wires are rapidly alternated in potential, and this, therefore, applies to the ends of the wires an alternating electromotive force; which in one wire may be represented by $V \cos pt$ and that to the other by $-V \cos pt$. These electromotive forces create electric waves of potential which travel along the wires, as above proved, with the velocity of light, and if the wires are of suitable length compared with the frequency of the oscillations, the interference of the direct and

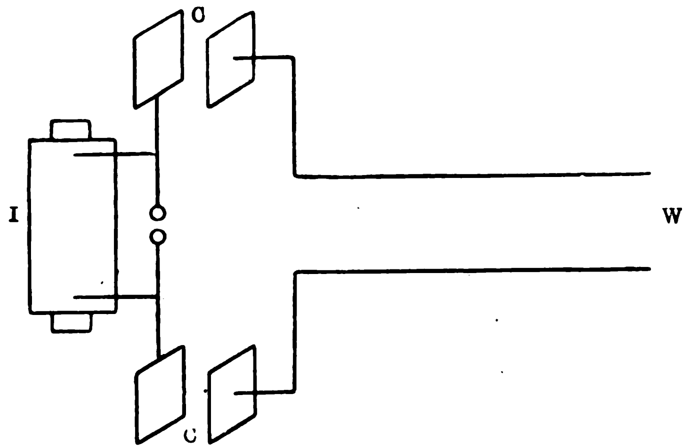


Fig. 14.—Lecher Arrangement for Creating Stationary Electric Waves on Parallel Wires. The two open circuits are coupled electrostatically. I, induction coil; C, C, condenser plates; W, parallel wires.

reflected waves establishes stationary waves of potential and current on the wires. Lecher, following a method suggested by Dragoumis,[12] employed a vacuum tube laid across the wires like a bridge to detect the position of the nodes. The tube may be without the usual electrodes, and contain rarefied nitrogen with a trace of turpentine vapour. The author has, however, found that a tube filled with rarefied neon is much better as an indicator.

When the vacuum tube is placed at a node of potential it remains dark, but when placed at an antinode it glows. Lecher also found that if the vacuum tube was placed permanently at the end of the parallel wires it could be caused to glow, or not to glow, by moving about on the parallel wires another transverse wire placed as a bridge across them. This, however, introduces a complication. At first sight it might appear that the positions at which the bridge wire must be placed not to affect the glow in the tube should depend solely upon the frequency of the

[9] See H. Hertz, "Electric Waves," English translation by D. E. Jones, p. 108, or *Wied. Ann.*, 1888, vol. 34, p. 551.

[10] See R. Blondlot, "Sur un nouveau procédé pour transmettre des ondulations électriques le long de fils metalliques," *Comptes Rendus*, 1892, vol. 114, p. 283.

[11] See E. Lecher, "Eine Studie über electrische Resonanzerscheinungen," *Wied. Ann.*, 1888, vol. 41, p. 850.

[12] See *Nature*, vol. 39, p. 548.

oscillator. Experiments by H. Rubens[13] showed, however, that the position of the bridge at which the glow of the vacuum tube was brightest or extinguished did not depend upon the time period of the oscillator.

This is only one instance out of a number in connection with this subject which shows that the phenomena cannot be rightly interpreted unless we bear constantly in mind that, as already explained, the oscillations of an open circuit radiator, like a Hertz oscillator, subside with great rapidity. They are damped chiefly owing to dissipation of energy by radiation. On the other hand, if a circuit is nearly closed, the oscillations in it are very persistent. Hence, if an open circuit radiator, like that of Hertz, acts on a nearly closed circuit, the radiator, when in action, merely administers to the receiver, or resonant circuit, a sort of blow or electromagnetic impulse at each discharge. The oscillations excited in the resonant circuit are those of its own free period, and not those forced on it by the radiator.

If we consider the bridge wire put across at any place transversely to the parallel wires, it creates two oscillation circuits. One of these consists of the two condensers, which are formed by the two plates of the Hertzian oscillator and the other two plates respectively in opposition to them, together with the rods of the Hertzian oscillator, and also the bridge wire and the included portion of the parallel wires. This circuit is denoted in Fig. 15 by the letters SC_1XYC_2. The

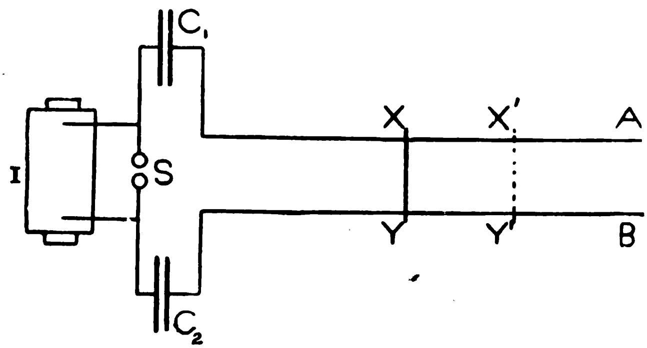


FIG. 15.—Lecher Wires bridged across and divided into Two Syntonic Circuits. I, induction coil; C_1, C_2, air condensers; A, B, Lecher wires; S, spark balls.

other circuit consists of the bridge wire and the remainder of the parallel wires, and is denoted by AXYB (see Fig. 15). The magnitude of these circuits is dependent upon the position of the bridge wire. Experiment shows, then, that what takes place is as follows: When the Hertzian oscillator is excited, oscillations take place in the circuit SC_1XYC_2, and these excite other oscillations in the circuit AXYB. The condition which must hold good for these last oscillations is, that the free extremities A and B of the wires must be antinodes of potential and of opposite sign. Hence, if we consider the wire AXYB stretched out straight, the oscillations of potential on it that are possible are indicated by the diagrams in Fig. 16. The length of the circuit AXYB must, therefore, be equal to some odd multiple of half the stationary wave-length, in order that we may have the necessary conditions fulfilled, which are, first, that the centre of the bridge wire XY, that is, the central point of the circuit AXYB, shall be a node of potential, and the two extremities A and B shall be antinodes, and at any instant have opposite potentials. Therefore the distance between two positions of the bridge wire XY, say at XY and X′Y′ (see Fig. 15), at which the vacuum tube shines equally brightly, is equal to one-half of the length of a stationary wave on the circuit AXYB. This wave-length is determined by the length of the wire itself, and not that of the other exciting circuit.

[13] See H. Rubens, *Wied. Ann.*, 1891, vol. 42, p. 154; also see J. J. Thomson, "Recent Researches in Electricity and Magnetism," p. 462.

On the other hand, to set up strong oscillations in the circuit AXYB, the frequency of the oscillations in the other circuit, SC_1XYC_2, must be so adjusted that the two circuits are in resonance. It follows, therefore, that to excite an oscillation on the wires, such that there shall be an antinode of potential at A and B, and one node only, at the centre of the bridge XY, the frequency n of the oscillations in the other circuit must be so adjusted that—

$$n = \frac{3 \times 10^{10}}{2\ (\text{the length of AXYB})} \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

The numeric which occurs in the denominator, viz. 2, is, in fact, a little more than 2, very nearly 2·5, because the length of the fundamental wave-length of a linear oscillator is 2·5 times its length nearly, and not simply twice its length.

If, then, the vacuum tube is placed across the ends AB of the parallel wires, and the bridge XY moved to different positions, the tube glows most brightly for

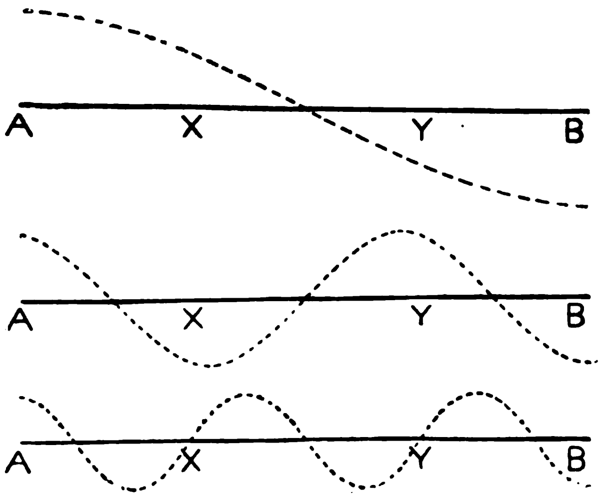


FIG. 16.—Possible Stationary Oscillations which can be created on the Section AXYB of a Lecher Circuit, as shown in Fig. 15. The distance of the dotted line from the firm line represents the potential amplitude at that point.

certain positions of the bridge. The condensers formed of the pairs of plates at the ends of each wire have a certain capacity, and this may be considered to be reckoned in its equivalent in length of straight wire. We might, in fact, replace the nearly closed oscillating circuit SC_1XYC_2 by an open circuit consisting of a wire bent twice at right angles.[14] The actual Lecher circuit is, therefore, equivalent to two pieces of wire, each bent twice at right angles, and having their central portions in common. Experiment, then, shows that if oscillations are set up in one part, they will create vigorous oscillations in the other part, if the lengths of the two circuits are in the ratio of any pair of the odd integer numbers. Experiments made by H. Rubens fully confirm this deduction,[15] and they show that we must not consider the phenomenon to consist simply in oscillations having the period due to the Hertz oscillator alone being forced on the long wires, and the bridge merely non-effective when placed at the nodes of potential so formed, but we have to consider the bridge as a common part of two circuits, in one of

[14] See E. Lecher, *Wied. Ann.*, 1890, vol. 41, p. 850.

[15] See H. Rubens, *Wied. Ann.*, 1889, vol. 37, p. 529. For an account of these experiments in English, see J. J. Thomson's book, "Recent Researches in Electricity and Magnetism," pp. 461-467.

which oscillations are set up with a certain period, whilst others are created in the adjacent circuit, provided this is made to be of such a length that one of its natural periods of oscillation is in agreement with those of the primary circuit.

Another method of setting up oscillations in wires is due to M. Blondlot.[16] In this a circular wire circuit, of which the inductance can be calculated, has inserted

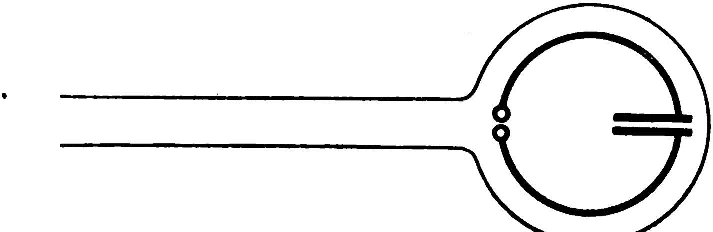


FIG. 17 —Blondlot's Mode of Inductive Coupling in an Open and Closed Oscillation Circuit.

in it a spark gap and a condenser of known capacity. Surrounding this circular circuit, and in close contact with it, but separated by an insulator, is a second circuit consisting of one long wire (see Fig. 17). The oscillations in the condenser circuit act inductively on the wire circuit, and if the length of this last is properly adjusted, create stationary oscillations in it. By means of the arrangement of this

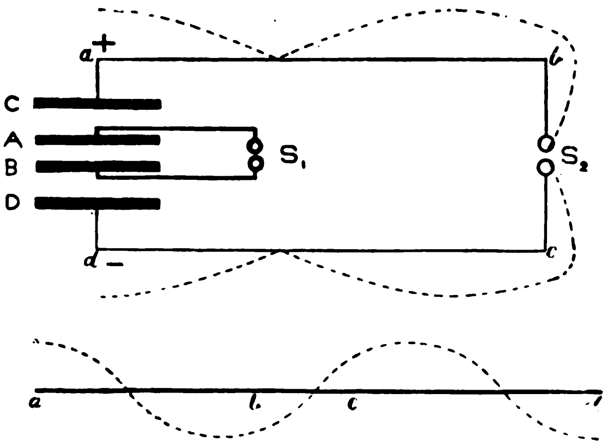


FIG. 18.—Trowbridge and Duane's Experiments on the Velocity of Electric Wave Propagation along Wires.

kind Blondlot was able to make a satisfactory determination of the velocity of propagation of an electric wave along a wire, and prove experimentally that it was identical with the velocity of light.

One of the most complete investigations on this matter was conducted by Professor Trowbridge and Mr. Duane.[17] In this work the authors set up a nearly

[16] Blondlot, *Journal de Physique*, ser. 2, vol. x. p. 549.

[17] See Trowbridge and Duane, "On the Velocity of Electric Waves," *Phil. Mag.*, August 1895, ser. 5, vol. 40, p. 211; also see J. A. Fleming, "The Alternate Current Transformer," vol. i. 3rd ed. p. 499.

closed wire circuit, terminating in condenser plates A, B (see Fig. 18), having a small spark gap, S_1, in a symmetrical position in it. The terminal plates were in apposition to two other condenser plates, C, D, which formed the condenser of a secondary oscillation circuit. Hence the two circuits were connected electrostatically. The frequency of the oscillations was then adjusted until stationary electric waves formed on the wire; the median point at the spark gap S_2 being a potential node, and one other node existing on each side of it between the central node and the terminal plates. This distribution of potential is indicated by the dotted lines in Fig. 18. By photographing the secondary spark the frequency of the oscillations was obtained, and by measuring the distance separating the two nodes lying on either side of the median node S_2 the semi-wave-length was obtained. The velocity of the wave then became known. The mean of a large number of closely concordant observations gave the velocity of the wave along the wire as $3{\cdot}003 \times 10^{10}$ cms. per second. This is very close to the best determination of the velocity of light.

It may be taken, therefore, as definitely proved, both by theoretical reasoning and by experiment, that the velocity with which an electric disturbance travels along a straight or slightly flexed metal wire is equal to the velocity of light.

If, however, the wire is closely coiled into a helix we have to treat the helix as if it were a linear conductor, and the velocity of the wave along it is inversely as the square root of the product of the capacity and inductance of the helix per unit of length.

7. Oscillations in an Earthed Aerial Wire.—A case of great practical importance arises when we consider the oscillations set up in a metal rod, like a lightning conductor, one end of which is in good connection with the earth and the rest of the wire is free, insulated and placed more or less vertically in the air. This wire is called an *aerial wire*, or *antenna*, or *Marconi aerial*, and is the essential element in telegraphy by electric waves on the Marconi system.

There are two ways in which we may set up the oscillations in this wire.

I. The wire may be cut at a point near the earth and two spark balls placed at this point. The secondary terminals of an induction coil are then attached to these balls. When the coil is in action the upper part of the wire is charged to a high potential and then discharged across the air gap. Just before discharge the upper portion of the wire has a certain capacity with regard to the earth and takes a certain charge. This discharge takes place across the spark gap, and as the spark has a low resistance the discharge is oscillatory, but greatly damped by reason of the rapid radiation of the energy.

The condition when the spark is passing is that the lower end of the aerial near the earth is at zero potential, or there is a potential node at this place. Since, however, there must be a current node at the upper end of the wire, there must be an antinode of potential there.

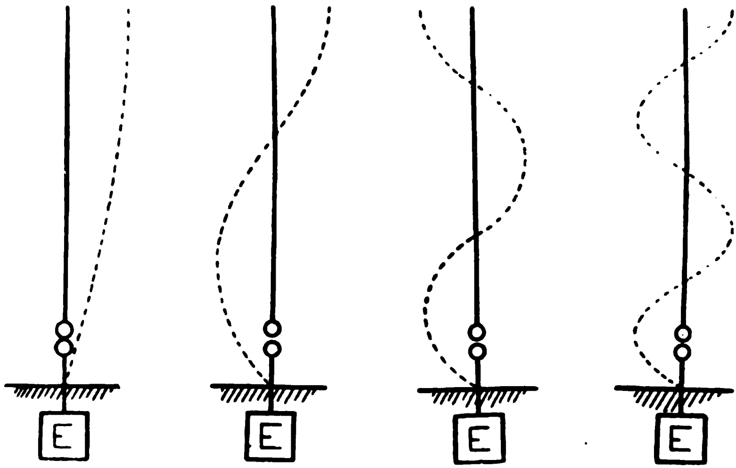
It is easily seen, therefore, without more calculation, in the light of explanations already given, that the fundamental electrical oscillation which can be excited on the wire is one in which the amplitude of the potential increases all the way up the wire from the earth end to the summit.

The first harmonic oscillation which can be excited is one having three times the frequency of the fundamental, and it has a node about one-third of the length of the aerial from the top. We may represent the amplitude of the potential variation by the ordinate of a dotted line, and the aerial itself by a firm line. In Fig. 19 the thick black vertical lines represent the earthed aerial wire, and the two small circles the spark balls, the lower one being connected to an earth plate, E. The horizontal distance of the dotted line from the firm line represents in diagrammatic form the potential variation up the aerial. If the oscillation is the fundamental oscillation, then the potential increases all the way up the aerial from the spark balls to the top. If the electric oscillation is the first harmonic, there is one potential node about one-third of the way from the top. If the oscillation is the second harmonic, there are two potential nodes and 2½ semi-waves of potential on the wire. Thus we have the distribution of potential, as follows:—

Oscillation taking place on the aerial.	Number of potential nodes, not including the one at earth.	Number of quarter waves of potential on the aerial.
Fundamental	0	1
1st harmonic	1	3
2nd ,, ...	2	5
3rd ,, ...	3	7
*n*th ,, ...	n	$(2n+1)$

The above distributions are represented in Fig. 19. The distribution of current in the aerial is such that antinodes of current occur at the same places where nodes of potential exist, and *vice versâ*.

Thus at the summit of the aerial, then, is a node of current or no current and



(*a*) (*b*) (*c*) (*d*)

FIG. 19.—Diagram representing the Fundamental (*a*) and Harmonic (*b*), (*c*), (*d*) Oscillations of Potential on a Vertical Earthed Antenna.

an antinode of potential or a maximum potential variation. At the base or earthed end there is a node of potential or no potential, but an antinode of current or a maximum value of the current. Electric current is, so to speak, pumped into and sucked out of the earth when the aerial is in oscillation. If there exist harmonic oscillations, then the current at different points in the aerial is not flowing in the same direction at the same time, but at two adjacent points may be moving in opposite directions.

Analytically, we may arrive at the above result as follows: Referring to equations (9) and (10), § 2 of this chapter, we have the expressions for the potential and current at any point in the wire having abscissa equal to x when traversed by electrical oscillations. Let us suppose that R and S are negligible in value compared with pL and pC, then we have—

$$V = a\epsilon^{+j\beta x} + b\epsilon^{-j\beta x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

$$I = \sqrt{\frac{C}{L}}(a\epsilon^{+j\beta x} - b\epsilon^{-j\beta x}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

If $x=0$, as at the lower end of the aerial, then $V=0$. Hence $b=-a$, and therefore—

$$I=\sqrt{\frac{C}{L}}a(\epsilon^{+j\beta x}+\epsilon^{-j\beta x}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

$$=2a\sqrt{\frac{C}{L}}\cos\beta x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

If, then, $x=l$, as at the upper end of the aerial, we have $I=0$, and therefore $\cos\beta l=0$.

Therefore, also, we must have $\beta l=\frac{m\pi}{2}$, where m is some odd integer. But $\beta=\frac{2\pi}{\lambda}$, where λ is the wave-length of the potential wave on the aerial. Accordingly, $\lambda=\frac{4l}{m}$, and the wave-lengths possible on the aerial are—

$$\lambda_0=4l, \quad \lambda_1=\tfrac{4}{3}l, \quad \lambda_2=\tfrac{4}{5}l, \text{ etc.} \quad . \quad . \quad (72)$$

Therefore the fundamental wave-length is four times the length of the aerial, and the higher harmonic oscillations have wave-lengths $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc., of the fundamental. If this simple theory held good, an aerial 100 feet high should radiate electric waves having a wave-length of 400 feet when the fundamental oscillations are set up on it, and waves of length 133 feet, 80 feet, 59 feet, etc., corresponding to the higher harmonic oscillations. Experiment, however, shows that the ratio $\frac{\lambda}{4l}$ is only unity for a single long very thin antenna wire, but that for a thicker wire or multiple wire the ratio $\frac{\lambda}{4l}$ is always somewhat greater than unity, and may reach 1·25 or more. According to the theory of stationary oscillations developed by Mr. H. M. Macdonald, the fundamental wave-length on the aerial should be 500 feet in length, or the quarter wave-length is 25 per cent. longer than the aerial. Also the wave-length of the first harmonic, instead of being equal to $\frac{4l}{3}$, is equal to $\frac{7l}{5}$ according to Macdonald's theory, and the wave-length of the second harmonic is $\frac{4l}{5}$ by both the simple and more complete theories. The value of the wave-length tends to become equal to $\frac{4l}{2m+1}$ for the mth harmonic.[18]

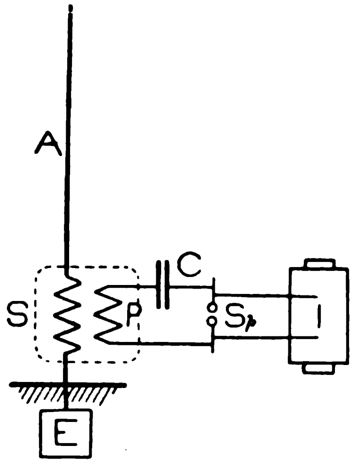


FIG. 20.—Inductively Coupled Antenna and Condenser Circuit. I, induction coil; S*p*, spark balls; C, condenser; P, primary coil of oscillation transformer; S, secondary coil; A, antenna.

II. The wire may have the oscillations induced in it by either a two coil or a single coil oscillation transformer.

In this case an air core transformer consisting, say, of two interlinked circuits has one circuit inserted in the aerial wire near the base between the aerial and the earth (see Fig. 20), and the other circuit has a condenser and spark balls included in it. When oscillations are set up in the condenser circuit they induce others in the aerial circuit, and the two circuits, open and closed, may be brought into

[18] See H. M. Macdonald, "Electric Waves," Adams Prize Essay, 1902, p. 112.

resonance with each other. This is called the *inductive coupling* of the aerial with an exciting circuit.

The electric oscillations must then be such that at the earthed end of the oscillation transformer circuit in series with the aerial we have a potential node, and at the summit of the aerial an antinode or maximum of potential.

If there is no other potential node the aerial has established on it its fundamental oscillation. The practical difficulty is to ascertain the equivalent length of the transformer secondary circuit in terms of the length of the aerial. If, for instance, the vertical aerial wire itself is 180 feet in height, and the oscillation transformer connected to it consists of a coil having a primary circuit of one turn of 4 feet in total length in circuit with the condenser and spark gap, and a secondary of ten turns of 40 feet in total length in series with the aerial wire, we require to know the length of the wave of the fundamental oscillation of such a complex aerial wire.

We cannot answer this question unless we can ascertain what length of simple straight aerial wire earthed at the bottom would have the same natural period of oscillation as the aerial with oscillation transformer inserted in it. This problem may be dealt with by a method suggested by Mr. Louis Cohen in an article published in the *Electrical World* of 30th January 1915, which is as follows :—

Let C_1 be the total distributed capacity of the aerial wire, and L_1 its total inductance. Let L_2 be the inductance of the oscillation transformer or other coil in series with it, the capacity of which may be neglected.

Then let C and L be the capacity and inductance per unit of length of the aerial wire, and let l be its length or height.

If then v and i are the potential and current at any point in the wire at a distance x reckoned from any origin, and if we neglect the resistance and leakance of the wire the equations (1) and (2) of this chapter become reduced to

$$L\frac{di}{dt} = -\frac{dv}{dx} \quad . \quad . \quad . \quad . \quad (73) \qquad C\frac{dv}{dt} = -\frac{di}{dx} \quad . \quad . \quad . \quad . \quad . \quad (74)$$

By differentiation of (1) and (2) we can separate the variables and arrive at the equations—

$$LC\frac{d^2i}{dt^2} = \frac{d^2i}{dx^2} \quad . \quad . \quad . \quad . \quad (75) \qquad LC\frac{d^2v}{dt^2} = \frac{d^2v}{dx^2} \quad . \quad . \quad . \quad . \quad . \quad (76)$$

A solution of these equations is—

$$i = (A_1 \cos pt + A_2 \sin pt)(B_1 \cos qx + B_2 \sin qx) \quad . \quad . \quad . \quad . \quad (77)$$

$$v = \sqrt{\frac{L}{C}}(A_1 \sin pt - A_2 \cos pt)(B_1 \sin qx - B_2 \cos qx) \quad . \quad . \quad (78)$$

$$\text{provided} \qquad p = \frac{q}{\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

This can easily be proved by differentiation of (77) and (78) and substitution in (75) and (76). The frequency (n) of the oscillations is given by the equation—

$$n = \frac{p}{2\pi} = \frac{q}{2\pi\sqrt{LC}} = \frac{ql}{2\pi\sqrt{L_1C_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

To obtain the values of the constants A_1, A_2, B_1, B_2 in (77) and (78), we take as origin the lower end of the aerial wire at which it is connected to the inductance coil and reckon x from this point upwards. Hence when $x = l$ we have $i = 0$, and when $x = 0$ we have $v = -L_2\frac{di}{dt}$, because then v is equal and opposite in sign to the reactance of the coil. Substituting these values in (77) and (78), we have—

$$B_1 \cos ql + B_2 \sin ql = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

$$B_2\sqrt{\frac{L}{C}} + B_1 pL_2 = 0$$

Hence, eliminating B_2/B_1 from the last two equations, we have—

$$\text{Cot } ql = pL_2\sqrt{\frac{C}{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

In virtue of (80) we can write (83) in the form—

$$\frac{\text{Cot } ql}{ql} = \frac{L_2}{L_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

We can now construct a table which shows the values of ql, cot ql, and (cot ql)/ql, as below—

ql.	Cot ql.	(Cot ql)/ql.
0	∞	∞
0·1	9·96	99·6
0·2	4·93	24·65
0·3	3·23	10·77
0·4	2·37	5·92
0·5	1·83	3·65
0·6	1·461	2·435
0·7	1·187	1·696
0·8	0·971	1·214
0·9	0·794	0·882
1·0	0·642	0·642
1·1	0·509	0·463
1·2	0·389	0·324
1·3	0·278	0·214
1·4	0·1724	0·123
1·5	0·0709	0·047
1·57 = $\pi/2$	0·	0·

From the above table we can construct, by interpolation, another table, which gives us the value of ql in terms of the ratio L_2/L_1—

L_2/L_1.	ql.	L_2/L_1.	ql.
0·	1·57 = $\pi/2$	1·50	0·735
0·05	1·51	2·00	0·650
0·10	1·426	2·50	0·59
0·20	1·314	3·00	0·545
0·30	1·219	3·50	0·510
0·40	1·142	4·00	0·475
0·50	1·078	5·00	0·430
0·60	1·022	6·00	0·40
0·70	0·968	7·00	0·370
0·80	0·925	8·00	0·350
0·90	0·892	9·00	0·325
1·00	0·855	10·00	0·31
		25·00	0·19

Hence we can obtain the natural frequency (n) of the whole antenna from the formula—

$$n = \frac{ql}{2\pi\sqrt{L_1C_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

if we have given the ratio L_2/L_1, and look out the corresponding value of ql in the previous table. The radiated wave-length λ in kilometres is obtained by dividing the frequency n by 300,000.

We cannot, in such a case, consider that the total equivalent inductance of the antenna is obtained by adding together that of the aerial wire (L_1) and that of the oscillation coil (L_2) in series with it, and then assume that the frequency would be given by the formula—

$$n = \frac{1}{2\pi\sqrt{C_1(L_1+L_2)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

or which is the same thing by $n = \frac{1}{2\pi\sqrt{C_1L_1}}\sqrt{\frac{L_1}{L_1+L_2}}$ . . . . (87)

For these formulæ, (86) and (87), would give a value for the frequency less than the true value given by (85). Thus, if we assume that the aerial wire has an inductance $L_1 = 100{,}000$ cms., and the oscillation transformer coil an inductance

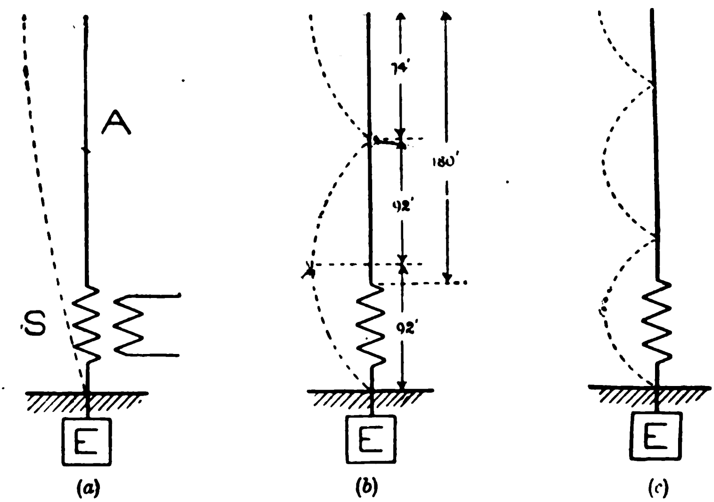


FIG. 21.—Fundamental and Harmonic Oscillations of Potential excited on an Inductively Coupled Antenna.

$L_2 = 50{,}000$ cms., then the value of L_2/L_1 is 0·5, and the value of ql is 1·078. On the other hand, the value of $\sqrt{L_1}/\sqrt{(L_1+L_2)}$ is 0·812, and the result would be nearly 27 per cent. too small.

Therefore a large error would be committed by employing formula (86) instead of (85).

Since we can determine experimentally by the cymometer[19] the wave-length of the wave emitted by any antenna, we can determine the frequency by dividing this wave-length, reckoned in metres, into the wave velocity in metres, viz. 3×10^8, and the reciprocal of this frequency will give us the value of $(2\pi\sqrt{C_1L_1})/ql$. Hence, if we calculate from the dimensions of the aerial wire its capacity C_1 and inductance L_1 by the aid of the formulæ given in Chap. II., we have the means of finding the value of ql, and therefore of L_2/L_1, from the above tables.

We can therefore determine the effective inductance of the jigger, or coil, in series with the aerial wire.

In making this measurement we must, however, be sure that the oscillations excited on the aerial wire are the fundamental ones.

In all cases in which we have distributed capacity and inductance along a wire it is possible, as we have already shown, to excite harmonic oscillations. If, however, one end of the jigger, or other coil in series with the aerial wire, is earthed,

[19] For a description of the cymometer and method of using it, see Chap. VI. of this Treatise.

then that end will always be at zero potential, or will be a potential node and current antinode or loop. On the other hand, the free or upper end of the aerial wire will be a potential antinode or loop, but a current node. Subject, however, to this condition, it is possible to excite on the aerial wire not only the fundamental oscillation, but also the first and higher harmonic oscillation. In Fig. 21 the amplitude of these potential oscillations for the fundamental and first two harmonics is represented by the distance of the dotted line from the black line, denoting the aerial wire.

An experimental examination of the relation of the lengths of wave emitted by a plain antenna to its length has been made by M. Ferrié (see M. C. Tissot, "Étude de la Résonance des Systèmes d'Antennes dans la Telegraphie sans fils." Gauthier-Villars. Paris, 1906).[20]

M. Ferrié has given some measurements taken for different antennæ. Thus, for instance, for single wires, 20 to 30 metres in length, the ratio of $\frac{\lambda}{4h}$, where λ is the wave-length and h the antenna length, is always rather less than unity. For branched antennæ it is greater than unity, 1·03 to 1·16 or more. The ratio increases with the number of branches and with their separation. It may amount to 1·27 or 1·3 for a many-branched antenna.

For a single antenna the above ratio tends to unity as the diameter of the wire decreases.

If the antenna is inductively excited and has the secondary circuit of an oscillation transformer inserted in it, then the ratio of $\frac{\lambda}{4h}$ may be very much greater than unity.

M. Tissot (*loc. cit.*) points out that the decrement or damping of the oscillations in an antenna in which they are excited by a spark discharge increases with the increase in the ratio of $\frac{\lambda}{4h}$. Hence, not only is the above fraction larger for a branched than for a single wire antenna, but the damping or radiation decrement of the branched antenna is larger than for the single wire antenna, both being excited in the same manner by charge and discharge across a spark gap.

The ratio between the wave-length λ emitted by a linear oscillator of Hertzian type of length l has been the subject of both theoretical and experimental investigations.

Professor H. M. Macdonald in 1902 deduced mathematically the relation $\lambda/l = 2{\cdot}53$ for the fundamental oscillation of a thin linear oscillator.[21]

Lord Rayleigh showed that for a thin wire the ratio must approximate to 2·0.[22]

J. A. Pollock gave a list of experimental measurements which vary between 2·5 and 2·0.[23] Thus Willard and Woodman give 2·48, Cole 2·52, Blake and Fountain 2·47, Webb and Woodman 2·3. Ives, however, found 2·04, and Anderson and Scarr 2·01.

The ratio, however, depends on the dimension ratio of the rod.

M. Abraham obtained a formula for the ratio λ/l for an ellipsoidal oscillator having a major axis of length l and minor axis of length $2b$ as follows:

$$\lambda/l = 2(1 + 5{\cdot}6e^2) \qquad \text{where} \qquad \frac{1}{e} = 4 \log_e \frac{l}{b}$$

For a thin rod the ratio approximates to 2·0 as stated by Rayleigh and by Pollock.

[20] See also M. Ferrié, *Comptes Rendus*, 1903, p. 128; or *Jour. Soc. Franc. Phys.*, April 8, 1904.
[21] See H. M. Macdonald, "Electric Waves," 1902, p. 111.
[22] See Lord Rayleigh, *Phil. Mag.*, 1904, vol. viii. p. 105, or 1913, vol. xxv. p. 1.
[23] J. A. Pollock, *Phil. Mag.*, 1916, vol. xxi. p. 96.

CHAPTER V

ELECTRIC RADIATION

1. The Electromagnetic Medium and its Properties.—The fact that electric oscillations produced in one circuit can set up secondary oscillations in another circuit at a distance forces upon us the consideration of the nature of the machinery by which this is effected.

Notable investigators of natural phenomena, from Newton to Maxwell, have strongly expressed their conviction that actions of this character make it necessary for us to postulate some interconnecting medium, or else we have to take refuge in the bare assumption, repugnant alike to common sense as well as philosophic thought, that physical effects can be produced at a distance without the aid of any intervening mechanism. In his well-known second letter to Bentley, Newton, writing on the attraction of matter, said

"that gravity should be innate and essential to matter, so that one body can act upon another at a distance through a vacuum without the mediation of anything else by which their action may be conveyed from one to another, is to me so great an absurdity that I believe no man who has a competent faculty of thinking in physical matters can ever fall into it."

The propagation of light with a finite velocity through interstellar space has compelled us to accept the hypothesis, with some considerable body of arguments in its favour, that space is occupied by a medium called the æther, capable of transmitting undulations, which, when falling on the retina of the eye, produce the sensation of light. Ampère, Faraday, Henry, and others, moreover, long ago arrived at the conclusion that electric and magnetic phenomena, especially the facts of electric and magnetic induction, demanded also the assumption of a special medium for their explanation. Maxwell has remarked that it is clearly unphilosophical to postulate more than one æther. Hence the work done by Huyghens, Arago, Fresnel and their followers in consistently deducing observed optical effects from the assumed properties of the luminous æther called for a corresponding definite effort on the part of electricians. The work began when Clerk Maxwell took up the study of Faraday's experimental researches, and endeavoured to discern whether the ideas of Faraday, which were then not in accord with current views, were capable of being translated into mathematical language, and made the foundation of a new method of regarding electrical facts.

The publication in 1865 of James Clerk Maxwell's paper, "A Dynamical Theory of the Electromagnetic Field," marks a great epoch in the history of scientific thought.[1] In that paper Maxwell applied to the facts of electromagnetism certain equations and methods of analysis which the French mathematician, Joseph Louis Lagrange, had employed in formulating the dynamical relations of the kinetic and potential energies, the velocities and momenta of various parts of any system of interconnected moving material masses. Maxwell saw that in electric and magnetic actions we have energy involved, and that this energy takes two forms, electrostatic and electrokinetic, which have a close similarity to energy of strain and motion. Moreover, the interconvertibility of various forms of energy, and the fact that we can invaluate them in their equivalent in motional energy, whilst indicating that all energy is probably in the ultimate issue kinetic in nature, affords at the same time logical ground for applying the methods of Lagrange to the phenomena of electricity and magnetism.

[1] Maxwell sent this paper to the Royal Society on October 12, 1864. It was read on December 8, 1864, and printed in the *Philosophical Transactions of the Royal Society* for 1865, vol. 155, p. 419.

The systematic examination that has been made of the relations of the electric and magnetic quantities shows us that we can co-ordinate them in a scheme of related magnitudes, each one corresponding to some well-known dynamical equivalent. Thus, corresponding to the fundamental dynamical quantities, *e.g.*, mass, velocity, acceleration, momentum, force, energy, and activity or power, we can place in contiguity such electrical quantities as inductance, current, rate of current change, total magnetic flux, electromotive force of self-induction, current energy, and rate of dissipation of current energy. In parallel with mechanical quantities such as stress, strain, elastic yielding, strain energy, we can place analogous electric and magnetic quantities such as electric and magnetic forces, displacement or magnetic flux, dielectric constant or magnetic permeability, and electric or magnetic energy.

We may, as Heaviside and other writers have shown, draw up many consistent schemes of analogy between mechanical and electromagnetic quantities, but we must beware of enslaving ourselves to any one particular set of mechanical similarities. Analogies of this kind are often like mountain paths, which begin in well-beaten routes, but sooner or later, if followed up too far, terminate in a barren region. There remains, however, the fact that corresponding to two well-recognized forms of mechanical energy, namely, motional energy measured by half the product of momentum and velocity, and configurational energy measured by half the product of stress and strain, we have a duplex system of electric and magnetic quantities which are for the most part circuital or manifested in circuits. Thus we have *two circuits*, the electric and magnetic; *two physical effects* produced in these, *electric strain* or *displacement* (D) and *magnetic flux* (B); *two agencies* producing these effects, the electric and magnetic forces; *two specific physical qualities* of the circuits corresponding thereto, namely, *dielectric constant* and *magnetic permeability*, or, as Mr. Oliver Heaviside calls them, *permittivity* and *inductivity*; *two line integrals* of electric and magnetic force called respectively *voltage* (V) and *gaussage* (G); *two forms of energy*, electric and magnetic; and *two* corresponding *forms of activity* or power.

Moreover, we have a curious interlinking of these quantities when circuital, best expressed by *two circuital laws* which symbolically and in rational units are stated as follows[2]:—

$$-\dot{B} = V \text{ and } \dot{D} = G,$$

where the *dot* over the symbol signifies time differentiation, or $\frac{d}{dt}$.

The first of these equations is merely the symbolical expression of Faraday's law, that the electromotive force or line integral of electric force round any circuit is numerically equal to the time rate of decrease of the magnetic flux through it ($-\dot{B}$); and the second is the simplest expression of Maxwell's principle that the time rate of change of electrical displacement ($\dot{D}$) through any circuit is measured by the gaussage or line integral of magnetic force round the circuit.

In cases when the circuits are formed of certain kinds of matter we have also to introduce two other conceptions, namely, *electric current* and *magnetization*, which are produced when the two circuits possess *conductivity* or *susceptibility*, and when these qualities are present the fundamental equations take a more general form, which, expressed by Heaviside in rational units, are—

$$-(\dot{B} + \dot{M}) = V \text{ and } \dot{Q} + \dot{D} = G,$$

where M stands for magnetization, and Q for a quantity of electricity moved non-elastically past any section of the circuit.[3]

Out of this double-stranded system of interlinked quantities and their fundamental relations, it follows that in considering the measurement of any one electric or magnetic quantity we can arrive at the same point by two paths, starting either

[2] For a fuller information of Mr. Heaviside's system of rational electric and magnetic units, the reader must be referred to his book, "Electromagnetic Theory," vol. i. chap. ii.

[3] The systematic formulation of these circuital laws, as well as a fuller appreciation and elucidation of Maxwell's views, have been assisted of late years in a remarkable degree by the writings of Mr. Oliver Heaviside.

from an electric or magnetic definition. Thus, we may consider an electric current to be due to a series of successive discharges of electric strain by a conductor, or we may consider it to arise from the movement of a magnet to or from a closed conducting circuit. In the one case our measure of current involves the quantity K, or the dielectric constant of the medium in which the electric strain takes place. In the other case it involves μ, or the magnetic permeability of the medium which encloses the magnet and the circuit.

In every case in which this double measurement of the same quantity can be carried out, the numerical ratio of the two measurements when conducted in absolute or dynamical units gives us a number which it can be shown represents either the geometrical mean of K and μ, viz. $\sqrt{\mu K}$, or its reciprocal $\frac{1}{\sqrt{\mu K}}$, or their squares μK and $\frac{1}{\mu K}$. In other words, it gives us a numerical value for the product of these two qualities, but it does not tell us their individual values for any medium.[4]

An immense number of investigations in the last fifty years have shown that the product μK in the centimetre-gramme-second system of absolute units for air or a good vacuum closely approximates to a value $\frac{1}{9 \times 10^{20}}$, and is identical numerically with the square of the reciprocal of the velocity of light. This numeric 3×10^{10} will be hereafter denoted by the symbol u. A list of some of the principal determinations of this unitary ratio called u, obtained prior to 1897, is given in Table I.

A résumé of all determinations of the value of u previous to 1900 was prepared by H. Abraham for the International Congress of Physics which met at Paris in that year (see *Congrès International de Physique*, 1900, *Rapports II.*, p. 247). Abraham states that he considers the most accurate results to be as follows :—

Himstedt	$3{\cdot}0057 \times 10^{10}$
Rosa	$3{\cdot}0000 \times 10^{10}$
Thomson and Searle	$2{\cdot}9960 \times 10^{10}$
H. Abraham	$2{\cdot}9913 \times 10^{10}$
Pellat	$3{\cdot}0092 \times 10^{10}$
Hurmuzescu	$3{\cdot}0010 \times 10^{10}$
Perot and Fabry	$2{\cdot}9978 \times 10^{10}$
Mean value	$3{\cdot}0001 \times 10^{10}$

Abraham considers that this mean of the best results, viz. 3×10^{10}, probably does not differ from the true value by more than 1 part in 1000. The most recent result is that of E. B. Rosa and N. E. Dorsey (see *Bulletin of the Bureau of Standards*, Washington, U.S.A., May 20, 1907, vol. 3), which gave the value $2{\cdot}9963 \times 10^{10}$.

The above values are all values in air, but if expressed for vacuum require to be increased by 55 parts in 1,000,000.

A glance at the above table shows that the numerical value of u or of $\frac{1}{\sqrt{\mu K}}$ for air or vacuum is nearly identical with that of the velocity of light through empty space when measured in centimetres per second.

The best measurements of the velocity of light are those of—

Michelson (1885)	$=2{\cdot}99853 \times 10^{10} \frac{\text{cms.}}{\text{sec.}}$
Newcomb (1883)	$=2{\cdot}99860 \times 10^{10}$,,
Perrotin (1902)	$=2{\cdot}99860 \times 10^{10}$,,

[4] For a more complete discussion of this matter, the reader is referred to the author's treatise on "The Alternate Current Transformer," vol. i. p. 354 (Messrs. Benn Bros., 8 Bouverie Street, London).

Weinberg, discussing the results, comes to the conclusion that the most probable value of the velocity of light in vacuo is $2{\cdot}99852 \times 10^{10} \frac{\text{cms.}}{\text{sec.}}$ with an accuracy of 1 part in 10,000.

Hence we may say that the unitary ratio expressed in the same units of length and time is identical with the velocity of light within 1 part in about 3000.

The fact that the ratio between electric and magnetic quantities measured on two systems is so closely connected with the numerical value of the velocity of light, is a strong argument that there must be some common basis for optical and electromagnetic phenomena.

TABLE 1

TABLE OF OBSERVED VALUES OF u IN CENTIMETRES PER SECOND

Year.	Name.	Reference.	Electric quantity measured.	u in centimetres per second.
1856	Weber and Kohlrausch	*Electrodynamische Maasbestimmungen und Pogg. Ann.*, xcix., Aug. 10, 1856	Quantity .	$3{\cdot}107 \times 10^{10}$
1867 1868	Lord Kelvin and W. F. King	"Report of British Association, 1869," p. 434; and "Reports on Electrical Standards," F. Jenkin, p. 186 .	Potential .	$2{\cdot}81 \times 10^{10}$
1868	Clerk Maxwell .	*Phil. Trans. Roy. Soc.*, 1868, p. 643	,,	$2{\cdot}84 \times 10^{10}$
1872	Lord Kelvin and Dugald M'Kichan	*Phil. Trans. Roy. Soc.*, 1873, p. 409	,,	$2{\cdot}89 \times 10^{10}$
1878	Ayrton and Perry .	*Journal of the Society of Telegraph Engineers*, vol. viii. p. 126	Capacity .	$2{\cdot}94 \times 10^{10}$
1880	Lord Kelvin and Shida	*Phil. Mag.*, 1880, vol. x. p. 431	Potential .	$2{\cdot}955 \times 10^{10}$
1881	Stoletow . .	*Soc. Franc. de Phys.*, 1881	Capacity .	$2{\cdot}99 \times 10^{10}$
1882	F. Exner . .	*Wien. Ber.*, 1882 . .	Potential .	$2{\cdot}92 \times 10^{10}$
1883	J. J. Thomson .	*Phil. Trans. Roy. Soc.*, 1883, p. 707	Capacity .	$2{\cdot}963 \times 10^{10}$
1884	Klemencic . .	*Proc. of the Soc. of Telegraph Engineers*, 1887, p. 162	,,	$3{\cdot}019 \times 10^{10}$
1888	Himstedt . .	*Electrician*, Mar. 23, 1888, vol. xx. p. 530	,,	$3{\cdot}007 \times 10^{10}$
1888	Lord Kelvin, Ayrton, and Perry .	British Association, Bath; and *Electrician*, Sept. 28, 1888	Potential .	$2{\cdot}92 \times 10^{10}$
1888	Fison . . .	*Electrician*, vol. xxi. p. 215; and *Proc. Phys. Soc. Lond.*, June 9, 1888	Capacity .	$2{\cdot}965 \times 10^{10}$
1889	Lord Kelvin. .	Royal Institution Lecture, Feb. 8, 1889	Potential .	$3{\cdot}004 \times 10^{10}$
1889	Rowland . .	*Phil. Mag.*, 1889 . .	Quantity .	$2{\cdot}981 \times 10^{10}$
1889	E. B. Rosa . .	*Phil. Mag.*, 1889 . .	Capacity .	$3{\cdot}000 \times 10^{10}$
1890	J. J. Thomson and Searle	*Phil. Trans.*, 1890 . .	,,	$2{\cdot}995 \times 10^{10}$
1897	M. E. Maltby	*Wied. Ann.*, 1897 . .	Alternating currents	$3{\cdot}015 \times 10^{10}$

Since light is propagated from place to place with a finite velocity, and as the facts of interference prove it to be a wave motion, theorists had been compelled to assume the existence of a space-filling medium possessing two qualities—first,

inertia, in virtue of which kinetic energy is exhibited by parts of the medium in motion, and secondly, an *elastic resistance* to strain or distortion of some kind, in consequence of which potential energy is stored up in the distorted medium, these two properties being the essential qualities of a medium capable of undulation.

The study of physical optics resolved itself, then, into a dynamical analysis of the phenomena, and efforts to explain them by the hypothesis of an æther possessing inertia and capable of some elastic distortion in virtue of which waves could be propagated through it.

Maxwell's electromagnetic theory starts from a more general point of view. We know nothing about the inner structure of the æther or the kind of distortions it can experience. We do, however, know that in a dielectric, even empty space, we have present at any point the two qualities permeability and dielectric constant or inductivity, and also that in electric and magnetic phenomena we are concerned with two physical effects, called respectively magnetic flux and electric displacement or strain. When these conceptions and fundamental relations of the electric and magnetic quantities had been mathematically expressed, Maxwell found that they led to equations of the same type and form as those which express the propagation of an undulation through a continuous medium, and they indicated that if the effects we call magnetic flux or electric displacement are created at one point in space, they are propagated in all directions with the velocity of light in that dielectric.

Starting from fundamental electric and magnetic facts, it has been found possible to build up a theory which embraces not only electrical but optical phenomena, and shows them to be manifestations of the properties of one single medium, modified, however, profoundly in certain localities by the presence of that which we will call gravitative matter. This comprehensive theory is generally known as Maxwell's theory, and it will be necessary to consider it at least in outline.

Broadly speaking, it may be held to be, that there exists a space-filling æther or medium, not, as far as we know, composed of gravitative matter, the principal qualities it possesses being those in virtue of which two physical states can be established in it, one called Electric Strain and the other Magnetic Flux. From the known relations between these states it can be shown that when either of them is established in one place it will spread or diffuse with a velocity equal to that of light. The inference is that optical phenomena are electromagnetic in nature, and must be interpreted in terms of the known electric and magnetic properties of dielectrics, and not by the assumption of mechanical qualities which cannot be verified.

2. Maxwell's Theory of Electromagnetic Phenomena.—Since electric and magnetic forces are vector quantities having direction as well as magnitude at every point in the electric and magnetic field, and since they are obviously related to each other, we must in the first place consider some qualities of vectors generally.

Let us suppose any closed curve described in a region in which there is a distribution of a certain vector quantity, E. Divide the curve up into elements of length, ds, and at every point of the curve resolve the vector E denoting the quantity considered at that point into components along these elements of length. Then the sum or integral of all such quantities as $E \cos \theta ds$, where θ is the angle between the direction of ds and the direction of the vector E at its centre, is called the *line integral of* E *along the curve*. In taking this integral, the sign of the product must be reckoned positive when the direction of the vector is in the same direction as the movement round the curve, and negative when it is against it. In many cases this line integral round a closed curve drawn in the field is zero, and the vector is then said *to have a potential.*

Thus if E denotes the electric force in the electric field near an electrified body, and if $\int E \cos \theta ds$ is zero for any closed curve drawn in the field, then the electric force is said to be derived from, or to have a potential.

In other cases this line integral may not be zero, but have a finite value independent of the form of the path, which is increased n times by taking the

line integral n times round the circuit. This is the case with the magnetic field round a conductor conveying an electric current; for if the conductor carries a current, C, the line integral of the magnetic force taken along a line embracing the circuit can easily be shown to be equal to $4\pi C$ for a single journey round, and to $4\pi n C$ for n journeys round the closed line. The vector is then said *to have a many-valued potential.*

On the other hand, the line integral may have a value which is dependent upon the form of the path. If the area enclosed by the path is small and lies in one plane, the ratio of the quotient obtained by dividing the line integral by the area of the path may have a finite limit, and in this case the limiting value is called the *curl of the vector in that plane.*

At any one point in the field there is some plane in which this ratio is a maximum, and this maximum value is generally called *the curl* of the vector.

A curl is itself a vector, and may be resolved into component curls. Very often the curl has a physical meaning with respect to the original vector which gave rise to it.

Thus it can be shown that if the vector considered is the velocity of the particles of a liquid mass at various points, then the curl denotes twice the angular velocity with which a very small sphere of the liquid, which may be supposed to enclose and coincide with the particle considered, is rotating.

Suppose we consider any vector, E, which has rectangular components, X, Y, and Z, along three rectangular axes, x, y, and z.

If, then, we describe a little rectangle on each co-ordinate plane, and take the line integral round it, we shall obtain the rectangular component of the curl. Thus on the plane of xy we have the line integral round the parallelogram $dx \,.\, dy$ with one corner at the origin given by—

$$Xdx + \left(Y + \frac{dY}{dx}dx\right)dy - \left(X + \frac{dX}{dy}dy\right)dx - Ydy$$

which is equal to $$\left(\frac{dY}{dx} - \frac{dX}{dy}\right)dx \,.\, dy$$

The above conclusion follows at once from Taylor's theorem that if y is the ordinate at any curve at abscissa x, then the ordinate corresponding to abscissa $x+dx$ is $y+\frac{dy}{dx}dx$. Hence the component curl on the plane xy is $\left(\frac{dY}{dx} - \frac{dX}{dy}\right)$, and similarly the component curls in the planes yz and zx are respectively—

$$\left(\frac{dZ}{dy} - \frac{dY}{dz}\right) \text{ and } \left(\frac{dX}{dz} - \frac{dZ}{dx}\right)$$

There is, then, a connection between a vector and its curl, the statement of which constitutes an important theorem. If we have any surface bounded by any line described in a field in which a certain vector quantity is distributed, we may cut up this surface into small elements of area. It follows by the above definition of the curl that the product of the curl for each element of area and the size of that area is equal to the line integral of the vector round the boundary of the element. Hence if we take the line integrals round all the elements, the line integral for each common boundary of any pair of elements of the area is taken twice, once negatively and once positively, and the products cancel each other. Accordingly, it is easy to see that *the line integral of a vector round the boundary of the whole of the surface is equal to the surface integral of its curl over the whole of the surface.*

Conversely, if this relation holds good between two quantities, viz. that the line integral of one is equal to the surface integral of the other, we are enabled to recognize by it that the one quantity bears to the other the relation of vector and corresponding curl.

Let us consider, then, the relation between the electric and magnetic forces and their effects, viz. the electric displacement and magnetic flux. Let **E** be the electric force at any point in a dielectric and **H** the magnetic force. Let **D** be

the corresponding electric strain or displacement and B the magnetic flux. Then in the ordinary system of units we have—

$$\mathsf{B} = \mu\mathsf{H} \quad . \quad . \quad . \quad . \quad . \quad (1) \qquad \mathsf{D} = \frac{K}{4\pi}\mathsf{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where μ is the magnetic permeability and K is the dielectric constant.[5]

If, then, we describe any closed line in a conductor, and make the magnetic flux through it vary with time, we have produced in the circuit an electromotive force. In accordance with Faraday's law, the time rate of change of the surface integral of the magnetic flux through this area is a measure of the electromotive force created in the circuit. This electromotive force is the line integral of the electric force E. Hence the line integral of E round the boundary is equal to the surface integral of $-\frac{d\mathsf{B}}{dt}$ (or of $-\dot{\mathsf{B}}$, as we may write it) over the area. Therefore it follows that $-\dot{\mathsf{B}}$ is the curl of E, or the time rate of decrease of the magnetic flux is the curl of the electric force.[6]

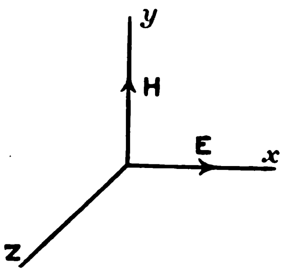


FIG. 1.—Electric and Magnetic Vectors at Right Angles.

But since $\mu\mathsf{H} = \mathsf{B}$, we may write the above equation in the form—

$$-\mu\dot{\mathsf{H}} = \text{curl } \mathsf{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Again, if round an electric current we describe any closed line, the line integral of the magnetic force along that line is equal to $4\pi C$, where C is the total electric current through the closed line. Maxwell laid down as a fundamental principle that when change of electric displacement through a dielectric takes place, the change, *whilst taking place*, produces all the magnetic effect of a current. Hence, if we denote the rate of change of electric displacement with time by the symbol $\dot{\mathsf{D}} = \frac{d\mathsf{D}}{dt}$, then the total displacement is the surface integral of D, and the effective current is the surface integral of $\dot{\mathsf{D}}$. Accordingly, when dealing with a pure dielectric, we may, in accordance with Maxwell's postulate, consider that the time rate of change of the total displacement produces a magnetic force embracing it, and that the line integral of this magnetic force is equal to 4π times the total displacement current surrounded. Hence the surface integral of $4\pi\dot{\mathsf{D}}$ is equal to the line integral of H, or $4\pi\dot{\mathsf{D}}$ must be the curl of H. But since $\mathsf{D} = \frac{K}{4\pi}\mathsf{E}$, it follows that $4\pi\dot{\mathsf{D}} = K\dot{\mathsf{E}}$, and accordingly $K\dot{\mathsf{E}}$ is the curl of H, or—

$$K\dot{\mathsf{E}} = \text{curl } \mathsf{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

[5] In Mr. Oliver Heaviside's system of rational units the 4π would be omitted and the relation between D and E expressed by the equation D = KE.

[6] Continental mathematicians frequently employ the symbol "rot," an abbreviation for rotation or rotator, to denote the operator above called the "curl." Thus rot E is equivalent to curl E.

Putting together equations (3) and (4), we see that there is a direct and a cross relation between **E** and **H**, as follows :—

$$\left.\begin{array}{ll} 4\pi\mathbf{D}=\mathrm{K}\mathbf{E} & \mathbf{B}=\mu\mathbf{H} \\ 4\pi\dot{\mathbf{D}}=\mathrm{K}\dot{\mathbf{E}}=\mathrm{curl}\ \mathbf{H} & -\dot{\mathbf{B}}=-\mu\dot{\mathbf{H}}=\mathrm{curl}\ \mathbf{E} \end{array}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

These equations are the fundamental equations connecting the so-called forces, fluxes, and qualities of the dielectric medium.

Suppose that we apply them to a very simple case. Let the vector **E** be everywhere parallel to itself and its direction taken as the x axis. Let the vector **H** be at right angles to **E** and its direction taken as the y axis (see Fig. 1).

Also let the value of **E** and **H** increase as we proceed along the axes away from the origin.

To calculate the curls of these forces we have to take line integrals of them round elementary areas, $dxdy$, $dydz$, $dzdx$, in the counter-clockwise directions.

Then the curl of **E** in the plane xz is $\frac{d\mathbf{E}}{dz}$, and in the plane of yx is $-\frac{d\mathbf{E}}{dy}$, and in the plane of yz it is zero. Similarly, the curl of **H** is zero for the plane xz. For the plane yz it is $-\frac{d\mathbf{H}}{dz}$, and for the plane yx it is $\frac{d\mathbf{H}}{dx}$.

Consider the plane perpendicular to the direction of **H**, viz. the zx plane. The curl of **E** for that plane is $\frac{d\mathbf{E}}{dz}$. Also consider the plane perpendicular to the direction of **E**, viz. the yz plane.

The curl of **H** for that plane is $\frac{d\mathbf{H}}{dz}$. Therefore, substituting in the general equations (5), we have—

$$\mathrm{K}\frac{d\mathbf{E}}{dt}=-\frac{d\mathbf{H}}{dz} \qquad -\mu\frac{d\mathbf{H}}{dt}=\frac{d\mathbf{E}}{dz} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Differentiate these equations with respect to z, and with respect to t, and equate results. We obtain—

$$\frac{d^2\mathbf{H}}{dz^2}-\mu\mathrm{K}\frac{d^2\mathbf{H}}{dt^2}=0 \quad . \quad . \quad . \quad (7) \qquad \text{and}\ \frac{d^2\mathbf{E}}{dz^2}-\mu\mathrm{K}\frac{d^2\mathbf{E}}{dt^2}=0 \quad . \quad . \quad . \quad . \quad (8)$$

The above differential equations have general solutions of the form—

$$\mathbf{H}=f_1(z-ut)+f_2(z+ut) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$
$$\mathbf{E}=f_3(z-ut)+f_3(z+ut) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where f_1, f_2, f_3, and f_4 are some functions of z and t and $u=\frac{1}{\sqrt{\mu\mathrm{K}}}$.

These are well-known equations which indicate that **E** and **H** are wave motions propagated through space with a velocity u, since they remain unchanged if for z we put $z+z'$ and for t we put $t+t'$, provided $\frac{z'}{t'}=u$. In other words, the $\frac{z'}{t'}=u$ electromagnetic disturbance reaches a point at a distance z' further on in a time t', such that $z'=ut'$, and u is therefore the velocity of propagation. The matter may be put verbally thus: The characteristic of a wave of any kind is that the same physical events are taking place at the same moment at places separated by a distance called a wave-length. Also the changes are periodic or cyclical both in space and in time. It is obvious from equations (9) and (10) that the periodic quantities **E** and **H** are in step or in phase with each other, both varying periodically and arriving at their maximum values at the same instant.

The above equations may be generalized for space of three dimensions, as follows :—

Let X, Y, and Z be the components of electric force **E** at any point, measured in electrostatic units, and let α, β, and γ be the components of the magnetic force **H** at the same point measured in electromagnetic units. Then, since the unit of electrostatic electromotive force is 3×10^{10} larger than the unit of electromagnetic

electromotive force, we can write the general equations connecting X, Y, and Z with α, β, and γ for any dielectric medium of dielectric constant K and permeability μ as follows: where A stands for $\frac{1}{u}$ and $u=3\times 10^{10}$, or is the velocity of light in centimetres per second and the unitary ratio. We have then—

$$\left.\begin{aligned} A\mu\frac{d\alpha}{dt}&=\frac{dZ}{dy}-\frac{dY}{dz}\\ A\mu\frac{d\beta}{dt}&=\frac{dX}{dz}-\frac{dZ}{dx}\\ A\mu\frac{d\gamma}{dt}&=\frac{dY}{dx}-\frac{dX}{dy}\end{aligned}\right\} \quad . \quad . \quad (11) \qquad \left.\begin{aligned} AK\frac{dX}{dt}&=\frac{d\beta}{dx}-\frac{d\gamma}{dy}\\ AK\frac{dY}{dt}&=\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\\ AK\frac{dZ}{dt}&=\frac{d\alpha}{dy}-\frac{d\beta}{dx}\end{aligned}\right\} \quad . \quad . \quad (12)$$

The above equations are in the form given by Hertz, and in writing them he follows conventions as to directions of axes, as follows. Suppose the origin of the co-ordinates to be within the head of the reader, then the x axis is directed straight away from you horizontally, the direction of the z axis is straight up, and the direction of the y axis is to the right hand. This plan differs from the usual English plan in that the z and y axes have changed places.

Suppose that we limit our consideration to space occupied only by æther, and take the permeability and dielectric constant to be unity. Then the equations (11) and (12) become—

$$\left.\begin{aligned} A\frac{d\alpha}{dt}&=\frac{dZ}{dy}-\frac{dY}{dz}\\ A\frac{d\beta}{dt}&=\frac{dX}{dz}-\frac{dZ}{dx}\\ A\frac{d\gamma}{dt}&=\frac{dY}{dy}-\frac{dX}{dy}\end{aligned}\right\} \quad . \quad . \quad (13) \qquad \left.\begin{aligned} A\frac{dX}{dt}&=\frac{d\beta}{dz}-\frac{d\gamma}{dy}\\ A\frac{dY}{dt}&=\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\\ A\frac{dZ}{dt}&=\frac{d\alpha}{dy}-\frac{d\beta}{dx}\end{aligned}\right\} \quad . \quad . \quad (14)$$

Also we have two equations of continuity—

$$\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which express the fact that there is no discontinuity in the electric and magnetic force in the region considered.

From the above equations it is easy to deduce by differentiation and substitution six others, viz.—

$$\frac{d^2\alpha}{dt^2}=\frac{1}{A^2}\left(\frac{d^2\alpha}{dx^2}+\frac{d^2\alpha}{dy^2}+\frac{d^2\alpha}{dz^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and similar ones for β and γ.

Also $$\frac{d^2X}{dt^2}=\frac{1}{A^2}\left(\frac{d^2X}{dx^2}+\frac{d^2X}{dy^2}+\frac{d^2X}{dz^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and similar ones for Y and Z.

This equation may be written symbolically thus—

$$A^2\ddot{X}=\Delta^2(X), \text{ where } \Delta^2 \text{ stands for } \left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)$$

and similar ones in β and γ and Y and Z. These equations are the general differential equations for the propagation of a disturbance of any type with finite velocity $\frac{1}{A}$ through a medium, and they are similar to those which can be obtained in the case of a disturbance or wave propagated through air or water.

It is quite easy to show that a solution of equation (18) is—

$$X = \frac{1}{r}F(t \pm Ar)$$

where $$r^2 = x^2 + y^2 + z^2$$

and $F(t \pm Ar)$ stands for any single-valued function of t and r.

This solution denotes a space wave moving outwards or inwards with a velocity $1/A$, the amplitude varying inversely as the distance from the origin. This is the characteristic of a normal space wave, and it indicates that the electric and magnetic forces do not make their appearance instantly at a distance, but are propagated through space with a finite velocity.

These equations are the simplest mathematical expression of the fact that the æther is a continuous medium, which can everywhere exhibit two physical effects, or can experience two correlated changes, one due to electric and the other to magnetic force. We may accept this as an ultimate fact, or we may try to picture to ourselves some form of mechanical movement or displacement constituting these changes. In any case these changes are not independent of each other. The occurrence of one brings into existence the other, and the creation of electric or magnetic force at one point results in its propagation through space with the velocity of light.

Thus, if we suppose that we have a steady electric current in a wire, then this involves a distribution of magnetic force throughout space, along certain closed lines. If we imagine this current suddenly reversed in direction, then the reversal of the direction of the magnetic force due to it at points in space about 3×10^{10} cms., or nearly 1000 million feet away, would not take place at the same moment as the reversal of the current, but one second later. During that time (one second) the reversal of the direction of the magnetic force would be travelling through space as a change in the medium. Hence it follows that, when we are concerned with currents which are changing their direction very often or quickly, as in the case of electric oscillations, we are also concerned with rapid changes in the surrounding medium, which are travelling through it with the velocity of light.

This at once suggested to Maxwell that what we call light is, in fact, an electromagnetic phenomena. On this hypothesis, along the path of a ray of light we must have electric and magnetic forces normal to each other and to the direction of propagation of the ray, which are varying rapidly in a periodic and connected manner, and hence giving rise to waves travelling with the electromagnetic velocity.

3. Maxwell's Law Connecting Dielectric Constant and Refractive Index for Electromagnetic Waves.—Maxwell's next step was to make a further deduction from these equations. We know that light moves through any transparent body, say water, more slowly than through empty space, as shown by actual experiment, and the ratio of the velocity in space to the velocity in water is called the *refractive index* of the water. Hence, if the velocity of electromagnetic waves in space is measured by $\frac{1}{\sqrt{\mu K}}$, where K and μ are respectively the dielectric constant and permeability of vacuous space, it is a legitimate deduction that the velocity through water will be represented by $\frac{1}{\sqrt{\mu' K'}}$, where K' is the dielectric constant and μ' the magnetic permeability of water. Accordingly, the refractive index of water for the electromagnetic waves will be numerically measured by the ratio of—

$$\sqrt{\mu' K'} \text{ to } \sqrt{\mu K}$$

Experiment shows that the permeability μ' in the space occupied by water is not sensibly different from the permeability μ of empty space. Hence, if we take the dielectric constant of space arbitrarily to have a value unity, we should have a relation between the dielectric constant of water and its refractive index (i) as follows :—

$$i = \frac{\sqrt{K'}}{\sqrt{K}}$$

or if K is taken as unity, then the dielectric constant of water should be equal to the square of its refractive index for electromagnetic waves.

The same argument applies to all other transparent and refractive dielectrics, and it therefore becomes a test of Maxwell's theory to examine how far the above law (called Maxwell's law) holds good. At the time when Maxwell published his theory there were very few data by which to test it, but in the last thirty years an immense number of methods of measuring dielectric constants have been invented, and a great number of numerical measurements have been made for various substances under different conditions of temperature and frequency, or time of application of the electric force.

Also direct measurements have been made of the refractive index of various substances for electromagnetic waves of various wave-lengths. But at that date (1865-1866), and for some years afterwards, the only measurements of refractive index which had been made were those for the very short wave-lengths constituting light or eye-affecting electromagnetic radiation. On comparing together the measured value of the dielectric constant of each of the few optically transparent dielectrics with the square of its refractive index for rays of light, it was found that the discrepancies were more numerous than the agreements. The dielectric constants had generally been measured by comparing the ratio of a steady or slow period alternating electric force, E, with the corresponding electric displacement, D, so as to obtain K from the equation—

$$D=KE, \text{ or } K=\frac{D}{E}$$

If this experiment is tried, for example, with water, with steady or even fairly rapidly reversed electric displacements, we get a value for K not far from 80, and for most varieties of glass we obtain values of K varying from 6 to 10. The optical refractive index of water, however, is 1·336, and that of most kinds of glass from 1·5 to 1·6 or more; hence it is clear that for these substances there is an enormous discrepancy between the square root of K (namely, 9 and 2·5 or 3·1) and the optical refractive indices (namely, 1·3 and 1·8).

More extensive research has shown that there are many substances for which we obtain, however, a fairly good agreement between the two numbers. Hence we may divide all dielectrics broadly into two classes, one including those substances which comply fairly well with Maxwell's law, and the other those cases in which there are great discrepancies between the value of the dielectric constant and the square of its optical refractive index.

In view of the extreme importance of the interconnection between refractive index and dielectric constant as a test of Maxwell's theory, it is desirable to discuss briefly the nature of these apparent exceptions to Maxwell's law. Investigation has shown that the values determined for dielectric constants are immensely affected in many cases by temperature and by the time of application of the electric force. Also it is known that refractive index is greatly affected by the frequency.

Dealing first with the effect of temperature on dielectric constant, a somewhat extensive examination has been made of the effect of low temperatures on dielectric constants. One of the substances examined with great care by Sir James Dewar and the author was liquid oxygen. Sir James Dewar long ago showed that this substance had remarkable magnetic qualities, and a preliminary measurement made by us showed that its magnetic permeability had a value exceeding that of saturated ferric chloride.

As liquid oxygen is transparent, and as its refractive index had been carefully determined by Professor Liveing and Sir James Dewar, it was evidently desirable to measure its dielectric constant carefully. This was done by means of the commutator method already explained, using a small aluminium condenser consisting of seventeen plates, which could be immersed in a vessel full of liquid oxygen. The result was to show that liquid oxygen has a dielectric constant 1·491.[7] The capacity of the small condenser with air as its dielectric at 15° C.

[7] See Fleming and Dewar on "The Dielectric Constant of Liquid Oxygen and Liquid Air," *Proc. Roy. Soc.*, 1897, vol. 60, p. 358.

was 0·001030 mfd. The above value has been substantially confirmed more recently by a measurement made by Dr. Fritz Hasenoehrl, at the University of Leyden, his value for the dielectric constant of oxygen being 1·465.

The refractive index of liquid oxygen for two cadmium lines having a wave-length respectively 4416 and 6438 was determined by Professor Liveing and Sir James Dewar to be 1·2249 and 1·2211.

Calculating from the above measurements, the value of the refractive index for waves of infinite wave-length, we obtain the value 1·2181. The magnetic permeability of liquid oxygen was determined by a direct method, consisting in the immersion of a small air core transformer under the surface of liquid oxygen.[8] The value thus obtained for the magnetic permeability of liquid oxygen was 1·00287.

A more recent and very careful measurement of the susceptibility of liquid oxygen, made by an entirely different method by Sir James Dewar and the author, showed that the value of the magnetic susceptibility of liquid oxygen is 323×10^{-6}, and that therefore its permeability is equal to 1·0041.

If we take the value of the square of the index of refraction of liquid oxygen for waves of infinite wave-length, we obtain the number 2·4837; if we take the product of the dielectric constant of liquid oxygen as determined by Fleming and Dewar, namely, 1·491, and the value of its permeability as obtained by the direct method, namely, 1·00287, the product of these numbers is 1·395. If we take the best value of the magnetic permeability as determined by the experiments on the susceptibility of liquid oxygen (namely, 1·0041), and take the mean value of the dielectric constants as determined by Fleming and Dewar and Hasenoehrl, which is 1·478, we find the value of the product of 1·478 and 1·0041 to be 1·484, which agrees almost precisely with the value of the square of the refractive index of liquid oxygen for waves of infinite wave-length, namely, 1·4837, as determined by the experiments of Liveing and Dewar.

This remarkable equality in the case of liquid oxygen between i^2, or the square of optical refractive index for waves of infinite wave-length, and the numerical product of the value of its dielectric constant K and the magnetic permeability μ, is a very interesting confirmation of Maxwell's theory.

We have in liquid oxygen a substance which possesses four qualities found together in no other substances, namely, optical transparency, almost perfect non-conductivity, a magnetic permeability greater than unity, and a dielectric constant nearly 50 per cent. greater than that of empty space.

We turn, then, again to the question of the discrepancies, and ask, How is it that such substances as water, alcohol, æther, and glycerine, which in their pure condition are all good insulators, and therefore dielectrics, and optically transparent, show such marked disobedience to Maxwell's law? A careful investigation of this point has shown that temperature is largely accountable for the discrepancy.

By means of the cone condenser described in Chap. II., Sir James Dewar and the author have measured the dielectric constant of ice, frozen alcohol, frozen glycerine, and numerous other organic or inorganic frozen liquids, and have discovered that in all cases cooling them to a very low temperature destroys entirely these high dielectric values.

Thus, for instance, if the dielectric constant of ice is measured with an electric force applied either continuously or alternating 1 to 200 times a second, the temperature of the ice being 0° C., the value of the dielectric constant found is represented by a number in the neighbourhood of 80. If, however, the ice is cooled down to the temperature of liquid air, the dielectric constant of the ice falls to a value near to 2·4.

In the same manner, if the dielectric constant of alcohol is measured at ordinary temperatures, the number is found not very far from 25, but if the alcohol is frozen and cooled to the temperature of liquid air we find by the above-described methods a value 3·12.

[8] See Fleming and Dewar on "The Magnetic Permeability of Liquid Oxygen and Liquid Air," *Proc. Roy. Soc.*, 1896, vol. 60, p. 283.

Again, the dielectric constant of glycerine determined at ordinary temperatures gives a value 56, but if determined at the temperature of liquid air a value 3·9.

If we gather into one table (see Table II., below) the results of a number of these low-temperature measurements of dielectric constants taken at a frequency of 120 per second, and arranged so as to show the values of the dielectric constant at 15° C. and at −185° C. (the temperature of liquid air), we see at once the immense influence which temperature has upon the fundamental qualities of a dielectric. For the sake of comparison, the values of the square of the optical refractive index (ι^2) for very long wave-lengths or for certain wave-lengths in the visible spectrum have been placed in contiguity.

The conclusions to which the figures in Table II. lead us is that, whereas at ordinary temperatures there is an enormous difference between the dielectric constants of certain substances and the square of their optical refractive index, a continual lowering of the temperature destroys a large part of this disagreement.

On the other hand, there are some substances for which, even at ordinary temperatures, Maxwell's law is very approximately fulfilled as shown in Table III. (see below).

TABLE II

DIELECTRIC CONSTANTS (K) AT DIFFERENT TEMPERATURES

Substance.	K at 15° C.	K at −185° C.	Square of refractive index (ι^2).
Water	80	2·4 to 2·9	1·779 (for D line)
Formic acid	62	2·41	...
Glycerine	56	3·2	...
Methyl alcohol	34	3·13	...
Mononitrobenzol	32	2·6	...
Ethyl alcohol	25·8	3·11	1·831
Acetone	21·85	2·62	...
Ethyl nitrate	17·72	2·73	...
Amyl alcohol	16	2·14	1·951
Aniline	7·51	2·92	...
Castor oil	4·73	2·14	2·153
Ethylic æther	4·25	2·31	1·805
Olive oil	3·16	2·18	2·131
Carbon bisulphide	2·67	2·24	2·01

TABLE III

Substance.	Dielectric constant, K at 14° C.	Square of optical refractive index (ι^2).
Sulphur	4·73	4·89 (for B line)
Paraffin	2·29	2·022
Petroleum	1·92	1·922
Petroleum old	2·07	2·075
Turpentine	2·23	2·128
Benzine	2·38	2·26 (for D line)

Exceptions, however, are more numerous than accordances, and we find no apparent fulfilment of the law in the case of the following substances:—

TABLE IV

Substance.	K.	μ^2.
Glass (light flint) . .	6·57	2·375 (for B line)
Glass (dense) . .	10·1	2·924
Calcite	7·7	2·734 (for A line)
Fluorspar . . .	6·7	2·05
Mica	6·64	2·526
Quartz	4·55	2·41
Tourmaline . . .	6·05	2·63
Rock salt . . .	5·85	2·26

In the case of gases there is a very fair agreement between K and μ^2.

Then, with respect to the question of frequency, it has been found that the rate at which the electromotive force is applied and removed, or reversed, has a great influence upon the dielectric constant. Generally speaking, we may say that the higher the frequency the lower the dielectric constant. On the other hand, many substances exhibit, so to speak, a great constancy under variation in frequency.

By the employment of electrical oscillations, it is possible to determine the dielectric constant with very rapid alternations of electric force. It appears, however, that whether we use a continuous electric force or an electric force slowly alternating, or even alternating 30,000 million times a second, the dielectric constant of water is still a number not far from 80. On the other hand, in the case of alcohol the same variation in frequency reduces the dielectric constant from 25 to about 6·6. Ice is more sensitive to change in frequency than water, and an increase in the frequency which does not affect the dielectric constant of liquid water reduces that of ice to a value between 2 and 5.

In considering the causes of the discrepancies, it is obvious that if light waves consist of alternations of electric force, then, since the visible spectrum is comprised between the limits of 400 and 800 billion vibrations per second, there is an enormous gap between the highest frequencies which give rise to optical effects. The whole of these effects give us reason to consider that the numerous discrepancies and exceptions to Maxwell's law are really dependent upon temperature and frequency.

It is obvious that in making comparisons we can hardly expect to find the law fulfilled unless the alternations of electric force, with which we determine the dielectric constant, are comparable with the number of vibrations per second in the ray by which the refractive index is measured.

Of late years it has been possible to test the matter in another way. We are now able, as will be explained below, to produce electric waves which are known to have all the properties of light, except visibility. Many recent investigations have had for their object the determination of the refractive index of water, alcohol, and other bodies for electric waves of great length lying far beyond the region of the ultra-red spectrum. For water the refractive index found for these electric rays is a number in the neighbourhood of 8·9.[9] The square of this number 8·9 is very nearly 80, and hence is in very good agreement with the values of the dielectric constant of water determined by purely static electrical methods, and either with continuous electric force or electric forces very slowly alternating.

It is impossible to dismiss this part of the subject without raising one question: Why is it that certain kinds of matter have such exceptionally large dielectric constants, which at ordinary temperatures are so different in value from the square of the optical index of refraction? The answer to this is, that electric force produces two effects when acting on any space occupied by dielectric matter. In the

[9] See Fleming and Dewar, *Proc. Roy. Soc. Lond.*, 1897, vol. 61, p. 2, "On the Dielectric Constants of Ice and Alcohol at very Low Temperatures."

first place, it creates an electric strain in the æther, or true electro-magnetic medium, which strain is immediately responsive to the stress.

In the next place, it operates on the molecules of the matter, producing an additional strain or displacement; and it is not a little remarkable that those substances which have high dielectric values are those which easily suffer chemical decomposition by displacement or removal of some radicle.

Some interesting facts connected with dielectric constants of solids and liquids have been noted by C. B. Thwing.[10] He has pointed out that, for a large number of substances, the dielectric constant is 2·6 times the density, and that the dielectric constant can be predetermined for many substances by calculation.

The dielectric constant of a body can be calculated by a summation law, in accordance with the following rule:—

The product of the molecular weight of the substance and its dielectric constant divided by its density is equal to a sum formed by multiplying 2·6 times the number of atoms of each kind by their atomic weight; except in the case when the molecule contains certain radicles, when each radicle has in addition a multiplying constant differing from 2·6.

Hence if K = dielectric constant;
M = molecular weight;
D = density;
a_1, a_2, etc. = atomic weights or elements of radicles;
n_1, n_2, etc. = number of atoms or radicles;
we have—

$$K = \frac{D}{M}(2\cdot6\, a_1n_1 + 2\cdot6\, a_2n_2 + \text{etc.} + ka_3n_3 + \text{etc.})$$

The factor 2·6 is employed if the element is an atom of hydrogen, oxygen, carbon, etc., and the factor k if it is a chemical radicle OH, CO, COH, NO_2, CH_2, CH_3, or S, having values as follows:—

Radicle.	Molecular weight.	Value of k.
OH . . .	17	80·6
CO . . .	28	52
COH . . .	29	33·8
NO_2 . . .	46	67 6
CH_2 . . .	14	2·86
CH_3 . . .	15	3·12
S	32	0·016

Thus the dielectric constant of water (H_2O), which is a hydride of hydroxyl, having molecular weight = 18 and density = 1, is given by the formula—

$$K = \frac{1}{18}(2\cdot6 \times 1 + 80\cdot6 \times 17) = 75\cdot4$$

and that of ethylic alcohol (CH^3, CH^2, HO) by the formula—

$$K = \frac{0\cdot815}{46}(3\cdot12 \times 15 + 2\cdot86 \times 14 + 80\cdot6 \times 17) = 25\cdot6$$

These values agree with the results of experiments. This remarkable rule supplies us with a clue to the meaning of these large dielectric constants. We see that the presence in a molecule of a chemical radicle, or portion more easily detached than other atoms, seems to indicate a line of easy cleavage in the molecule of which the electric force takes advantage. It appears, therefore, that the simple properties of the electromagnetic medium filling space are profoundly modified by the presence of ordinary matter in the same place.[11]

[10] See C. B. Thwing, *Zeitschrift Phys. Chem.*, 1894, vol. xiv. pp. 286-300.

[11] The reader may be referred to an article by Sir J. J. Thomson on "Electromagnetic Waves," in the Supplement to the 10th edition of the *Encyclopædia Britannica*, for a mathematical discussion of the cause of these large dielectric constants, and an explanation of the abnormality as due to the presence of free ions or electrons in the mass of the dielectric.

Briefly, then, it may be stated that Maxwell's theory consists in the assumption that the effects we call electric displacement, or otherwise electric charge, and that which we call magnetic flux or magnetic induction, when they exist in a space free from ordinary gravitative matter, are affections of a medium capable of storing up energy in two different forms. The dielectric constant of the medium or the displacement per unit of electric force, and the magnetic permeability of the medium, or the magnetic flux per unit of magnetic force, are both altered by the presence of matter. The first quality is always increased, the second may be increased or diminished, and is enormously increased by the ferromagnetic substances.

These two qualities determine the speed of transmission of a disturbance or an electric wave through the medium, and an electric wave is created whenever a very sudden electric displacement is made or released. The moment, however, that we attempt to resolve the processes into mechanics, or the simple movement of matter possessed of inertia, and resisting some kind of change of configuration, we are met with many difficulties. The first question that presents itself is as to the nature of the elastic reaction of this medium against stress. What is the kind of deformation the medium resists? It cannot be a simple compressional elasticity or resistance to change of volume, as in the case of air. That would imply that the ray could not be polarized, whereas in the case both of light and electric rays they can be or are polarized, or made non-symmetrical with respect to the direction of propagation. Can the elasticity, then, be a simple resistance to shearing or change of form? This elastic solid or jelly theory of the æther fails to meet requirements in many points.

Then a third hypothesis is that the elementary portions of this medium do not resist either compression or shearing, but resist absolute rotation round any axis. This rotational theory of the æther, due originally to MacCullagh and Kelvin, has been developed in great detail by Sir Joseph Larmor, who has shown that it meets in many remarkable ways the demands of physical theory.[12] The temptation to try and construct a purely mechanical theory of the æther, in which displacements and fluxes are visualized as changes of configuration or motions, is very great.

We are unable to make for ourselves a mental picture of any physical processes which we cannot in the ultimate issue resolve into motion, either past or present. If we could resolve all the operations in the electromagnetic medium into mere motions of some substance possessing the single attribute of inertia, it would in one sense satisfy our minds.

But the æther, if it exists at all, must have many more functions (some, perhaps, yet unsuspected by us) than those of merely conveying vibrations. If that is the case, we may do well to refrain from attempting too much mechanical interpretation, and, whilst resting on the fact that the definite changes we call electric displacement and magnetic flux are directed, or vector changes in a universal medium, admit that the ultimate analysis of the nature and structure or æther, energy, and matter, will carry us far beyond the region of the ideas of motion, inertia, or force.

It may, however, be asked, How do the above statements afford proof that the optical æther is identical with the electromagnetic medium? So far all that we have passed under review has been proof that if electromagnetic effects are propagated from place to place with finite velocity, that velocity will be measured by the reciprocal of the quantity $\sqrt{\mu K}$, and it has been shown that this electromagnetic velocity in vacuum, air, other gases, also in certain liquids and solids, is equal to the measured velocity of light rays through that material.

Our conviction that the propagation of light through transparent matter is not an effect wholly or entirely due to matter alone, is based for one thing on the fact that the mean velocity of light coming to us from Jupiter's satellites is the same as the actually measured velocity of light in air at the earth's surface.

In like manner, the dielectric constant and magnetic permeability of the very best vacuum we can produce differ so exceedingly little from the same qualities of an air-filled space at ordinary pressure and temperature, that we cannot well

[12] For an exposition of Sir Joseph Larmor's views, we must refer the reader to his book, "Æther and Matter," University Press, Cambridge, 1900.

believe these properties of the space are wholly due to the matter, if taking out all but one-millionth of the gravitative matter makes so little difference.

The demonstration that light has an undulatory nature rests upon all the well-known facts of interference. The creation and similar properties of undulations having an electrical origin travelling through space with equal velocity, and exhibiting all the properties of visible light, has afforded more than ground for a suspicion; it has given an almost perfect proof that the basis, the undulating material, and the nature of that undulation must be similar in the two classes of phenomena.

4. **Electromagnetic Waves.**—We must next turn attention to the production of electromagnetic waves, or, as they are shortly called, electric waves, in dielectrics by means of electric oscillations.

There are one or two questions connected with wave production in general concerning which a little preliminary discussion may be useful. One physical characteristic of wave motion is that by it energy is conveyed entirely away from the wave-creating body and exists for a time stored up in a surrounding medium. Consider, for instance, the production of a compressional wave in air. If the hand or a fan is moved to and fro in the air, the mere production of this motion or change of motion in the material body absorbs energy. When it is so moved in a fluid such as air, the moving solid sets up vortex or rotational motions in the surrounding air, similar to those whirls which are seen on moving an oar or the hand through water, and these fluid motions also take up energy to produce them. If a fan is moved slowly through the air, all that happens is that the air in front passes round behind it, and in so doing air vortices are created. Energy is therefore absorbed not only in making changes in motion of the solid, but also is taken up in the surrounding medium in creating this vortex motion or movements in the air which cling to and surround the moving body.

A large part of the resistance of motion which a solid body experiences in passing through a fluid is due to this form of energy absorption by the fluid. A perfect fluid, or one without any quality of viscosity, could not have these vortex motions so set up in it by a body entirely submerged and moving steadily so as to create no waves. Hence, a perfect fluid offers no resistance to the motion through it of a solid.

If the solid oscillates or moves slowly through a fluid, the energy never dissociates itself entirely from the moving solid or the fluid in its neighbourhood. The energy, so to speak, travels with the vibrating body and exists where it is, or in proximity to it, and when its motion ceases the energy of motion of the fluid is frittered away into heat.

It is quite different, however, if a body is moved or vibrated very rapidly, so as to bring into play the inertia quality of the fluid. If, for instance, instead of moving somewhat slowly through the air, the fan or other body, such as a tuning-fork, is made to vibrate with considerable speed, and inertia and compressibility of the air come into play, with the result that we have a true wave produced, the air has not time to get out of the way of the moving solid, and thus, instead of moving round to the back of the vibrating body, it is suddenly compressed, and subsequently rarefied and startled into oscillations. Each portion of the fluid takes up successively the oscillatory motion or changes of pressure, and energy is conveyed entirely away from the moving body and its neighbourhood, and continues to exist in the medium as a wave long after the vibrating body which started it has come to rest.

Some at least of the energy imparted to the solid to set it in vibration is taken from it and handed on from point to point through the air.

The characteristic of a true wave is that in each portion of the medium the energy so being conveyed exists alternately as energy of strain or configuration and energy of motion, or in some form equivalent to these types of energy. Moreover, at a distance called a *wave-length*, similar energy changes are taking place at the same time. The mathematical expression for a wave is merely a symbolical statement of this fact. Thus the expression—

$$y = Y \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

is the algebraical method of denoting a wave of wave-length λ and periodic time T, and it tells us that a periodic disturbance or oscillation travels forward with a velocity $\frac{\lambda}{T}$, since the value of y remains the same if for x we substitute $(x+x')$ and for t, $(t+t')$, provided that $\frac{x'}{t'}=\frac{\lambda}{T}$.

Accordingly, at two places separated by a distance x', the same motion will take place after a time t'.

This is easily seen if we note that—

$$\frac{x+x'}{\lambda}-\frac{t+t'}{T}=\frac{x}{\lambda}-\frac{t}{T}$$

provided that $\frac{x'}{t'}=\frac{\lambda}{T}$.

The total energy of a wave can be shown to be at any moment half potential or configurational and half kinetic or motional. At each point in the medium cyclical changes of energy take place, and the disposition of either kind is periodic in space and time.

The term *wave motion*, therefore, has reference to this peculiar mode of transferring energy from place to place, and, as we have already seen, waves can exist in any medium which possesses two essential qualities. The first of these qualities is that some kind of vector or directed change made in it must tend to disappear if left to itself, and not only so, but in being created must call forth an opposition or resistance to its creation. In the second place, in disappearing, the change, whatever its nature, must tend to overshoot the mark and be reproduced in the opposite direction; in other words, there must be a persistence or inertia-like quality in connection with the change of deformation.

There may, therefore, be as many different kinds of waves as there are possible modes of deformation in extended media.

Take, for instance, the case of water. If the water has a free surface, this is a level surface, and tends to remain level. If the water is heaped up in one place and left to itself, it begins to regain its level; but it possesses inertia, and in so doing it overshoots the mark and creates a depression in the surface.

From this point, therefore, surface waves spread out which are changes in level, periodic in time and space. Again, a free water surface possesses what is called *surface tension*. The surface of any liquid offers a resistance to stretching like a sheet of india-rubber. If, therefore, a surface of water is slightly heaped up the surface is stretched, and tends again to become level in virtue of this surface tension. Hence we can have, not only what are called *gravitational waves* on the free surface of water, but ripples or *surface tension waves*. These latter may be seen to be formed when a fishing-line or thin rod is moved through water perpendicularly to the surface. Furthermore, water resists compression, and hence we can have produced in it *compressional waves*, not on the surface, but in the mass. Such waves are produced in water by an explosion taking place beneath the surface.

In every case, however, the velocity of propagation of the wave is measured by the square root of the ratio of two quantities, one being of the nature of an elasticity, and the other the density or mass per unit of volume. Moreover, in all wave motion the velocity of the wave is measured by the product of the wave-length and the number of complete oscillations per second executed by any part of the medium through which the wave motion is travelling.

If V represents the wave velocity, n the frequency, and λ the wave-length, then we have the relation $V=n\lambda$ as a fundamental equation connecting wave-length with frequency.

In the case of solid bodies we can have another kind of wave not capable of being produced in liquids, namely, a *distortional wave*. The special characteristic of a solid substance is that it resists shearing or being changed in shape. If, for instance, we give a twist to a rod of steel, it resists this type of distortional

deformation, but we cannot put a twist of the same kind upon a thread of honey or column of water.

Accordingly, we can have a great variety of waves in material media depending upon the fact that their parts possess inertia, and that they resist some kind of relative displacement. Thus, for example, we may have—

Gravitational or surface waves in liquids—due to the resistance of the surface to being made unlevel.

Capillary waves or ripples on the free surface of liquids—due to the resistance of the surface of the free liquid to stretching.

Compressional waves in the mass of gas, liquid, or solid—due to the resistance to change of bulk or volume elasticity.

Distortional waves in solid bodies—due to the resistance to shearing, twisting, or other changes of a form of any element; in other words, to shape elasticity.

These preliminary remarks will pave the way for a consideration of the nature of *electromagnetic waves*, or, as they are generally called, *electric waves*.

Every dielectric possesses, as we have seen, two properties. It can have a physical state produced in it at any point called the electric displacement, and this corresponds to the production of a deformation or strain in an elastic solid. The medium resists by an elastic reaction the creation of this displacement, and when the electric force creating it is withdrawn, the displacement disappears; but as a displacement requires an energy expenditure to produce it, the law of conservation of energy necessitates that the displacement in disappearing shall give rise to energy in some other form. This it does by the creation of magnetic flux in a direction at right angles to itself, and the flux in turn in disappearing gives rise again to a displacement at neighbouring points in the same direction as that displacement, the vanishing of which gave rise to the flux. Hence we detect in this operation an analogy with the case of a vibrating solid where mechanical stress gives rise to elastic strain, and strain in disappearing creates velocity or sets matter in motion, and hence reproduces the strain energy in a kinetic form. This, again, in virtue of inertia, recreates a new strain in an opposite direction.

The process of alternating electric displacement and resulting magnetic flux repeated cyclically in space and time from point to point through the dielectric constitutes an electric wave, and the velocity of this wave is measured by the value of $\frac{1}{\sqrt{K\mu}}$ for that dielectric. By the velocity of the wave is meant the quotient of wave-length by the periodic time.

In considering these matters, the question necessarily arises: What is it that constitutes an electric displacement in a dielectric? Maxwell never committed himself to any opinion as to the exact nature of the physical change which he called the electric displacement. Mr. Oliver Heaviside remarks paradoxically that the more general or more vague a physical theory, in one sense the more likely it is to be true, or perhaps we should say, the less likely it is to be untrue. This vagueness, however, is felt by some students to be unsatisfactory; they want to know whether an electric displacement is to be considered as an actual motion, or a stretch, squeeze, or rotation of an æthereal medium or of the material dielectric. If told that not only do we not know, but that all theories on this matter are most probably wide of the mark, they are apt to feel a degree of disappointment. We are on safer ground when we are content not to demand too much detail at present, provided that our hypothesis is sufficiently definite to enable it to become the foundation of a mathematical analysis of the phenomena.

Mechanical analogies are helpful as a guide, but we may easily become slaves to an analogy or a catch phrase.

In order that we may create an electric wave, we have, however, to create a state, called, for the sake of definiteness, electric displacement in a dielectric, and to release that constraint very suddenly, just as to produce a compressional wave in air we have to produce or release very rapidly an air compression.

5. **Hertz's Researches.**—These ideas had been grasped with some degree of clearness prior to the publication of the celebrated memoirs in Wiedmann's

Annalen der Physik, in which Hertz announced his discoveries to the world.[13] It is to him we are indebted for a new departure on the subject which brought it at one stroke within the region of experiment. Hertz equipped the secondary terminals of an induction coil with a species of Leyden jar or condenser which is now known as a Hertz radiator. This consists of a pair of metallic plates, or sometimes balls, having attached to them short rods ending in knobs placed a fraction of a centimetre apart (see Fig. 2). These knobs are connected to the secondary circuit of the coil. Hence, as the secondary electromotive force

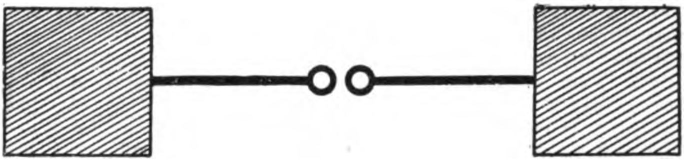


Fig. 2.—Hertz Radiator or Oscillator.

accumulates, the plates are brought to a difference of potential, and lines of electrostatic displacement stretch out from one part of the oscillator, which we will call the positive side, to corresponding points on the negative side. We thus have a strong electric displacement created along certain lines of electric force.

Corresponding to a critical value of the potential difference, the air insulation between the balls breaks down, and it becomes highly conductive. Then the whole radiator becomes one conductor for the moment, and the potential difference begins to equalize itself, that is to say, a current flows from one side to the other,

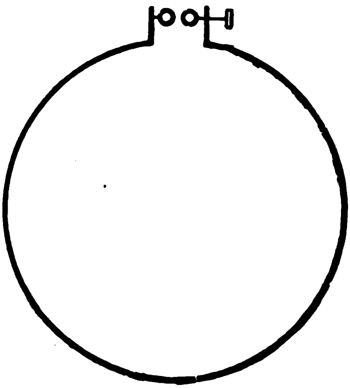


Fig. 3*a*.—Hertz Resonator or Receiver.

creating in the space around a magnetic flux, the direction of which is everywhere normal to the direction of the electric displacement. The electrostatic energy is thus transformed into electrokinetic energy. The flux then persists, and recreates in an opposite direction electric displacement. We may consider an illustration of the process as follows :—

Let a flat stretched steel spring represent the oscillator, and on it let a heavy disc be keyed like a wheel. Let the ends of the spring be fixed and the disc

[13] Heinrich Rudolf Hertz was born at Hamburg, February 22, 1857, and died at Bonn on January 1, 1894. He graduated at the University of Berlin, and was a favourite pupil of Von Helmholtz. In 1885 he became professor at the Technical College of Karlsruhe, and it was there that his epoch-making investigations were begun. In 1889 he received a call to succeed Clausius at Bonn. In July 1888 his most important memoir on electro-magnetic waves in air was published, and at once attracted general attention to his work.

turned round, the spring thus being twisted. If then the wheel is released, it begins to move under the action of the torsional force. It acquires kinetic energy, and when the twist of the spring has disappeared, the wheel is possessed of all the energy as rotational energy. This then expends itself in reproducing the twist of the spring in the opposite direction.

If the electric oscillation in the oscillator is started sufficiently suddenly, some of the energy is thrown off in the form of a displacement wave, and as a consequence the oscillations of the radiator, as Bjerknes has shown, are quickly damped out. Accordingly, when the induction coil is kept going we have groups of intermittent oscillations, and therefore trains of electric waves thrown off which travel off or spread out through the dielectric.

Hertz furthermore devised a form of *resonator* for detecting these electric waves at any point in space. In its simplest form this consists merely of a nearly closed ring of wire, the ends being provided with metallic balls placed very close together (see Fig. 3*a*). The ring may be a rectangle, and it may have a condenser inserted in its circuit, as in the arrangement due to Blondlot (see Fig. 3*b*). In order that we may secure the sharpness of breakdown in the air insulation which is necessary to obtain the oscillations, three things seem necessary.

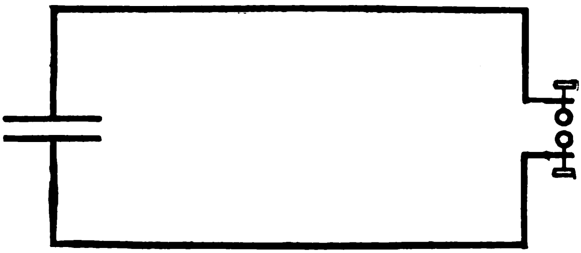


FIG. 3*b*.—Blondlot Resonator.

First, the spark-ball surfaces must be bright and clean; secondly, no ultra-violet light rays must fall on the balls, especially on the negative terminal; and, thirdly, the balls must be at a certain distance apart best determined by experience.

In describing experiments with the Hertz oscillator, we shall call the *axis of the radiator* the direction of the line joining the centres of the spark balls, and the line through the spark, perpendicular to this axis, will be called the *base line*. Also the line joining the spark balls of the resonator will be called the *spark axis of the resonator*. If the resonator is set in front of the oscillator with its centre on the base line, then there are three principal positions which the resonator may occupy. First, its plane may be parallel to the axis of the radiator and perpendicular to the base line: we shall call this the first position (see Fig. 4*a*). Secondly, the resonator may have its plane in the plane containing the radiator axis and the base line: we shall call this the second position (Fig. 4*b*). Thirdly, the resonator may have its centre on the base line and its plane perpendicular to the plane containing the radiator axis and the base line, and placed so that its plane passes through the spark gap: this will be called the third position (Fig. 4*c*).

Hertz found that when the resonator is placed in each of these three positions respectively, but not too close to the radiator, and if at the same time the resonator is turned round in its own plane so as to bring the spark axis of the resonator into various positions, different phenomena present themselves.

In the first place, if the resonator is placed in the first position, and with the spark axis of the resonator parallel to that of the radiator, then when the radiator is sparking, small sparks also occur between the spark balls of the resonator; but if the resonator is turned round in its own plane, so that the spark axis of the resonator is perpendicular to that of the radiator, then no sparks occur at the resonator.

In the next place, if the resonator is placed in the third position, with its plane

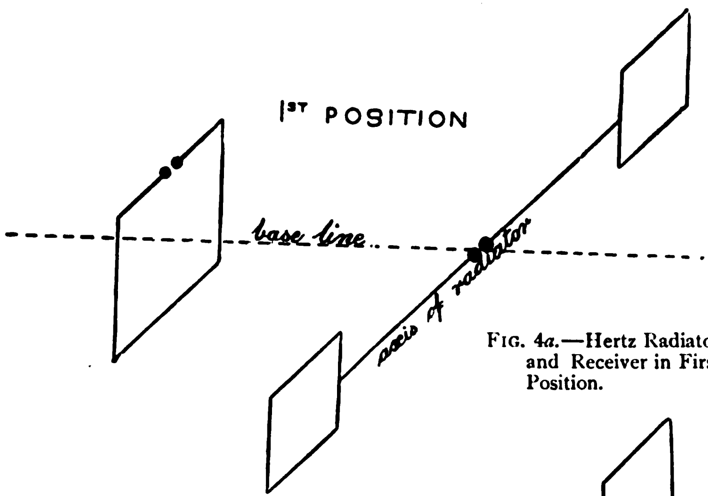


Fig. 4a.—Hertz Radiator and Receiver in First Position.

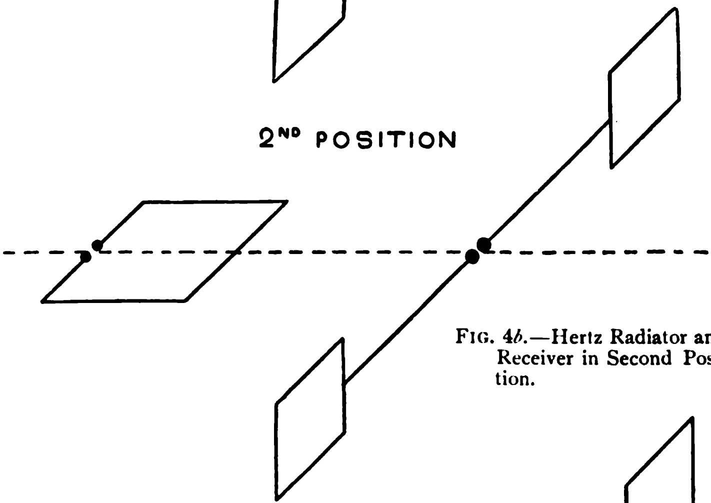


Fig. 4b.—Hertz Radiator and Receiver in Second Position.

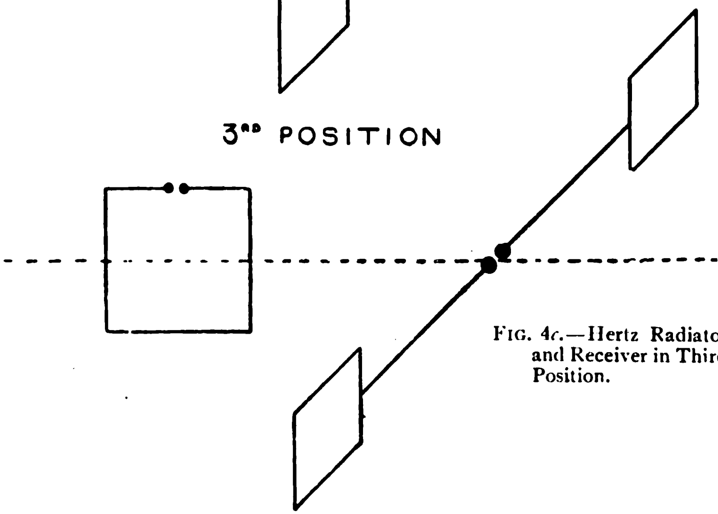


Fig. 4c.—Hertz Radiator and Receiver in Third Position.

perpendicular to the axis of the oscillator, then no sparks are seen, whatever the position of the air gap of the resonator.

When the resonator is placed in the second position, with its plane parallel to and passing through the axis of the radiator, then sparks are seen in the resonator air gap when that gap is turned towards the oscillator, but they become less and less bright as the resonator is turned round in its own plane until when the air gap is turned away as far as possible from the oscillator they cease altogether.

In order to explain this spark production in the resonator, it is necessary to make reference to a fact early discovered by Hertz.

If the resonator is attached by a wire to one terminal of the induction coil, then when the coil is in action, vigorous sparking is seen at the spark balls of the resonator, unless the connecting wire is attached to the resonator at a point symmetrical with respect to the spark balls. This is due to the inductance of the resonator circuit (see Fig. 5).

If the lengths of path measured along the resonator from the point of attach-

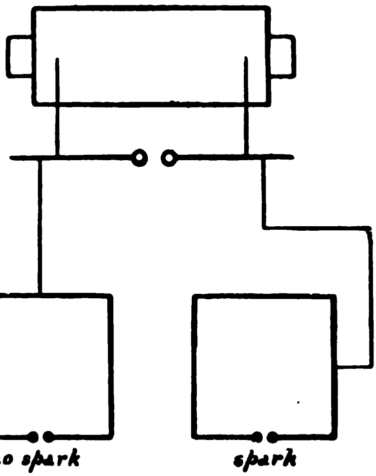


FIG. 5.—Hertz Resonator attached to One Terminal of the Secondary Circuit of an Induction Coil.

ment of the wire to the spark gap are unequal, then, owing to their unequal inductance, the rise or fall of potential produced by the coil terminal takes effect first at the spark ball attached to the branch of smaller inductance.

One might at first be inclined to suppose that no difference of potential could be created between two balls connected by a short loop of wire, but although this is the case when low frequency oscillations are used, it is not so when the frequency is very high.

The same thing holds good when the resonator is not connected with the induction coil by a wire, but placed at a distance from the oscillator. In this case electric displacement produced by the radiator travels to the resonator through the dielectric. If the spark gap of the resonator is held parallel to the spark gap of the radiator, then the displacement of electric force arriving at the resonator fills the spark gap of the resonator and creates there an alternating displacement and an alternating potential difference between the balls. When this reaches a certain amplitude the air insulation breaks down, and a small spark is produced between the ball terminals of the resonator. Even although the resonator and the spark balls are connected by the resonator wire, this does not hinder the creation of the spark, as the inductance of that wire makes it a practically perfect insulator to very suddenly applied potential differences.

If, however, the resonator is held in a position, so that the line joining the spark balls is in a direction at right angles to the spark axis of the oscillator, then no spark will occur in the resonator, because the electric force arriving there is not in a direction to create potential difference between the balls. If, however, the plane of the resonator is in the plane containing the base line and the spark axis of the radiator, and if the spark gap of the resonator is so placed that its direction is perpendicular to the axis of the vibrator, then feeble sparking is seen in the resonator. This, however, is because the electric force distribution is disturbed by the metallic circuit of the resonator.

The direction of the electric force, and therefore the displacement travelling through space, in the neighbourhood of the spark balls of the resonator is then no longer parallel to the spark axis of the radiator, but is slewed round so as to be inclined in a direction to the spark axis of the resonator. Hence the effect is to

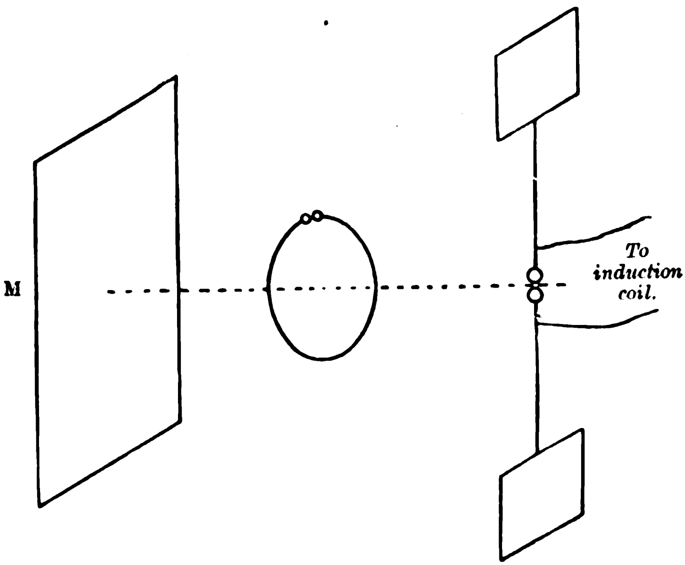


FIG. 6.—Hertz Resonator used to detect Electric Waves reflected from a Metal Sheet M.

cause a displacement across the air gap of the resonator, and therefore to create a spark.

We may ask, then, what are the functions of the wire of the resonator if the spark formation is due to the action of electric force propagated from the oscillator? To answer this, we must analyze a little more closely what takes place in the resonator when the spark passes.

The resonator is a circuit possessing capacity and inductance, the spark balls forming, so to speak, the condenser portion of the circuit; hence it has a natural free period of electrical vibration. If in the space between the balls alternating electric displacement is produced, being propagated to that point through the dielectric, this displacement may or may not synchronize in period with the free period of vibration of the resonator. If it does time in with it, then the amplitude of the displacement oscillations is increased, and a point is reached at which the air insulation breaks down and a spark then passes.

Owing to the fact that the resonator is a nearly closed circuit, it is a very bad radiator, and, as Bjerknes has shown (*Wied. Ann.*, 1891, vol. 44, p. 74), such a resonator has a very small coefficient of damping. If it is a circular resonator 35 cms. in diameter, as used by Hertz, it may even execute 1000 vibrations before the electric oscillations are practically damped out.

It is obvious, therefore, that oscillations can be most easily set up in the resonator circuit when the vibrations of electric displacement which give rise to these oscillations, propagated to the spark gap, are in a direction parallel to the spark axis of the resonator.

In the case in which the resonator is placed with its plane lying in the plane containing the axis of the radiator and the base line, the distribution of electric displacement is disturbed, as already explained, by the metallic circuit of the resonator, and the advancing wave surface of displacement has a component parallel to the spark axis of the resonator, and therefore the conditions are such as to be favourable to the production of at least feeble sparking.

Hertz's most famous discovery with the above described simple resonator was the proof that he was able to give of the existence of stationary electric waves set up in a dielectric or in space bounded by a sheet of metal. He attached to his induction coil terminals a radiator composed of two square sheets of metal 40 cms. inside, having fixed to them rods ending in brass balls. These plates were arranged with the rods in one line and the balls about a centimetre apart, the direction of the rods being vertical (see Fig. 6). As a resonator he used a circular wire 35 cms. in diameter, with the ends nearly meeting and furnished with spark balls. A large sheet of metal was set up at the end of the room, and the radiator with axis vertical to it placed in front of this sheet. The resonator was held with its plane parallel to the metal sheet, and its spark gap parallel to the spark gap of the radiator.

Under these conditions, if held near the metal sheet, no sparking occurred; but if moved away from it, sparks were seen, and at a certain distance these sparks had a maximum brilliancy; but if the resonator was removed still farther from the metal sheet, a position could be found in which the sparks again ceased.

All along the base line, therefore, perpendicular to the metal sheet, it was found that there were positions of maximum and minimum sparking indicating a periodicity in the distribution of electric force in that space.

A very important discovery in connection with this phenomenon was made by Sarasin and De la Rive (*Comptes Rendus*, March 31, 1891), who found that the distance between two non-sparking places essentially depended upon the size of the resonator, and was approximately equal to four times the diameter of the circular resonator.

The earliest view taken of the effect was that the radiator creates stationary dielectric waves of definite wave-length, and that the resonator indicates this wave-length by sparking when held as described at places of maximum electric force. But it is found that the size of the radiator very little affects the result.

Another hypothesis was that the radiator sends out waves of all wave-length, resembling, therefore, white light, and the resonator picks out and responds to its own particular wave-length. But this hypothesis is not justified by any facts. The most probable explanation was that given by M. Poincaré, in 1891, and also by Professor Sir J. J. Thomson ("Recent Researches on Electricity and Magnetism," p. 402). The Hertz radiator, as shown by Bjerknes, is a very strongly damped system, and at each discharge hardly makes more than a dozen oscillations, even if so many, before its electrical vibrations are damped out.

Suppose the resonator, then, held at a distance from the metal wall equal to a quarter the wave-length corresponding to this particular resonator, then, as the electric force passes over it, it will create a displacement between spark balls. This displacement travels on, is reflected from the wall, and returns. If it returns at such a moment as to assist the displacement then being made between the spark balls of the resonator, the amplitude of this displacement is increased, and a succession of such assistances will break down the insulation of the air and a spark will occur. It is clear, therefore, that this reinforcement of the displacement amplitude will occur when the distance of the resonator from the metallic wall is a quarter of its own wave-length. Sarasin and De la Rive used resonators of various diameters (D), as shown in the table below, and measured the distance $\frac{\lambda}{2}$ between places of maximum sparking in the field.

D.	4 D.	Distance between two adjacent points of maximum sparking = $\lambda/2_1$.
100 cms.	400 cms.	406 cms.
75 ,,	300 ,,	282 ,,
50 ,,	200 ,,	222 ,,
35 ,,	140 ,,	152 ,,
25 ,,	100 ,,	120 ,,
20 ,,	80 ,,	86 ,,
10 ,,	40 ,,	38 ,,

Accordingly, the distances between the positions of the resonator when the maximum sparking takes place in its air gap, reveal, not the wave-length of pre-existing stationary waves, but the oscillation period or wave-length corresponding to the resonator itself. Nevertheless, they prove the existence of stationary dielectric waves in the space between the metal sheet and the radiator, and therefore that the electromagnetic impulses travel through space with a finite velocity. On referring to the last table, it will be seen that the wave-length observed was

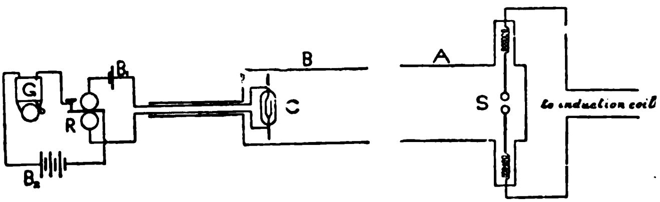


FIG. 7.—Apparatus for producing and detecting Electric Waves. S, spark balls in metal box, A, with open mouth; C, coherer in metal box, B, with open mouth; R, relay; G, electric bell; B_1, relay battery; B_2, electric bell battery.

very nearly equal to eight times the diameter of the circular resonator. Professor H. M. Macdonald has shown, in his book on "Electric Waves" (see p. 112), that by theory the fundamental wave-length proper to a circular resonator is 7·95 times its own diameter. The singular agreement between theory and experiment shows that the resonator does not indicate the wave-length of a train of waves of definite wave-length passing through space, but that it is set in vibration by an electric impulse administered to it, and this calls forth its own natural proper vibration. The only satisfactory explanation of the phenomena is that which is based upon Bjerknes' discovery, that the oscillations sent out by the Hertz radiator are, as we have already seen, highly damped, whilst the oscillations of the nearly closed resonator are very slightly damped; hence the radiation proceeding from the radiator consists, at most, of half a dozen rapidly damped oscillations constituting each train, whereas the resonator, when set in vibration, may execute 1000 oscillations before they are extinguished. This fact has an important bearing upon the theory of the arrangements used in wireless telegraphy, as we shall see later on.

The Hertz resonator resembles the simple Marconi aerial in possessing a large radiation decrement, that is, its oscillations are highly damped by reason of radiation, whereas the receiving circuits employed are generally circuits having very small radiation decrement.

6. Repetition of Hertz's Experiments on Electric Radiation.—It is a difficult matter to repeat Hertz's own experiments on this subject even in a laboratory, and almost impossible to show them to a large audience. Nevertheless, the facts are so important, and an experiment shown is so much more valuable than a statement, that the author has devoted much attention to devising apparatus suitable for lecture purposes by which the principal facts of electro-optic can be

shown even to large audiences. For this purpose he constructed a special form of radiator and receiver. The radiator consists of a zinc box, A, with one end closed, but open at the opposite end (see Fig. 7). From the sides of the box protrude zinc tubes. In these zinc tubes are fixed ebonite tubes, each of which contains a rod of brass 4 inches long, ending in a brass ball 1 inch in diameter. The rods are attached to long spirals of guttapercha-covered wire, which fill up the rest of the ebonite tube.

The rods are so fixed that the balls are held about a millimetre apart in the interior of the zinc box. The outer end of the wire spirals are connected with the secondary circuit of an induction coil. When the coil is in action sparks pass between the balls and create electric waves about 8 inches in wave-length, which issue from the open mouth of the zinc box. The use of the wire spiral at the end of the rod is to prevent the waves from travelling out at the side tube.

The receiver B (see Fig. 7) consists of a similar box containing a simple form of nickel-filings coherer, or electric wave detector. For the details and description of the mode of action of this device, called the coherer, C, the reader must be referred to the next chapter. The wires in connection with the coherer are brought out through a metal pipe, which must be screwed or soldered into the box. This pipe is a couple of yards in length, and leads to an open metal box, in which is placed an electric bell, G; battery, B_2; relay, R; and relay battery, B_1, so joined up that when the metal filings in the sensitive tube become conductive, the relay is traversed by a current and sets the electric bell in action. The sensitive tube is restored to non-conductivity by giving the receiver box a smart knock with the fingers. The radiator box is held on a stand, so that it can be placed with its axis at any angle.

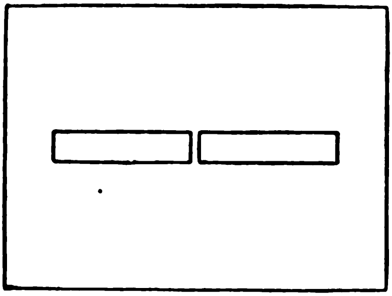


FIG. 8.—Righi Resonator.

Furnished with this apparatus, we can generate a nearly parallel beam of electric radiation, the wave-length of which is only about 8 inches. By its aid we can follow out a series of demonstrations, proving, as Hertz first showed, that this electric radiation is capable of reflection, refraction, and interference, and that various substances are opaque to it and others transparent. Moreover, this radiation, he showed, was stopped by a grating of fine wires placed with their direction parallel to that of the electric force or axis of the radiator. Since Hertz's experiments were made, many have traversed the same ground, and gleaned much additional knowledge.

It is now well known that to produce successfully on a moderately small scale optical effects with electrical radiation, it is necessary to employ radiators of small dimensions.

Professor A. Righi, in 1894, described investigations made with an oscillator consisting of two metallic spheres 3·75 cms. in diameter, immersed in oil. These, when actuated by a large induction coil, produced electric waves 10·6 cms. in length. The resonator consisted of a piece of glass silvered along a certain strip 4 cms. in length, and one-fifth of a centimetre (see Fig. 8). Across the centre of this strip a minute scratch was made, forming the spark gap, and a microscope was employed to observe the tiny sparks in this spark gap.

With this apparatus, or with another circular or ring-shaped resonator formed in the same way of silver deposited upon glass, Righi obtained electrical equivalents of all the familiar optical facts, the resonator acting as an eye to detect the invisible radiation. Since that time other workers, such as Lebedew, Bose, and Lampa, have, by reducing the dimensions of the apparatus yet further, decreased the wave-length of electrical waves to about 4 cms., and obtained electrical radiation the wave-length of which is only fifty to sixty times longer than that of the longest heat rays which have been sifted out by

repeated reflection from a luminous source of radiation, such as the Welsbach gas radiator.

This electrical radiation penetrates easily through dielectric bodies. It is completely reflected from metallic surfaces, and is also more or less reflected from the surface of insulators.

These facts can be easily exhibited with the above-described apparatus. If the radiator box and the receiver box are placed with their open ends towards each other and about a couple of feet apart, the axes being in the same straight line, we find that on pressing a key in the primary circuit of the induction coil the bell in the receiver circuit rings. If, however, a sheet of tin, or tinfoil, or even of silvered paper, is interposed, the radiation is cut off. A sheet of perforated zinc, a wet duster, and even the human hand or body, are found to be perfectly opaque. On the other hand, a slab of wood, paraffin, wax, pitch, glass, ebonite, leather, dry cloth, and all other insulators are transparent. Conductors of any kind are opaque. Amongst liquids, water, alcohol, glycerine, and amyl alcohol are also opaque; whilst paraffin oil, turpentine, bisulphide of carbon, and creosote are very transparent.

If we turn the radiator so that its open mouth is not directly towards that of the receiver, we find that the receiver is not affected, showing that the radiation is not entering it. We can, however, reflect the radiation into the receiver by using as a reflector a sheet of metal, a wet cloth, the hand, or a moist sheet of glass. We can easily prove that this radiation obeys the optical law, and that the angle of incidence is equal to the angle of reflection. All good reflectors are opaque to the radiation. It is curious to notice how much of the radiation is reflected from a sheet of window glass unless carefully dried. This is due to the film of moisture generally present upon it.[14]

By examining the reflection from dielectrics such as glass and paraffin, Professors FitzGerald and Trouton were enabled to settle the long-disputed question as to the direction of the vibration in relation to the plane of polarization in plane polarized light.

According to Fresnel, the luminous vibration is at right angles to the plane of polarization, that is, to the plane of reflection when light is polarized by reflection, whilst according to MacCullagh it is coincident with that plane.

The theory of electric waves indicates, as we have seen, that we are concerned with two vectors, one the magnetic force and the other the electric force, and that both these periodically vary. Theory indicates that the electric force is perpendicular to the plane of polarization. This conclusion was verified by FitzGerald and Trouton, for electric waves were found not to be reflected at the polarizing angle from the surface of a dielectric when the electric force is parallel to the plane of polarization; but reflection occurs at all angles when the electric force is perpendicular to that plane. In the electric ray, therefore, the electric force is perpendicular to, and the magnetic force parallel to or in, the plane of polarization.

Some of the most interesting results in the study of electric waves are those which have flowed from experiments made on the refraction of these electric rays. By the use of a colossal prism of pitch, having a refracting angle of 30°, Hertz was able to discern a refraction of 22° when long electric waves were incident on the prism, indicating a refractive index of 1·69. It is convenient to call the refractive index so determined the electrical refractive index, and the refractive index for luminous or visible light the optical refractive index.

With the author's apparatus, it is very easy to exhibit the power of insulators to refract this radiation with a prism of quite small dimensions. A paraffin prism having a refracting angle of 60°, the length of each side being about 6 inches, is constructed. If we set the radiator and receiver boxes in such positions that the electric ray emerging from the radiator just escapes the receiver, and so does not directly affect it, we shall find that on introducing the above-mentioned paraffin prism in the path the electric ray is refracted just as would be a ray

14 Prof. Trouton has shown that in this case the reflection is really due to a film of moisture on the glass. There is no reflection from a sheet of perfectly dry glass.

of light by a glass prism (see Fig. 9). With a little care, it is easy to measure the deviation of the ray produced by the prism, and hence to calculate the electric index of refraction of the material. The author has, in this manner, measured with his apparatus the refractive index of paraffin wax and also of dry ice, employing for this purpose a large ice prism, cut with the saw out of a block of ice. The refracting angle r of the paraffin prism was 60°, and the minimum deviation d of the electric ray produced by it was 45° to 50°. In the case of the ice, the refracting angle of the prism was 50°, and the minimum deviation of the ray was also 50°. Hence, by the formula—

$$i = \frac{\sin \frac{r+d}{2}}{\sin \frac{r}{2}}$$

we have for the paraffin a value of the electric refractive index—

$$i = \frac{\sin \frac{60^\circ + 50^\circ}{2}}{\sin \frac{60^\circ}{2}} = \frac{\sin 55^\circ}{\sin 30^\circ} = 2 \sin 55' = 1 \cdot 64$$

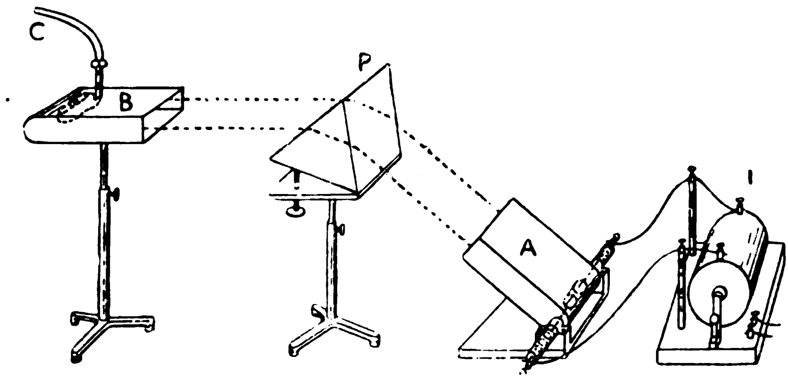


FIG. 9.—Refraction of an Electric Beam by a Paraffin Prism.

and for the ice—

$$i = \frac{\sin \frac{50^\circ + 50^\circ}{2}}{\sin \frac{50^\circ}{2}} = \frac{\sin 50^\circ}{\sin 25^\circ} = 1 \cdot 83$$

By Maxwell's law the squares of these indices should be equal to the dielectric constants. The square of 1·64 is nearly 2·7, and the square of 1·83 is nearly 3·34.

The values obtained by electrostatic methods for the dielectric constant of paraffin wax give numbers not far from 2. The values obtained for ice at or near 0° C., by low frequency on electrostatic methods, give values near 80. If, however, the ice is taken at very low temperatures (−190° C.), then for low frequency we find values of the dielectric constant near 3·0 and under. (See Fleming and Dewar, *Proc. Roy. Soc. Lond.*, 1897, vol. 61, p. 2, on the "Dielectric Constant of Ice at Low Temperatures.")

It is interesting to notice that M. C. Gulton (*Comptes Rendus*, 1900, vol. 130, p. 1119; or *Science Abstracts*, vol. 3, p. 545) has by another electric wave method determined the electric refractive index of dry ice at a little below 0° C. He found the ice did not perceptibly absorb electric waves. He determined the refractive index to be 1·76, corresponding to a dielectric constant 3·1. The wave-length used was 14 mms. He also measured the refractive index for waves of 25 cms. in length,

and up to 2000 cms. He discovered that the electric refractive index progressively decreases from 1·76, corresponding to the 14-mm. waves, down to 1·50 for waves 2088 cms. in length. This last gives a dielectric constant of 2·25, which is not far from the value 2·0, found by M. Blondlot for still greater wave-lengths. Hence the rather rough experiment made by the author with an ice prism gives a result for the dielectric constant which is not greatly different from those found by other electrical methods when the disturbing influence of temperature is eliminated. The observed values of the deviation of the ray by the prism used by the author are unquestionably only approximate values, as the radiation emitted from the radiator is far from being a well-defined ray. It is remarkable, in fact, that when dealing with radiation, the wave-length of which is so large compared with the dimensions of the prism, one should be able to obtain any well-marked refraction at all.

The author has also succeeded, with the same apparatus, in showing the total internal reflection of the ray by a right-angled prism of paraffin. Most interesting of all, however, is the concentration of the electric ray by paraffin lenses. It is easy to cast a plano-cylindrical lens of paraffin wax. The radius of curvature of the curved side may be 6 inches, and the focal length is then 12 inches. Two conjugate foci exist for such a lens (made of a material of refractive index 2), at

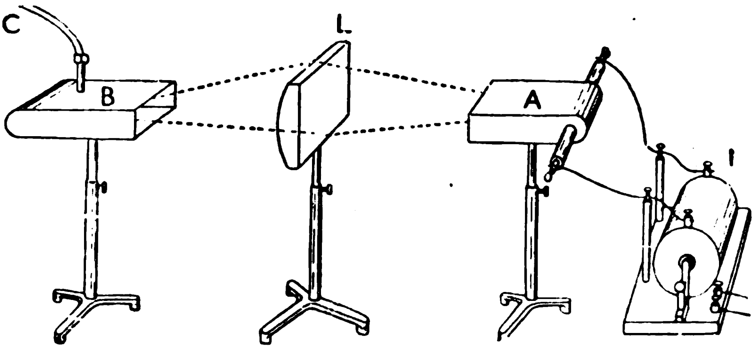


FIG. 10.—Convergence of an Electric Beam by a Paraffin Lens.

equal distances of 24 inches on either side of the lens. If we place the radiator box and receiver box at a distance of 4 feet, we may so adjust the receiver that the direct radiation is too weak to cause the bell to ring. If we interpose the paraffin lens halfway between, it converges the radiation on to the receiver and creates an electrical focus at or near the sensitive tube or box, and the bell of the receiver at once rings (see Fig. 10). This shows clearly that the paraffin lens gathers up the diverging electric radiation and focusses it on to the receiver.

With the same apparatus, interesting experiments can be shown illustrating the action of gratings on this electric radiation.

If we interpose in the path of the ray a grid made by winding wire over a frame (see Fig. 11), it is found that this grid is opaque to the radiation if the wires are held parallel to the electric force of the ray, but transparent if they are held parallel to the magnetic force. The reason for this seems to be that in the former case secondary electric currents are set up in the wires, and these shield the receiver from the original radiation, because the magnetic force of the induced current is exactly opposite in phase to the magnetic force of the original ray at that point where the wire is situated, and hence at the point where the coherer is situated, and accordingly a complete shielding takes place.

The author has found that a set of large pins, arranged parallel to each other at a little distance apart on a sheet of paper, acts in a similar manner; but a set of very small or midget pins similarly arranged is not an effective screen. The use of the small pins simply amounts to the cutting up of a large wire into

very short lengths, and this effectually prevents the induction in it of any sensible current.

A large number of different methods have been employed for determining the electrical refractive index of dielectrics. One of the most simple of these is to employ a Hertz resonator of rectangular form, having spark balls at the centre of one side, and a wire attached to the centre of the opposite side, this wire being connected to the secondary terminal of an induction coil. When the coil is set in operation, no sparks would then be found to occur at the spark balls of the resonator, because the electrical oscillations, starting from the point of origin, arrive at the spark balls by two different routes of equal length. If, however, one side of the rectangle is immersed in paraffin, sulphur, or any other dielectric, the equality is broken down and sparking would occur. This sparking can only be stopped by lengthening the opposite side of the rectangle so as to increase its inductance, and when this is the case the product of inductance and capacity of each side must be equal. Hence we can deduce the dielectric constant, and therefore the refractive index of the material in which one side of the rectangle is immersed.

Experiments of this kind made by Sir J. J. Thomson, to determine the electrical index of refraction of paraffin and sulphur, gave values respectively of 1·35 and 1·7, indicating dielectric constants equal to 1·8 and 2·9.

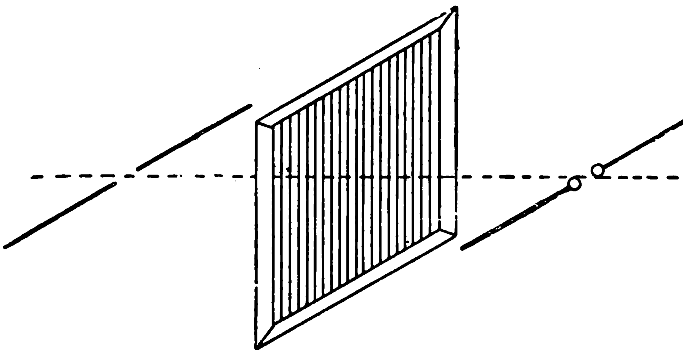


FIG. 11.—The Interposition of a Grid between Radiator and Receiver Rods to show Opacity or Transparency according to Position.

By a similar and more sensitive arrangement, Arons and Rubens found the electrical refractive indices of certain substances to be as follows :—

Castor oil	2·05
Olive oil	1·71
Xylol	1·50
Petroleum	1·40

The values for the electrical refractive index were found to be in fair agreement with the dielectric constants of the same substances, as determined by slow alternations of electric force.

Similar measurements have been made by A. D. Cole, by determining the reduction in wave-length which occurs when the parallel wires of a Lecher arrangement are passed through a trough containing liquid. Cole has measured in this way the electrical refractive index of water and alcohol (*Wied. Ann.*, 1896, vol. 57, p. 290).

In the first experiment, waves having a wave-length of 300 to 600 cms. length in air were used. The wave-length in water for the same frequency was about one-ninth part of the wave-length in air, the exact ratio being 8·9, which is therefore the electrical refractive index of water. This number agrees very well with similar measurements by Drude, using waves of 60 cms. in length, which

gave the value 8·7 for the electrical refractive index of water. The square of 8·9 is 79·21, which is almost identical with the value of the dielectric constant of water as determined by electrostatic methods, such as that employed by Heerwagen.

Electrical waves having a wave-length of 209 cms. in air have been found to give for alcohol an electrical refractive index of 5·24, and the square of this last number agrees very well with the electrostatic or low frequency determinations of the dielectric constant of alcohol.

By employing short electric waves, 5 cms. or so in wave-length, Cole was able to measure the electrical refractive indices of water and alcohol by an indirect method. A sheet of zinc, 1 mm. thick, is found to reflect the electric ray practically without loss at 45°, when the electric component is perpendicular to the plane of incidence. Measurements of the reflective power of a water surface at the same incidence (45°) show that the reflective power is 71·8 per cent. when the electric component is perpendicular to the plane of incidence. In this latter case the zinc surface would reflect 92 per cent.

By applying two formulæ due to Fresnel, the index of refraction can be determined from these data. For water the value deduced was 8·85, for alcohol the electrical refractive index lies between 3·15 and 3·25.

Hence it appears that in the case of alcohol there is a rapid diminution in the refractive index as the wave-length is shortened from 300 or 600 cms. to 5 cms., but for wave-length variation over the same range little or no such diminution occurs in the case of water.

The above facts, however, show that in the case of both these fluids there must be considerable anomalous dispersion. It is well known that within the limits of ordinary visible spectrum a decrease in the wave-length of the refracted light is accompanied by an increase in refractive index in the case of most transparent bodies.

For instance, when light passes through water, alcohol, or bisulphide of carbon, the waves which produce the sensation of violet light are shorter in wave-length and have a larger refractive index, and are therefore more refracted than those which produce the sensation of red light. But this is not universally the case. Many substances are known, such, for instance, as an alcoholic solution of fuchsine, which possess anomalous dispersion, and for these substances the red rays are not less refracted than the violet; but the order of the colours in the spectrum is entirely changed. If light is passed through a thin prism formed with the above solution, the violet rays are found to be less refracted than the red. This anomalous dispersion always accompanies great local absorption in the spectrum; and as Kundt has pointed out, wherever there is a strong absorption band in the spectrum the refractive index is abnormally increased below the band and abnormally diminished above the band in going up the spectrum from the red to the violet.

In the case of water, the optical refractive index for waves having a wave-length within the limits of the visible spectrum is a number lying between 1·4 and 1·3, a decrease in the refractive index within these limits corresponding to an increase in wave-length. If, however, the incident wave-length is increased in length up to 5 cms. or upwards by employing electrical waves, the refractive index rises to a number not far from 8·9, and all experiments show that when using electric waves having wave-lengths between the limits of 6 metres and 6 mms., the electrical refractive index of water is a number not far from 8·9.

Hence there must be a large fall in refractive index in passing from the frequency 6×10^{10}, corresponding towaves of 5 mms. in length, to the frequency of 400×10^{12}, corresponding to the waves which give rise to red light which have a wave-length of about $\frac{1}{1300}$ mm.

Accordingly it is clear that, in the case of water, when we select a sufficiently wide range of vibrations, there must be a marked anomalous dispersion. This may be connected with the strong absorption band which is known to exist in the case of water in the ultra-red spectrum.

For alcohol, it has been found that in passing from electric waves having a wave-length of 8 or 9 metres to waves having a wave-length of about 8 mms., the electrical refractive index drops from a value of 5 or thereabout to a value of 2·5.

In other words, the refractive index diminishes with the wave-length ; hence it is clear that here also there must be anomalous dispersion.

One of the results which has emerged from these investigations is the proof that is afforded by them of the fact that a change in frequency has a very much greater effect upon the electrical refractive index of some substances than others. Thus, as regards ice, it has been shown by M. E. Bouty that when using low frequency alternations of electric force, the dielectric constant of ice at – 23° C. and upwards has a value 78·8.[15] Dr. J. Hopkinson and Professor E. Wilson also made determinations of the same constant, and found that for alternations lying between 10 and 100 a second the dielectric constant of ice is a number of the order of 80.

M. Blondlot (*Comptes Rendus*, 1894, vol. 119, p. 595), using electric waves, has measured the electrical refractive index of ice and found a value of 1·41 for it, corresponding to a dielectric constant 2. The experiments of Dr. Hopkinson and Professor Wilson showed that the dielectric constant of ice measured with a frequency of a million is a number less than 3. Blondlot's value for the electrical refractive index of ice has been confirmed by A. Perrott (*Comptes Rendus*, 1894, vol. 119, p. 601), who found the value of 1·43 of the electrical refractive index.

We see, therefore, that for even, comparatively speaking, very moderate increase in frequency the electrical refractive index of ice falls to a value not far from that of its optical refractive index, whereas over the same range of frequency the electrical refractive index of water still maintains a value 8·9, which is far above the value of its optical refractive index. This and many other similar facts appear to show that when liquid dielectrics of high dielectric constant pass into the solid state, these abnormally large values of electrical refractive indices are more easily reduced to an approximation in value to the optical refractive indices by increased frequency than are those of the corresponding liquids.[16]

As regards glass, Bose has measured the index of refraction of glass for electric waves by a method resembling the optical method of total reflection due to Terquem and Trannin, using electric waves having a frequency of 10^{10} or a wave-length of 3 cms. By four different methods he found a value for the electrical refractive index of glass close to 2·04 ; the value of the optical refractive index for the D rays for the same glass was 1·53. The dielectric constant of this glass, when determined by static methods, would probably have yielded a number not far from 6 ; the square root of its dielectric constant would probably have been a number lying between 2·5 and 3. Hence the electrical refractive index has a number approximating more closely to the optical refractive index than does the square root of the static dielectric constant.

Leaving out of account questions of the absorption of energy, the facts show then that dielectric waves travel very much more slowly through dielectrics than through empty space. In the case of water, the velocities in space and water are in the ratio of 9 to 1, for any electric wave-lengths yet produced ; whilst for visible light waves the ratio is more nearly 1·3 to 1. We find that for alcohol the wave velocity ratio is 5 to 1 for long electric waves, and 2·5 to 1 for the shortest electric waves yet produced ; whereas for visible light waves the ratio is only about 1·3 to 1.

When, however, we select such substances as paraffin oil, turpentine, many hydrocarbons, liquid oxygen, or bodies of simple chemical constitution, we find no such great difference between the velocities of the electrical and light waves of very different wave-length. Then, again, it has been shown that very low temperature annuls this difference in the velocity ratios for electric and eye affecting radiation.

An interesting question then presents itself for solution. We ask, Why is it that water reduces the velocity of non-visible electric waves passing through it so much more, relatively speaking, than it does the velocity of visible light waves of much higher frequency? The answer to this question is, no doubt, to be found in the variation of dielectric constant with frequency. There are a large number of substances of simple symmetrical chemical constitution, such as the liquid gases, paraffins, saturated hydrocarbons, etc., which have all dielectric constants,

[15] *Journ. de Physique*, 1892, vol. 1.

[16] See Fleming and Dewar, "Note on the Dielectric Constant of Ice and Alcohol at Very Low Temperatures," *Proc. Roy. Soc.*, vol. 61, pp. 2 and 316.

lying in value between 2 and 3, and optical and electrical refractive indices lying between 1·4 and 1·7, and these values are but little disturbed by any change in frequency varying between zero and billions per second. It would seem as if the *matter* of which these bodies consist merely had the power of about doubling the dielectric constant of empty space or æther, without much changing the qualitative characteristics of the dielectric constant of the æther. We have seen that, according to Thwing's law, the dielectric constant of these bodies is nearly 2·6 times their density.

On the other hand, all bodies the molecules of which contain those little groups of easily removed atoms which chemists call radicles, such, for instance, as hydroxyl, nitryl, etc., have dielectric constants more or less sensitive to change in frequency according to their temperature. An increase in the frequency generally, but not always, decreases the dielectric constant. Hence, as a rule, in these cases the electric displacement is larger for a given electric force the longer the time during which the force is applied. To elucidate these anomalies, Sir J. J. Thomson suggests the following theory.

The large dielectric constant of certain bodies, even under high frequency electromotive forces, implies that there is a corresponding large refractive index for waves of the same frequency. As a rule refractive index decreases as frequency decreases, hence large dielectric constants of such substances as water implies an abnormal refraction or a range of frequencies for which there is intense absorption. Thomson shows (see article, "Electromagnetic Waves," *Encyclopædia Britannica*, 10th edition) that this is due to the presence of a relatively small number of molecules in a special state. He expresses the opinion that the experiments of Dewar and Fleming on the effect of low temperature on the dielectric constants of numerous bodies are in harmony with this supposition, and that the effect of very low temperature on ice, glycerine, ethylic alcohol, etc., must be to prevent that dissociation of some molecules which produce the bodies, the presence of which, when mixed with the ordinary molecules, gives rise to the abnormally large dielectric constants. Whatever may be the cause, the existence of these large dielectric constants is always dependent on the presence of certain chemical radicles in the molecules of the substance.

7. The Production of Electric Waves by Oscillations in an Open Circuit.— No method has yet been discovered by which an electric wave can be produced, except by means of the excitation of electric oscillations in an electric circuit possessing capacity and inductance. We must, therefore, study a little in detail the actions which take place when high frequency oscillations are set up in a linear circuit.

Let us consider the simple case of a pair of rods placed in one line with their ends (which should be smoothly rounded) placed a millimetre or two from each other (see Fig. 12). Let these rods be connected to the secondary terminals of an induction coil, or in any way brought to a difference of potential just sufficient not to pierce the air between their contiguous ends and make a spark. Then these rods are charged, one with positive and the other with negative electricity. There is, therefore, a distribution of electric strain in the space round the rods which very roughly may be represented as to direction by the dotted lines in Fig. 12. In the next place, suppose the difference of potential increased until a spark passes. The rods just before that moment are in the condition of the two coatings of a condenser; in fact, they have a certain capacity with respect to each other, and a certain charge determined by that capacity and by their difference of potential. When the spark passes, this condenser begins to discharge with oscillations. We may regard these oscillations as due to the rapid movement to and fro in the wire of *electrons* or point-charges of negative electricity and the lines of electric strain as starting from and terminating on these electrons. Accordingly the oscillations are oscillations of the ends of the lines of electric strain where these abut on the wire or rod. The whole of the facts discovered by Hertz show that when a change is made in the direction or amount of the electric strain at any one point in the medium this change is not instantly felt at all points, but is propagated through the medium with a finite velocity equal to that of light. Accordingly we must consider the lines of electric strain as possessing inertia

which, in fact, is the inertia in the medium in which the lines themselves are a peculiar state. If the end of a line of electric strain has a sudden movement given to one end, this movement being in a direction at right angles to the direction of the line, the result is to create in the line a *kink* which travels outwards, just as would a *kink* on a stretched rope if the end were given a jerk at right angles to the direction of the rope. If the end of a line of electric force terminates on a point-charge of electricity or so-called electron, then a sudden movement of this electron, say from A to B (see Fig. 13), will be accompanied by the outward propagation of *kinks* or places of sudden bend or flexion along the lines of electric strain. It will be seen that we are here treating the lines of electric force as if they were objective realities, and not merely curved or straight directions in space.

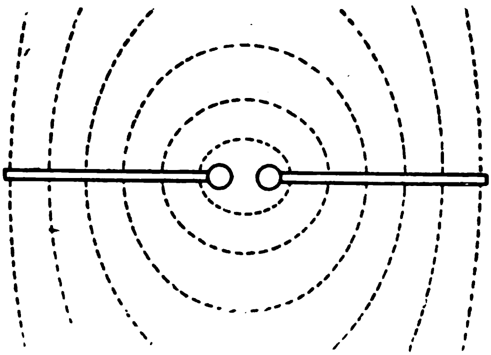


FIG. 12.—Lines of Electric Strain round a Hertz Linear Oscillator.

The question whether a line of electric force is to be regarded merely as a convenience of thought similar to a line of latitude or longitude, or whether it has an objective reality, is too large to discuss here fully. There are some good reasons for considering that in a space occupied at least by air or other gas in which electric force exists, that force may not be distributed absolutely without discontinuity through the space, but the reader must be referred to Faraday's "Experimental Researches on Electricity," vol. iii. ser. xxix. §§ 3273, 3297, and

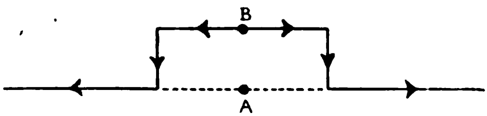


FIG. 13.

3299, and also to his memoirs "On Physical Lines of Magnetic Force" and "Thoughts on Ray Vibrations" (*Phil. Mag.*, ser. iii. vol. xxviii., 1846) for arguments in favour of this view. A confirmatory argument has been put forward by Sir J. J. Thomson in his book, "Electricity and Matter," p. 63, for the physical existence or objective reality of lines of electric force based on the ionization of gases by Röntgen rays. The reader may also be referred to a suggestive article by the same author in the *Phil. Mag.*, ser. 6, vol. 19, p. 301, February 1910, confirming this view of the field. It may therefore be taken that there are some valid reasons for thinking of lines of electric force as if they were discrete entities or bodies capable of being moved through space, or at least displaced in a continuous medium like vortex filaments in a liquid. These filaments behave as if they possessed inertia. In reality this inertia is the inertia of the medium in which they exist. When moved through space laterally these

lines or tubes of electric force create magnetic force, this magnetic force being proportional to the velocity of the line of electric force perpendicular to itself. Since the lines or tubes possess inertia, a sudden displacement made at one place does not appear everywhere at once, but is propagated along the line of electric force as a kink in a rope travels along a rope.

Consider, then, a right angle kink travelling outwards along a line of electric strain. It moves with the velocity of light or with the velocity $u = \frac{1}{\sqrt{\mu K}}$.

The portion which constitutes the kink is therefore moving sideways or at right angles to its own direction through space. When a portion of a line of electric strain so moves through space it gives rise to a magnetic force at right angles to its own direction and to the direction in which it is moving. If a portion of a line of electric strain in a medium of dielectric constant k along which the electric force is E, moves with a velocity V in a direction at right angles to itself, it gives rise to a magnetic force $H = kEV$ at right angles to the electric force and to the direction of motion.

This is an important principle which must be clearly grasped. Suppose that a line of electric strain exists, which is represented by the firm line AB in Fig. 14, and that it moves parallel to itself from the position AB to the position CD. This movement may be considered to be produced by the creation of a closed line of electric strain of equal strength in the direction represented by the dotted rectangle. If such a closed line is created, it would, so to speak, annihilate the strain line AB, because the circuital strain is in the opposing direction on that side, and would create a strain in the same direction along CD. This circuital strain is equivalent by Maxwell's principle to a closed electric current *whilst it is increasing*, and hence in coming into existence it must have a magnetic force, which in this case is towards the reader and perpendicular to the paper. Accordingly, during the time the movement of the original line of electric strain is taking place it is accompanied by the production of a magnetic flux, which is in a direct normal to itself and to its direction of motion. We can easily remember this directional relation by a *hand rule* as follows:—

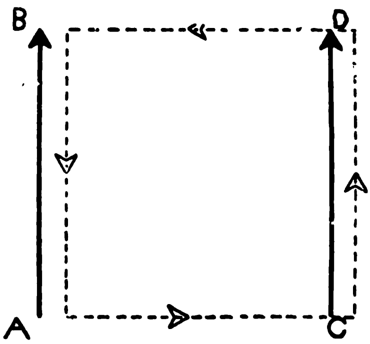


FIG. 14.—Lateral Displacement of a Line of Electric Strain, producing Magnetic Force at Right Angles.

Hold the forefinger, middle finger, and thumb of the *right hand* as nearly as possible in the direction of three co-ordinate axes mutually at right angles (see Fig. 15).

Let the direction of the *forefinger* represent that of the line of Electric *Force*, and the direction of the *thumb* the direction of its *motion* in space, then the direction of the *middle finger* will represent the direction of the resulting *Magnetic Flux*. If μ is the permeability of the medium, and V is the velocity of the line of electric strain perpendicular to itself, and if E is the electric force along the line, then the magnetic force H produced by its motion is such that $H = \frac{E}{\mu V} = kVE$.

The proof of this will be found in Oliver Heaviside's treatise on "Electromagnetic Theory," vol. 1, p. 56.

If, then, we consider a very long line in which electric oscillations are taking place, we can think of the electrons in the wire as oscillating backwards and forwards, and the lines of electric strain which abut perpendicularly on the wire as having their ends wagged backwards and forwards rapidly. The result is to propagate outwards along these lines right-angled kinks or discontinuities, as represented in Fig. 16, in which the dots represent the end-on view of the accompanying lines of magnetic force.

It will be seen that this is equivalent to the continual motion outwards from the wire of lines of electric strain which are parallel to the wire, these lines being made up of the union of the portions of the kinks which lie parallel to the wire and move outwards in a direction normal to their length.

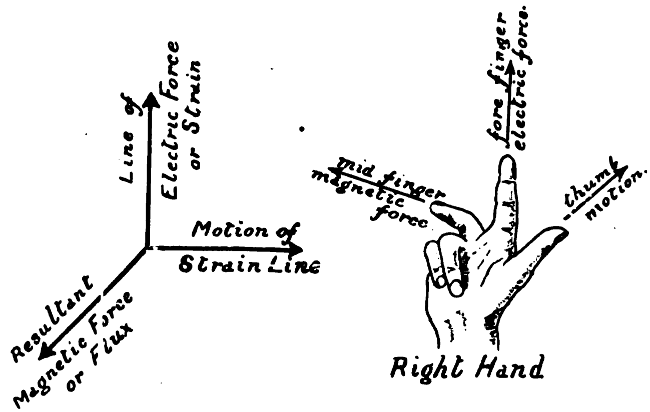


FIG. 15.—Mnemonic Rule for the Relation between Electric Strain, Motion of Strain Line, and Resulting Magnetic Force.

It will be clear, then, that these moving electric strain lines give rise to magnetic force lines which must be distributed in circles round the wire with centres on its axis; and these also must be considered to expand outwards, like the ripples on a lake when a stone is thrown on its surface.

As the oscillations continue we have a procession of lines of electric strain in which the electric force is alternately directed one way and the other, which move outwards radially from the wire. Mingled with these, and at right angles to them, we have the circular lines of magnetic flux also alternately directed.

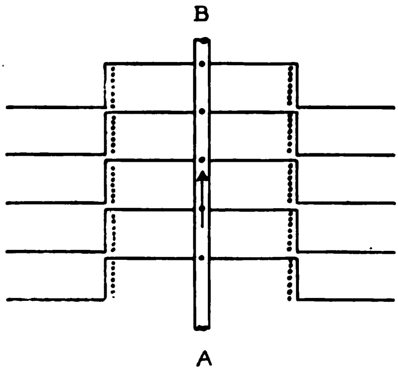


FIG. 16.

At any point in space not too near the wire there is an alternating electric force which is parallel to the wire in direction and an alternating magnetic force which is at right angles to it. The two forces are periodic and pulsate together, coming to their maximum values at the same instant at places not very near the wire. The result is to propagate outwards a cylindrical electromagnetic wave.

If the wire in which oscillations are taking place is finite in length, then the ends of the alternately directed electric strain lines will be united so as to form closed loops of electric strain which will move outwards in radial planes from the oscillator. We have then to find the form of the lines of strain and the magnitude of the forces at any point in the space round the oscillator.

We shall proceed then to discuss the effects produced when high frequency electric oscillations take place in a linear oscillator, such as that shown in Fig. 12. In a strikingly original paper Hertz considered mathematically the case of such oscillations produced in a pair of very short rods terminating in balls called a

Hertz doublet, or dumb-bell oscillator.[17] His analysis proceeds on the following lines.

We have seen that the components of the electric and magnetic forces, and hence all the quantities with which we are concerned in considering the changes of electric and magnetic force, propagated through the electromagnetic field must satisfy a certain differential equation of the type— h 304

$$A^2\frac{d^2\phi}{dt^2}=\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+\frac{d^2\phi}{dz^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

or, as it may be written, $A^2\ddot{\phi}=\nabla(\phi)$, where ϕ is any function of x, y, z, and t, and A is the reciprocal of the velocity with which the effect is propagated through space (see equation (18), Chap. V. p. 304). This equation, as we have shown, is satisfied by the components of the electric and magnetic forces and potentials in the electromagnetic field.

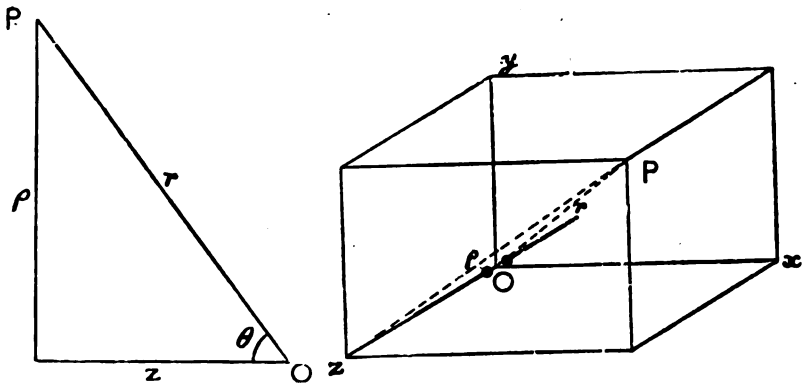


FIG. 17.

We then consider a small Hertz oscillator or doublet, at the centre of which is a spark gap which is taken as the origin of co-ordinates. Let the doublet be placed horizontally, and its direction taken as the axis of z (see Fig. 17). Let the axis of x and the axis of y be taken in directions perpendicular to z, and in a plane perpendicular to the axis of the oscillator and at right angles to each other. Then the position P of any point in the field may be specified by stating its vertical distance ρ from the axis of z and its distance r from the origin. Everything being symmetrical with respect to the axis of z, the above system of co-ordination is sufficient.

Therefore $\rho=\sqrt{x^2+y^2}$. Also we may define the position in polar co-ordinates where r is the distance of the point considered from the origin and θ is the angle between the directions of r and the z axis (see Fig. 17). Then $\rho=r \sin\theta$ and $z=r\cos\theta$. We desire to find expressions for the electric and magnetic force at all points in the field.

Suppose, then, that we consider the case of a small Hertzian oscillator consisting of two short metal rods placed in line with each other, the inner ends separated by a small spark gap and the outer ends terminated in small metal spheres. If we start oscillations in this linear circuit the balls will be alternately charged positively and negatively, and as these charges change in magnitude and sign an alternating current is produced in the rods.

To bring the problem within the grasp of simple analysis we must limit the

[17] See H. Hertz, *Wied. Annalen*, 1889, vol. **36**, p. 1, "The Forces of Electric Oscillations treated according to Maxwell's Methods." See also "Electric Waves," by H. Hertz, English translation by D. E. Jones, p. 137.

conditions by assuming that the current in the rods has at all points the same value, and that the electric charges are located entirely on the balls. These assumptions are not completely in accordance with actual facts, but are not greatly in error when we are considering a short oscillator. We have, then, two effects produced in the surrounding space—

(i.) The electric charges on the spheres produce a distribution of electric force and *scalar electric potential* in the space; and

(ii.) The current in the rods produces a magnetic force and a *vector potential* in the space.

This term "vector potential" was a name given by Maxwell to a mathematical quantity such that its curl gives us the magnetic force at any point due to a current, just as the term "scalar potential" is the name for a quantity such that its space variation or gradient in any direction gives us the electrostatic force when the charges are steady. If we have any current i in an element of length of a circuit ds, then it can be shown that the vector potential due to this element at any point P at a distance r from the element is $\frac{ids}{r}$, and the vector potential due to any circuit is the vector sum of the vector potentials due to the several elements (see Fig. 18).

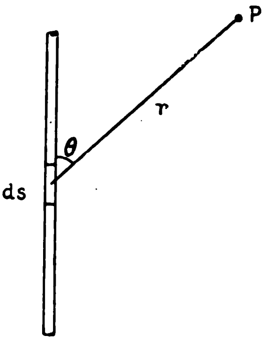


FIG. 18.

The vector potential being a directed quantity can be resolved into axial components, and must be added according to the rules of vector addition. It is usual to denote the components of the vector potential in the directions of the axes of x, y, and z by the letters F, G, and H.

On the other hand, the scalar potential is not a directed quantity, and the scalar potential due to any distribution of electricity is the algebraic sum of that due to each element of charge dq separately. The scalar potential due to the charge dq at a point at a distance r being $\frac{dq}{r}$ and the total scalar potential being $\frac{\Sigma dq}{r}$. We shall denote the electric force at any point by E and its axial components by the letters X, Y, and Z as before, the scalar potential by ψ, and the axial components of the vector potential by F, G, and H, and the axial components of the magnetic force M by α, β, and γ. Then Maxwell showed that when we are considering operations in pure æther, for which we may take the dielectric constant and permeability to be unity, we have the relations between the above quantities expressed by the six equations:—

$$\left.\begin{aligned} X &= -\frac{dF}{dt} - \frac{d\psi}{dx} \\ Y &= -\frac{dG}{dt} - \frac{d\psi}{dy} \\ Z &= -\frac{dH}{dt} - \frac{d\psi}{dz} \end{aligned}\right\} \quad . \quad (20) \qquad \left.\begin{aligned} \alpha &= \frac{dH}{dy} - \frac{dG}{dz} \\ \beta &= \frac{dF}{dz} - \frac{dH}{dx} \\ \gamma &= \frac{dG}{dx} - \frac{dF}{dy} \end{aligned}\right\} \quad . \quad . \quad . \quad (21)$$

These last being the expression of the fact that the magnetic force is the curl of the vector potential.

We have then to find for the small Hertzian oscillator in question expressions for the scalar and vector potential at all points around it. Since everything must be symmetrical with respect to the axis of the oscillator, it will be an advantage to assume the oscillator to be placed with its axis in the direction of the z-axis, and we shall, following Hertz, consider the y-axis to be drawn to the left, and the x-axis towards the reader. Everything is then symmetrical with respect to the axis of z, and we need only concern ourselves with the calculation of the forces in and at right angles to the plane yz.

Suppose, then, that a small sphere charged with a quantity of positive electricity $+q$ is placed at the origin O, and that we consider a point T of which the co-ordinates are y, z in the plane yz, and at a distance $OT=r$ from the origin. The potential at the point due to this charge is $-\frac{q}{r}$. Suppose the sphere to be displaced along the z-axis in a positive direction by a distance $\frac{1}{2}\delta z$. The potential then created by it at the same point yz is—

$$-\left\{\frac{q}{r}+\frac{1}{2}\frac{d}{dz}\left(\frac{q}{r}\right)\delta z\right\}$$

Again, if an equal quantity of negative electricity $-q$ is placed on a small sphere at the origin, its potential at the point in question is $+\frac{q}{r}$, and if this is displaced downwards through a distance $\frac{1}{2}\delta z$, the potential due to it at the point T is—

$$-\left\{-\frac{q}{r}-\frac{1}{2}\frac{d}{dz}\left(-\frac{q}{r}\right)\delta z\right\}$$

If then we form a small oscillator of two spheres separated by a distance δz, and place the axis along with the z-axis and the centre at the origin, then when the charges on the spheres are $+q$ on the top ball and $-q$ on the bottom ball, the scalar potential ψ due to these two electrostatic charges will be the sum of those due to each sphere separately, or will be—

$$\psi=-\frac{d}{dz}\left(\frac{q}{r}\right)\delta z \qquad \text{(22)}$$

Suppose then that q varies with the time in a simple harmonic manner, so that $q=Q\sin nt$ where $\frac{2\pi}{n}$ is the periodic time of the oscillation, and let the product of the length of the oscillator and the maximum charge on each sphere $=Q\delta z=\phi$ be called the *electric moment* of the oscillator, then we have—

$$\psi=-\phi\frac{d}{dz}\left(\frac{\sin nt}{r}\right) \qquad \text{(23)}$$

This expression, however, is obtained on the assumption that the electric effects are propagated through space instantaneously. Hertz's experiments prove, however, that this is not the case, but that the forces and potentials are propagated with a finite velocity. Accordingly if the charges on the spheres of the oscillator are rapidly oscillating, the potential at any point in the field will not depend upon the state of the oscillator at the same instant, but upon a state at a time earlier by the time taken for the effect to be propagated the distance r. If, then, u denotes the velocity, the time taken by the effect to travel to the point in question is $\frac{r}{u}$.

Therefore the potential ψ at the selected point in the field is, in fact, given by—

$$\psi=-\phi\frac{d}{dz}\left(\frac{\sin n(t-Ar)}{r}\right) \qquad \text{(24)}$$

where $A=\frac{1}{u}$. Hence the effect is propagated as a wave motion. It is cyclical in space as well as in time. If we call λ the wave-length or shortest distance between two places at which the effect is a maximum at the same instant, and T the periodic time, then $\lambda=uT$. It is convenient to write m for $\frac{2\pi}{\lambda}$ and $n=\frac{2\pi}{T}$. Also, since Ar is usually larger that t, we may write $\sin(mr-nt)$ instead of $\sin n(t-Ar)$. It is convenient to write Π for $\frac{\sin(mr-nt)}{r}$.

We then have for the potential at the point yz, due to the oscillating electric charges of the oscillator, the expression—

$$\psi=-\phi\frac{d\Pi}{dz} \qquad \text{(25)}$$

We have, in the next place, to find the vector potential. At the moment when the charges on the spheres of the oscillator have a value q, the current along the connecting-rod has a value $-\frac{dq}{dt}$ in electrostatic measure, or $-\frac{1}{u}\frac{dq}{dt}$ in electromagnetic measure. Hence the vector potential due to this current existing in an oscillator of length δz at a point at a distance r from its centre is $\frac{1}{u}\frac{1}{r}\frac{dq}{dt}\delta z$. Since r is independent of t, we can write the above expression in the form—

$$\frac{1}{u}\frac{d}{dt}\left(\frac{q}{r}\right)\delta z$$

and then, as in the case of the expression for the scalar potential, we must put instead of q the function Q sin $(nt-mr)$ to obtain the component H of the vector potential at the point in the field which is parallel to the direction of the current element. Finally, using the same notation as in the case of the scalar potential, we have—

$$H=\frac{\phi}{u}\frac{d\Pi}{dt} \qquad (26)$$

Since there is no current parallel to the axis of x or y, the components F and G of the vector potential are zero. Hence, inserting these values of ψ and H in the equations (25) and (26) for the electric and magnetic force components, we have—

$$\left.\begin{aligned} X&=\phi\frac{d^2\Pi}{dxdz}\\ Y&=\phi\frac{d^2\Pi}{dydz}\\ Z&=-\phi\left(\frac{d^2\Pi}{dx^2}+\frac{d^2\Pi}{dy^2}\right)\end{aligned}\right\} \quad (27) \qquad \left.\begin{aligned} \alpha&=A\phi\frac{d^2\Pi}{dydt}\\ \beta&=-A\phi\frac{d^2\Pi}{dxdt}\\ \gamma&=0\end{aligned}\right\} \quad (28)$$

In these equations we assume that the dielectric constant and magnetic permeability are both taken as having unit value.

We have then to find the various differential coefficients of—

$$\Pi=\frac{\sin(mr-nt)}{r}=\frac{\sin\chi}{r}$$

and to introduce these into the above equations. We have then—

$$\left.\begin{aligned} X&=\frac{\phi}{r^3}\left\{3\sin\chi-m^2r^2\sin\chi-3mr\cos\chi\right\}\frac{x}{r}\frac{z}{r}\\ Y&=\frac{\phi}{r^3}\left\{3\sin\chi-m^2r^2\sin\chi-3mr\cos\chi\right\}\frac{y}{r}\frac{z}{r}\\ Z&=\frac{\phi}{r^3}\Big\{(2\sin\chi-2mr\cos\chi)+\\ &\qquad(m^2r^2\sin\chi+3mr\cos\chi-3\sin\chi)\frac{x^2+y^2}{r^2}\Big\}\end{aligned}\right\} \qquad (29)$$

$$\left.\begin{aligned} \alpha&=\frac{A\phi n}{r^2}\left\{mr\sin\chi+\cos\chi\right\}\frac{y}{r}\\ \beta&=-\frac{A\phi n}{r^2}\left\{mr\sin\chi+\cos\chi\right\}\frac{x}{r}\\ \gamma&=0\end{aligned}\right\} \qquad (30)$$

In the above equations the electric forces are expressed in electrostatic units, and the magnetic in electromagnetic units.

These equations show that the magnetic force is distributed in circles whose centres lie on the axis of z, because $\alpha^2+\beta^2$ is a constant, and $\frac{\alpha}{y}+\frac{\beta}{x}=0$. Hence

there is no magnetic force along the line perpendicular to the axis z. If we write $\sin\theta$ for $\frac{y}{r}$ and $\cos\theta$ for $\frac{z}{r}$, and consider only the forces at points at a large distance from the origin in the plane yz, at which mr is therefore much greater than unity, and therefore m^2r^2 much greater than mr, we can reduce the above six equations (29) and (30), to three, viz.—

$$\left.\begin{aligned} Y &= -\frac{\phi m^2}{r}\sin\chi\sin\theta\cos\theta \\ Z &= \frac{\phi m^2}{r}\sin\chi\sin^2\theta \\ \alpha &= \frac{A\phi mn}{r}\sin\chi\sin\theta \end{aligned}\right\} \qquad (31)$$

These last equations are those given by Hertz as his solution of the problem in his original memoir.[18] They show that at distances large compared with the dimensions of the oscillator the magnetic and electric forces vary inversely as the distance, a fact which has been approximately confirmed by experiment. Also since $\beta=\gamma=0$ and $Z\cos\theta+Y\sin\theta=0$, it is clear that at large distances the electric and magnetic forces are both perpendicular to the radius vector.

To obtain the equations to the lines of electric force, it is necessary to transform the above expressions for X, Y, and Z into another form. Let the distance of the point or space at which we are considering the force from the axis of z be denoted by ρ. Then $\rho^2=x^2+y^2$. Let us denote the component of the electric force in the direction of ρ by R. It is then quite easy to show by differentiation that—

$$\frac{d^2\Pi}{dx^2}+\frac{d^2\Pi}{dy^2}=\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\Pi}{d\rho}\right) \qquad (32)$$

And hence we have— $$Z=-\frac{\phi}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\Pi}{d\rho}\right) \qquad (33)$$

Also $$R=X\frac{x}{\rho}+Y\frac{y}{\rho} \qquad (34)$$

But $$X=\phi\frac{d^2\Pi}{dx\,dz},\text{ and } Y=\phi\frac{d^2\Pi}{dy\,dz} \qquad (35)$$

Hence it follows that— $$R=\frac{\phi}{\rho}\frac{d}{dz}\left(\rho\frac{d\Pi}{d\rho}\right) \qquad (36)$$

Now the equation to a line of electric force in the plane of ρ and z is $Zd\rho-Rdz=0$; for this is the mathematical expression of the fact that the resultant force is directed along the line of force.

Substituting, then, in this last equation the values of Z and R given above, we have as the differential equation of the lines of electric force—

$$\frac{d}{d\rho}\left(\rho\frac{d\Pi}{d\rho}\right)d\rho+\frac{d}{dz}\left(\rho\frac{d\Pi}{d\rho}\right)dz=0 \qquad (37)$$

and integrating this last we have— $$\rho\frac{d\Pi}{d\rho}=c=\text{a constant} \qquad (38)$$

as the equation to the lines of electric force of the oscillator in a meridional plane. If that plane is the plane yz, then we have $\rho=y$, and hence—

$$\rho\frac{d\Pi}{d\rho}=y\frac{d\Pi}{dy}$$

and the equation to the lines of force transposes into—

$$\cos(mr-nt)-\frac{\sin(mr-nt)}{mr}=\frac{c}{m\sin^2\theta} \qquad (39)$$

18 See Hertz's book, "Electric Waves," English translation by D. E. Jones, "The Forces of Electric Oscillations treated according to Maxwell's Theories," chap. ix. p. 137.

we can then give to t any required value and determine a family of curves for a given epoch which represents the lines of electric force of the oscillator. If mr is a quantity large compared with unity, that is, if the field at a large distance from the oscillator is considered, then $\frac{\sin(mr-nt)}{mr}$ becomes a small quantity, and we may take as the equation to the lines of force—

$$\cos(mr-nt)=\frac{c}{m\sin^2\theta} \quad \quad \quad \quad (40)$$

To plot the form of the lines we may then select any values of m, θ, and c such that $\frac{c}{m\sin^2\theta}$ is not greater than unity. We then find the angle a which has this fraction as its cosine, and it follows that—

$$mr=nt\pm a$$

Hence for every value of t and θ there are two values of r, except in the case when $\frac{c}{m\sin^2\theta}=1$, and then $a=0$. This shows that the form of the line of electric force

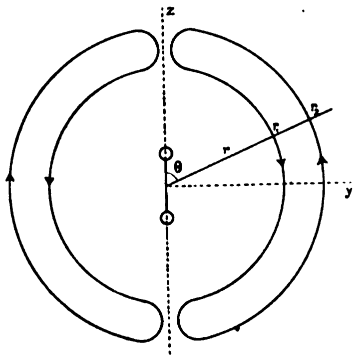


Fig. 19.

at a considerable distance from the oscillator is a closed curved loop, as shown in Fig. 19. If we take any fixed values of t, c, m, and θ, then a is determined. If then t increases, whilst c, m, and θ remain the same, there are still a pair of values of r derived from the equation $r=\frac{(nt+a)}{m}$, but their absolute values are larger. Hence if we consider that any particular loop of force is identified by the particular value of the constant c, which is used to calculate the value of the angle a, whose cosine is $\frac{c}{m\sin^2\theta}$, we may say that as time increases the loop moves outwards away from the oscillator. In a certain sense we may then say that the oscillations in the oscillator result in the throwing off of closed loops of electric force which move outwards away from the oscillator with the velocity of light. An objection has sometimes been raised to this mode of regarding the phenomenon, on the ground that we cannot identify or ear-mark any particular loop of electric force so as to follow its progress. If, however, we consider each particular loop as characterized by the constant c which belongs to it, we can, as it were, watch the movement of one particular loop. In the celebrated paper in which Hertz first discussed the theory of the oscillator on the lines above given, he gave a series of diagrams representing the distribution of the electric force, that is, the lines of electric force round such an oscillator or doublet when in action. He considered four stages separated in point of time by one-eighth part of a complete period T, or to $\frac{T}{8}$, and delineated the electric field at moments corresponding to 0, $\frac{T}{8}$, $\frac{T}{4}$, $\frac{3T}{8}$, starting from the time when the current in the oscillator was a maximum.[19] These diagrams are not reproduced here, but they are very similar

[19] In interpreting Hertz's diagrams, it must be remembered that his T is our $\frac{T}{2}$ and his λ is our $\frac{\lambda}{2}$. In the diagrams as drawn in Plate VI. the λ and T have the signification of a complete wave-length and complete period. In Hertz's diagrams the lines of electric force are not continued right up to the oscillator, because the equations by which he determines their form are not valid at places very near the oscillator, but only at and beyond a certain distance, which is rather more than a quarter wave-length.

to those given in Figs. 1, 2, 3, 4, Plate VI. (see end of this chapter), which are taken from a paper by Dr. F. Hack. Hertz's diagrams are given in the English translation of his papers by D. E. Jones, entitled "Electric Waves," pp. 144, 145 (Macmillan & Co.).

In Fig. 1, Plate VI., we see the oscillator is not a source of lines of electric force. The current in it is at its maximum value at that moment, and there are no electric charges at the ends of the rods.

In Figs. 2 and 3 we see the lines of electric force increasing as the charges accumulate, and the fourth diagram (Fig. 4) shows us the state of affairs as the discharge is beginning to take place. The lines of electric strain are bending inwards, and in one place a line has already crossed, or decussated, and formed a little detached loop or circle of electric strain. As this process continues the result is to detach or throw off closed loops of electric strain which are represented by the closed lines lying outside a certain boundary line. This boundary is the region within which the lines of strain are, as it were, giving birth to the closed loops, and it is only outside this area that we have electric radiation in the true sense. Then, in addition to these lines of strain, we have to imagine other closed lines of magnetic flux which lie in planes perpendicular to the paper, and have their centres on the two axes or axis of the oscillator.

The result of the operations is then to detach from the oscillator a successive series of closed lines of electric strain, the strain being oppositely directed round successive loops. As these move outwards they are accompanied by expanding rings of magnetic flux in planes at right angles, and at a distance from the oscillator we have a spherical wave of electric radiation, the electric force everywhere being tangential to the surface, directed, so to speak, along lines of latitude, and magnetic flux directed along lines of longitude if we suppose the z axis to be the axis of the rotation of the earth. The magnetic flux and electric strain are periodic, or fluctuate harmonically in space and in time, and at large distances the flux and force are in step with each other. The energy is propagated outwards along radial lines, and therefore in a direction at right angles to the lines of electric and magnetic force.

At very great distances the spherical wave becomes practically a plane wave. The electric and magnetic forces are at right angles to each other, and in the plane of the wave. Along the line at right angles to the axis of the oscillator, the electric force is parallel to the axis of the oscillator, and the magnetic force is at right angles to it. The energy is transmitted at right angles to the electric and magnetic forces.

The reader should particularly notice that Hertz's and our assumption as to the form of the function which is represented by the symbol Π, viz.—

$$\Pi = \frac{\sin (mr - nt)}{r}$$

is equivalent to assuming that the electric oscillations in the radiator are persistent or *undamped*, in other words, are continuously maintained. We know, however, from Bjerknes' researches that this is very far from being the case, and that the oscillations of such a radiator are highly damped.

Accordingly, various investigators have considered the modification of the form of the magnetic and electric force lines when a train of highly damped oscillations is emitted. The effect of the damping has been considered in an important memoir by Professor K. Pearson and Dr. Alice Lee.[20]

Assuming that Π is a function of the form—

$$\frac{Et}{r}\epsilon^{-p(t-Ar)} \cdot \sin q(t - Ar) \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Professor Pearson and Miss Lee have discussed the whole question afresh in their above-mentioned paper, and draw the following general conclusion from their analysis :—

[20] See Prof. Karl Pearson, F.R.S., and Miss Alice Lee, "On the Vibrations in the Field round a Theoretical Hertzian Oscillator," *Phil. Trans. Roy. Soc.*, 1900, vol. 193, A, p. 159.

(I.) The effect of damping makes itself very sensible in modifying the form of the wave surface as propagated into space from a theoretical oscillator. The typical Hertzian wave diagrams require to be replaced by the fuller series shown in Plates II., III., IV., and V. at the end of this chapter.

(II.) *Three* waves of electromagnetic force may be considered as sent out from the oscillator, and these waves are capable of physical identification.

(i.) A component wave of transverse electric force.

(ii.) A component wave of electric force parallel to the axis.

(iii.) A wave of magnetic force.

The waves of magnetic force and of component axial electric force both move outwards with the same velocity at all points, and this velocity is identical for all points at the same distance from the oscillator. The intensity of the first force for points on the same sphere varies as the cosine of the latitude, but that of the second force is constant. The wave of component transverse electric force moves outward with equal velocity for all points at the same distance from the oscillator, and its amplitude varies as the cosine of the latitude. Its velocity after it has reached a certain distance from the origin is always greater than that of the waves of component axial electric force and of magnetic force, and its excess over the velocity of light tends to become three times the excess of the velocity of the wave of magnetic force over the velocity of light.

(III.) The velocities of these waves undergo remarkable changes in the neighbourhood of the oscillator, even within such distances as Hertz employed.

(IV.) The point of zero phase for both transverse and axial component electric waves does not coincide with the centre of the oscillator, so that these waves appear to start from a sphere of small but finite radius round the oscillator. A fourth wave, dealt with by Hertz, the wave of magnetic induction, does not, as he supposes, start from the centre of the oscillator with zero phase, but in the case of a damped wave train with a small but finite phase.

(V.) The analysis of these waves and of their singular points in the neighbourhood of the oscillator appears to add something to Hertz's discussion; it is possible that it may throw light on the difficulties which arise in connection with some of his interference experiments. It seems that all interference experiments ought to be made at distances greater than 6 to 7 from the centre of the oscillator, roughly about a wave-length from the oscillator, whereas Hertz rather terminated than started his experiments at this distance. At such distances the phase curves are approximately parallel to their asymptotes. To exhibit the form of the electric strain lines at various epochs thrown off from a damped linear oscillator, Professor Pearson and Miss Lee delineated a series of 56 diagrams (see Plates II., III., IV., and V.), covering a period of time equal to seven complete periods of the oscillator. The oscillator was assumed to be a small linear oscillator of such *moment* that the quantity $\frac{Q\lambda}{2\pi El}$ had values 50, 30, 10, 1, -1, -10, -30, -50. In the diagrams the oscillator is represented by the small dumb-bell within the inmost circle. The fine continuous curves correspond to the intensity $\pm$ 50, the fine dotted curves to the intensity $\pm$ 30, the heavy continuous curves to the intensity $\pm$ 10, and the heavy dotted curves to $\pm$ 1. The outermost circle is the boundary of the field explored, and the small inner circle surrounds the space within which it is not legitimate to consider the oscillator a double point. These curves show us the distribution on one meridional plane of the strain lines at various epochs. These diagrams of Professor Pearson and Miss Lee are very instructive. They show us the whole process of creating an electric wave. If the diagrams are cut out and placed round a zoetrope, or "wheel of life," the operation of a linear oscillator can be made visible to the eye. If reproduced on a film for a kinematograph they provide the means of showing an electric oscillator at work generating electric radiation.

In this paper it is assumed that the epoch from which the time is measured is that at which the vibrations begin, so that the field considered is confined within the sphere of which the radius is $\frac{t}{A}$, where A is the reciprocal of the radiation velocity.

Professor A. E. H. Love has pointed out, in another interesting paper on this subject, that the front of the advancing wave is a surface of discontinuity in regard to the electric and magnetic forces.[21] Within this surface the forces are expressed by the formulæ given by Hertz, which may be generalized in the following form :—

$$\left.\begin{aligned} A\frac{\delta}{\delta t}(X, Y, Z) &= \text{curl}\,(\alpha, \beta, \gamma) \\ -A\frac{\delta}{\delta t}(\alpha, \beta, \gamma) &= \text{curl}\,(X, Y, Z) \end{aligned}\right\} \qquad (42)$$

The only difference between these formulæ and those given by Hertz is that Hertz used a left-handed system of axes, x, y, and z; and it is more convenient to employ the normal or right-handed system.

To adapt the analysis to the case of damped oscillations, Love, following Pearson and Lee, takes as the expression for Hertz's quantity Π the expression—

$$\Pi = \frac{C}{r}\epsilon^{-\frac{\delta}{\lambda}(ut-r)}\sin\frac{2\pi}{\lambda}(ut-r-\phi) \qquad (43)$$

where C is a constant which determines the amplitude, δ is the logarithmic decrement of the oscillations per complete period, u is the velocity of radiation, and ϕ is a constant expressing the phase.

According to the experiments of Bjerknes already quoted, δ (for one complete period) has a value of about 0·4 for an oscillator sending out waves 10 metres in length.

If we put $\delta=0$ and $\phi=0$ in the last expression for Π, it reduces to that used by Hertz.

Love has delineated (*loc. cit.*) the form of the lines of electric force round a Hertz doublet or ideal dumb-bell oscillator in action, taking into account the discontinuity which exists at the surface of the wave front. These diagrams (see Plate V.) are modifications of those given by Pearson and Lee. In these diagrams four lines of electric force are drawn for different epochs, which are respectively denoted by heavy firm, heavy dotted, light firm, and light dotted lines.

The diagrams given in Figs. 4-11, Plate V., represent, according to Professor Love, the state of the electric field within and without the wave front surface at various epochs, and these, he says, should replace the diagrams 4-11 given in Plate II. by Professor Pearson and Miss Lee. They have only been commenced on the outside of a small sphere drawn round the oscillator. The lines above mentioned have been drawn in Love's diagrams corresponding to values of $\frac{Q\lambda}{2\pi El}$ in Hertz's notation, equal respectively to $\pm 0{\cdot}01$, $\pm 0{\cdot}1$, $\pm 0{\cdot}3$, $\pm 0{\cdot}5$. The fine continuous circular line enclosing the oscillator is a surface for which $Q=0$, or the electric force has no radial component.

These diagrams show in a striking manner the discontinuity in the direction of the lines of electric force at the wave front surface represented by the fine continuous circle. Before the discharge begins we must regard the electric force lines as stretching out to infinity in all directions, and when the discharge happens, a discontinuity or kink in these lines flies outwards through space with the velocity of light. The diagrams show also the gradual formation and detachment from the oscillator of the closed loops of electric force, and their enlargement and the formation of others within them.

There are many points of interest involved in the examination of the force of the field near to, or in the direction of, the axis of a small oscillator to which space cannot here be given. From the point of view of radiotelegraphy we are not much concerned with the field in close proximity to the oscillator, but the reader may be referred for an exposition of some of them to a "Treatise on Magnetism and Electricity," by Andrew Gray, vol. i. p. 400, where a discussion is given of the field of the oscillator at various points near to it.

[21] See A. E. H. Love, "The Advancing Front of the Train of Waves emitted by a Theoretical Hertzian Oscillator," *Proc. Roy. Soc. Lond.*, 1905, vol. 74, p. 73.

8. Poynting's Theorem.—We owe to Dr. J. H. Poynting an important theorem concerning the energy transmission through the electromagnetic field.[22] If a small volume is marked off by a closed surface in the field, and the energy of electric strain and magnetic flux contained in it be varying, Poynting proved that the amount of energy which enters each element of the surface is measured by the sum of the product of the electric and magnetic forces resolved along each element of the surface, multiplied by the sine of the angle between their directions and divided by 4π.

Maxwell had previously shown that the energy of the electromagnetic field is made up of two parts, due respectively to the electric strain and to the magnetic flux. The part due to the electric strain is equal, per unit of volume, to $\frac{K}{8\pi}\mathbf{E}^2$, where $\mathbf{E}$ is the electric force assumed constant throughout the unit of volume, and K is the dielectric constant.[23]

If we consider any finite space throughout which there is a disposition of electric force $\mathbf{E}$, and if the rectangular components of that force at any point are X, Y, and Z, then, to obtain the whole electrostatic energy contained in the given volume, we have to find the value of the integral—

$$\frac{K}{8\pi}\int (X^2+Y^2+Z^2)dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

where dv is an element of volume.

This expression follows at once from the fact that if D is a displacement produced by an electric force, $\mathbf{E}$, in the same direction, the two being uniform throughout the space of a unit of volume, then the energy of strain per unit of volume (T_s) is equal to half the product of the force and the displacement.

$$\text{But } D=\frac{K}{4\pi}\mathbf{E}, \text{ hence } T_s=\frac{K}{8\pi}\mathbf{E}^2 \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Again, Maxwell shows that another part of the energy of the field is magnetic, and that if $\mathbf{H}$ is the uniform magnetic force throughout a unit of volume, the magnetic energy (T_m) contained therein is equal to $\frac{\mu}{8\pi}\mathbf{H}^2$. Hence, to obtain the magnetic energy contained in any finite space, we have to find the value of the integral—

$$\frac{\mu}{8\pi}\int (\alpha^2+\beta^2+\gamma^2)dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

where dv is a unit of volume, and μ is the magnetic permeability of the material filling it.

Accordingly, in the æther, where Hertz takes $\mu=1$ and $K=1$, the total energy stored up in any volume is the sum of the two energies given by the two expressions, viz.—

$$(1) \text{ The electrostatic energy} = \frac{1}{8\pi}\int (X^2+Y^2+Z^2)dv \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

$$(2) \text{ The magnetic energy} = \frac{1}{8\pi}\int (\alpha^2+\beta^2+\gamma^2)dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Starting from these expressions, and considering a reduced case, we may follow the method which Hertz employed in proving the theorem due to Poynting.

We take the fundamental equations connecting the electric and magnetic forces in the electromagnetic field, viz.—

[22] See Prof. J. H. Poynting, F.R.S., *Phil. Trans. Roy. Soc.*, 1884, part ii. p. 343, "On the Transfer of Energy in the Electromagnetic Field."

[23] See Maxwell's "Electricity and Magnetism," vol. ii. p. 253, § 638.

$$(1)\begin{cases} A\dfrac{d\alpha}{dt} = \dfrac{dZ}{dy} - \dfrac{dY}{dz} \\ A\dfrac{d\beta}{dt} = \dfrac{dX}{dz} - \dfrac{dZ}{dx} \\ A\dfrac{d\gamma}{dt} = \dfrac{dY}{dx} - \dfrac{dX}{dy} \end{cases} \qquad (2)\begin{cases} A\dfrac{dX}{dt} = \dfrac{d\beta}{dz} - \dfrac{d\gamma}{dy} \\ A\dfrac{dY}{dt} = \dfrac{d\gamma}{dx} - \dfrac{d\alpha}{dz} \\ A\dfrac{dZ}{dt} = \dfrac{d\alpha}{dy} - \dfrac{d\beta}{dx} \end{cases} \quad . \quad . \quad (49)$$

In these equations X, Y, and Z represent the rectangular components of the electric force in electrostatic units, and α, β, and γ the rectangular components of the magnetic force, and $A = \frac{1}{u}$ is the reciprocal of the electromagnetic velocity.

Multiply equations (1) by α, β, and γ, and equations (2) by X, Y, and Z, respectively, and add the results. Then multiply each side by an element of volume $dx \,.\, dy \,.\, dz$, and integrate, and we arrive at the equation—

$$A\iiint\left(\alpha\frac{d\alpha}{dt} + \beta\frac{d\beta}{dt} + \gamma\frac{d\gamma}{dt}\right)dx \,.\, dy \,.\, dz + A\iiint\left(X\frac{dX}{dt} + Y\frac{dY}{dt} + Z\frac{dZ}{dt}\right)dx \,.\, dy \,.\, dz$$

$$= \iiint\frac{d}{dx}(\gamma Y - \beta Z)dx \,.\, dy \,.\, dz + \iiint\frac{d}{dy}(\alpha Z - \gamma X)dx \,.\, dy \,.\, dz + \iiint\frac{d}{dz}(\gamma X - \alpha Y)dx \,.\, dy \,.\, dz$$

$$= \iint(\gamma Y - \beta Z)dy \,.\, dz + \iint(\alpha Z - \gamma X)dx \,.\, dz + \iint(\beta X - \alpha Y)dx \,.\, dy. \quad . \quad . \quad . \quad . \quad (50)$$

Let dS be an element of the surface of the element of volume, and let l, m, n be its direction cosines. Then, by a well-known theorem in solid geometry—

$$ldS = dy \,.\, dz, \qquad mdS = dx \,.\, dy, \qquad mdS = dx \,.\, dy. \quad . \quad . \quad . \quad (51)$$

This simply amounts to saying that the projection of the element of volume $dx \,.\, dy \,.\, dz$ on the three co-ordinate planes gives us three surfaces, having respectively areas equal to $dy \,.\, dz$, $dx \,.\, dz$, $dx \,.\, dz$, respectively.

Again, in order to interpret the above equation we must remind the reader of a simple theorem in geometry of three dimensions. If any plane area dS is projected on the three co-ordinate planes, we have as above—

$$ldS = S_1, \qquad mdS = S_2, \qquad ndS = S_3,$$

where S_1, S_2 and S_3 are the projections on the planes of reference, and l, m, and n the direction cosines of the normal to the surface. If we multiply each of these last expressions by l, m, and n respectively, and remember that $l^2 + m^2 + n^2 = 1$, we have—

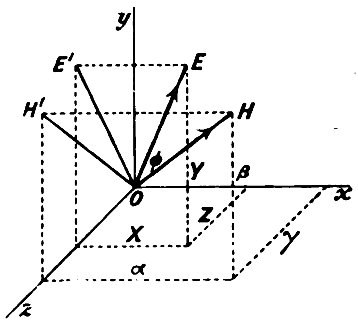


FIG. 20.—Diagram illustrating Poynting's Theorem.

$$dS = lS_1 + mS_2 + nS_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Consider now the lines meeting at the origin (see Fig. 20), one of which represents the electric force **E** in the field with its three axial components X, Y, and Z, and the other one represents the magnetic force **H** with components α, β, and γ. Joining the outer extremities of lines **E** and **H**, we have a triangle OEH, of which the area is $\frac{1}{2}$**EH** $\sin\phi$, where ϕ is the angle between the lines OE, OH. If we project this triangle on the three co-ordinate planes, it is not difficult to show that on the yz plane this projected area OE′H′ is equal to the difference between a triangle whose area is $\frac{1}{2}\beta\gamma$, and the sum of two other areas $\frac{1}{2}(\beta + Y)(\gamma - Z)$ and $\frac{1}{2}YZ$. Hence the area of the projection of $\frac{1}{2}$**EH** $\sin\phi$ on the yz plane is $\frac{1}{2}YZ + \frac{1}{2}(\beta + Y)(\gamma - Z) - \frac{1}{2}\beta\gamma = \frac{1}{2}(\gamma Y - \beta Z)$. In the same manner we can show

that the projections on the two other planes xz and xy are $\frac{1}{2}(\alpha Z-\gamma X)$ and $\frac{1}{2}(\beta X-\alpha Y)$. Hence, by the theorem just mentioned, we have—

$$\mathbf{EH}\sin\phi=(\gamma Y-\beta Z)l+(\alpha Z-\gamma X)m+(\beta X-\alpha Y)n$$

where l, m, and n are the direction cosines of the normal to the triangle OEH. Returning then to the equation (50), we can write it in the following form. Since—

$$\alpha\frac{d\alpha}{dt}+\beta\frac{d\beta}{dt}+\gamma\frac{d\gamma}{dt}=\frac{1}{2}\frac{d}{dt}(\alpha^2+\beta^2+\gamma^2)=\frac{d}{dt}(\tfrac{1}{2}\mathbf{H}^2) \quad . \quad . \quad . \quad (53)$$

and $$X\frac{dX}{dt}+Y\frac{dY}{dt}+Z\frac{dZ}{dt}=\frac{1}{2}\frac{d}{dt}(X^2+Y^2+Z^2)=\frac{d}{dt}(\tfrac{1}{2}\mathbf{E}^2) \quad . \quad . \quad . \quad (54)$$

we can write the left-hand side of (50) in the form[24]—

$$4\pi A\frac{d}{dt}\int\int\int\left(\frac{\mathbf{E}^2+\mathbf{H}^2}{8\pi}\right)dv$$

where dv is an element of volume, and by the theorems just stated the right-hand side of (50) can be written—

$$\int\int(\mathbf{EH}\sin\phi)dS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where dS is an element of surface and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic forces resolved along it. Hence, dividing $4\pi A$, we have—

$$\frac{d}{dt}\int\int\int\left(\frac{\mathbf{E}^2+\mathbf{H}^2}{8\pi}\right)dv=\frac{1}{4\pi A}\int\int(\mathbf{EH}\sin\phi)dS$$

The interpretation of the above equation is as follows: It tells us that the rate at which the total electromagnetic energy in any space is changing with time is measured by the sum or integral of the products of the electric and magnetic forces resolved along each element of surface of the volume multiplied by the sine of the angle between the directions of these resolved parts, and divided by $4\pi A$. As, therefore, the right-hand expression is a surface integral, it implies that the energy enters or leaves the interior of the space by passing inwards or outwards through the bounding surface.

This remarkable theorem is consistent with the law of conservation of energy which asserts that if the energy in any region is increased or diminished it is not due to the creation or annihilation of energy, but to the arrival or departure of energy in some form which must come in through the surface.

9. **Radiation from an Oscillator.**—Hertz applied Poynting's theorem to calculate the radiation of energy from an electric oscillator or doublet when in action.

Describe round the oscillator a sphere of radius r, where r is large compared with the wave-length, and apply Poynting's theorem to this sphere. Take any point on the surface of this sphere. Then the polar co-ordinates of this point are r and θ, the angle θ being measured from the axis of the oscillator.

At the point so defined the electric force resolved tangentially to the spherical surface is $Z\sin\theta-Y\cos\theta$, and the magnetic force at right angles to this is the component α. If we substitute for Z, Y, and α the values already formed in equations (31), we have for the product $\mathbf{EH}\sin\phi$ the quantity $\frac{A\phi^2m^3n}{r^2}\sin^2\chi\sin^2\theta$, since $\sin\phi=1$. The element of area of the sphere may be taken to be the zone of area $2\pi r^2\sin\theta\, d\theta=dS$ lying between small circles of polar distance θ and $\theta+d\theta$, and of mean radius $r\sin\theta$.

Accordingly, the whole energy sent out in a time dt through this zone is equal to $\frac{1}{4\pi A}(\mathbf{EH}\sin\phi)dS$, which by substitution of the above values is found to be—

$$\frac{\phi^2m^3n}{2}\sin^2\chi\sin^3\theta\, d\theta dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

[24] The factor A, which is the reciprocal of v, the electromagnetic velocity, comes in here because Hertz supposes that the electric force is measured in electrostatic units.

Hence the whole energy escaping through the whole sphere *per half period* is obtained by taking the integral of the above quantity between the limits 0 and π, and 0 and T. The reader should again note that Hertz uses the symbol T for the half period, and λ, therefore, for the half wave-length.

The integral—

$$\int \sin^3 \theta \, d\theta = \int \sin \theta \, d\theta - \int \cos^2 \theta \sin \theta \, d\theta = \tfrac{1}{3} \cos^3 \theta - \cos \theta$$

Hence we have $$\int_0^\pi \sin^3 \theta \, d\theta = \tfrac{4}{3}$$

Also $$\int \sin^2 (mr - nt) dt = \int \frac{dt}{2} - \frac{1}{2}\int \cos 2(mr - nt) dt$$
$$= \frac{t}{2} + \frac{1}{2} \frac{\sin 2(mr - nt)}{2n}$$

Now, $m\lambda = nT$. Hence $(mr - nT) = m(r - \lambda)$, and since by supposition r is large compared with λ, we have $(mr - nT) = mr$.

Therefore $$\int_0^T \sin^2 (mr - nt) dt = \frac{T}{2}$$

Collecting these results, we find that the whole energy sent out through the sphere per half period is given by—

$$\frac{4\phi^2 m^3 n T}{12} = \frac{\phi^2 m^3 n T}{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

But $m\lambda = nT$ and $m = \frac{\pi}{\lambda}$ according to Hertz's notation.

Therefore the whole energy sent out per half period is given by the expression—

$$\frac{\phi^2 \pi^4}{3\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

If, however, we remember that Hertz uses λ for the *half wave-length*, we may change the formula into our usual notation by writing $\frac{\lambda}{2}$ instead of λ, and we then have—

$$\textit{The energy sent out by the oscillator per half period} = \frac{8\phi^2\pi^4}{3\lambda^3} \quad . \quad . \quad (59)$$

or

$$\textit{The energy sent out by the oscillator per complete period} = \frac{16\phi^2\pi^4}{3\lambda^3} \quad . \quad . \quad (60)$$

where λ has the ordinary signification of the complete wave-length.

This is the formula (30) we have used in § 8 of Chap. III. We shall now apply this result to calculate the energy sent out per half period by the Hertz oscillator, described in § 8 of Chap. III. We have there seen that an oscillator described by Hertz was of such dimensions that each half had with reference to the other a capacity of 10 cms. Also, he employed a spark gap 1 cm. in length, which corresponds to a spark potential of 30,000 volts, or 100 C.G.S. electrostatic units. Hence the charge E on each half of the oscillator was 1000 electrostatic units. The length l was 100 cms., and the wave-length λ was 480 cms. Also, $\pi^4 = 97{\cdot}4$.

Therefore the energy sent out per half period is—

$$\frac{8 \times 97{\cdot}4 \times (1000)^2 \times (100)^2}{3 \times (480)^3} = 23{,}636 \text{ ergs (nearly)}$$

But the energy imparted to the oscillator at starting was equal to $\frac{1}{2} \times 10(100)^2 = 50{,}000$ ergs. Hence, we see that in about two half oscillations, or one complete period, the energy is dissipated. The frequency of the oscillator is nearly 50×10^6 (see § 8, Chap. III. p. 280). Hence the half period occupies 10^{-8} of a second, and the radiation of the energy is 23×10^{11} ergs per second. Hence, to maintain this radiation continuously would necessitate the expenditure of 300 horse-power. It will be seen, therefore, that even a small oscillator might

ε S cmp = v^2 ε M cmp = $(3 \cdot 10^{10})^2 \times 10^{-9}$
∴ pot = $\frac{1}{v}$ pot = $\frac{1}{3 \cdot 10^{10}} \times 10^{8}$

require to be supplied with an immense power to keep its electric radiation going continuously.

We may apply the above equation (60) to calculate the radiation from an oscillator which will be useful later on.

We may put the equation (60) first into a form more useful for calculation. If C is the capacity in microfarads of each sphere or half of the oscillator with respect to the other, and V is the maximum P.D. in volts before discharge, then $\frac{C \times 9 \times 10^5 \times V}{300}$ is the maximum charge of the oscillator in electrostatic units. Hence, if l is the length of the rod in centimetres we have $3000\ CVl = \phi$ as the
35 maximum electric moment in electrostatic units. Accordingly the radiation per period is given by—

$$W = \frac{17}{10^{23}} C^2 V^2 l^2 N^3 \text{ ergs}$$

C being the capacity in microfarads, V the P.D. in volts, l the length in centimetres, and N the frequency.

The radiation per period for a Hertzian oscillator increases, therefore, as the cube of the frequency and as the square of the potential just before discharge.

For a given oscillator the initial energy imparted to it is—

$$\frac{CV^2}{2 \times 10^6} \times 10^7 = 5CV^2 \text{ ergs}$$

and from this and the previous expression for the energy radiated per period we can tell how many oscillations take place in a train.

Suppose a large Hertzian oscillator is constructed by taking two rods each 2 metres long and attaching to one end of each rod a spark ball and to the other a disc of metal 1 metre in diameter. These, when placed in line, give us an oscillator in which oscillations can be set up in the Hertzian manner. The capacity of a circular disc is $\frac{d}{\pi}$ electrostatic (E.S.) units, where d is the diameter in centimetres. Hence the capacity of each disc of the above oscillator with respect to the other is—

$$\frac{100}{2\pi} = \frac{100}{6 \cdot 28} = 15 \text{ E.S. units}$$

or

$$\frac{15}{9 \times 10^5} = \frac{1}{60{,}000} \text{ mfd.}$$

Let the connecting rod be 0·5 cm. diameter. Now the inductance in centimetres L of a straight rod of length l and diameter d, is—

$$L = 2l\left(\log_e \frac{4l}{d} - 1\right)$$

and for the above rod is $800\ (8 \cdot 05 - 1) = 5640$ cms. Hence for the above oscillator the oscillation constant $\sqrt{CL} = 0 \cdot 3$ nearly, and the frequency N of the oscillation is—

$$\frac{5 \cdot 033 \times 10^6}{\sqrt{CL}} = 17 \times 10^6 \text{ (nearly)}$$

The wave-length λ of the fundamental wave is 1760 cms. Suppose, then, that the spark gap of the oscillator is 1 cm., then the spark potential V corresponding to this is 30,000 volts, or 100 E.S. units, and the radiation in ergs per period is given by—

$$W = \frac{17}{10^{23}} C^2 V^2 l^2 N^3 = \frac{17}{10^{23}} \frac{9 \times 10^8 \times 16 \times 10^4 \times (17)^3 \times 10^{18}}{36 \times 10^8} = \frac{17 \times 9 \times 16 \times (17)^3}{36 \times 10} = 33{,}410 \text{ ergs.}$$

Hence this oscillator radiates nearly 33,000 ergs per period. The original charge of energy is—

$$\frac{1}{2} CV^2 = \frac{9 \times 10^8 \times 10^7}{2 \times 6 \times 10^{10}} \text{ ergs} = 75{,}000 \text{ ergs}$$

Accordingly the initial energy is all radiated in about two complete periods.

Suppose, however, we were to maintain this rate of radiation by creating in the oscillator persistent oscillations, the rate of radiation would be 56·1 kw., or nearly 75 horse-power.

This example shows us the enormous radiative power of open or Hertzian oscillators.

One more point in connection with them is of considerable interest.

Let the maximum value of the current reckoned in amperes in the centre of the antenna be denoted by A and its electrostatic measure by I. Then—

$$\frac{A}{10}=\frac{I}{u} \text{ and } I=CV2\pi N$$

Accordingly we may transform the expression for the energy W radiated in ergs per period as follows :—

$$W=\frac{16\pi^4 \cdot C^2V^2l^2}{3\lambda^3}=\frac{16\pi^4A^2u^2l^2}{3\lambda^3 100 \cdot 4\pi^2N^2}=\frac{4\pi^2}{300} \cdot \frac{l^2}{\lambda^2}\frac{u}{N}A^2$$

If the oscillations are continuous and of frequency N, then, since $\pi^2=9{\cdot}87$ or nearly 10, the power in watts radiated is given by—

$$P=394{\cdot}8\frac{l^2}{\lambda^2}A^2$$

Approximately we can say that the radiation from such an oscillator reckoned in ergs per period is given by the expression—

$$W=\frac{2}{15}\frac{l^2A^2}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

If we suppose a sphere described round the oscillator of radius r large compared with the dimensions of the oscillator, then the surface of this sphere is $4\pi r^2$, and the mean density of the radiation in ergs per square cm. is—

$$\frac{W}{4\pi r^2}=\frac{1}{30}\frac{lA^2}{\pi r^2}\frac{l}{\lambda}$$

We shall return to this question of the radiation from an antenna in Chap. IX.

10. Connection between the Logarithmic Decrement and the Radiation of an Oscillator.—We can establish a connection between the expression as above obtained by Hertz for the radiation of energy per period from an oscillator and the radiation logarithmic decrement, and thus obtain a means of predetermining the value of the radiation decrement. For since the radiation in ergs per complete period is given by the expression—

$$W=\frac{16\pi^4\phi^2}{3\lambda^3}$$

it follows that the mean rate of radiation of energy, which we may denote by $\frac{dW}{dt}$, is given by—

$$\frac{dW}{dt}=\frac{16\pi^4\phi^2}{3\lambda^3 T} \quad . \quad . \quad . \quad . \quad (62)$$

But $\phi=Ql$, where Q is the maximum charge on each sphere of the oscillator, and l is its length or the distance between the spheres, and the original energy of the oscillator W is equal to $\frac{Q^2}{2C}$, where C is the capacity of one-half of the oscillator with respect to the other half. If we consider the oscillator as consisting of two spheres, each of radius R, and neglect the capacity of the short rods between the spheres and the spark balls, then the capacity of each sphere is equal to R electrostatic units, and the capacity of one-half of the oscillator with respect to the other $\frac{R}{2}$. Hence $C=\frac{R}{2}$, and $W=\frac{Q^2}{R}$. Therefore—

$$\frac{dW}{dt}=\frac{16\pi^4 l^2 RW}{3\lambda^3 T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

or
$$\frac{dW}{W}=\frac{16\pi^4 l^2 Ru}{3\lambda^4}dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Accordingly
$$W=\epsilon^{-ht}W_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

where W_0 is the original charge of energy, and—

$$h=\frac{16\pi^4 l^2 Ru}{3\lambda^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

u being the velocity of radiation, and $uT=\lambda$. Therefore the time t in which the energy of the oscillator falls to $\frac{1}{\epsilon}$ of its original value is given by $t=\frac{1}{h}$, and the time in which the amplitude of the oscillations falls to $\frac{1}{\epsilon}$ of the original amplitude is given by $t=\frac{2}{h}$.

If we then define the logarithmic decrement, as we may do, to be the logarithm of the ratio of two successive oscillations in opposite directions or separated by half a complete period, and if we call I_1 and I_m the first and the mth oscillations respectively, we have—

$$I_m=I_1\epsilon^{-(m-1)\delta}$$

and hence
$$I_m^2=I_1^2\epsilon^{-2(m-1)\delta}$$

where δ is the log. dec. is defined as above.

The energy of an oscillation varies as the square of the amplitude, and accordingly the time t in which the energy falls to $\frac{1}{\epsilon}$ of its initial value is such that—

$$2(m-1)=\frac{1}{\delta}, \text{ but } (m-1)\frac{T}{2}=t$$

Therefore
$$\frac{T}{4\delta}=\frac{1}{h} \text{ or } \delta=\frac{hT}{4}$$

But we have found (see equation (66)) that—

$$h=\frac{16\pi^4 l^2 Ru}{3\lambda^4}$$

Hence
$$\delta=\frac{16\pi^4 l^2 R}{12\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

or
$$\delta=\frac{16\pi^4 l^2 C}{6\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

where C is the capacity of one part of the oscillator with respect to the other.

This last expression gives us a value for the radiation decrement δ_r in terms of the quantities l, C, and λ. The time τ in which the amplitude of the oscillation falls to $\frac{1}{\epsilon}$ of its original is twice that in which the energy falls to $\frac{1}{\epsilon}$ of its original value, and is therefore equal to $\frac{2}{h}$ or to $\frac{T}{2\delta}$. Hence the time τ in which the *amplitude* of the oscillations falls to $\frac{1}{\epsilon}$ of its original value is given by—

$$\tau=\frac{T}{2\delta}=\frac{6\lambda^3 T}{32\pi^4 l^2 C}=\frac{6\lambda^3}{16\pi^4 l^2 R}T \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

since $C=\frac{R}{2}$.

Thus in the case of the Hertz oscillator already mentioned, consisting of two spheres, each 15 cms. radius, placed at the ends of a rod 100 cms. in length with

spark gap in the centre, Hertz found by experiment that this radiator emitted a wave having a wave-length of 560 cms. Hence $\lambda=560$ cms., and—

$$\tau=\frac{6\times(560)^3\times T}{16\times 97\cdot 4\times(100)^2\times 15}=4\cdot 4T \quad . \quad . \quad . \quad . \quad . \qquad (70)$$

For an oscillator of nearly equal size, Bjerknes found experimentally $\tau=3\cdot 8T$.

11. Radiation of Electromagnetic Waves from a Marconi Earthed Oscillator.—G. Marconi made a remarkable improvement in the practical means for the production of electric waves by his invention of the earthed vertical oscillator (see Chap. VII.). Although Hertz had employed oscillators as above described, both in horizontal and vertical positions, it had not occurred to anyone before the time when Marconi began to experiment on this subject to bury a Hertz radiator partly in the earth with its axis vertical.

Marconi did that which was equivalent to this when he connected an insulated elevated cylinder or plate suspended in the air by a wire, with one spark ball attached to the secondary circuit of an induction coil, and connected the other spark ball to a plate buried in the earth. On bringing the spark balls near together and starting the coil in action we set in operation an oscillator, one-half of which is buried in the earth. By so doing an oscillator is constructed which is equivalent or nearly so in radiative power to a complete or Hertzian oscillator of double the total length. The novelty of such a suggestion is to be measured rather by its non-obviousness to experts than by the simplicity of the device in itself, and its value is proved by its utility.

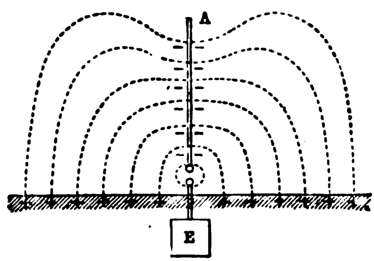


FIG. 21.—Rough Representation of Lines of Electric Strain round a Simple Marconi Antenna before Spark Discharge.

Since the earth is a fairly good conductor, we may consider the insulated aerial wire to form with the earth, and the space in between, a condenser. The aerial wire has a certain capacity with respect to the earth. Hence, when the aerial is charged with electricity, there must be lines of electric strain stretching from it to the earth, in all directions around it symmetrically, as shown roughly in Fig. 21. If we now consider the aerial to be suddenly discharged across the spark gap, we may, in accordance with principles already explained, consider that the ends of the lines of strain terminate on electrons in the aerial, and these electrons will receive a sudden displacement or be set in oscillation. Hence, in accordance with the explanation already given in § 7, the inertia quality of the lines of electric force will come into play, and kinks or displacements be propagated along them. These kinks or discontinuities unite into loops of electric force, which are detached from the antenna. In the case, however, of the Marconi aerial, these loops must be semi-loops, with their feet or ends resting on the earth. As each loop is formed it is pushed outwards by others, and the process may be diagrammatically indicated as in Figs. 22 and 23. Accompanying this outward movement of the semi-loops of electric strain, there will be an expansion of circular lines of magnetic flux in circles with their planes parallel to the earth and centres in the aerial wire. These lines of flux are alternately directed in a right-handed and left-handed direction, as seen from above. If we can imagine a being endowed with a kind of vision enabling him to see the lines of electric strain and magnetic flux in space, he, standing at any spot on the earth's surface, would see, when the radiator was in action, bunches or groups of lines of electric strain fly past. Near the earth's surface these strain lines would be vertical. Alternate groups of lines of strain would be oppositely directed, and the spectator would also see groups of lines of magnetic flux fly past, directed in a horizontal direction or parallel to the earth's surface. These strain and flux lines would move with the velocity of light, and the distance between two successive maxima of electric strain directed in the same direction would be the wave-length of the wave. It

will be seen, therefore, that the process is one which necessitates a perfectly free movement of electricity into and out of the earth at the base of the aerial, and experience shows that a "good earth," that is, a good low resistance and low inductance connection between the earth and the lower spark ball, is important. Also, it has been found that a good conducting earth surface is required.

If we imagine a Hertz oscillator, consisting of a rod severed in the middle and having at that point a spark gap, to be bisected by a plane, so that the rods are

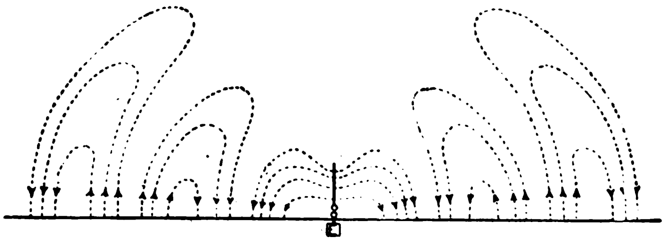


FIG. 22.—Diagrammatic Representation of the Detachment of Semi-Loops of Electric Strain from a Simple Marconi Antenna or Rod Oscillator.

perpendicular to the plane, then, since the electric force due to the oscillator is everywhere perpendicular to this medium plane, we can make this plane conducting without affecting the distribution of the force on either side. The force systems on the two sides are then independent. We may consider this plain conducting sheet to be at zero potential, and we may imagine the force system on one side suppressed ; still the distribution of electric force on the other side will not be affected.

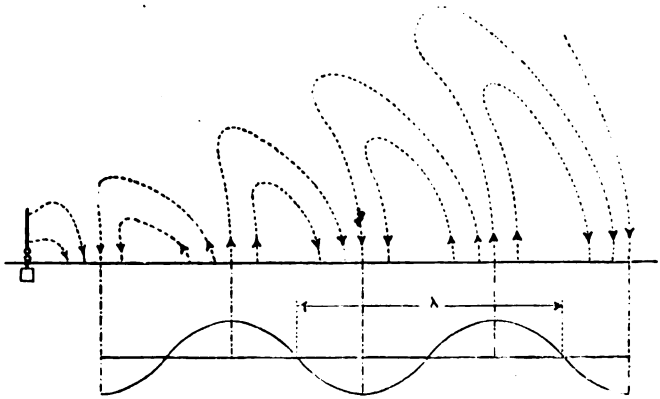


FIG. 23.—Diagram illustrating the Meaning of the Term Wave-Length in Connection with the Electric Radiation from a Rod Oscillator.

We thus arrive at the conclusion that if a vertical rod is set up in the air, and at its lower end there is a spark ball in apposition to another spark ball connected to an earth plate, this arrangement constitutes electrically one-half of a Hertz oscillator. The antenna above the earth is said to be electrically "reflected" in the earth's surface, and the electromagnetic effect at any distant point is that due to the electrical oscillations in the antenna itself and to its "image" reflected in the earth's surface.

The assumption here made is that the earth is a good conductor, and this is valid for damp soil or sea surface. We shall return again, in a later chapter,

to the consideration of the influence of the earth and of the atmosphere above it on electric wave propagation, in considering the actual apparatus used in wireless telegraphy by electric waves.

12. **Theory of a Rod-Shaped Oscillator.**—The theory given by Hertz applied to an ideal oscillator in which two equal and opposite electric charges were supposed to reside in two small spheres separated by a short linear conductor with a spark gap in it. The electric moment was taken as the product of either charge and the length of the oscillator. This ideal case, however, does not quite correspond with the practical case as exhibited in wireless telegraphy. In this latter case we have as oscillator in its simplest form a vertical wire or rod having a spark gap at its lower end, and the lower spark ball connected to a good earth. This, as we have seen, may be regarded as half a complete linear oscillator, consisting of two rods placed in line with each other, their inner ends provided with spark balls and placed in apposition. We require, therefore, the theory of a linear or rod oscillator. This has been given by several writers, particularly in complete form by M. Abraham [25] and by H. M. Macdonald.[26]

Abraham's memoir on the subject is long and abstruse, and almost impossible to abstract adequately. His method of treatment, however, is as follows: To bring the problem within the grasp of analysis, he considers the rod to be an ellipsoid of revolution symmetrical round its major axis, the eccentricity of this ellipsoid being very large. The ratio of the semi-minor axis to the semi-major axis is therefore a very small fraction, the square of which may be neglected. As the external effects will be symmetrical with respect to the major axis, it suffices to consider the problem as one in two dimensions. The outline of the conductor is therefore taken as an ellipse, and the half distance between the foci is taken as the unit of length. A system of elliptical co-ordinates is then adopted in which confocal ellipses are described round the elliptical conductor, and confocal hyperbolas cut these ellipses orthogonally. The electric and magnetic forces in the space outside the aerial must therefore satisfy the equations of Maxwell, and the lines of electric force must terminate on the conductor normally to its surface. These equations are then written down in terms of the system of elliptical co-ordinates selected.

It is then shown that the free time period of oscillation of such a rod-shaped oscillator varies as the square root of the dielectric constant of the surrounding medium, but that the logarithmic decrement is independent of the nature of the medium.

It follows that the wave-length of the waves sent out into the surrounding medium is independent of the dielectric constant of that medium, for the wave-length is the product of the velocity and time period. Now the wave velocity varies inversely as the square root of the dielectric constant, and since the period varies directly as the same quantity, the wave-length is constant. Again, Abraham shows that the time periods of geometrically similar oscillators are proportional to their length, whilst their logarithmic decrements are the same.

He takes as the meridian section of his oscillator an ellipse having a semi-minor axis, b, and a semi-interfocal distance, 1, such that b^2 may be neglected in comparison with unity.

The quantity $\dfrac{1}{4 \log_e \left(\dfrac{2}{b}\right)}$, or $\dfrac{1}{4 \log_e \left(\dfrac{2l}{d}\right)}$, where l is the length of the wire and d its diameter, is then denoted by e, and it is then shown that for such an oscillator the fundamental wave-length is approximately equal to twice the length of the rod, also that the damping by radiation diminishes as the thickness of the rod decreases, and, moreover, that the damping is less for the higher harmonics than for the fundamental.[27]

[25] M. Abraham, "Electrischen Schwingungen um einen stabförmigen Leiter behandelt nach der Maxwell'schen Theorie," *Annalen der Physik*, 1898, vol. 66, p. 435.

[26] H. M. Macdonald, "Electric Waves," Chap. X.

[27] Abraham shows, as also does Macdonald, that the length of the wave is rather greater than twice the length of the rod. Macdonald shows it to be 2·5 times nearly. See H. M. Macdonald, "Electric Waves," p. 111. See also end of § 7 in Chap. IV.

Abraham denotes the fundamental frequency n by unity, and the harmonics by $n=2$, $n=3$, etc.

He then shows that the logarithmic decrement *per complete period* (δ_n), where n is the order of the oscillation, viz. whether fundamental or higher, is given by the expressions—

$$\delta_1 = 9{\cdot}74\, e = \frac{2{\cdot}44}{\log_e \frac{2l}{d}}, \quad \delta_2 = 6{\cdot}23\, e,$$

and generally,
$$\delta_n = \frac{9{\cdot}66 + 4 \log_e n}{n} e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

Thus, for instance, if we consider a vertical Marconi aerial wire of which the height is 180 feet and diameter 0·2 inch, we may consider the vertical section of this wire as the meridional section of a semi-ellipse, of which the semi-interfocal distance is 2160 inches, which is the length of the wire. The semi-diameter is then 0·1 inch, and the value of b or the semi-minor axis of the ellipse is $\frac{1}{21600}$, or $2l/b = 43200$. Hence we have—

$$e = \frac{1}{4 \log_e \left(\frac{2}{b}\right)} = 0{\cdot}0234$$

Therefore $\quad \delta_1 = 0{\cdot}23, \quad \delta_2 = 0{\cdot}146$

Accordingly, the fundamental decrement per *half period* would be 0·115, and this agrees with the results of the calculation given in Chap. III. § 8.

A very interesting paper has been published by F. Hack,[28] which supplements that of M. Abraham by delineating graphically the form of the lines of electric force round a linear or rod oscillator.

Hack takes the expressions derived by Abraham and applies them in the case of an infinitely thin rod for which the quantity denoted by $e=0$, and deduces an equation for the lines of electric force due to the fundamental oscillation in the form—

$$\cos \frac{\pi y}{2} \cos \frac{\pi(ut - x)}{2} = C_1$$

where x and y are the elliptical co-ordinates of a point in the meridional plane, u is the velocity of radiation, and C_1 is a constant.

The diagrams in Figs. 1 to 4, Plate VI., represent the form of the lines of electric force round the linear oscillator for epochs $t=0$, $t=\frac{1}{2u}$, $t=\frac{1}{u}$, $t=\frac{3}{2u}$. In this case the fundamental wave-length $\lambda_1 = 4$, unity representing the half length of the rod.

As in the case of the diagrams given by Hertz, the above diagrams by Hack show that the wave-making process consists in the detachment of loops or closed lines of electric force, and that the true wave state is not established within a distance equal to about half a wave-length.

If we suppose these diagrams traversed by a horizontal line, then all that part of the diagrams above that horizontal line will represent the distribution of electric force round a Marconi aerial wire or antenna at various stages during the oscillation.

Hack has also (*loc. cit.*) given an additional very interesting series of diagrams showing the distribution of the electric force round the rod oscillator when the oscillations are harmonics (see Figs. 5 to 12, Plate VI.). Thus, for the first harmonic ($n=2$) the equation to the lines of electric force is given by—

$$\sin \pi y \sin (ut - x) = C_2$$

and Hack gives a series of four diagrams showing the distribution of the electric force corresponding to the times—

$$t=0, \quad t=\frac{1}{4u}, \quad t=\frac{1}{2u}, \quad t=\frac{3}{4u}$$

where u is the velocity of radiation.

[28] See F. Hack, "Das Elektromagnetische Feld in der Umgebung eines linearen Oszillators," *Annalen der Physik*, 1904, vol. 14, p. 539.

These are shown in Figs. 5 to 8, Plate VI.

In this case the wave-length $\lambda_2=2$, unity representing the half length of the rod.

Again, for the second harmonic ($n=3$) he also gives the electric force distribution. This case is important, because the second harmonic for the finite rod is the first harmonic for the rod earthed at one end, so that the case when the frequency is three times that of the fundamental is a practical case which concerns us in wireless telegraphy. Hack shows that the Abraham equations reduce in this last case to the form—

$$\cos\frac{3\pi y}{2}\cos\frac{3\pi}{2}(ut-x)=C_3$$

The wave-length λ_3 is $\frac{4}{3}$ of the half length of the rod.

This force system is represented by the four diagrams in Figs. 9 to 12, Plate VI., for the epochs $t=0$, $t=\frac{1}{6u}$, $t=\frac{1}{3u}$, $t=\frac{1}{2u}$.

It will be seen that the wave production consists in sending out as usual closed loops of electric force.

If we take the force distribution in the upper half of each diagram, we have a representation of the system of lines of electric force sent out by a Marconi aerial when the oscillations are the first or first odd harmonic. The force system then consists partly of closed loops of electric force and partly of semi-loops of electric force with their ends on the earth surface. The full discussion of the problem of the transmission of these electric waves round the earth's curved surface is reserved for Chap. IX. in Part III. of this book. The satisfactory explanation of it involves many difficulties, some of which are by no means cleared up.

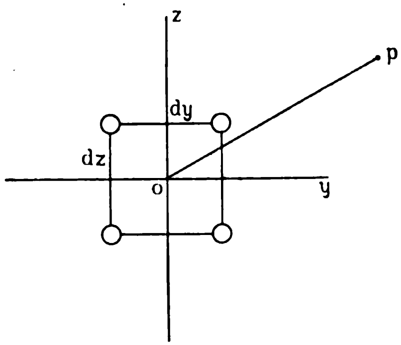


FIG. 24.

By means of a cinematograph it is possible to throw on the screen a representation of the moving lines of electric force of a Hertzian oscillator or Marconi aerial in operation. In this case a series of diagrams have to be prepared similar to those in Figs. 1 to 56, Plates II., III., IV., and V. (see end of this chapter), only delineated for much closer intervals of time. The whole periodic time must be divided into twenty or thirty parts, and diagrams delineated, representing the exact state of the field of electric force for these instants. When such a series of diagrams is photographed on a celluloid strip and sent through a cinematograph lantern, we see on the screen a "living picture" of the Hertzian oscillator or Marconi aerial in electrical oscillation, and can witness the pulsation of the lines of electric force, and the radiation or throwing off of the loops or semi-loops of electric strain.

13. The Radiation from Open and Closed Oscillators.—As already explained, a closed oscillation circuit is one which consists of a condenser, the plates of which are very near together, and are also connected by a loop or circuit of wire. If oscillations are set up in this circuit then, although the plates of the condenser are charged alternately with charges of opposite sign, yet being near together their charges practically neutralize each other in external space as far as regards the production of electrostatic potential. The effect which is produced is nearly all due to the current in the nearly closed circuit. Hence there is a vector potential but no scalar potential distribution. Nevertheless, such a closed oscillator can create both electric and magnetic forces, and radiate electromagnetic waves into surrounding space. An extreme case of such a closed oscillatory circuit is a small, square, closed circuit formed by placing in contiguity four

Hertzian oscillators with spheres or ends in contact, and assuming that the oscillations in each separate oscillator are simultaneous and directed in the same direction. This does not differ in effect from a simple closed conductive circuit assumed to be the seat of a high frequency current. It is interesting to note that the mathematical problem of ascertaining the external effect of such a circuit was considered by the late Professor G. F. Fitzgerald prior to the date of Hertz's researches, and he showed that such a circuit could radiate electromagnetic energy.[29] We can easily obtain expressions for the electric and magnetic forces produced by such an oscillator by considering a small square circuit of side $\delta z = \delta y$ placed with its centre at the origin (see Fig. 24), each side consisting of a small Hertzian oscillator of electric moment ϕ. Let the oscillator be traversed by an alternating current of maximum value, I, and let n stand for 2π times the frequency as before. Let M denote the value of $I\delta y\delta z$ or the product of the maximum current, and the area of the oscillator, and let this product be called the *magnetic moment* of the oscillator. Then, since $\phi = Q\delta y = Q\delta z$, and since $I = Qn$, we have $\frac{M}{n} = \phi\delta y = \phi\delta z$. Also, since the oscillator produces no scalar potential, and since the currents in it are wholly in the plane of yz, we have $V=0$, $F=0$, but we have components of the vector potential G and H parallel to the currents in the two sides of the square, which can easily be shown to have values—

$$G = \phi\frac{d^2\Pi}{dzdt}\delta z,$$

$$H = -\phi\frac{d^2\Pi}{dydt}\delta z$$

where Π stands as before for $\frac{\sin(mr-nt)}{r}$.[30] Accordingly, when we substitute these values in the Maxwellian equations for the electric and magnetic forces given as in equations (20) and (21), we have—

$$\left.\begin{aligned} X &= 0 \\ Y &= -\frac{AM}{n}\frac{d^3\Pi}{dzdt^2} \\ Z &= \frac{AM}{n}\frac{d^3\Pi}{dydt^2} \end{aligned}\right\} \quad . \quad . \quad (72) \qquad \left.\begin{aligned} \alpha &= -\frac{M}{n}\left(\frac{d^3\Pi}{dy^2dt} + \frac{d^3\Pi}{dz^2dt}\right) \\ \beta &= \frac{M}{n}\frac{d^3\Pi}{dxdydt} \\ \gamma &= \frac{M}{n}\frac{d^3\Pi}{dxdzdt} \end{aligned}\right\} \quad . \quad . \quad (73)$$

The coefficient $A = \frac{1}{u}$ appears in the expressions for the electric force, because M being in electromagnetic units we must divide by 3×10^{10} to obtain the values of Y and Z in electrostatic units.

If these equations (72) and (73) are compared with the corresponding equations (27) and (28) in § 7, it will be seen that the two sets differ as follows. For the closed oscillator we have the same expressions for the electric force components as for the magnetic components of the linear oscillator, with the exception that in the closed circuit everything is symmetrical with respect to the x axis, and in the case of the open oscillator with respect to the z axis. Again the expressions for the magnetic force components of the closed oscillator are identical with those for the electric force components of the open oscillator, with the exception that z takes the place of x.

Hence it is clear that the oscillations in the closed oscillator give rise to electromagnetic radiation, as in the case of the open oscillators with the difference that the electric forces and magnetic forces change places. The open oscillator has rings or circles of magnetic force surrounding its symmetrical or z axis, and

[29] See the scientific writings of the late Prof. G. F. Fitzgerald, edited by Sir Joseph Larmor, F.R.S., p. 128.

[30] See J. A. Fleming, "A Note on the Theory of Directive Antennæ or Unsymmetrical Hertzian Oscillators," *Proc. Roy. Soc. Lond.*, vol. 78, A, 1906, p. 3.

the closed oscillator has rings or circles of electric force surrounding its symmetrical or x axis, whilst the lines of magnetic force of the closed oscillator are similar in form to the electric lines of the open one.

There is, however, an immense difference between the two cases, viz. in the energy radiated outwards in each case per unit of time. It is clear, from the symmetry of the two cases, without any long detailed proof, that since the energy sent out by the open oscillator per complete period is expressed by $W=\frac{16\phi^2\pi^4}{3\lambda^3}$, where ϕ is the electric moment of the open oscillator, the energy sent out per period by the closed oscillator must be equal to $\frac{16M^2\pi^4}{3\lambda^3}$, where M is the magnetic moment of the closed circuit. If detailed proof is required, the reader may be referred to a series of three articles by the author on "The Elementary Theory of Electric Oscillators," published in *The Electrician*, vol. lix., Sept. 27, Oct. 4, 11, 1907, pp. 936, 976, and 1016.

For the purposes of comparison we can put these formulæ in a more convenient form. Let us suppose the open oscillator to have a length l, and that the capacity of each end sphere, or half of the oscillator with respect to the other, be denoted by C, and the maximum potential difference of the spheres is V. If V is reckoned in volts, then $\frac{V}{300}$ is the P.D. in electrostatic units, and if C is in microfarads, then 9×10^5C is the capacity in electrostatic units. Hence the electric moment $\phi=3000CVl$.

Let the current in the centre of the oscillator have a maximum value A, and let it be in the form of undamped sinoidal oscillations, so that $A=\frac{nCV}{10^6}$, where $n=2\pi$ times the frequency N. Then, if a is the R.M.S. value of the current, $a=\frac{A}{\sqrt{2}}$, and the energy e radiated per period in ergs is given by—

$$e=4\pi^2 10^8\frac{l^2}{\lambda^2}\frac{A^2}{N} \qquad (74)$$

and the radiation in watts w is given by—

$$w=40\pi^2\frac{l^2}{\lambda^2}A^2 \qquad (75)$$

Remembering that $\pi^2=9\cdot87$, and $N\lambda=3\times10^{10}$, we have—

$$e=0\cdot2632\frac{l^2}{\lambda^2}a^2 \qquad (76)$$

$$w=789\cdot6\frac{l^2}{\lambda^2}a^2 \qquad (77)$$

In the case of the closed circuit, if we assume it to be a square circuit, having a length of side l, and therefore an area l^2, and if the current in it is an alternating current of frequency N, maximum value A amperes, and root-mean-square value $a=\frac{A}{\sqrt{2}}$, we have for the magnetic moment M, $M=\frac{Al^2}{10}$, and hence for the energy in ergs sent out per period—

$$e=10\cdot4\frac{l^4a^2}{\lambda^4} \qquad (78)$$

and the radiation in watts is—

$$w=31200\frac{l^4a^2}{\lambda^4} \qquad (79)$$

We can then write the formulæ (77) and (79) for the radiation in the two cases open and closed, as follows :—

$w=87\times10^{-20}a^2l^2N^2$ (for the open or electric oscillator) . . . (80)

$w=4\times10^{-38}a^2l^4N^4$ (for the closed or magnetic oscillator) . . . (81)

In the formula (77) we see that the expression for the radiation in watts is the product of the mean square value of the current (a^2) in the centre, and a factor $789{\cdot}6l^2/\lambda^2$, which corresponds therefore to a resistance. This last quantity is therefore called the *radiation resistance* of the oscillator. Since λ is approximately equal to $2{\cdot}5l$ for a simple rod oscillator, it follows that the radiation resistance for such an oscillator is not far from 128 ohms.

These formulæ show us that for the same mean-square-current, linear dimensions, and frequency, the radiation of the open oscillator is immensely greater than that of the closed oscillator, provided that the frequency is not very high. Also they show us that the radiation of the closed oscillator increases very much faster with the frequency than that of the open oscillator.

In the case of an open oscillator consisting of a simple straight rod with spark gap in the centre, there is a definite relation between the length l and the wave-length emitted, which is such that $\frac{l}{\lambda}$ is approximately 0·4. If the rod is a thin wire it is nearer 0·5. Hence for such an oscillator we have $w=128a^2$, or the radiation in watts depends only on the mean square value of the current at the centre.

The formula would, however, require some correction in the constant before applying it to a real linear antenna, because it has been obtained on the assumption that the oscillator is short, and the current at all points in it the same. This, however, is not the case, for the current is a maximum at the centre and zero at the ends.

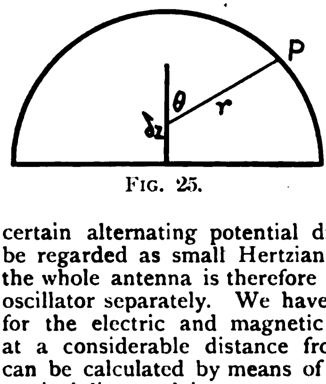


FIG. 25.

We may consider the problem of determining the total radiation of a single wire antenna theoretically from first principles thus:—We have seen that in an antenna the current varies from point to point, and also the potential. We may then consider the antenna, however complicated, as made up of a number of elements, in each of which the current is approximately constant. Each of these elements will have a certain alternating potential difference between their ends, and may therefore be regarded as small Hertzian oscillators. The electric and magnetic force of the whole antenna is therefore the resultant of the forces due to each elementary oscillator separately. We have already given in Chap. V. § 7, the expressions for the electric and magnetic forces of a small Hertzian oscillator at points at a considerable distance from it, and also shown how the total radiation can be calculated by means of Poynting's theorem. Suppose we now consider a vertical linear plain antenna, and divide it up into Hertzian elements, each of length δz, and suppose a hemisphere described around it, the radius of which is very large compared with the height of the antenna (see Fig. 25). Then at any point P on the surface of this hemisphere we can calculate the electric and magnetic force due to the whole antenna as follows:—

We shall assume that the hemisphere is so large that lines drawn from any point on it to all points on the antenna make the same angle θ with the vertical. The expressions for the forces due to a small oscillator of electric moment ϕ at a point P, at a great distance r when the radius vector r makes an angle θ with the vertical are (see equations (31))

$$\left.\begin{aligned} Y &= -\frac{\phi m^2}{r}\sin\chi\sin\theta\cos\theta \\ Z &= \frac{\phi m^2}{r}\sin\chi\sin^2\theta \\ a &= \frac{\phi mn}{\mu r}\sin\chi\sin\theta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

Now, by Poynting's theorem it is known that the whole energy sent out in a time dt from the element is (by (56), Chap. V.) equal to—

$$\frac{\phi^2 m^3 p}{2} \sin^2 \chi \sin^3 \theta \, d\theta \, dt$$

where $p = 2\pi$ times the frequency. Hence the energy sent out by the same oscillator per period is—

$$\frac{16\phi^2\pi^4}{3\lambda^3}.$$

Suppose that i is the maximum current in any element of length δz of the antenna, and that ϕ is the maximum electric moment of this element during the period. Then it is easily seen that $i\delta z = \phi p$.

The maximum current at any point in the linear simple antenna may be expressed as a sine function of the position of δz in the form—

$$i = I \sin \frac{\pi}{2h}(h - z)$$

where h is the height of the antenna, and z is the distance of the point from the lower end, and I is the maximum current at the base or earthed end of the antenna. This expression gives us $i = I$ when $z = 0$, and $i = 0$ when $z = h$, as it should do.

If we then substitute the above values of ϕ and i in the equations for the electric and magnetic forces of the elementary oscillator, we have—

$$\left.\begin{aligned} Y &= -\frac{Im^2}{rp} \sin \frac{\pi}{2h}(h - z) \sin \chi \sin \theta \cos \theta \, \delta z \\ Z &= \frac{Im^2}{rp} \sin \frac{\pi}{2h}(h - z) \sin \chi \sin^2 \theta \, \delta z \\ a &= \frac{IAm}{r} \sin \frac{\pi}{2h}(h - z) \sin \chi \sin \theta \, \delta z \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (83)$$

To obtain the radiation of the whole antenna, we have to integrate the above expressions for the forces due to an element, δz, of the antenna, along the entire length of the antenna or from 0 to h, and use these integral forces in obtaining by Poynting's theorem the total radiation.

But in so doing we must bear in mind that the earthed linear antenna having its base on good conducting soil and of height h is only equivalent, as far as radiation goes, to half of a Hertzian oscillator. For the electrical image of the linear wire reflected in the earth does not radiate.

If then we call l the total length of a simple Hertzian oscillator or dipole in which all the capacity is at the ends, and the current has the same value all along the rod, and if ϕ denotes the electric moment or product Ql where Q is the total maximum charge on either capacity, we have seen that the total radiation in ergs per period is given by the expression—

$$w = \frac{16\phi^2\pi^4}{3\lambda^3}$$

Hence the radiation from one-half of this oscillator is $\frac{8\phi^2\pi^4}{3\lambda^3}$ per period.

We have then to substitute in the last expression the proper equivalents for ϕ and λ for the linear oscillator. Now $Q = \frac{2I}{\pi p}$ and $l = 2h$. Hence $\phi = Ql = \frac{4hI}{\pi p}$ and $\lambda = \frac{u}{n}$, where $p = 2\pi n$ as usual.

Accordingly, the energy radiated in ergs per period by the whole linear antenna is—

$$w = \frac{32h^2 n}{3u}\left(\frac{I}{u}\right)^2$$

where I is the current at the base of the antenna in electrostatic units, and n is the frequency of the oscillations.

If we reckon the current in amperes, and denote it by A, we have $\frac{A}{10}=\frac{I}{u}$, and the radiation per complete period in ergs is—

$$w=\frac{32}{300}\frac{h^2n}{u}A^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

Hence the rate of radiation, or radiation per second reckoned in *watts*, is—

$$\left.\begin{aligned} W&=\frac{320}{u^2}h^2n^2A^2 \\ &=320\frac{h^2}{\lambda^2}A^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

In the case of single or multiple wire antenna oscillating freely there is a relation between the radiated wave-length λ and the antenna length h, such that $\frac{h}{\lambda}$ varies between 0·25 and about 0·15, and may approximately be expressed by the equation $hn=\frac{u}{5}=6\times10^9$, or $h^2n^2=\frac{u^2}{25}$. In this case then—

$$W=\frac{320}{25}A^2=12{\cdot}8A^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

If a is the root-mean-square value of the current at the base of the linear antenna, then $A^2=2a^2$, and we have $W=25{\cdot}6a^2$.

We are therefore led to the curious result that the radiation from an antenna of the above type is independent of its height, and depends only upon the square of the current at the base or into the earth.

The formula (86) is quite consistent with that given above (see § 13 (75)) for the radiation from a Hertzian linear oscillator. It was there proved, on the assumption that the current is the same at all points in the oscillator, that the radiation in watts is given by the formula—

$$W=40\pi^2\frac{l^2}{\lambda^2}A^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

where l is the total length of the Hertzian oscillator.

Taking the half of this last expression (87), we have $W=20\pi^2\frac{l^2}{\lambda^2}A^2$ as the radiation from a semi-dipole or half-Hertzian oscillator of total length $l=2h$.

In this case, however, the current has the same value at all points of the length, whereas in the case of the simple linear antenna it varies in accordance with a simple sine law. To obtain from (87), therefore, the radiation for a linear antenna or simple Marconi aerial wire of height h and maximum current A at the base, we have to substitute $\frac{2}{\pi}A$ for A and $2h$ for l in half the formula (87).

We then obtain the expression—

$$W=320\frac{h^2}{\lambda^2}A^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

which is the same as formula (85).

The same method may be applied to find an expression for the radiation from a T-shaped or flat-top antenna, the vertical part of which has a height h, and the top or horizontal part such a large capacity that the current in the vertical part is practically the same at all points. In this case the radiation is the same as that from half of a simple dipole or Hertzian oscillator whose total length is $2h$.

The radiation from half of a Hertzian oscillator of total length $2h$ is—

$$W=20\pi^2\frac{4h^2}{\lambda^2}A^2$$

$$W=800\frac{h^2}{\lambda^2}A^2 \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

This last expression is therefore the total radiation from a flat-top aerial of vertical height h when A is the maximum current at the base.

It is best, however, to express the current by its root-mean-square value, which would be read by a hot-wire ammeter inserted in the base of the antenna near the earth.

Let a be this root-mean-square (R.M.S.) value. Then $2a^2=A^2$. Hence we have the following expressions for the radiation in watts (W) from an antenna of height h measured from the ground to the top, or to a certain point in it. The current at the base of the antenna is a amperes, as read on a hot-wire ammeter.

For a plain Marconi aerial wire of total height h—

$$W=640\frac{h^2}{\lambda^2}a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

For a flat-top antenna with vertical part of height h—

$$W=1600\frac{h^2}{\lambda^2}a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

where λ is the wave-length of the radiation.

In the case of an antenna of any form the radiation in watts can be expressed in the form—

$$W=C\frac{h^2}{\lambda^2}a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

where C is some constant lying between 400 and 1600, h is the height measured up to a certain point, λ is the wave-length, and a the R.M.S. current at the base. The quantity $C\frac{h^2}{\lambda^2}$ is of the nature of a resistance, and is called the *radiation resistance* of the antenna. In the case of the plain single wire antenna, the ratio $h^2/\lambda^2=0{\cdot}04$, and hence the radiation resistance is 25·6 ohms. In the case of other forms of antenna, it may fall to a small fraction of an ohm. For a flat-top or T antenna the constant C in the above expression for the radiation resistance is 1600.

Dr. Austin has given a useful Table, calculated by the above formula, showing the radiation resistance in ohms for flat-top antennæ of various heights.[31]

By the aid of this Table we can calculate at once the radiation in watts corresponding to any given antenna current at the base of the antenna, for it is equal to the product of this radiation resistance and the mean square value of the current, or square of the current, as read on a hot-wire ammeter inserted near the base of the antenna. This Table is in part reproduced on p. 360.

In the case of umbrella antennæ, the height h may be measured from the ground up to a point half-way along the descending arms of the umbrella.[32]

This height h may be called the equivalent height of the antenna, meaning the length of the semi-Hertzian oscillator or dipole to which it is equivalent.

Since the image of the antenna in the conducting ground does not radiate, the length l of the Hertzian dipole, which will radiate the same amount of energy, is given by the equation $l^2=\frac{1}{2}(2h)^2$ or $l=h\sqrt{2}$.

There is, however, in some cases an uncertainty as to how h should be measured. In the case of ships the wireless cabin may be 30 feet above the water and enclosed in a steel hull, and the centre of capacity of the antenna is generally lower than the top of the vertical portion of it. In this case h should really be measured from the wireless room to just below the top of the vertical portion of the antenna.[33]

The above formulæ show that the antenna radiation resistance should decrease as the wave-length increases, being inversely as the square of the wave-length.

Experimental work by Dr. Austin[34] at the United States Naval Radio-

[31] See L. W. Austin on "Radiotelegraphy," *Journal of the Washington Academy of Sciences*, vol. 1, No. 7, November 1911.

[32] See R. Ruedenberg, *Annalen der Physik*, vol. 25, p. 446 (1908).

[33] See L. W. Austin, "A Ship's Antenna as a Hertzian Oscillator," *Journal of the Washington Academy of Sciences*, vol. 1, No. 10, December 19, 1911.

[34] See W. L. Austin, "The Work of the United States Naval Radiotelegraphic Laboratory," *Journal of American Society of Naval Engineers*, vol. 24, Feb. 1912.

A Table of Antenna Radiation Resistances for Flat-Top or T. Antennæ.

h = height of antenna to centre of capacity of aerial wire system in feet.
λ = wave-length in metres.

λ.	h = 40 ft.	= 60 ft.	= 80 ft.	= 100 ft.	= 120 ft.	= 160 ft.
m.	ohm.	ohm.	ohm.	ohm.	ohm.	ohm.
200	6·0	13·4	24·0	37·0	54·0	95·0
300	2·7	6·0	10·6	16·5	23·8	42·4
400	1·5	3·4	6·0	9·3	13·4	23·8
600	0·66	1·5	2·7	4·1	6·0	10·6
800	0·37	0·84	1·5	2·3	3·4	6·0
1000	0·24	0·54	0·95	1·5	2·1	3·8
1500	0·106	0·24	0·42	0·66	0·95	1·7
2000		0·134	0·24	0·37	0·54	0·95
2500			0·15	0·24	0·34	0·61
3000			0·106	0·17	0·24	0·42
4000			0·06	0·093	0·134	0·24

λ.	h = 200 ft.	= 250 ft.	= 300 ft.	= 450 ft.	= 600 ft.
m.	ohm.	ohm.	ohm.	ohm.	ohm.
600	16·4	25·8	37·4	84·0	149·0
800	9·2	14·5	21·0	47·0	84·0
1000	6·0	9·3	13·5	30·0	54·0
1500	2·6	4·1	6·0	13·4	24·0
2000	1·5	2·3	3·4	7·5	13·4
2500	0·95	1·49	2·2	4·8	8·6
3000	0·66	1·03	1·5	3·4	6·0
4000	0·37	0·58	0·84	1·9	3·4
5000	0·24	0·37	0·53	1·2	2·2
6000	0·16	0·26	0·37	0·84	1·49
7000	0·12	0·19	0·27	0·61	1·09

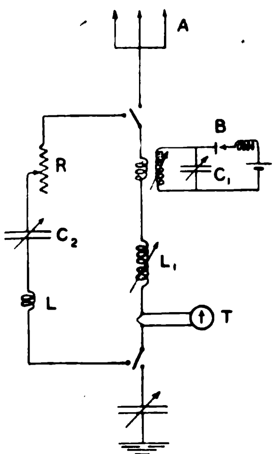


Fig. 26.

telegraphic Laboratory has shown, however, that when an earth connection is used the above law is not fulfilled, but that there is a certain wave-length for which the antenna radiation resistance is a minimum.

The experiments were made by creating in the antenna circuit by a buzzer oscillations of known frequency and measuring the resulting current by a thermo-galvanometer. The circuits were so arranged that the antenna-loading inductance L_1 and thermo-junction T could be switched over from the antenna A and earth to an air condenser circuit R, C_2, L with adjustable resistance in it which could be varied until the current became the same for the two cases (see Fig. 26). The capacity in the air condenser circuit C_2 was made equal to the antenna capacity. Hence the resistance R introduced into the condenser circuit represents the total antenna resistance, and by deduction of the true high frequency ohmic resistance of the antenna we obtain the antenna radiation resistance.

It was found that the curve (see Fig. 27) repre-

senting the radiation resistance in terms of the wave-length had a minimum ordinate and then rose rapidly again. This implies that there must be some additional source of energy loss which increases with the wave-length or diminishes as frequency increases. It appears probable that this additional source which thus comes in is earth plate resistance, or else due to small conductivity in neighbouring bodies. See J. M. Millar, *Washington Bureau of Standards*, Bull. 13, 1916.

Even when the antenna is not earthed conductively there appears to be an energy waste in the earth around the antenna and balancing capacity due to currents induced in the earth, and this creates a damping of the oscillations

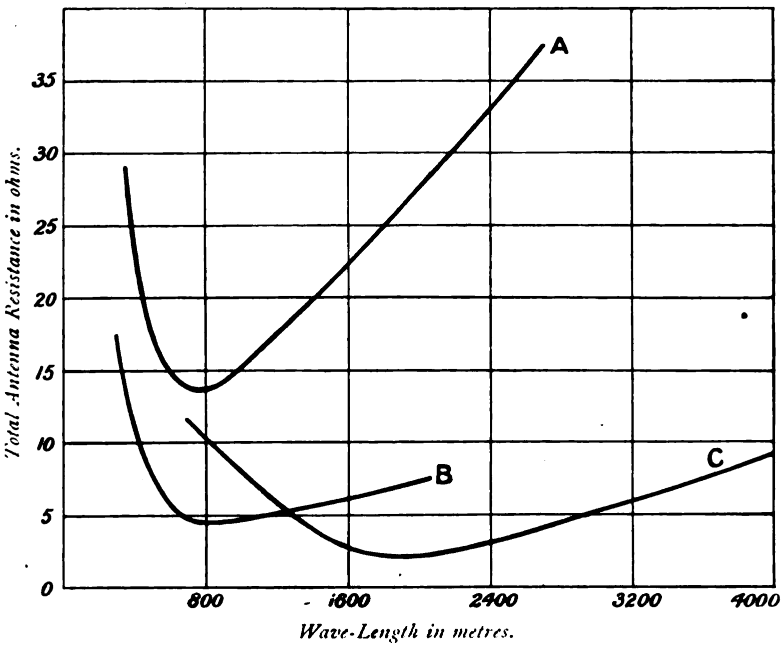


FIG. 27.—Curves representing the Variation of Antenna Resistance in Terms of Wave-Lengths (Austin).

even in the case of antenna with insulated balancing capacity (see M. Reich, *Jahrbuch der Drahtlosen Telegraphie*, vol. v. p. 176, 1911, or *Science Abstracts*, vol. 15, B, 1912, No. 444).

In connection with antennæ, it is important to notice that the radiated energy is not sent out equally in all directions.

If we describe round a Hertzian oscillator a spherical surface of radius r, large compared with the length of the oscillator, then we have shown that the radiant energy sent out by the oscillator per period, which passes through the surface of this sphere, is given by the formula $E = \frac{16\phi^2\pi^4}{3\lambda^3}$, where ϕ is the maximum electric moment of the oscillator and λ the radiated wave-length.

The energy sent out through a zone of width $d\theta$ in time dt is equal to—

$$\frac{\phi^2 m^3 n}{2} \sin^3\theta \, d\theta \sin^2(mr - nt)dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

(see equation (56), § 9, Chap. V.). Hence the energy sent out through the zone per period is equal to—

$$\frac{\phi^2 m^3 n}{2} \sin^3\theta\, d\theta \cdot \frac{T}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (94)$$

Bearing in mind that $m=\frac{2\pi}{\lambda}$ and $n=\frac{2\pi}{T}$, and that the area of the zone is $2\pi r^2 \sin\theta\, d\theta$, it is easily seen that the energy radiated per period through each such elementary zone is expressed by—

$$\frac{4\phi^2\pi^4}{\lambda^3} \sin^3\theta\, d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (95)$$

Accordingly there is no energy radiated in the direction of the antenna itself, and it is a maximum in the equatorial plane.

If we plot out a curve such that its radii vectors drawn from a point are proportional to $\sin^3\theta$, where θ is the colatitude angle, we obtain a curve having the shape shown in Fig. 28, which may be called a radiation curve. A curve of this

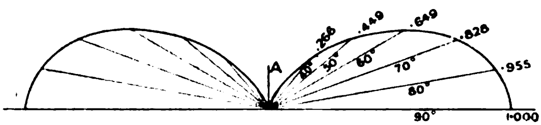


FIG. 28.

kind was first given by Professor A. Blondel in 1903.[35] The numbers on the radii are the cubes of the sines of the angles θ taken from the Table below.

θ.	Sin θ.	Sin^2 θ.	Sin^3 θ.
0	0·0	0·0	0·0
10	0·173	0·03	0·005
20	0·342	0·117	0·040
30	0·500	0·25	0·125
40	0·643	0·413	0·266
50	0·766	0·587	0·449
60	0·866	0·749	0·649
70	0·939	0·882	0·828
80	0·985	0·970	0·955
90	1·000	1·000	1·000

Now, the surface of the enclosing sphere is $4\pi r^2$, and hence the mean spherical energy radiated per period is given by—

$$\frac{E}{4\pi r^2}=\frac{4\phi^2\pi^3}{3r^2\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (96)$$

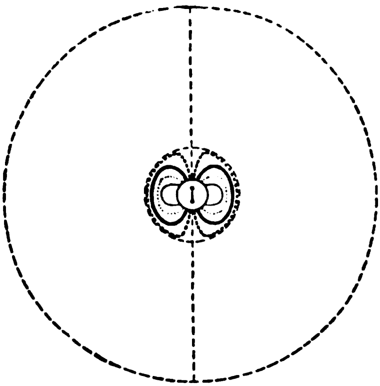
This corresponds to the mean spherical candle-power of an arc lamp.

This energy is not, moreover, radiated equally in all directions. If we divide the surface of the sphere into narrow zones, bounded by lines of latitude having angular distances or colatitudes θ and $\theta+d\theta$ respectively, we have seen that the energy passing through each such zone per period is given by equation (66). Accordingly, the radiation per square unit of area for each zone is obtained by

[35] In a paper entitled "Quelques Remarques sur les Effets des Antennes de Transmission," at the meeting of the French Association for the Advancement of Science at Angers in 1903.

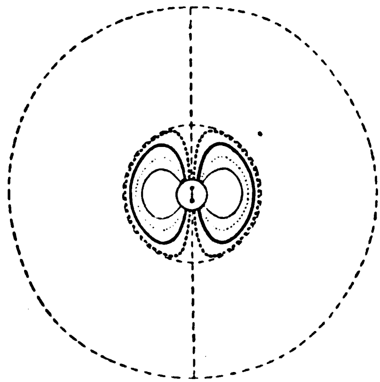
PLATE II.—DIAGRAMS SHOWING THE FORM OF T

1.



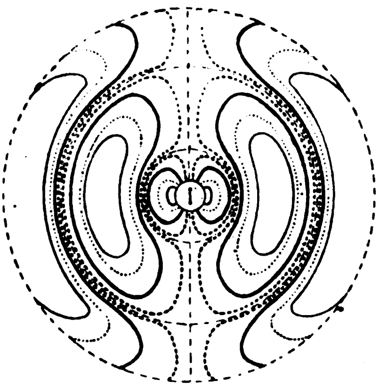
Time = $\frac{1}{8}$ Period = $\frac{1}{8}$ (2T)

2.



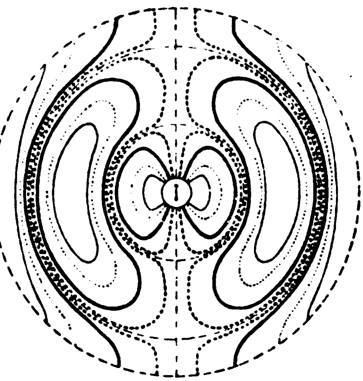
= $\frac{1}{4}$ (2T)

9.



Time = $1\frac{1}{8}$ (2T)

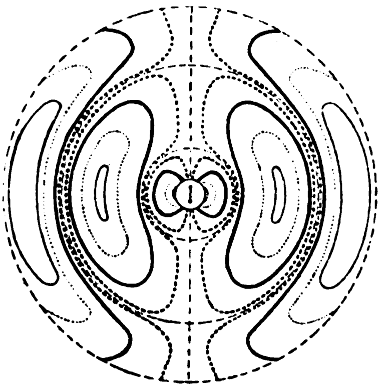
10.



= $1\frac{1}{4}$ (2T)

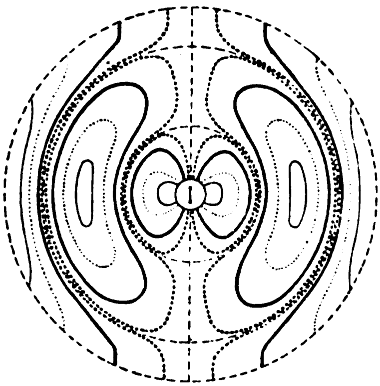
PLATE III.—DIAGRAMS SHOWING THE FORM OF TIDES

17.



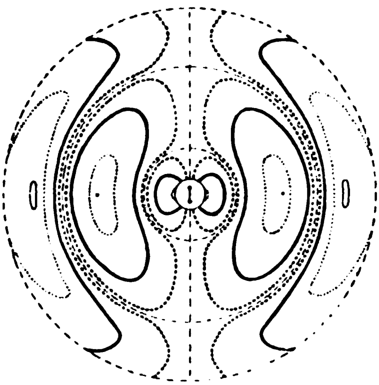
Time = $2\frac{1}{8}$ (2T)

18.



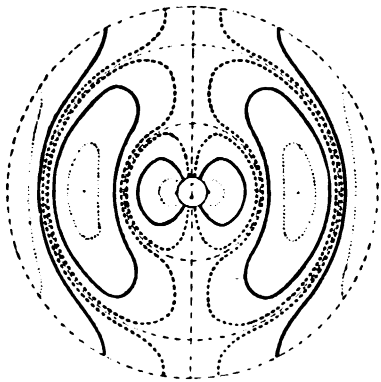
= $2\frac{1}{4}$ (2T)

25.



Time = $3\frac{1}{8}$ (2T)

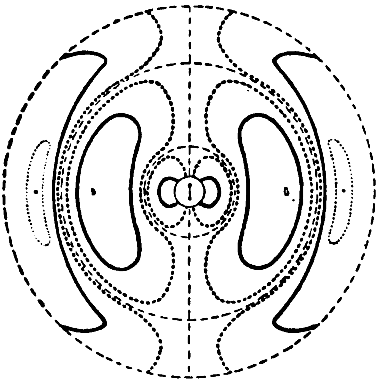
26.



= $3\frac{1}{4}$ (2T)

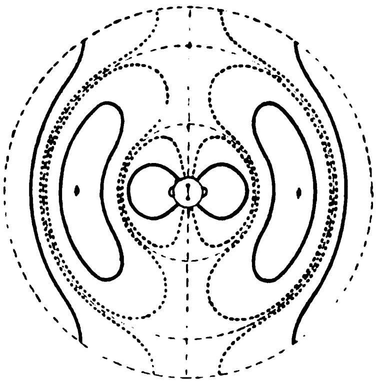
PLATE IV.—DIAGRAMS SHOWING THE FORM OF TH

33.



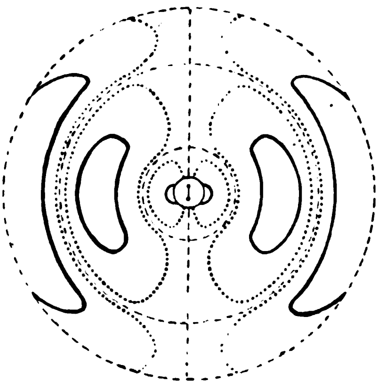
Time = $4\frac{1}{8}(2T)$

34



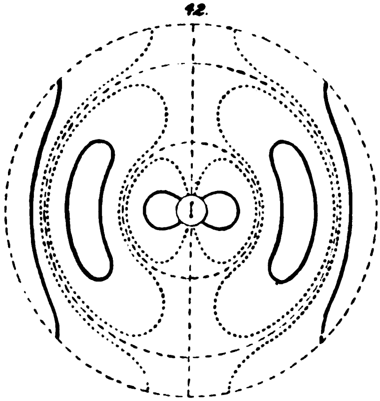
= $4\frac{1}{4}(2T)$

41.



Time = $5\frac{1}{8}(2T)$

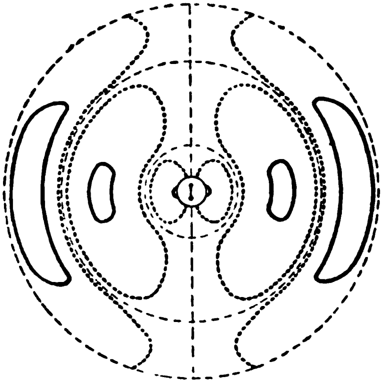
42.



= $5\frac{1}{4}(2T)$

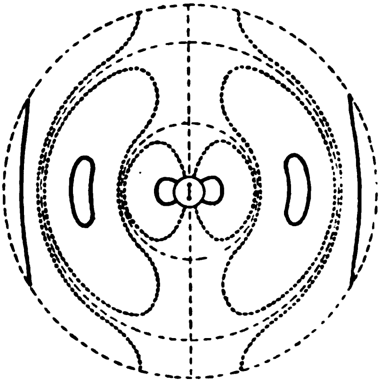
PLATE V.—DIAGRAMS SHOWING THE FORM OF T

49.



Time $= 6\frac{1}{8}\,(2T)$

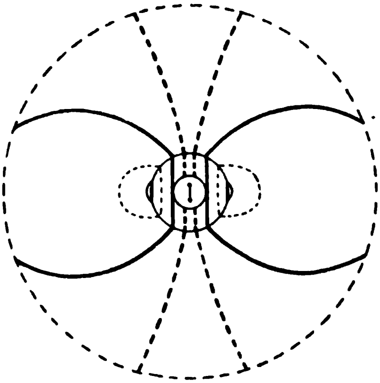
50.



$= 6\frac{1}{4}\,(2T)$

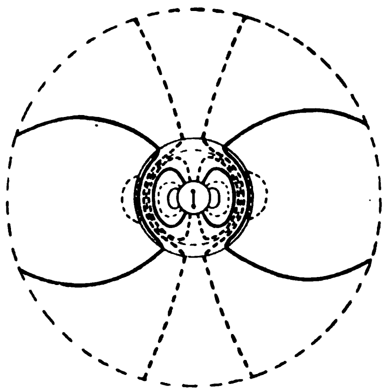
DIAGRA

FIG. 4.



$t = (0{\cdot}26)\,2\tau.$

FIG. 5.



$t = (0{\cdot}385)\,2\tau.$

PLATE VI.—DIAGRAMS SHOWING ‖S DURING THE FUNDAM

M. ABRAHAM.

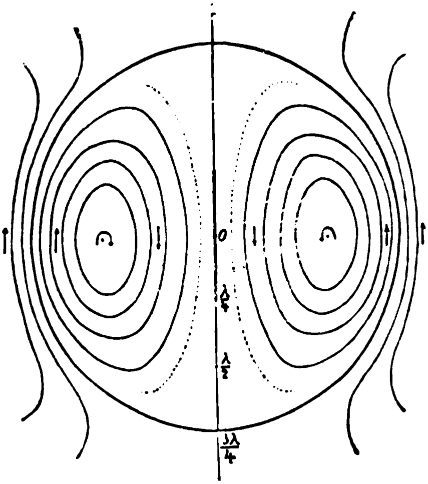
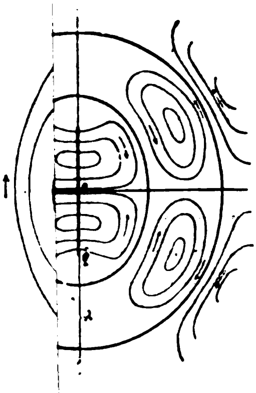


FIG. 1.—$t=0$.



‖G. 6.—$t=\frac{1}{4u}$.

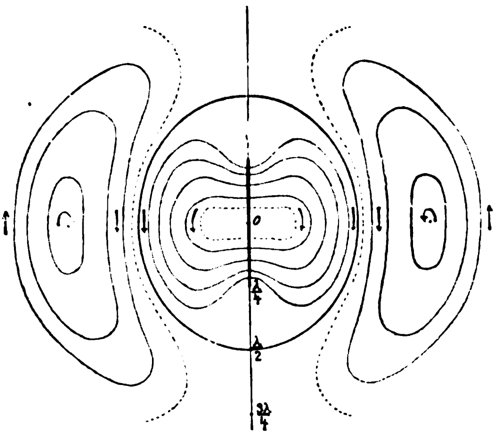
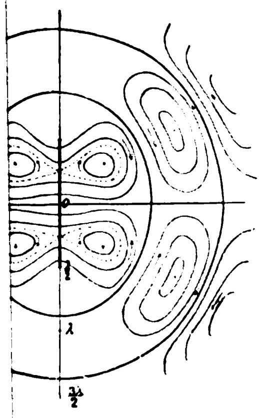


FIG. 3.—$t-\frac{1}{u}$.



‖. 8.—$t=\frac{3}{4u}$.

Fundamental Os

‖ Oscillator.

dividing $\frac{4\phi^2\pi^4}{\lambda^3}\sin^3\theta\,d\theta$ by the area of the zone$=2\pi r^2\sin\theta\,d\theta$, and is therefore equal to—

$$\frac{2\phi^2\pi^3}{r^2\lambda^3}\sin^2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

Since the mean value of $\sin^2\theta$ for uniform distribution over a sphere is $\frac{2}{3}$, the formula (97) agrees with (96), viz. that for the mean spherical energy radiation.

If in (97) we put $\theta=90°$, we find the mean horizontal energy radiation per period per unit of surface to be $\frac{2\phi^2\pi^3}{r^2\lambda^3}$.

It is seen, therefore, that the mean horizontal energy radiation which passes through unit of area of the sphere per period in the equatorial plane is equal to 1·5 times the mean spherical energy radiation per period per unit of area.

Accordingly, in considering the energy available for absorption by the receiving antenna, we have to deal with this horizontal radiation density, and not with the mean spherical radiation density as many writers have done.

It is therefore a peculiarly valuable quality of the linear antenna, or earthed vertical antenna as used in radiotelegraphy, that it sends out its energy chiefly in the direction in which it is desired to be sent out, viz. along the surface of the earth, and not up into the sky or equally all round the hemisphere. Professor Blondel pointed out in 1903 (*loc. cit.*) that this quality of vertical antenna, however, renders it incapable of communicating with a balloon nearly overhead, and limits very much the power of such an antenna to radiate usefully to a receiving station on any aerial vessel overhead or high up above the horizon.

Dr. B. van-der-Pol, jun., has shown, however, that if an antenna is so loaded with capacity at the top as to produce a current node half-way up the vertical part, it will then radiate chiefly in a direction at about 45° to the vertical and not at all horizontally, and hence might be used to explore the atmosphere for reflecting ionic clouds.[36]

[36] See *Proc. Phys. Soc. Lond.*, vol. 29, p. 269, 1917.

CHAPTER VI

DETECTION AND MEASUREMENT OF ELECTRIC WAVES

1. Appliances for Detecting Electric Waves.—When the length of electric waves falls within a certain limited range, they are able to affect directly organs of sensation with which we are provided. Thus, if their wave-length is anything between $0{\cdot}43\mu$ and $0{\cdot}75\mu$ (where μ denotes 1 micron or 0·001 of a millimetre) they affect the retina of the normal human eye and produce the sensation of light, the wave-length determining the sensation of colour which we experience. This range of wave-lengths barely covers one octave of radiation.

If the waves are sufficiently strong and have wave-lengths rather greater than about $0{\cdot}5\mu$, they produce a sensation of heat when falling upon the skin. It is not yet known precisely how far down the gamut of electric waves this power extends, but it is certain that when the waves have a wave-length of even a few millimetres they excite no sensation of heat when falling upon the human skin. Hence we may say that electric waves of the length with which we are chiefly concerned in this treatise do not directly affect any of our bodily organs. If, then, we are to detect their presence, it can only be in virtue of some change or action which they produce upon material substances or some device or apparatus arranged for this purpose.[1]

All devices for detecting electric waves of wave-length too long to affect our eyes or nerves as light or heat are, in fact, instruments for detecting high frequency currents or high frequency oscillations of potential. To make them effective as wave-detecting appliances, they have to be associated with a collecting wire called an antenna, or aerial wire, placed in the path of the wave.

As the wave sweeps across this wire, the fluctuating electric and magnetic force which constitute it, create in the wire high frequency currents, provided that wire forms part of a circuit suitably tuned to the length of the wave, or at least approximately in resonance with it. The wave-detecting device tells us whether feeble electric oscillations are set up in this wire, or feeble oscillations of potential generated across any capacity in it.

Roughly speaking, we may classify the devices that have been used as oscillation detectors into the following groups :—

1. Spark detectors.
2. Contact detectors, or coherers.
3. Magnetic detectors.
4. Electrolytic detectors.
5. Thermal detectors.
6. Rectifying crystal, or contact detectors.
7. Thermionic detectors.
8. Physiological detectors.

The above classification is not exhaustive, but it is sufficient for the present purposes of description. Examples of each of these types will be briefly described.

2. Spark Detectors.—If a pair of metal wires or strips of tinfoil attached to a glass plate are placed in line with each other, and their inner ends terminated in spark balls or sharp points placed close together, we have an appliance with which some adjustment will detect the presence of strong electric waves.

[1] Collectively these devices may be termed *cymoscopes*, from κῦμα, a wave.

If the rods or strips are so placed that when the electric wave traverses them the electric force is parallel to the rods, then these metal conductors integrate or add up the electric force existing all along their direction at any one instant, and the ends in opposition are at a difference of potential which depends on the length of the rods and the strength of the waves. As the rods have capacity with respect to each other, and also inductance depending on their length, they have a certain natural time period of oscillation.

If this is adjusted by shortening the rods so as to agree with that of the incident wave, then resonance will come into play to increase the potential difference between them when a wave train sweeps over the rods having the same time period. Such resonant rods may be made by making one or more tubes fitting telescopically into each other so as to be lengthened or shortened within limits, the ends in apposition being provided with small spark balls capable of having their distance from each other adjusted by a screw (see Fig. 1).

These rods of adjustable length may be carried on an insulating fixture. When held with the rods parallel to the electric force component of an electric wave, minute electric sparks will make their appearance at the spark balls if these are sufficiently near together.

The arrangement can, however, only detect the presence of relatively powerful electric waves. In order to make any visible spark in air between balls, however near, but not in contact, there must be a certain maximum potential difference, which for air at normal pressure is not far from 300 volts.

This was shown particularly by experiments made in the Cavendish Laboratory, Cambridge, by Mr Peace.[2] Sir J. J. Thomson observes that in this respect gases resemble electrolytes, in that a certain definite difference of potential between

FIG. 1.—Sliding Rod Spark Cymoscope adjustable as to Time-period.

the electrodes has to be exceeded before any current or discharge will pass. No matter, therefore, how near the spark balls may be placed, the electric force in the interspace, and along the region occupied by the rods, must be such that there is a difference of potential between the rods equal to 300 volts, or to 1 electrostatic unit of potential difference, at least if a visible spark is to occur.

The spark detector can, therefore, never be a very sensitive instrument.

Hertz's ring resonator with micrometer spark gap belongs to the same category and suffers under the same limitations. In fact, considering its comparative insensibility, it is marvellous that Hertz was able to accomplish with it all that he did.

There are, however, some occasions when a somewhat insensitive wave detector is needed, and in these cases a spark detector is useful. Since the spark length corresponds to the maximum potential difference of the electrodes between which it occurs, it follows that the maximum spark length which can be obtained, other things remaining the same, is a measure of the maximum value of the wave amplitude during the passage of the wave train.

The spark detector, though not very sensitive, has accordingly some valuable qualities. We can by it determine approximately the direction of the electric force in the wave—in other words, its plane of polarization—since it gives its maximum indication when the rods are placed parallel to the direction of that force. We can also roughly determine comparative maximum wave amplitudes, since, other things being equal, the wave with greatest maximum amplitude will create the longest spark.

3. Contact Detectors or Coherers.—The electric wave-detecting device, commonly known as *a coherer*, has been the subject of much research. Many experimentalists in past years noticed that powdered metals or conducting

[2] See "Recent Researches in Electricity and Magnetism," J. J. Thomson, p. 89.

substances in a state of fine division, or mixtures of metallic particles and other semi-conducting substances, were practically non-conductors under small electromotive forces when the mass was loosely compressed, but suddenly became possessed of good conductivity when a large electromotive force was applied to it.

Dr. K. E. Guthe traces this knowledge as far back as 1835, to Munk of Rosenschoeld, who clearly described the permanent increase in the electric conductivity of a mixture of tin filings, carbon, and other conductors resulting from the passage through it of the discharge of a Leyden jar.[3]

In 1852 the high resistance of a mass of loose metallic powder was observed by S. A. Varley, and it is said that four years later he had noticed a remarkable fall in the resistance of such material during a thunderstorm.[4]

In a British Patent Specification (No. 165 of 1866), C. and S. A. Varley described a device for protecting telegraphic instruments from lightning, which consisted of two copper points, nearly touching each other, set in a small box filled with powdered carbon. They say that the box may or may not be exhausted of its air. Also they observe that "powdered conducting matter offers great resistance to a current of moderate tension, but offers little resistance to a current of high tension."

This observation, however, did not attract the attention it deserved. In 1878 Professor D. E. Hughes was engaged on researches on the microphone, and in some of his experiments he employed a tube of glass, filled loosely with filings of zinc and silver, placed in series with a telephone and a single voltaic cell.[5] He appears to have discovered the important fact that such a tube, when so used, was sensitive to electric sparks at a distance as indicated by its sudden changes of conductivity. He subsequently stated that in these experiments he used a carbon-steel microphone which also proved to be very sensitive to an electric spark. He showed these experiments privately at the time to many scientific friends, but was discouraged from publishing the results, and it was not until twenty years afterwards that he publicly mentioned them.[6] Meanwhile the same facts had come to the notice of other observers. In Italy, Professor T. Calzecchi-Onesti made experiments on the changes in electric conductivity of metallic powders, loosely aggregated, under the action of various electromotive forces. These he described in the Italian journal, *Il Nuovo Cimento*, 1884, vol. 16, p. 58, and 1885, vol. 17, p. 35, also in the *Journal de Physique*, 1886, vol. 5, p. 573.

He did not, however, carry knowledge much beyond the point at which it was left by the brothers Varley, viz. that whereas loosely compressed metallic filings constitute a poor electrical conductor under low electromotive forces, yet under the operation of the high electromotive forces such as that of an induction coil the conductivity is remarkably increased. These observations, moreover, attracted at the time no particular attention.

In 1890, Professor E. Branly, of Paris, published an account of a very extensive series of observations on the same subject. Whilst confirming the work of previous observers, he added much new knowledge.[7] He made the extremely important and novel observation that an electric spark *at a distance* had the power of suddenly changing the electric conductivity of loose masses of powdered conductors. In some cases he observed that this change was an increase in conductivity, and in other cases a decrease. The majority of common metals exhibit the increase in conductivity, but the contact between lead and peroxide of lead becomes less conductive. To Professor Branly belongs the honour of giving to science a new weapon in the shape of a tube or box containing metallic filings rather loosely packed between metal plugs (see Fig. 2).

[3] See paper read before the St. Louis International Electrical Congress, 1904, by K. E. Guthe, on "Coherer Action"; see also *The Electrician*, 1904, vol. 54, p. 92.

[4] See *The Electrician* (Leader), vol. 40, p. 86.

[5] See D. E. Hughes, *Proc. Roy. Soc. Lond.*, May 9, 1878, vol. 27, p. 36.

[6] See a remarkable letter from Prof. Hughes in *The Electrician* of May 5, 1899, vol. 43, p. 40. An epitome of these important experiments made in 1879-1886 by Hughes is given in Fahie's "History of Wireless Telegraphy," p. 296. For a further reference to Hughes' work, see Chap. VII. § 1, of this treatise.

[7] See E. Branly, *Comptes Rendus*, 1890, vol. 111, p. 785; also 1891, vol. 112, p. 90; or *La Lumière Electrique*, 1891, vol. 40, pp. 301, 506; or *The Electrician*, 1891, vol. 27, pp. 221, 448.

He showed that such a tube may be a conductor of very high resistance when the metallic filings are loosely arranged, but that if a discharge from a Leyden jar or other electric spark was made in its vicinity, the conductivity of the metallic powder was suddenly increased. He detected this change by connecting a galvanometer and single cell in series with the tube, and adjusting the pressure on the powder until no current would pass through it. Under the influence of a spark at a distance, the galvanometer needle then made a sudden deflection, showing the acquirement of conductivity by the mass of metallic filings.

Branly also found that the same effect occurred in the case of two slightly oxidized steel or copper wires laid across each other with light pressure. This loose or "imperfect contact" was found by him to be extraordinarily sensitive to a distant electric spark, dropping in resistance from some thousands of ohms to a few ohms, when an electric spark was made many yards away.

Branly's work did not, however, secure the notice it demanded until 1892, when Dr. Dawson Turner described Branly's experiments at a meeting of the British Association in Edinburgh, and his own additions to them.[8]

Dr. Dawson Turner's paper raised a discussion on the subject, and drew from Professor George Forbes an important question. He asked whether it was not possible that Hertz waves might in a similar manner break down the resistance of a tube of loose metallic filings. This question showed that the real cause of the phenomena noted by Branly had not yet been fully comprehended, and even then its importance was not appreciated. In the following year Mr. W. B. Croft exhibited Branly's experiments at a meeting of the Physical Society in London, and read a short paper on the action of electric radiation on copper filings.[9] He exhibited a glass tube containing copper filings joined in series with a galvanometer and a battery. When the filings were loosely arranged no current passed, but immediately an electric spark was made by an electrical machine at a little distance, the galvanometer was deflected, and remained deflected until the tube was tapped. He stated that he had tried different kinds of metallic filings. Aluminium and copper he found equally good, but iron not so good, and with carbon he obtained no effect at all. These facts themselves had already been observed by Branly, but the advance appeared to be a more definite recognition of the cause of the phenomena to be electric radiation falling on the tube. In this discussion Professor Minchin distinctly said the change was due to electric radiation, and not to the light of the spark. He stated that he had found his impulsion cells to be rendered sensitive to light by an electric spark 140 feet away. Mr. Croft called attention to the fact that the filings tube passed into a conductive state *before* the actual spark passed when the static electrical machine was set in motion.

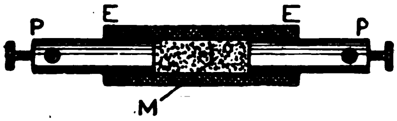


FIG. 2.—Branly Metallic Filings Tube or Cymoscope. E, tube of insulating material; P, P, metal plugs; M, metallic filings loosely packed.

This paper was followed shortly after by another by Professor G. M. Minchin, entitled "The Action of Electromagnetic Radiation on Films containing Metallic Powders."[10] He exhibited films of gelatine and collodion, impregnated with metallic powders. These were inserted in the circuit of a battery and galvanometer. He found that contact with an electrified body rendered the film con-

[8] See Dr. Dawson Turner, *The Electrician*, 1892, vol. 29, p. 432, "Experiments on the Electrical Resistance of Powdered Metals."

[9] See W. B. Croft, *Proc. Phys. Soc. Lond.*, vol. xii. p. 421.

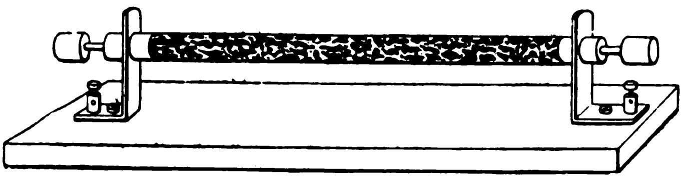
[10] See Prof. Minchin, *Proc. Phys. Soc. Lond.*, November 24, 1893, vol. xii. p. 455. Also *Phil. Mag.*, January 1894, vol. 37, p. 90. A paper by Prof. Minchin will also be found in *The Electrician*, November 27, 1891, vol. 28, p. 85, on the "Detection of Electromagnetic Disturbances at Great Distances," in which he describes a form of cell, consisting of pieces of tinfoil specially prepared, placed in methyl alcohol, which generates an E.M.F. when exposed to light. It can be rendered insensitive by small shocks. When in this insensitive condition, Prof. Minchin found that the cell again became sensitive to light by the action of an electric spark at a distance.

ductive. In this paper Professor Minchin made definite reference to the Branly tube, and says that "the waves sent out from the spark at once render the column (of metallic filings) a conductor."

It is clear, therefore, that at the end of 1893 a few physicists, pre-eminently Professor Minchin, had clearly recognized that the action discovered by Branly had its origin in electric waves sent out from the spark.

This paper of Professor Minchin was followed by another from Sir Oliver Lodge, entitled "On the Sudden Acquisition of Conducting Power by a Series of Discrete Particles."[11] In this paper he alludes to an observation he had frequently made in connection with his experiment of the Syntonic Leyden jars, viz. that if the two metal knobs of the receiver were very close together, a battery and electric bell being in circuit, the occurrence of an electric oscillation in the circuit caused the knobs to come into good contact and made the electric bell ring. Four years previously, in 1889, Lodge had noticed that the passage of a spark between two metal plates in microphonic or imperfect contact caused them to weld together and make good contact. There is, however, no clear proof that at this date (1889) Lodge had recognized that this action could be produced by electric radiation alone.

In June 1894 Lodge gave a lecture at the Royal Institution, entitled "The Work of Hertz." In this lecture he described the Branly tube, and showed an



[*From* "*The Electrician.*"

FIG. 3.—Lodge Coherer or Electric Wave Detector.

instrument in which a light microphonic or imperfect electrical contact was made a good conducting one by the impact on it of electric waves. He also employed a tube loosely full of iron borings closed at the ends with metallic plugs, and this in the same manner exhibited improved conductivity when electric radiation fell upon it. Lodge then gave the name *coherer* to any device in which a loose or imperfectly conducting contact between pieces of metal was improved in conductivity by the impact on it of electric radiation. This lecture was the means of drawing attention strongly to the discoveries of Branly, and to the fact that a new and highly sensitive means of detecting electric radiation had been evolved.[12]

In the next year or two no very notable improvement took place in the construction of the coherer. In the form in which it was used by Lodge in 1894, the coherer consisted of a glass tube about 1 cm. or less in diameter, and 6 or 8 cms. in length, filled loosely with coarse filings or borings of iron or other metal contained between two metal plugs (see Fig. 3). Brass borings were tried, and various other metals, and the tube filled with air, hydrogen, or even exhausted. Lodge experimented with various forms of light contact between plates and points, such as a steel sewing-needle lightly resting on an aluminium plate, and with slightly oxidized steel rods lightly resting on each other.

The appliance in the above described forms was generally found to be a somewhat capricious instrument to use; in some conditions highly sensitive to distant

[11] See *Proc. Phys. Soc. Lond.*, 1893, vol. xii. p. 461. See also *Phil. Mag.*, January 1894, vol. 37, p. 24.

[12] See *Proceedings of the Royal Institution*, 1894, vol. xiv. p. 321; also *The Electrician*, 1894, vol. 33, pp. 153, 186, 204. Lodge republished his 1894 Royal Institution lecture as a book, the first edition bearing the title "The Work of Hertz and Some of his Successors." A second edition appeared in 1897, under the modified title, "Signalling across Space without Wires."

electric sparks, and then without apparent reason becoming far less sensitive. It had in general been found that the metals most suitable were those which were slightly but not very oxidizable. Iron, steel, nickel, copper, or zinc filings or borings worked fairly well, but coherers made with gold, silver, platinum, or noble metals proved more difficult to handle.

Meanwhile, in 1894 and 1895, G. Marconi began his work in Italy, and turned his attention to the improvement of Branly's coherer. He carefully investigated the relative advantages of the various metals in regard to their suitability for making a metallic filings coherer, and he modified the form and size of the tube. Whereas others had used rather large tubes, filled with somewhat coarse filings or borings, he adopted a much smaller size of glass tube[13] (see Fig. 4), about 3 or 4 cms. long and about 5 mms. internal diameter. He placed in this two silver plugs fitting the tube tightly, and attached to these platinum wires which were sealed through the glass. The ends of these plugs were polished and slightly amalgamated with mercury. The ends of the plugs were brought within a couple of millimetres of each other. The interspace was about half filled with a small quantity of nickel and silver filings, 95 per cent. nickel, and 5 per cent. silver, carefully sifted so as to be of a certain degree of fineness. The glass tube was then exhausted and sealed. Subsequently he bevelled the edges of the silver plugs so as to make the interspace wedge-shaped. So improved and carefully

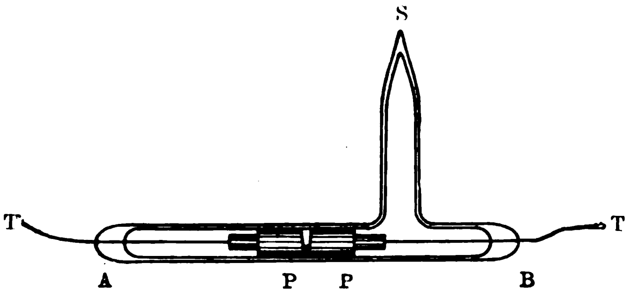


FIG. 4.—Marconi Sensitive Tube or Cymoscope. (*Full size.*) A, B, glass tube exhausted; T, T, platinum terminal wires; P, P, silver bevelled plugs; S, side tube for exhaustion.

made, the Marconi coherer proved to be a most sensitive electric wave-detecting device or cymoscope, far more certain in its action than anything which had previously been designed.

Marconi then proceeded to invent the devices for employing his sensitive tube as a relay upon a relay in a telegraphic instrument, and to use it for conducting wireless telegraphy in the manner described in the next chapter. The news of his success in this important practical application caused widespread interest in the subject of coherers.

Branly reasserted his claim to be the inventor of the metallic filings coherer, but he seems to have underrated the importance of Marconi's improvements.[14] Lodge, about this time, wrote a paper on the "History of the Coherer Principle,"[15] in which he described his own discoveries and those of others leading up to Marconi's work. In January 1896 Professor A. S. Popoff, of Cronstadt, Russia, communicated a paper to the *Journal of the Russian Physical and Chemical Society*, in which he described certain experiments made with a metallic filings coherer.[16] He made his sensitive tube of glass with two platinum leaves down opposite sides, the interspace being loosely filled with iron filings (see Fig. 5).

13 See British Patent Specification of G. Marconi, No. 12,039 of June 2, 1896.

14 See E. Branly, "On the Electrical Conductivity of Discontinuous Conducting Substances," *Comptes Rendus*, December 6, 1897, vol. 125, p. 939; also *The Electrician*, 1897, vol. 40, p. 333.

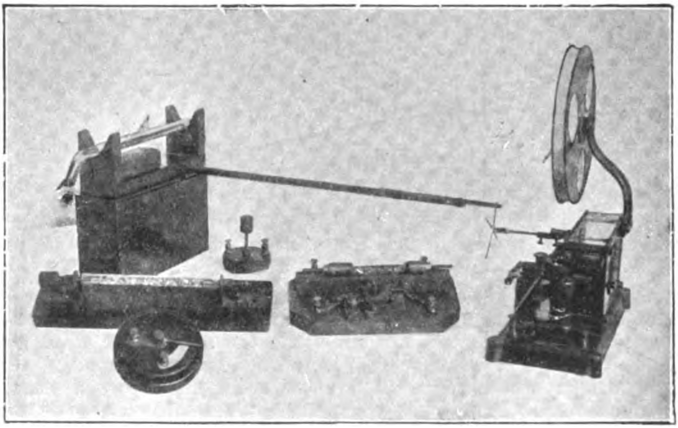
15 See *The Electrician*, 1897, vol. 40, p. 87.

16 *Journal of the Russian Physical and Chemical Society*, January 1896, vol. 28.

Popoff combined with this tube an arrangement for automatically tapping back the filings to a sensitive condition. As already stated, Branly observed that a'ter the loose aggregations of imperfectly conducting metallic filings had been exposed to an electric spark and rendered thereby conductive, a slight tap or mechanical shock brought them back again into the high resistance or non-conductive condition.[17] Lodge had also noticed that when two brass knobs in microphonic contact had been caused to cohere by the passage of a small spark between them the contact was destroyed by a slight mechanical shock.

FIG. 5.—Popoff's Coherer.

Lodge had found that if a battery and electric bell were connected between the knobs and placed on the same stand, the mere mechanical vibration of the bell when the circuit was completed was sufficient to destroy the light contact of the knobs. When he subsequently came to employ the metallic filings coherer of



[*From* "*The Electrician.*"

FIG. 6.—Lodge's Arrangement for tapping back his Coherer Tube to Wave Sensitiveness by means of a Clockwork-driven Tapper.

Branly as an electric wave detector, he arranged a "clockwork tapper consisting of a rotating spoke wheel driven by the clockwork of a Morse instrument, and giving to the filings tube or to a coherer a series of jerks at regular intervals"[18] (see Fig. 6). He states, however (*loc. cit.*), that "an electric bell mounted on the

[17] See a paper by E. Branly in *La Lumière Electrique*, May 16, 1891, translated in *The Electrician*, 1891, vol. 27, pp. 221, 448. The restoration of non-conductivity by a tap or shock is particularly mentioned. Also it is mentioned in the paper read by Dr. Dawson Turner to the British Association at Edinburgh, in August 1892; see *The Electrician*, 1892, vol. 29, p. 432.

[18] See O. J. Lodge, "On the History of the Coherer Principle," *The Electrician*, 1897, vol. 40, p. 90. See also *The Electrician*, vol. 39, p. 687, for a photograph of this mechanical tapper, as exhibited at a meeting of the British Association at Oxford, in 1897.

base of the filings tube was not found very satisfactory, because of the disturbances caused by the little sparks at its contact breaker to which the previous coarser knob arrangement had failed to respond."

Popoff, however, arranged an electric bell so that the hammer was made to tap the coherer tube lightly and administer to it the small shock required to make the filings cohere.

Popoff employed his filings tube to close the circuit of a telegraphic relay, and this, in turn, when actuated, set the electric bell in operation so that its hammer struck the coherer tube [19] (see Fig. 7). The purpose which Popoff had in view was to study the phenomena of atmospheric electricity, and he employed the Branly filings tube as a means of detecting and making records of atmospheric electrical discharges at a distance. He states that the above-described apparatus was set

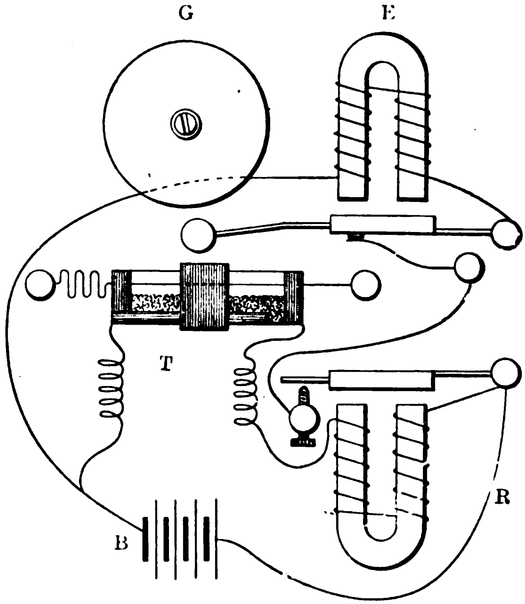


FIG. 7.—Popoff's Electromagnetic Tapper for tapping back the Metallic Filings Tube to Wave Sensitiveness. T, coherer; R, relay; E, electromagnetic tapper; B, battery; G, gong.

up in July 1895 at the Meteorological Observatory of the Forest Institution in St. Petersburgh. One end of the coherer was connected to a lightning conductor and the other end to the earth, and he further remarks that from July 1895 to December 1897 his apparatus worked well as a lightning recorder. He made the arrangement record by connecting in parallel with the electric bell an instrument in which a pen was caused to make marks on a moving band of paper when the circuit of an electromagnet was closed. Popoff's apparatus was in fact intended to record only what are now called atmospheric stray waves, but not intelligible signals.

Marconi had meanwhile filed an application for a British patent,[20] in which the details of his electric wave detector were particularly described. He also associated

[19] See a letter from Prof. A. Popoff, *The Electrician*, December 10, 1897, vol. 40, p. 235.

[20] No. 12,039 of June 2, 1896. "Application of Guglielmo Marconi for Improvements in Transmitting Electrical Impulses and Signals and in Apparatus therefor."

with its sensitive metallic filings tube a telegraphic relay and a single cell, so that when the tube passed into its conductive condition the circuit of the relay was closed. He points out that the metallic filings tube must not be traversed at any time by a current greater than about 1 milliampere. Hence his relay was wound with a wire of high resistance, and one having a resistance of 1000 ohms is usually employed. The relay, in turn, is made to close the circuit of an electromagnet which operates a tapping arrangement to bring back the tube continually to a sensitive condition. The details and adjustments of his tapper were designed with peculiar care. The tapper consists of an electromagnet having a vibrating armature like an electric bell, the armature carrying a hammer consisting of a round brass knob on a stem. The electromagnet is so fixed on an inclined plane that the blow administered to the sensitive tube is from below upwards (see Fig. 8). He added delicate screw adjustments by means of which the exact strength of the blows, and the rate at which they were given, could be precisely controlled. In addition to this, he overcame the difficulty mentioned by Lodge, by inserting in the circuit of the sensitive tube two choking coils of wire, the effect of which was to prevent the oscillations started by the minute sparks at the relay or vibrating hammer contacts, travelling back along the wires and causing coherence in the filings. The vibrating tapper and any other telegraphic printing or recording instruments were worked in parallel by the relay by a set of dry cells, one separate cell being used in the circuit of the coherer and relay.

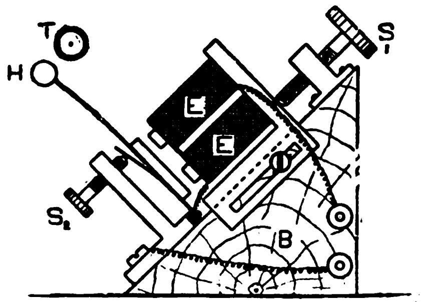


FIG. 8.—Marconi's Electromagnetic Tapper for tapping back his Sensitive Tube to a Receptive Condition. E, E, electromagnet; H, hammer; T, sensitive tube shown in section; S_1, S_2, adjusting screws.

The cell in series with the relay and sensitive tube passed a current not exceeding a milliampere through the tube when the filings cohered under the action of an electric wave, and the circuit of the relay thereby closed caused the current from about eight other dry cells to work the tapper and Morse printer or recorder.

Marconi mounted the whole of this apparatus on one base board and enclosed it in a metal box to preserve it from the action of stray waves or sparks.

In this manner, towards the end of 1896 Marconi produced a compact self-contained cymoscope of greater sensitiveness and certainty of action than any which had previously been designed.

The results which he obtained with it when associated with his other inventions stimulated research in a remarkable manner. He came over from Italy to England and made known his improvements to Sir W. H. Preece, then the engineer-in-chief of the British Government Telegraph Department of the General Post Office. Sir W. H. Preece exhibited it at a lecture he gave at the Royal Institution on June 4, 1897, and stated that "Marconi has produced from known means a new electric eye, more delicate than any known electrical instrument, and a new system of telegraphy that will reach places hitherto inaccessible."[21]

4. Methods of Detecting and Recording the Passage of Electric Waves by Contact Detectors.—It has already been mentioned that Branly discovered the fact that whilst some kinds of loose or imperfect electrical contact are lowered in resistance by the creation of an electric spark near them, other descriptions of contact are caused to increase in resistance. This change is the result of an electric wave created by the spark. The alteration, whether to greater or less conductivity, is essentially due to the creation of an electric force at the imperfect contact. The change has, however, to be detected in some manner by, so to

[21] See *Proc. Royal Institution*, 1897, vol. xv. p. 461, or *The Electrician*, vol. 39, p. 217.

speak, testing the contact or contacts before and after the passage of the wave. We do this by applying to the imperfect contact a small external electromotive force in conjunction with some means for detecting an electric current. This external electromotive force must not exceed a certain small value, generally a fraction of a volt, or at most 1·5 to 2 volts, or else it will itself produce the change in conductivity without any assistance from a passing electric wave.

There are three methods by which we may explore and reveal the change in conductivity (if any) which has been produced in the sensitive conductor consisting of a loose or microphonic contact or contacts of some kind.

We may simply connect the coherer in series with a single cell, or with a shunted cell, and a detector, such as a galvanometer, or other direct telegraphic recorder, such as a syphon recorder, as used in submarine telegraphy. In the next place, we may, as already described, connect the sensitive device in series with a single cell and a delicate telegraphic relay. Such a relay should be one which operates with a current of not more than 1 milliampere, and, better still, with one-tenth of a milliampere. This relay can then be made to close the circuit of any form of ordinary printing or recording telegraphic instrument, such as a Morse inker or printer, which works with a single current. This plan is generally called the telegraphic receiver method.

In the third place, we may employ an ordinary magnetic or Bell telephone in series with the sensitive device, coherer, or contact detector, a single voltaic cell and a high resistance, or else a shunted cell, being placed in a circuit. Then, when the sensitive device suddenly changes in conductivity, it either increases or diminishes suddenly the current through the circuit, and a *click* is heard in the telephone by a listener. Generally speaking, this last method, called the telephonic method of receiving, enables us to estimate smaller changes in the conductivity of the sensitive contact than the method with the relay. Each of these methods has, however, its own peculiar advantages. The direct or galvanometric method appeals to the eye, and is suitable for lecture or demonstration purposes. The telephonic method appeals to the ear, and is the most delicate. The method with the relay enables a permanent record of the signals or changes to be obtained on a Morse tape or in printed signs, and has much to recommend it. On the other hand, the numerous adjustments require more skill when a recording instrument is used, and hence, for the great bulk of ordinary radiotelegraphic messages the telephonic method is now exclusively employed. The particular arrangements employed in the receiving circuit will be described in a later chapter, but as regards the appliances in use in the generality of cases they comprise the following elements.

(i.) Some source of electromotive force, viz. a voltaic cell or cells, which may be shunted by a resistance if required.

(ii.) Some device for indicating the presence or change in strength of an electric current, as, for instance, a galvanometer, telephone, or telegraphic relay or recorder.

(iii.) Some appliance for creating a variation in the strength of the above current, which is set in operation by electric oscillations, so that it either varies the resistance of the detector circuit, or rectifies the electric oscillations, or creates electromotive force in that circuit. These appliances collectively form the receiving circuit, and it is connected either directly or inductively with an *antenna* or wire in which the incident electric waves set up electric oscillations.

5. Various Forms of Coherer and Materials for their Construction.—The simplest form of contact detector is the crossed needle or single contact, originally described by Branly.[22] Lodge also found that a steel sewing-needle having its point lightly pressed against an aluminium plate made a fairly regular coherer.[23] When an electric wave passes over it, the point is welded to the plate, and the loose or imperfect contact becomes a good one, and will pass a current from a

[22] See E. Branly, "Variation of Electric Conductivity under Electrical Influence," *The Electrician*, vol. 27, p. 222.

[23] See O. J. Lodge, "The History of the Coherer Principle," *The Electrician*, vol. 40, p. 90.

single cell through a galvanometer. It may here be noted that the passage of the current is not necessary to create the coherence, it merely reveals it.

Branly found in 1891 that if a pair of slightly oxidized copper wires rest across one another, the contact resistance, when no pressure is applied, is very high (8000 ohms or so). It falls, however, to a few ohms (6 or 7) on the impact of an electric wave.

The objection, however, to a single contact from the point of view of telegraphy is the small detecting current which can be passed through the junction without damaging the sensitiveness of the contacts by welding them together too much. In this case it requires a considerable shock to effect severance, and the junction becomes less sensitive to electric waves. Subsequently, however, Branly found that a series of metal balls in light contact formed a good coherer.[24] He tried using small balls of soft iron placed in a glass tube. Thus ten balls showed an initial resistance of 990 ohms, which dropped to 60 ohms on passing a 1·5-mm. spark at a distance of 10 metres. He found hard steel balls still better, varying in contact resistance from 2000 to ·100 ohms on passing a spark. A coherer of six hard steel balls, such as are used for bicycle bearings, is about as sensitive as a gold filing coherer, but to work well the decohering shock must be carefully regulated.

Subsequently he devised a tripod coherer, consisting of a small metallic stool with three slightly oxidized legs.[25] This tripod stands on a polished steel plate, and arrangements are made to decohere it by tilting the little stool by an electromagnet when the legs become cohered to the plate under the action of a wave. The ends of the legs must not be too heavily oxidized, and the stool must be very light in weight, so as to make bad contact with the plate until the impact of the wave improves it (see Fig. 9).

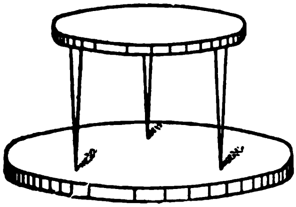


FIG. 9.—Branly's Tripod Coherer.

In the case of the ball coherer, variation of sensibility may be made by tilting the glass tube containing the balls to various inclinations, so as to make the balls press more or less heavily on each other.

In the case of metallic filings coherers, a variation in sensibility may be obtained by bevelling off the electrodes obliquely so as to make the gap wedge-shaped, as in the Marconi coherer.[26] The present form of Marconi telegraphic coherer is attached to a bone holder, consisting of a round bone rod with squared end, to which the coherer is lashed by silk threads. By taking hold of the glass "tail" of the coherer, or sealed-off glass end by which the coherer is exhausted, it can be turned on its axis into various positions, so that the filings lie in a broader or narrower portion of the bevelled gap between the silver plugs. The sensibility is thereby altered within certain limits (see Fig. 10). Other ways of adjusting the sensibility of filings coherers have been devised. Thus M. Blondel constructed a coherer with a side tube or pocket (see Fig. 11) in which was placed a reserve of filings, and by shaking more or less of these into the gap between the electrodes the desired variation could be produced.[27] A somewhat similar arrangement was tried by Lodge.

As regards materials for the construction of coherers, it seems to be generally agreed that the non-oxidizable or noble metals, such as gold, platinum, and silver, taken alone, are not well suited for making telegraphic cymoscopes. All metals

[24] See E. Branly, "Ball Coherer," *Comptes Rendus*, 1899, vol. 128, p. 1089, or *Science Abstracts*, vol. ii. No. 1164.

[25] See Prof. E. Branly, "A Sensitive Coherer," *Comptes Rendus*, 1902, vol. 134, p. 1197, or *Science Abstracts*, 1902, vol. 5, p. 852.

[26] This device was very early employed by Marconi, but it has been patented again and again by various inventors. See German Patent, No. 116,113, Class 21a, 1900. It has also been claimed by M. Tissot.

[27] See a letter from M. A. Blondel, *The Electrician*, 1899, vol. 43, p. 277.

in the state of filings or fine borings exhibit the phenomena of coherence, but the non-oxidizable metals are too sensitive. On the other hand, the very oxidizable metals are too insensitive, and in some cases, such as potassium and arsenic, Professor J. C. Bose found a reversal of effect, viz. that under the action of electric waves the constant resistance between particles of these metals was increased, and not diminished, by the impact of an electric wave.

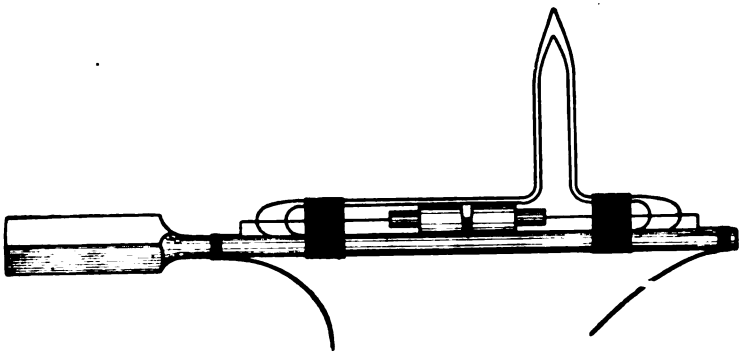
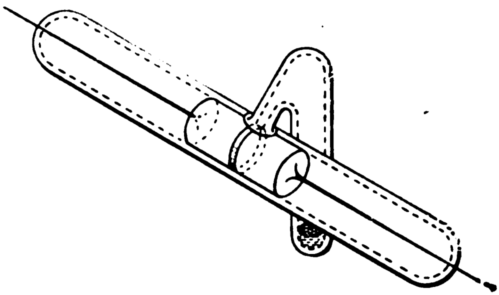


FIG. 10.—Marconi Sensitive Metallic Filings Tube or Cymoscope supported on a Bone Holder. (*Full size.*)

It is a curious fact that the magnetic metals, nickel, iron, cobalt, in the order named, give better results than non-magnetic metals whether used as filings in a tube or as ball or rod coherers. The best results are obtained when employing in a metallic filings tube a mixture of filings of a magnetic and a noble or unoxidizable metal; for example, a mixture of iron, nickel, or cobalt filings with a small percentage of silver, gold, or platinum filings, the mixture of nickel and silver giving, as Marconi has shown, a particularly good result.



[*From "The Electrical Review.*

FIG. 11.—Blondel Side-pocket Metallic Filings Coherer.

The use of carbon for the construction of coherers has been much discussed. A British Patent Specification of Messrs. A. C. Brown and G. R. Neilson, No. 28,955 of December 17, 1896, one of the earliest taken out for improvements in electric wave telegraphy, mentions the employment of carbon granules or powder in a coherer in series with a telephone and a voltaic cell as a means of detecting electric waves.

Some persons at one time declared that carbon could not be used in a Branly

tube as a Hertzian wave receiver, and Branly himself makes no mention of it. Sir Oliver Lodge, in his book on "The Work of Hertz," mentions in a footnote that Professor Fitzgerald had succeeded in employing carbon, but no details of the experiments are given.

Mr. F. J. Jervis-Smith showed in 1897 that powdered carbon could be used in place of metallic filings in a coherer tube and found it very sensitive. He employed graphitic carbon in a glass tube with pointed metallic electrodes, which could be screwed more or less into the carbon powder.[28] He stated that placing the carbon powder in a vacuum does not improve it.

On the other hand, an iron or nickel filings coherer will not work well for long unless the filings are in a fairly good vacuum. The particles probably become too much oxidized on the surface.

In the latter part of 1897, Mr. F. J. Jervis-Smith, following Brown and Neilson, employed a combination of carbon coherer and telephone as a detector of electric waves. In 1898 he described the arrangement as follows:—

"A Hertz resonator is usually adjusted by altering the length of the two conductors on either side of the spark gap till the best results are obtained; this alteration of length has been effected by cutting off portions of the conductors, and observing the length of the spark in the gap in a dark room. Adjustment by means of this method is by no means easy, and when the primary oscillations are feeble, it is difficult to accomplish. In the new form of resonator, two ribbons of copper foil or a flexible metallic conductor of equal length are symmetrically coiled on to two cylinders, geared together by means of non-conducting cog-wheels; the cylinders are carried on insulated bearings, and the ribbons are kept in a state of tension by means of two weights attached to silk cords running over two pulleys; the length of the ribbons is regulated by the milled head."

These ribbons form extensions of metallic caps or ends to a small tube, which is filled with powdered carbon, in form of grains.

"This carbon detector also forms part of a circuit, including a telephone, and a battery and an adjustable resistance. To adjust the resonator, it is placed so that it may be influenced by an oscillator or radiator of Hertz waves, the milled head is slowly turned until a clear, sharp click is heard by means of the telephone. Unlike the metal filings detectors of Branly, this powdered carbon detector allows a very minute current to flow continuously through the telephone circuit, but when the carbon is subjected to a Hertz wave, a click is heard in the telephone."

T. Tommasina also constructed in 1899 a carbon coherer with carbon particles, which he says was as sensitive as a metallic filings coherer, and required less mechanical shock to decohere it. One efficient form consisted of two arc lamp carbons placed in a glass tube held by rubber stoppers, so that the ends were in light contact. He found this form of carbon coherer extremely sensitive and not easily put out of order.[29] Mercury has also been used with great success in making coherers. T. Tommasina in 1899 made a coherer with a drop of mercury contained between two brass electrodes in a glass tube.[30] He does not say, however, whether it required tapping to make it decohere. It is a peculiar and valuable property of carbon in certain forms in combination with mercury and iron that it enables us to construct coherers which are self-restoring and return spontaneously and immediately to a high resistance condition after the impact of an electric wave.

As already mentioned, Professor D. E. Hughes in 1879 found that a carbon-steel microphone was sensitive to electric sparks at a distance, and he subsequently stated that he had at that time found it to be self-restoring. T. Tommasina discovered that a certain variety of graphitic carbon used in the microphones of certain Swiss telephones had the same property.[31] An interesting form of self-

28 See F. J. Jervis-Smith, *The Electrician*, 1897, vol. 40, p. 85, or *Science Abstracts*, vol. i. No. 166. Mr. A. A. C. Swinton (see *The Electrician*, 1897, vol. 40, p. 133) had previously noticed the reduction in resistance of a carbon tube filled with carbon granules by a neighbouring electric spark.

29 See *Comptes Rendus*, 1899, vol. 128, p. 666; also *Science Abstracts*, vol. ii. No. 1023.

30 See *Comptes Rendus*, 1899, vol. 128, p. 1092; or *Science Abstracts*, vol. ii. p. 521.

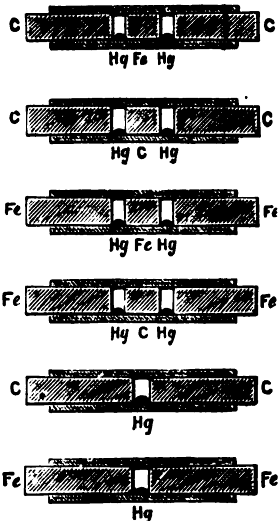
31 See T. Tommasina, *Comptes Rendus*, 1900, vol. 130, p. 904.

restoring cymoscope is one the invention of which has been attributed by Captain Quintino Bonomo to P. Castelli, a signalman in the Italian Navy.[32] This coherer has been made in many forms, several varieties being described by Captain Bonomo in an official Report, published by him in 1902, in which he gave an account of work done in wireless telegraphy for the Italian Ministry of Marine between September 1900 and May 1901. In a glass tube of 3 mms. internal diameter are placed electrodes or rods of iron or carbon, fitting the tube closely, the ends of the iron or steel rods being well polished. If carbon rods are employed, it should be in the form used for arc lamp carbons with smooth ends. These rods nearly meet in the centre of the tube, and a drop of clean mercury is placed between them (see Fig. 12, Diagrams 1, 5, or 6). Alternatively there may be two drops of mercury with a short block of iron or carbon interposed between them (see Fig. 12, Diagrams 2, 3, or 4).[33] The size of the drop of mercury should be between 1·5 and 3 mms. in diameter. If less than 1·5 mm. the coherer is insensitive, if larger than 3 mms. it is not sharp in action. The distance between the electrodes of iron or carbon must be carefully adjusted. This is done by inclining the tube to 35° or 40° to the horizon, and displacing the upper electrode until there is a little space of 0·2 to 0·5 mm. between the mercury drop and the end of the upper electrode. The adjustment will then be correct when the tube is in a horizontal position.

The tube so prepared is placed in series with a single voltaic cell and a telephone, a resistance being added if necessary (see Fig. 13). When an electric wave falls on the tube, it causes a sudden decrease in the resistance between the electrodes, and a sharp click is heard in the telephone. The tube, however, if properly adjusted, returns immediately to its original high resistance. Hence, if waves continue to arrive, the sound in the telephone is almost continuous.

In a slightly modified form, with one fixed plug or electrode of carbon, the other being an adjustable one of iron, and a globule of mercury included between (see Fig. 14) the inner ends, the arrangement was claimed in 1902 as the invention of the Marquis Luigi Solari of the Italian Navy, and denominated the Italian Navy Coherer.[34]

The hygrometric state of the air is said to affect these tubes unfavourably if they are not sealed.



From "The Electrical Review."

FIG. 12.—Various Forms of Castelli Coherer.

6. Restoration of Coherers to the Sensitive Condition.—The use of a metallic filings coherer made with most ordinary metals necessitates also the employment of some means for tapping the tube to restore it to the sensitive condition after the coherence has been produced. These mechanical shocks must be capable of very nice adjustment. In telegraphic work, the possibility of sending and receiving a *dash* signal as well as a *dot* signal on the Morse alphabetic code is essentially dependent upon this delicate setting of the tapper to administer a series of blows of just the right strength. The original clockwork tapper of Lodge is not sufficiently adjustable. The arrangements of Marconi, though admirable,

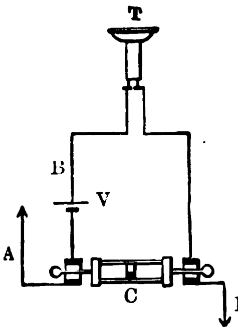
[32] See "Telegrafia Senza Fili," Rome, 1902; or *L'Elettricista*, ser. ii. vol. i. pp. 118, 173.

[33] See *The Electrical Review*, 1902, vol. 51, p. 968.

[34] See a letter to *The Times* of July 3, 1902, by the Marquis Luigi Solari, claiming this invention; also see a Royal Institution Friday Evening Discourse by Senator G. Marconi, June 13, 1902, reported in *The Electrician*, 1902, vol. 49, p. 490; also British Patent Specification, No. 18,105 of September 1901, amended July 16, 1902, granted to G. Marconi, communicated by the Marchese Luigi Solari, for "Improvements in Coherers or Detectors for Electrical Waves."

require some dexterity to manage them. Inventors have therefore sought for simpler means of effecting the decoherence of the filings or surfaces, and also for forms of contact cymoscope which should be self-decohering or continually self-restoring to a sensitive condition.

H. Rupp found that rotating the metallic filings tube continually but slowly was sufficient to keep the filings in a sensitive condition.[35] T. Tommasina discovered that, when using coherers made with filings of magnetic metals, decoherence could be effected by a magnet placed a little distance above the tube. He accordingly fixed an electromagnet above a nickel, iron, or cobalt filings coherer, and caused the action of the electric wave on the tube to close the circuit of a single cell through the coherer and a relay. The relay in turn closed another cell circuit through the electromagnet, and so effected the decoherence. He says the arrangement worked perfectly.[36]



From "The Electrical Review."

FIG. 13.—Mode of Employing a Mercury-iron-carbon Detector with a Telephone, T, and Auxiliary Cell, V, as an Electric Wave Detector.

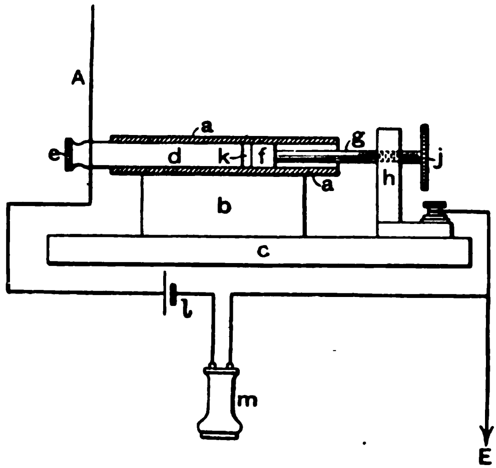


FIG. 14.—Italian Navy or Solari Coherer. A, antenna; E, earth connection; *d*, carbon plug; *f*, iron plug; *m*, telephone; *k*, mercury globule; *g*, adjusting screw; *l*, voltaic cell.

The explanation of this action seems to be that the chains of filings, which Tommasina contends are formed under the action of the waves, are torn apart.

Other inventors have attached the coherer to the armature of an electromagnet, the circuit of which included a voltaic cell, and was closed through a relay by the current sent from a separate cell through the coherer when the latter became conductive. The play of the armature of the electromagnet can be limited by screw stops. The author has used for many years a device of this kind in a lecture apparatus, which was most easy to adjust and efficient in action. The form of coherer used was one suggested by the author some time ago.[37] It consisted of two L-shaped pieces of silver, *l*, *l*, which were bound on either side of a thin slip of ivory or fibre, *d*, in which a U-shaped gap was cut. This formed a small box, not more than 2 or 3 mms. wide, with metallic sides. In this box is placed

[35] See H. Rupp, *Elektrotechnische Zeitschrift*, April 14, 1898; or *Electrical Review*, 1890, vol. 42, p. 535.

[36] See T. Tommasina, *Comptes Rendus*, 1899, vol. 128, p. 225; or *Science Abstracts*, vol. ii. No. 1166.

[37] See *Journal of the Institution of Electrical Engineers*, 1899, vol. 28, p. 292, remarks by J. A. Fleming in "Discussion on Mr. Marconi's Paper on Wireless Telegraphy."

a very small quantity of freshly made nickel filings, and the box is closed by a wooden wedge (see Fig. 15). This coherer is attached to the vibrating armature of an electromagnet, E, made like an electric bell, except that two screw stops limited the play of the armature. The coherer is placed in series with a single dry cell and a relay which closes the circuit of another cell through the electromagnet above mentioned, and also closes another circuit of a large electric bell or Morse printing telegraphic instrument. When an electric wave falls on the coherer, the vibrating armature of the electromagnet gives one or two quick motions, and shakes the filings coherer back to a non-conductive condition.

7. **Self-Restoring Contact Detectors.**—The disadvantage of all these arrangements from a telegraphic point of view is that the train of mechanism necessary to administer the shock to the coherer has so much mechanical and electrical inertia, and hence is limited in speed. Also, generally speaking, more than one blow must be applied. A series of light taps is more effective than one violent blow. All this means time, and therefore loss of speed in receiving signals.

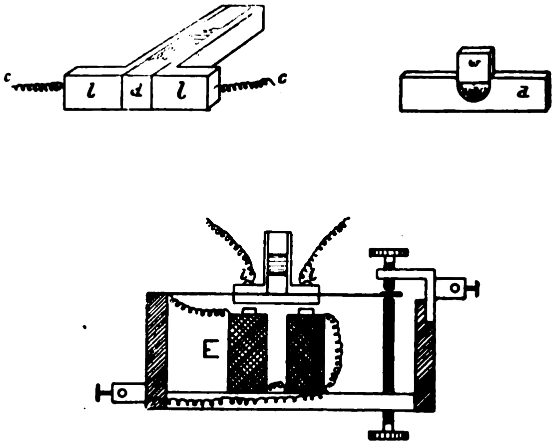


FIG. 15.—Fleming Coherer carried on the Armature of an Electromagnet.

Amongst other methods which have been tried is one due to Mr. S. G. Brown.[38] The pole pieces of the coherer tube are made of iron, and enveloped in magnetizing coils traversed by an alternating current. Between these pole pieces a small quantity of nickel or iron filings is placed, and under the action of an electric wave these loose filings cohere. The moment the wave ceases the alternating magnetism causes the filings to fall apart. Mr. Brown found that revolving a permanent magnet near an ordinary nickel or iron filings coherer tube had the same effect.

Of all these substitutes for tapping one of the most effective and simple is that due to Sir Oliver Lodge, Dr. Muirhead, and Mr. Robinson.[39] In this arrangement a steel disc is caused to revolve slowly by clockwork. The disc just touches a globule of mercury, the surface of which is covered with a layer of paraffin oil (see Fig. 16). Under these conditions there is no good electric contact between the steel disc and the mercury. If a fraction of a volt difference of potential (0·3 of a volt or less) is created, by the use of a shunted voltaic cell between the steel and the mercury, then when an electric wave falls on this coherer the film of oil

[38] See S. G. Brown, British Patent Specification, No. 19,710 of 1899.

[39] See Sir Oliver Lodge, "A New Form of Self-restoring Coherer," *Proc. Roy. Soc. Lond.*, 1903, vol. 71, p. 402; also British Patent of Lodge, Muirhead, and Robinson, No. 13,521 of June 14, 1902.

is perforated, and a current passes which is sufficient to work a syphon recorder placed in series with the cymoscope without the interposition of any relay. The rotation of the steel disc continually restores the cymoscope to a sensitive condition. The edge of the disc is continually kept clean by a pad of felt or leather.

An ingenious form of combined telephone and coherer has been designed by T. Tommasina. In this instrument the diaphragm of a Bell telephone carries on it a small carbon or metallic filings coherer, which is also in series with a bell and a relay. When the coherer becomes conductive it closes the circuit of the relay, and the latter in turn closes the circuit of another cell in series with the telephone coil. The jerk given to the vibrating diaphragm of the telephone resets the coherer. The arrangement works with more precision if the coherer contains iron or nickel filings, as then the magnetization of the telephone core assists the decoherence.

All these arrangements, however, in which a sensitive relay is employed, involve continued adjustment and some considerable dexterity to obtain the best results. Hence of late years practice has tended in the direction of receiving arrangements which do not involve the use of an electromagnetic relay with a coherer, as this last method requires the employment of two sets of cells and a number of minute adjustments to secure uniformly good results.

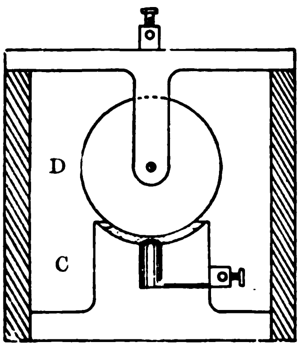


FIG. 16.—Lodge-Muirhead-Robinson Rotating Steel Disc Mercury Coherer. D, steel disc; C, cup containing mercury covered with oil.

Owing to the preference now given to methods of aural reception, many efforts have been made to find simple forms of contact detector capable of being used with a telephone which requires no tapping, rotating, or other operations, but restore themselves immediately to their original state of resistance after the action of electric oscillations upon them. An example of this type is the tantalum detector of Mr. L. H. Walter.[40]

The construction of this detector is as follows: A platinum wire is sealed through the wall of a glass bulb and dips into a pool of mercury contained in it. A second platinum wire, also sealed through the glass, carries at its end a short length of tantalum wire which is part of a filament of a tantalum incandescent lamp. This wire is only 0·05 mm. in diameter, and just touches the mercury surface (see Fig. 17). The detector is used by joining the terminals of the tantalum and mercury in series with a telephone, and with a potentiometer arrangement consisting of a shunted cell which applies a fraction of a volt 0·2 to 0·4 electromotive force in the circuit, so that the tantalum point is negative. The terminals of the detector are then also connected to the circuit in which oscillations are being set up. The arrangement employed in radiotelegraphy as a receiver with this detector is shown in Fig. 18, in which A is the antenna, S, P the oscillation transformer, C, C condensers. The tantalum detector has its electrodes joined across the terminals of the condenser. An electromotive force of about 0·3 of a volt is applied by means of a shunted voltaic cell B, and the battery circuit includes a telephone T.

The form of detector just described, while serving very well for use in fixed stations where a firm support can be obtained, is not so satisfactory when the detector is liable to be subjected to shaking or mechanical shocks during the reception of messages. Various devices have been tried without success, but one satisfactory solution is arrived at by constructing the detector in the following manner:—The tantalum wire is fastened in a platinum clip and the end of the

[40] See L. H. Walter, "On a Tantalum Wave Detector and its Application in Wireless Telegraphy and Telephony," *Proc. Roy. Soc. Lond.*, vol. 81, A, p. 1, 1908.

tantalum encased in glass by a special method, necessitated by the impossibility of sealing in tantalum in the ordinary way as is done with platinum. The platinum wire is sealed into a minute glass bulb, B (see Fig. 19), blown on one end of a glass

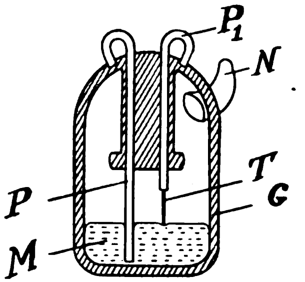


FIG. 17.—Walter's Tantalum Detector. *P*, platinum wire; *T*, tantalum wire; *M*, mercury; *G*, glass vessel.

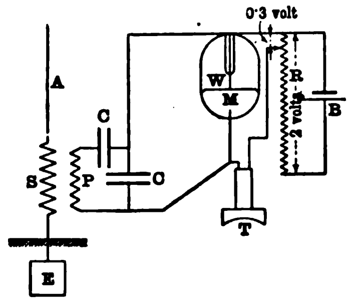


FIG. 18.—Arrangement of Circuits employed in using the Walter Tantalum Detector as a Cymoscope in Radiotelegraphy.

tube; the other end of the tube is connected to an air pump and the interior exhausted. The glass tube is next heated, when the vacuum causes it to collapse on to the tantalum wire. The end of the glass-sheathed wire can then be ground down so that the tantalum surface is just flush with the glass (simply breaking off the glass end usually suffices). The mercury is contained in a glass tube, G, having a bore of $\frac{6}{32}$ inch. A larger tube would be better, but the sensitiveness to shaking then reappears; a smaller tube gives a less sensitive and more variable detector. An ivory plug, I, through which a platinum or nickel wire passes and projects, is placed at one end of a length of a few inches of such glass tube with thick walls. A few drops of mercury—enough to form a pellet, M, about $\frac{1}{20}$ inch long—are then put in and a second ivory plug, I_1, this one with the sheathed tantalum wire passing through it and projecting about $\frac{1}{20}$ inch, inserted so that the tantalum glass surface just dips into or under the mercury surface. The best (most sensitive) position is that shown in Fig. 18, with the glass tube vertical and the tantalum electrode at the top, and this gives a detector which may be roughly shaken or tapped during the reception of signals, without affecting their sound in any way. For sealing up, the whole arrangement is encased in an ebonite tube, E, and the ends filled in with insulating compound. The device is then permanent, though experience (time) is wanted to decide whether it is as inalterable as the first form.

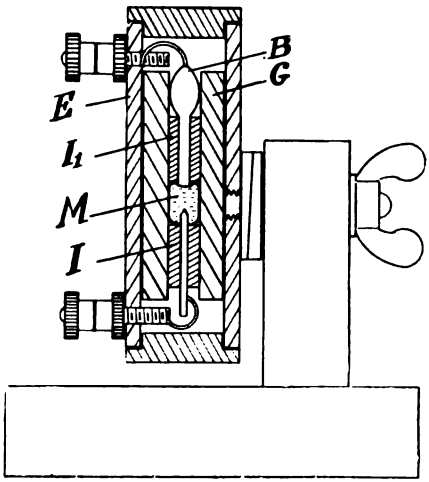


FIG. 19.—Walter's Adjustable Tantalum Detector.

[*Figs.* 17 *and* 19 *are reproduced from the "Proceedings of the Royal Society" by permission of the Author and the Society.*]

8. Theories of Coherer Action.—At this point it is desirable to consider some of the theories which have been put forward to account for the phenomena of coherence under the impact of electromagnetic waves.

At an early stage Lodge advanced the opinion that the metallic surfaces in light contact were welded together, and his expressions have been interpreted to mean that he took the view that the action was in part, at least, a thermal one. The surfaces in loose contact being at different potentials, were assumed to be drawn together and then autogenously welded or caused to cohere, like clean surfaces of lead when strongly pressed together. Many observers have asserted that this process may be witnessed, through the microscope, actually taking place. Thus D. van Gulik observed through the microscope the action of electric waves upon the rounded ends of the two platinum wires almost in contact, and saw them drawn together when the wave fell on them. Also in another experiment he used mercury drops separated by a thin surface contamination of chalk, and said that he saw them coalesce under the action of radiation. Other observers assert that they have seen under the microscope minute sparks passing between the filings in a filings coherer when a wave was allowed to act on it. These observations may be correct as far as they go, but it is clear that very powerful radiations must have been employed, far in excess of that necessary to produce the true coherer phenomena.

Other physicists have examined particularly the filings coherer. T. Sundorph states that under the action of electric radiation chains of conducting particles are formed, and that the process may be examined by laying some iron filings on a glass plate between the ends of two rods. Then, on making electric sparks in proximity, some of the filings cling together and form connected chains from rod to rod. The loose or disconnected filings may be removed by a feeble magnet and the chains exposed to view.[41]

T. Tommasina supports this opinion, and says that these chains of particles stretching between the electrodes are more easily formed when the surrounding medium is distilled water or some dielectric other than air.[42]

R. Malagoli, in referring to Tommasina's assertions, states that this process of the creation of chains of conducting particles between the electrodes can be witnessed in the case of brass filings placed between two plates of metal immersed in vaseline oil, when a difference of potential is made between the plates.[43]

In some of these experiments, however, considerable potential differences must have been employed. The actual electric or electromotive forces which come into play under the action of electric waves are very small, and experiments such as the above do not necessarily explain the real coherer effect.

E. Aschkinass very properly observes, moreover, that any theory of the coherer must take into account not only the coherence and increased conductivity of the magnetic and ordinary metals, but the decreased adherence and reduced conductivity which takes place between such substances as peroxide of lead, arsenic, potassium, etc. In other words, no theory of the coherer can be complete which does not include an explanation of the two kinds of effect on imperfect contacts discovered by Branly, which can be produced by an electric wave.[44]

Furthermore, we have to take into account that highly oxidized particles of metal operate in many cases as effectively in making a coherer as perfectly clean surfaces. Welding, in the ordinary sense of the word, cannot then occur.

The welding theory also fails to account for the power of carbon granules to form a good coherer, since carbon cannot be welded at any such temperature as can then exist at the points of contact of the carbon particles ; and, moreover, the coherence is not permanent.

On the general subject of the sensitiveness of loose aggregations of metal filings to electric waves, the researches of Professor J. C. Bose are of particular

[41] See *Wied. Annalen*, 1899, vol. 68, p. 594 ; or *Science Abstracts*, vol. ii. No. 1717.
[42] See *Comptes Rendus*, 1899, vol. 129, p. 40 ; or *Science Abstracts*, vol. ii. No. 1718, 1899.
[43] See *Il Nuovo Cimento*, 1889, vol. 10, p. 979.
[44] See E. Aschkinass, "On the Coherer," *Wied. Ann. der Physik*, 1898, vol, 66, p. 284.

interest.[45] He states that the sensitiveness of any form of contact cymoscope consisting of conducting particles depends upon the proper adjustment of the pressure between the particles and the value of the external electromotive force which is in waiting, so to speak, to send or increase the current through the contacts.

Bose discovered other substances which, like the peroxide of lead mentioned by Branly, exhibit the phenomenon of decreased coherence under the action of electric waves, such as metallic arsenic, potassium in petroleum oil, and some forms of silver. The best instance, however, of this so-called anti-coherer action is a light contact between a mass of compressed peroxide of lead and metallic lead.

S. G. Brown has made a so-called automatic *anti-coherer* by pressing lightly a lead point against a surface of lead which has been peroxidized. The electrical resistance of this contact becomes greater, and not less, under the action of electric waves, and it is also self-restoring. The fact can easily be shown as a lecture experiment.

Any theory of the action of electric waves or loose or imperfect metallic contacts, to be complete, must embrace both the cases of coherence and those of increased resistance at the junction. It must explain why a copper wire lightly touching an oxidized copper wire experiences a great reduction in resistance at the contact when an electric wave falls upon it, whilst a lead wire lightly touching a peroxidized lead wire exhibits under the same conditions an increase in resistance at the contact.

Branly had recourse, therefore, to a theory in which the dielectric between the particles is considered to play an important part. He calls all the substances which when in light contact have their conductivity altered one way or the other by an electric wave, *radio-conductors.*

Guthe, however, gives reasons, derived from the experiments by himself and others, for doubting whether the interposed dielectric has the functions ascribed to it by Branly.[46]

J. C. Bose, after extensive examination of the coherer phenomena, divided all substances into positive and negative, the first and largest class reducing their resistance, and the second and smallest class increasing their resistance. He used the term *electric touch* to signify sensitiveness to electric radiation, and concluded that the effect of radiation was to produce a molecular change or allotropic modification of the substance acted upon, so that a positive substance becomes less positive, and a negative less negative; in some cases reversal taking place.[47] These descriptions, terms, and hypotheses have not, however, much increased our real insight into the matter.

It has been asserted that for every particular Branly tube there is a critical electromotive force in the neighbourhood of two or three volts, which causes the tube to break down and pass instantly from a non-conductor to a conductive condition, and that this critical electromotive force may become a measure of the utility of the tube for telegraphic purposes. Thus C. Kinsley (*Physical Review*, 1901, vol. 12, p. 177) made measurements of this supposed critical potential for different "coherers," and subsequently tested the same as receivers of a wireless telegraph station of the U.S.A. Signal Corps. The average of twenty-four experiments gave in one case 2·2 volts at the breaking down potential of one of these coherers or Branly tubes, 3·8 volts for a second, and 5·5 volts for the third. These same instruments tested as telegraphic cymoscopes showed that the first of the three was most sensitive.

On the other hand, W. H. Eccles (*Electrician*, 1901, vol. 47, pp. 682, 715) conducted some very instructive experiments with Marconi nickel-silver sensitive tubes, using a liquid potentiometer made with copper sulphate to apply the

[45] See J. C. Bose, *Proc. Roy. Soc. Lond.*, 1899, vol. 65, p. 166; or *Science Abstracts*, vol. ii. No. 1716.

[46] See K. E. Guthe, "Coherer Action," paper read September 1904 before the St. Louis International Electrical Congress; see also *The Electrician*, 1904, vol. 54, p. 92.

[47] See J. C. Bose, *Proc. Roy. Soc. Lond.*, 1900, vol. 66, p. 450.

potential, so that the infinitesimal spark at the sliding contacts might be avoided, and the changes in potential made without any abruptness. He states that if the coherer tube is continually tapped at the rate of fifty vibrations per second, whilst at the same time an increase in potential is applied to its terminals, and the current passing through it measured on a galvanometer, there is no abrupt change in current at any point. He found that when the current and voltage were plotted against one another a regular curve was obtained, which after a time becomes linear. A decided change occurs in the conductivity of the mass of metallic filings when treated in this manner at voltages lower than the critical voltages obtained by previous methods. He ascertained that there was a complete correspondence between the sensitiveness of the tubes used as telegraphic instruments, and the form of the characteristic curve of current and voltage drawn by the above-described method.

In the same manner, K. Guthe and A. Trowbridge (*Physical Review*, 1900, vol. 2, p. 22) investigated the action of a simple ball coherer formed of half a dozen steel, lead, or phosphor-bronze balls in slight contact. They measured the current i passing through the series under the action of a difference of potential, v, between the ends, and found a relation which could be expressed in the form—

$$v = V(1 - \epsilon^{-ki})$$

where v and k are constants.

The current through this ball coherer is, therefore, a logarithmic function of the potential difference between its ends of the form—

$$i = \frac{1}{k} \log \left(\frac{V}{V - v} \right)$$

and exhibits no discontinuity. The inference was drawn that the "resistance" is due to films of water adhering to the metallic particle, through which electrolytic action occurs. On the whole, the theory of a critical potential is not upheld by the general facts.

It is clear, however, that the agency which actually causes coherence is electromotive force, and that the matter to be explained is the reason electromotive force, acting on an imperfect contact, brings into better conducting contact the surfaces of certain materials which are in light or imperfect contact which constitute part of the circuit, whilst in a few other instances it is made worse. Lodge has shown that two conductors separated by a film of air one ten-thousandth of a millimetre in thickness, and having a difference of potential of 1 volt, are drawn together by electrostatic attraction with a force of 44 atmospheres per square centimetre of contact surface. Hence this pressure would be sufficient to force out a film of gaseous dielectric between the surfaces, and bring them into closer contact. Applying to this fact the electron theory, K. E. Guthe[48] has expressed the opinion that the electrostatic pressure is sufficient to bring the surfaces into such contact that electrons can pass over from mass to mass, thus establishing a current through the discontinuous substance. This theory, however, gives no explanation how it comes to pass that there is coherence in some cases and decoherence in others. Even if we grant that the passage over the electrons from one surface to the other brings the opposed surfaces to such a potential difference that a practical welding of them takes place, we have yet to explain why there should be such a marked contrast between the behaviour of various substances, and why there should be such a difference between substances in the degree of mechanical shock necessary to rupture the contact thus formed.

On the whole, it cannot be said that our insight into the matter is very complete. Our knowledge of molecular processes is still far too imperfect to enable us to describe the actual atomic conditions at the surfaces of a loose contact between different pure or oxidized metallic masses.

The only facts which seem clear are that the phenomenon of coherence is essentially an electrical process, that it depends upon the creation of a small

[48] See paper on "Coherer Action," *loc. cit.*

difference of potential between the surfaces in light contact, and as such is not directly an effect of radiation *per se*, but merely of the electromotive force set up when electric waves are incident upon the conductors in light contact, or upon others connected with them.

A further study of these instances of *anti-coherence* or interruption of continuity is needed before we can possibly evolve a theory which will satisfactorily meet all the known facts concerning the effect of high frequency alternating electromotive force upon an imperfect or high resistance contact between substances of various kinds. It is possible that friction itself, generally speaking, is wholly an electric phenomenon.

There is a well-marked phenomenon of "fatigue" in the case of metallic filings coherers which also deserves mention and requires explanation. It has also been noticed that rise of temperature promotes or favours decoherence in the case of the positive class of radio-conductors.

It is clear that any theory of the operation of coherers must be in close touch with the theory of electric conduction generally. According to the electronic theory of electricity, the conduction of electricity in conductors is due to the motion of free corpuscles or electrons or so-called negative ions in them. In each conductor there is a certain number of these free ions in each unit of volume. It has been shown by Sir J. J. Thomson (see "Conduction of Electricity through Gases," p. 144) that an ion cannot fly off spontaneously and leave the conductor, since the moment it attempts to depart from the surface it is subjected to a mechanical force which is numerically equal to $\frac{e^2}{4d^2}$, where e is the ionic negative charge, viz. $4{\cdot}8 \times 10^{-10}$ electrostatic units, and d is the distance from the surface. Suppose, however, that two metal surfaces are very near together, and at a difference of potential of V volts, or $\frac{V}{300}$ electrostatic units. Let the distance between these surfaces be very small, and equal to x cms. Then the electric force in the interspace is $\frac{V}{300x}$ electrostatic units, and if this is comparable with or greater than $\frac{e^2}{4x^2}$, negative ions may be drawn out of one mass of metal and pass over to the other.

If, then, we have such a value of x and V that—

$$\frac{Ve}{300x} = \frac{e^2}{4x^2} \text{ or } x = 75\frac{e}{V}$$

this transference of ions can happen. Suppose that $V = 2$ volts, then the above equation is satisfied if $x = 1{\cdot}2 \times 10^{-8}$. This is a distance comparable with atomic diameters. If, then, two metallic surfaces are in *close* contact, the creation of a certain small difference of potential between them will result in the passage of negative ions from one to the other, and therefore in electric conduction. Moreover, this transference will increase the potential difference, and this will operate to draw the surfaces still closer by electrostatic attraction. The phenomena of coherence between loose or imperfect contacts between metals can thus be explained on the electronic hypothesis, since when subjected to the action of an electric wave small differences of potential are created between the conductors in loose contact, owing to the electromotive forces set up by the wave in these conductors, or others to which they are connected. Where very great differences in conductivity exist between the two surfaces in contact, the action may result in an accumulation of negative ions at the bounding surface in such number as to stop the flow of a current across the junction, and thus explain the decreased conductivity of a junction between such substances as lead and peroxide of lead when traversed by electric oscillations.

In connection with the theory of coherers the reader may be referred to an interesting paper by Dr. W. H. Eccles, in which he gives an account of experiments on coherers and a method of investigating them.[49]

[49] See *Proc. Phys. Soc. Lond.*, vol. 22, p. 289, 1910.

9. Magnetic Detectors.—It was well known long before the middle of the last century that the discharge of a Leyden jar had a magnetizing power. Sir Humphry Davy magnetized sewing-needles with Leyden jar discharges in 1821. Joseph Henry, in the United States, between 1842 and 1850, explored many of the puzzling facts connected with this subject, and only obtained a clue to the anomalies when he realized that the discharge of a condenser through a low resistance circuit is oscillatory in nature.[50] Amongst other things, Henry noticed the power of condenser discharges to induce secondary currents which could magnetize steel needles even when a great distance separated the primary and secondary circuits. He employed this magnetization to test the direction of the secondary currents, and he was followed in the same field of research by Abria, Marianini, Riess, and Matteucci.

In 1870 Lord Rayleigh, in discussing some electromagnetic phenomena, pointed out that the resultant magnetic effect of an oscillatory discharge depends upon the direction of the maximum value of the current during the oscillation, and also that there may be superimposed magnetic effects in the same needle.[51]

In 1895 the subject was again taken up by Professor E. Rutherford, and in a very able paper, published in 1896, he described experiments he had made on the subject.[52]

It is a familiar fact that if a soft iron bar is magnetized and then removed from the field, it preserves, in virtue of retentivity, its magnetization after the magnetizing force is withdrawn. Also it is known that if the iron is pure and annealed, its coercivity is small, that is to say, a very small mechanical shock or twist is sufficient to destroy its magnetization. Physicists were also aware that discharges of a Leyden jar passed through the iron could act like mechanical shocks and remove the feebly held residual magnetization.

Rutherford found that electric oscillations sent through a coil surrounding a very small steel or iron needle which had been magnetized to saturation could more or less demagnetize it, and after examining with care the best conditions, he made a detector for electric oscillations as follows :—

About twenty pieces of fine steel wire 0·007 cm. in diameter, each about 1 cm. long, and insulated from each other by shellac varnish, formed the detector needle used. A fine copper wire insulated with silk was wound directly over the needle in two layers, and making in all eighty turns. As the solenoid was of very small diameter, about 15 cms. of wire served to wind the coil. This small detector was fixed at the end of a glass tube, which was itself fixed on a wooden base, the terminals of the detector coil being brought out to mercury cups. To the ends of this solenoid were attached two long rods which served as electric wave collectors, and in which the oscillations were set up. The detector needle was strongly magnetized and then placed inside the oscillation coil, and a small magnetic needle with attached mirror set up near its end. The residual magnetism in the bundle of steel wires caused a deflection of the magnetometer needle. Some distance away a Hertz oscillator was set up, and when this was in action the oscillations created in the receiver rods caused a partial demagnetization of the steel detector needle, and a corresponding deflection of the magnetometer needle. These experiments were conducted at Cambridge (England) in 1895, and Rutherford found he was able to affect the above described detector when the Hertz oscillator was at a distance of half a mile away across the town. Rutherford employed this magnetic detector for examining many phenomena connected with electric oscillations, and in particular investigated by its aid the damping of electric oscillations as already described (see Chap. III. § 4).

[50] See "The Scientific Writings" of Joseph Henry, vol. i. pp. 203, 293; also *Proceedings of the American Assoc. for Advancement of Science*, 1850, vol. iv. pp. 377, 378, Joseph Henry, "On the Phenomena of the Leyden Jar." The effect of the oscillatory discharge on a magnetized needle is clearly described in this paper.

[51] See Lord Rayleigh (Hon. J. W. Strutt), "On Some Electromagnetic Phenomena," *Phil. Mag.*, ser. 4, vol. 38, p. 8; also *Phil. Mag.*, 1870, ser. 4, vol. 39, p. 431.

[52] Prof. E. Rutherford, "A Magnetic Detector of Electrical Waves and Some of its Applications," *Phil. Trans. Roy. Soc. Lond.*, 1897, vol. 189, A, p. 1; also *Proc. Roy. Soc. Lond.*, 1896, vol. 60, p. 184.

In 1897 Professor E. Wilson took up the subject and constructed a detector consisting of a bundle of fine steel wires, wound over with two helices of insulated wire, one to convey the electric oscillations and the other to carry a magnetizing current. His object was to be able to magnetize the detector by a battery current without removing it from its place, and he also patented an arrangement whereby the deflection of the magnetometer needle closed a circuit which remagnetized the detector needle and left it ready to detect another wave or oscillation.[53]

Success in these experiments depends upon attention to the details of construction of the detector needle. The steel wire used must be exceedingly thin. As the demagnetizing oscillations are very rapid, their magnetizing effect penetrates but a very little way into the mass of the metal, and therefore the proportion of the magnetism removed will be very small unless the wires are exceedingly thin. In

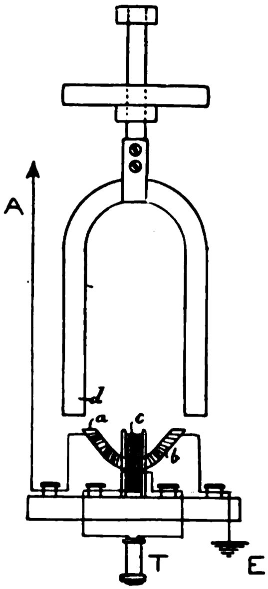


FIG. 20.—Marconi's Magnetic Cymoscope. (*First Form.*)

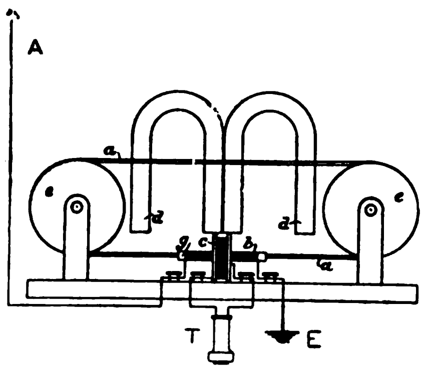


FIG. 21.—Marconi Magnetic Detector or Cymoscope. (*Second Form.*)

the next place, the bundle must be short, so that the self demagnetizing force is large, and under these conditions the residual magnetism is easily wiped out.

The effect observed is that due to the first oscillation, the magnetizing direction of which is such as to tend to annul the existing residual magnetization of the iron.

In 1902 Mr. Marconi described two other forms of magnetic cymoscope, one of which he has since used extensively for long-distance electric wave telegraphy.[54] These instruments depend upon the fact that when electric oscillations take place in a coil surrounding an iron wire which is placed in a varying magnetic field they change its magnetization. The first form of detector described by Marconi is as follows: On a rod or core consisting of thin iron wires are wound one or two layers of thin insulated copper wire. On this winding insulating material is placed, and over this again another longer winding of insulated copper wire. The inner core

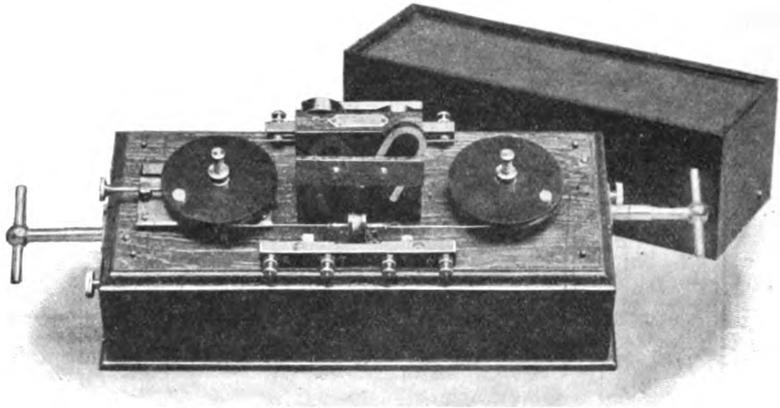
[53] See British Patent Specification, E. Wilson and C. J. Evans, No. 30,846 of 1897; also *The Electrician*, June 12, 1903, vol. 51, p. 330.

[54] See G. Marconi, "Note on Magnetic Detector of Electric Waves which can be Employed as a Receiver in Space Telegraphy," *Proc. Roy. Soc. Lond.*, 1902, vol. 70, p. 341; or British Patent Specification, No. 10,245 of 1902.

is traversed by the electrical oscillations, and when used as a telegraphic cymoscope is connected in between the aerial wire and the earth.

The other coil is connected to a telephone. Near the ends of the core is placed a horseshoe permanent magnet, which is made to rotate slowly by clockwork (see Fig. 20). If then the inner coil is traversed by a train of electrical oscillations, the magnetic state of the iron-wire bundle is suddenly altered, and a sudden click or sound is heard in the telephone. If trains of oscillations are sent for longer or shorter periods, these sounds in the telephone run together into a continuous sound, and long and short sounds may be arranged into a code of audible signals on the Morse system.

Marconi found that a better and more convenient plan was one in which the iron moves and the magnet remains fixed. In this second arrangement (see Fig. 21) there are two wooden discs, *e*, *e*, grooved on the edge, and these are driven round slowly by clockwork. An endless band, *a*, made of a bundle of fine silk-covered iron wire, is arranged like a belt over these wooden pulleys, and the



[*By permission from Marconi's Wireless Telegraph Co. Ltd.*

FIG 22.—Marconi Magnetic Detector. Double-coil type.

multifold iron band moves forward at the rate of 7 or 8 cms. per second. At one or more places the iron band passes through glass tubes, *g*, *b*. These are wound over with a coil of insulated wire, through which the electric oscillations pass, whilst over this is wound a longer coil of insulated wire, *c*, connected to a telephone, T. A pair of horseshoe magnets are placed with similar poles together opposite the last-mentioned coil.

When the band is driven forward the portion of the band nearly opposite to the magnet poles becomes magnetized, but, owing to magnetic retentivity or hysteresis, that portion, in virtue of the motion of the band, is shifted forward in the direction of rotation, and is not therefore situated symmetrically with respect to the poles. If an electric oscillation passes through the oscillation coil, it causes a sudden change in the magnetic state of that part of the iron band lying within it. This change, in turn, whether it be an increase or a decrease of magnetization, generates an induced current in the embracing coil connected to the telephone. This creates a sound in the telephone. The extreme sensitiveness of the telephone to induced currents bestows upon the whole apparatus a very great power of detecting feeble electrical oscillations. When used to detect electric waves, the oscillation coil is connected in between two aerial wires or between one aerial wire and the earth.

The sensitiveness of the instrument greatly depends upon the setting of the

magnets. Several demagnetizing coils may be used on the same band of iron, each overwound with a telephone coil, and these latter may be joined in either series or parallel.

Mr. Marconi states that this magnetic detector is more sensitive and certain in its action and much more easy to adjust than any coherer, and more suitable for use in syntonic telegraphy.

This magnetic detector has many practical advantages, as it does not require any local battery to actuate it, and is portable and easy to adjust. Its invention was not only a stroke of genius, but involves as well a very interesting scientific principle. A view of the most recent form of this Marconi magnetic detector is shown in Fig. 22.

Professor E. Wilson[55] also constructed a magnetic detector, consisting of a bundle of iron wires carried through a cycle of alternately reversed magnetism by a periodic electric current. On this bundle was also wound a coil, through which

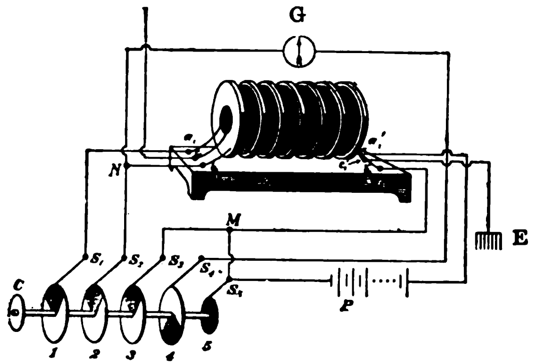
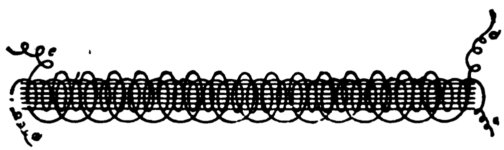


FIG. 23a.



[*From "The Electrical Magazine.*"

FIG. 23*b*.

Fleming Magnetic Cymoscope. Fig. 23*a*.—Bobbin, Cores, and Commutator. Fig. 23*b*.—A Single Iron Wire Core overwound with Magnetizing and Demagnetizing Solenoids.

the oscillations passed, and a third coil in series with a telephone. Owing to retentivity, the magnetic changes in the iron lag behind the magnetizing force. The action of the oscillatory field is to assist the magnetic changes when the magnetism is changing along the steep part of the cyclical curve, and this makes a change in the induction or flux linked with the secondary coil, and this, again, makes itself felt as a sound in the telephone.

The author has also devised a form of magnetic detector suitable for quantitative work, made as follows[56]:—

On a pasteboard tube, about 0·75 of an inch (18 mms.) in diameter and 5 or 6 inches long (15 cms.), are placed six bobbins of hard fibre, each of which contains about 6000 turns of No. 40 silk-covered copper wire (see Fig. 23*a*). These bobbins

[55] See Prof. E. Wilson, *British Association Report*, 1902; or *The Electrician*, vol. 49, p. 917' September 26, 1902; or British Patent Specification, No. 14,829 of 1902.

[56] See J. A. Fleming, "A Note on a Form of Magnetic Detector for Hertzian Waves adapted for Quantitative Work," *Proc. Roy. Soc. Lond.*, 1903, vol. 74, p. 398.

are joined in series, and form a well-insulated secondary coil, having a resistance of about 6000 ohms. In the interior of this tube are placed seven or eight small bundles of iron wire, each about 6 inches in length, each bundle being composed of eight wires, No. 26 S.W.G. in size, previously well paraffined or painted with shellac varnish. Each little bundle of iron is wound over uniformly with a magnetizing coil formed of No. 36 silk-covered copper wire in one layer, and over this, but separated from it by one or two layers of gutta-percha tissue, is wound a single layer of No. 26 wire, forming a demagnetizing coil. This last coil is in turn covered over with one or two layers of gutta-percha tissue (see Fig. 23*b*).

The magnetizing or inner coils are connected in series with one another, so that when a current passes through the whole of them, it magnetizes the whole of the wires in such a manner that contiguous ends have the same polarity. The outer or demagnetizing coils are joined in parallel. Associated with this induction coil is a rotating commutator, C, consisting of a number of hard fibre discs, secured on a steel shaft, which is rotated by an electric motor about 500 times a minute. There are four of these fibre discs, and each disc has let into its periphery a strip of brass, occupying a certain angle of the circumference. These wheels may be distinguished as Nos. 1, 2, 3, and 4. The brass sector of No. 1 occupies 95° of its circumference; and brass sectors of Nos. 2 and 3 occupy 135° of their circumference; and that of No. 4 disc 140° of its circumference. Four little springy brass brushes, S_1, S_2, S_3, S_4, make contact with the circumference of these wheels, and therefore serve to interrupt or make electric circuits as the disc revolves. The function of the disc No. 1 is to make and break the circuit of the magnetizing coils placed round the iron bundles, and thus to magnetize them during a portion of one period of rotation of the disc and leave them magnetized during the remaining portion. The function of discs 2 and 3 is to short-circuit the terminals of the secondary coils of the bobbin during the time that the magnetizing current is being applied by disc No. 1. A sensitive movable coil galvanometer, G, is employed in connection with the secondary coil, one terminal of the galvanometer being permanently connected to one terminal of the secondary coil, and the other terminal connected through the intermittent contact made by the disc No. 4. This disc No. 4 is so set that during the time that the secondary coil is short-circuited, and whilst the battery current is being applied to magnetize the iron-wire bundles, the galvanometer circuit is interrupted by the contact on disc No. 4.

The operations which go on during one complete revolution of the discs is as follows:—First the magnetizing current of a battery of secondary cells is applied to magnetize the iron bundles, and during the time the terminals of the fine-wire secondary coil are short-circuited, and the galvanometer is disconnected. Shortly after the magnetizing current is interrupted, the secondary bobbin is unshort-circuited, and an instant afterwards the galvanometer circuit is completed, and remains completed during the remainder of one revolution. Hence, during a large part of one revolution, the iron-wire bundles are left magnetized, but the magnetizing current is stopped, and the galvanometer is connected to the secondary coil. If during this period an electrical oscillation is passed through the demagnetizing coils, an electromotive force is induced in the secondary bobbin by the demagnetization of the iron, and causes a deflection of the galvanometer coil. Since the interrupter discs are rotating very rapidly, if the electrical oscillation continues, these intermittent electromotive impulses produce the effect of a continuous current in the galvanometer circuit, resulting in a steady deflection, which is proportional to the demagnetizing force being applied to the iron, other things remaining equal. If the oscillation lasts only a very short time, the galvanometer will make a small deflection; but if the oscillation lasts for a longer time, then the galvanometer deflection is larger, and tends to become steady.

In the numerous experiments which finally resulted in the construction of the above-described form of wave detector, it was found to be essential to have the iron core in the form of a number of small bundles of iron wire, each wound over with its own magnetizing and demagnetizing coil. No good results could be obtained when the iron core was in the form of a large bundle, say half an inch in diameter, and enveloped by a single magnetizing and demagnetizing coil.

Another condition of success is the short-circuiting of the fine-wire secondary

coil during passage of the current which effects the magnetization of the iron core. The core can be indefinitely increased in size, provided the augmentation of mass is obtained by multiplying small individual cores, each consisting of not more than eight or ten fine iron wires, and each wound over with a separate magnetizing and demagnetizing coil. The electromotive force in the secondary coil can in this manner be increased as much as is desired, and a very sensitive electric wave detector suitable for quantitative work constructed. The commutator can be driven either by an electric motor or clockwork, or any other source of power.

This detector has been employed by V. Buscemi (see *Nuovo Cimento*, February 1905, vol. 9, p. 105) for quantitative measurements on the transparency of various dielectrics. An oscillator was placed in a metal box having a rectangular opening 35 × 40 mms. in size, and over this was placed a glass trough filled with various liquids to the depth of 6 mms. The following Table shows the deflection of the galvanometer, which was connected to a Fleming magnetic cymoscope as above described :—

Liquid or dielectric in trough.	Galvanometer deflection in millimetres.
Air	21
Vaseline	22
Petroleum	16
Benzine	17
Æther	12
Sulphuric acid	0
Hydrochloric acid	0
Nitric acid	0
Distilled water	7
Sea water	0
Sodium chloride in water, 0·5 per cent. solution	1·5
Ditto, over 1 per cent. solution	0

Professor Wilson states [57] that Rutherford employed a moving band of iron wire in a magnetic detector in 1900 or 1901. Also, it has been asserted that Professor Fessenden, in the United States, was an early worker in this field of research.

A very large amount of research has been conducted of late years directed to the more complete elucidation of the action of the magnetic detector. There is a close similarity between the action of mechanical shock or torsion upon magnetic state or properties of iron and the effects produced by electrical oscillations. An explanation at one time given of Marconi's second form of magnetic detector was based upon the assumption that magnetic retentivity is reduced by a high frequency oscillation.

A reduction in magnetic hysteresis does not invariably accompany the action of the electric oscillations on iron or steel. Walter and Ewing discovered that in hard steel an increase of hysteresis results when oscillations are sent through the metal. Their experiments were made with an apparatus, described below, in which a steadily revolving magnetic field tends to cause rotation in an iron or steel specimen suspended in it owing to the magnetic hysteresis. The torque so produced is resisted by the control of an elastic spring. When electric oscillations were passed through a closed coil of hard-drawn insulated steel wire, used as a specimen in such a manner, it was found that the hysteresis of the metal was increased, and that it tended to twist more in the direction of rotation of the magnet. We take the following description of their investigations from a paper read before the Royal Society [58] :—

"A small bobbin was wound with insulated soft iron wire, and the end soldered to the upper and lower halves of a spindle which was itself divided at the centre, the upper half bearing a controlling spring, and the lower dipping into mercury, from which a connection led to the other terminal. On passing oscillations through this winding a remarkable and

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[57] See E. Wilson, "On Magnetic Detectors in Space Telegraphy," *Illustrated Scientific News*, August 1903.
[58] See L. H. Walter and J. A. Ewing, *Proc. Roy. Soc. Lond.*, 1904, vol. 73 (p. 120).

unexpected result was obtained. The change of deflection was much more marked than in the former experiments, and was in the opposite sense, indicating an increase of hysteresis while oscillations were present. Afterwards, hard steel wire was substituted for the soft iron, and a very great increase in the effect was observed, still in the same direction—that of increase of hysteresis.

"Owing to these encouraging results, it was decided to continue the experiments in this direction, abandoning the older form, in which a decrease of hysteresis was dealt with. The first bobbin constructed was about $\frac{5}{16}$ inch in external diameter, and had a vertical wire space of $\frac{1}{4}$ inch. The winding was a single No. 32-gauge iron wire, double cotton-covered, wound straight round from beginning to end. Later, No. 40 and No. 46 steel wires were employed, of which the latter gave the best results.

"It was soon noticed that any method of increasing the oscillatory current in the wires, as by winding the bobbin with two wires having a slightly unequal number of turns, was of advantage in giving a larger deflection. Later, a fine copper wire secondary, wound on the bobbin parallel to the magnetic wire, was tried, first with the ends insulated and then with the ends soldered together. A marked increase in deflection was observed when the secondary was closed, showing that the magnetic nature of the wire itself was influential. Accordingly, a bobbin was then wound with insulated steel wire, doubled back on itself. This non-inductive winding gave by far the best results hitherto attained, and is now used, except when special results are required.

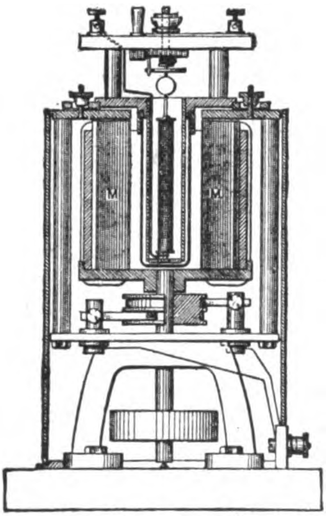


FIG. 24--Walter and Ewing Magnetic Detector.

"The instrument, though described as a detector of electrical oscillations, may be said to measure rather than detect, giving quantitative as well as qualitative results, and being capable of regulation from a sensibility of the same order as that of an average coherer, down to practical insensibility to powerful sparks in the same room.

"In the instrument (see Fig. 24) the electro-magnet takes the form of a ring capable of moving round a vertical axis, and is provided on the interior with two long wedge-shaped pole pieces, M, M, the current to the winding being supplied through brushes bearing against insulated rings below. The magnet is made to revolve by an electro-motor, the best speed being about five to eight revolutions per second, but the electromagnet may be replaced by a permanent magnet system giving a similar field. A structure is built up, external to the magnet, to support the vessel containing the pivoted bobbin and its centring arrangements. The bobbin itself is made of bone, and is about 2 inches long. It is provided with a steel spindle at each end bearing in a jewelled hole, the two halves of the spindle being insulated from one another. The winding, which is, as far as possible, non-inductive, consists of about 500 turns of No. 46-gauge hard-drawn steel wire, insulated with silk. The bobbin is immersed in petroleum, or a mixture of petroleum with thicker mineral oil, which serves the double purpose of fortifying the insulation, and giving the damping effect necessary to steady the deflection due to the drag of the revolving magnet. Readings are taken by means of a spot of light, as with speaking mirror galvanometers, but a siphon-recording attachment has been fitted, and any form of contact for working a relay can be employed.

"The detector, as before mentioned, gives quantitative readings, and in some cases the deflection may be too large to be easily read by the scale. For this purpose a variable shunt is provided, by which the deflection can be regulated.

"For the purpose of wireless telegraphy, the instrument has the advantage of giving metrical effects. The benefit of this in facilitating tuning, and in other respects, need not be insisted upon.

"From the physical point of view, the augmentation of hysteresis is interesting and unlooked for. It is probably to be ascribed to this, that the oscillatory circular magnetization facilitates the longitudinal magnetizing process, enabling the steel to take up a much larger magnetization at each reversal than it would otherwise take, and thus indirectly augmenting the hysteresis to such an extent that the direct influence of the oscillations in reducing it is overpowered. The net result appears to be dependent on two antagonistic

influences, and, in fine steel wire, under the conditions of our experiment, the influence making for increased hysteresis, as a result of the increased range of magnetic induction, is much the more powerful."

10. References to Other Work on Magnetic Detectors.—Experiments have been made by A. L. Foley[59] to ascertain the effect of substituting other magnetic metals for iron in the Marconi form of magnetic detector. He found that nickel wires could be used in place of iron, and states that a mixed core or band composed partly of nickel and partly of iron wires acted better than one of either metal alone. Probably in view of the well-known fact that some varieties of tungsten steel possess very large hysteresis constants, it may be found that some iron alloys will do better for this purpose than pure iron wire, even if hardened. MM. H. T. Simon and M. Reich have made interesting experiments with a combination of magnetic wave detector and a Poulsen telegraphone.[60]

If a steel wire is uniformly magnetized by passing it over a magnetic pole, and if this wire is then sent through a short glass tube, on which is wound a coil of insulated wire, through which trains of electric oscillations are sent at intervals for a longer or shorter time, each train wipes out the magnetism of the iron wire in that part which is at the moment within the coil. Hence, if the wire passes uniformly through the coil we can, so to speak, obliterate the magnetism for long or short spaces, in accordance with the signals of the Morse alphabet, by so regulating the duration of the trains.

If this steel wire is then passed uniformly through the receiving or repeater part of a Poulsen telegraphone, the listener in the attached telephone *hears* these signals as sounds in the telephone, and the wire becomes a record of the message, like the Morse tape of a printing telegraph.

Other investigations have also been made by M. C. Tissot, on forms of magnetic detector suitable for the detection of electrical oscillations (see *Comptes Rendus*, 1903, vol. 136, p. 361 ; or *Science Abstracts*, 1904, vol. 7, A, 107). Also we may note a paper by M. Maurain, on the "Suppression of Magnetic Hysteresis by the Action of an Oscillatory Magnetic Field" (*Comptes Rendus*, 1903, vol. 137, p. 917 ; or *Science Abstracts*, 1904, vol. 7, A, p. 108).

M. P. Duhem has also discussed the changes in the hysteresis by an oscillatory magnetic field (*Comptes Rendus*, 1903, vol. 137, p. 1022 ; or *Science Abstracts*, 1904, vol. 74, p. 108).

A. Sella (*Accad. Lincei Atti*, 1903, vol. 12, p. 340 ; or *Science Abstracts*, 1904, vol. 7, p. 344) has noticed that electric oscillations can also annul the magnetic hysteresis due to deformation by twisting, or, as it is called, the magneto-elastic hysteresis.

A good general account of the various forms of magnetic detector which were devised up to 1905 was given by L. H. Walter in *Technics* for August 1905, and also in the *Electrical Magazine* for December 1905, vol. 4, p. 359.

Forms of magnetic detector for wireless telegraphy have been devised by Lee de Forest, Shoemaker, and others, for descriptions of which the reader must consult the following United States Patent Specifications :—Lee de Forest, No. 772,878, June 20, 1903. This describes a magnetic detector with a divided core similar to the one previously described by the author (see p. 389).

H. Shoemaker, No. 711,182, September 5, 1902, and No. 734,476, January 8, 1903.

The theory of the magnetic detector in its various forms has been discussed by L. H. Walter,[61] W. H. Eccles,[62] J. Russell,[63] and C. Maurain,[64] and

[59] See A. L. Foley, *Physical Review*, 1904, vol. 18, p. 349; also *Science Abstracts*, 1904, vol. 7, A, p. 460.

[60] See *Elektrotechnische Zeitschrift*, 1904, vol. 22, p. 180; or *Science Abstracts*, 1904, vol. 7, B, p. 426.

[61] See L. H. Walter on "The Effect of Electric Oscillations on Magnetism," *The Electrician*, vol. 55, p. 83, 1905.

[62] W. H. Eccles, "The Effect of Electrical Oscillations on Iron in a Magnetic Field," *Proc. Phys. Soc. Lond.*, vol. 20, 1906; or *Phil. Mag.*, August 1906.

[63] J. Russell, "Note on the Effect of Electric Oscillations on the Magnetic Properties of Iron," *Proc. Roy. Soc. Edin.*, vol. 26, p. 53, 1905-1906.

[64] C. Maurain, "Magnetic Detectors and the Effect of Electric Oscillations on Magnetism," *Journ. de Physique*, vol. 6, p. 25, 1907; or *Science Abstracts*, A, vol 10, 1907, No. 479.

contributions to the discussion have also been made by Ascoli, Arnò, Piola, and P. Duhem.

Russell has carefully distinguished between the two conditions under which we can work.

(i.) Iron or steel may be placed in a constant magnetic field, and then subjected to the action of electric oscillations.

(ii.) Iron or steel may be subjected to continuing electric oscillation, and then the magnetic field around it changed.

In the case of Rutherford's experiment, hard iron or steel having considerable retentivity is subjected to a magnetic force, which is then removed, leaving remanent magnetization in the iron. The action of oscillations taking place round the iron is then always to remove or diminish this magnetization, and this can be detected by a change in the position of a suspended magnetic needle in the neighbourhood.

In Marconi's first form of magnetic detector, a horseshoe magnet is rotated slowly (about one turn in two seconds) over a thin bundle of hard iron wires, which are surrounded by two separate coils of wire. The iron is thus carried slowly through a cycle of magnetizing force of equal positive and negative values. The magnetization induced in the iron lags behind the magnetizing force in virtue of so-called hysteresis, and, therefore, if ordinates representing the magnetization are plotted out in terms of the magnetizing force as abscissæ, we obtain a magnetization curve of the well-known looped form. The area of this loop is proportional to the work expended in carrying the iron through one complete magnetic cycle. Russell states, as the result of his experiments, that if oscillations act continuously on the iron whilst it is being carried round the magnetic cycle, the area of this loop is greatly increased, thus showing an increase in hysteresis loss. If the oscillations come intermittently, as they would do in radiotelegraphic signalling, then the effect depends upon the particular point in the cycle at which the oscillations arrive. The result, in any case, is to produce a sudden change in the magnetization of the iron. Hence, if the oscillations are sent through one coil wound round the iron, the sudden change in magnetization produced by them creates induced or secondary electric currents in another coil wound over the oscillation coil, and therefore causes a sound in the telephone in series with the latter coil.

In Marconi's iron band form of magnetic detector the action is somewhat different. The band is passing through a magnetic field, so as to be always subject to a longitudinal magnetizing force, which is first in one direction and then is quickly reversed, because the two horseshoe magnets are placed with their poles near the wire and have similar poles in contact. Under these conditions, Russell found that the effect of a longitudinal oscillatory magnetic force is to increase the magnetization due to the steady force by an amount which is greater for an increasing than for a decreasing field. If the double north poles of the magnets are in the centre, and the iron moves from left to right, then the moving iron band distorts the field, and the effect of the oscillations passing round the iron is to increase the magnetization of the iron more on the left hand than on the right of the north poles. Both these effects alter the number of lines of magnetic flux through the secondary coil in series with the telephone, and therefore cause an induced current to flow through it, and the telephone in series emits a sound. Hence, Russell considers that Marconi's second form of magnetic detector acts in virtue of the increase of magnetization in iron which occurs when an oscillatory field is superimposed upon a slowly changing or stationary field near a cyclic extreme, whereas Rutherford's form of detector operates in virtue of a decrease of magnetization produced when the magnetizing force has been applied and has been removed.

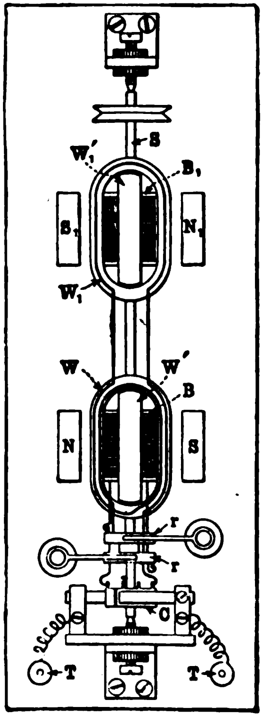
The function of the moving band is two-fold: it supplies the hard iron or steel in a condition of low permeability to be raised by the oscillations to a condition of higher permeability, and it distorts the field in the direction of motion. This view is rather different from that taken by Marconi himself and others, who have expressed the opinion that the action of the oscillations is to annul the hysteresis of the iron.

According to W. H. Eccles (*Phil. Mag.*, August 1906), the effect produced when a bundle of iron wires is taken slowly round a magnetization cycle, and an

oscillatory magnetizing force applied at any point, is to bring the iron back to the condition of magnetization it would have under the final steady impressed magnetic force acting on it if the hysteresis was suddenly annulled. The action of the oscillations is therefore to cause a return to the normal curve of magnetization.

In the Walter-Ewing form of detector we have different magnetic conditions. The field is then revolving somewhat rapidly, so that a drag is produced on the suspended iron, due to the so-called rotational hysteresis. Oscillations increase this hysteresis, and therefore the deflection of the suspended iron, at least in fairly strong fields.

L. H. Walter considers that all magnetic detectors may be divided into two classes. First, those in which the oscillations act on the iron after the magnetizing force has been applied and withdrawn. Examples of this type are the original Rutherford detector and the Fleming quantitative detector above described. In this case the available energy is limited to the remanent magnetism in the core, and the action of the oscillations is to reduce or destroy this remanent magnetism. The second class, represented by Marconi's moving band detector, derive their energy from an external magnetic field and from the motive power driving the band, and the action of the oscillations is merely to release some of this energy. If the iron is moving through a field of increasing magnetic force, it is on the lower side of the hysteresis loop, and the action of the superimposed oscillatory field is to increase the magnetization when not actually at the peak of the curve, the increasing effect being, as E. Wilson first showed, greatest at or near the point of inflection of the lower branch of the hysteresis curve. This increase in magnetization of the portion of the iron band partly enclosed by the coil in series with the telephone creates the induced current in the latter. We may, therefore, in a sense, speak of this increase in local magnetization of the iron as due to an annulment of hysteresis. The action of the moving band detector is, however, essentially dependent on the supply of energy from an external source to magnetize the iron and move it against the magnetic force. The action of the oscillations is only a trigger action, which creates a sudden increase in the magnetic flux in a part of the iron embraced by the secondary or telephone coil. This causes in turn induced currents to flow through it, first one way and then the other. Accordingly, with the band detector, the only possible signal receiving instrument is a telephone, unless we provide some means of sifting out the direct from the inverse induced current in the telephone coil. This has been achieved, however, by means of one of the author's oscillation valves or glow-lamp detectors, described in a subsequent section, and by its use it is possible to obtain from a Marconi moving band magnetic detector, associated with a Fleming oscillation valve described in a subsequent section, intermittent but unidirectional currents, which can operate a relay, and therefore work any ordinary telegraphic printing instrument.



[Reproduced from "The Electrician" by permission of the Proprietors.

FIG. 25.

Another method has been devised by L. H. Walter,[65] by which a detector of the Walter-Ewing type, depending upon rotational hysteresis, can be made to furnish continuous currents. In this case oscillations are made to act on a magnetic mass undergoing reversals of magnetism in a rotating field in such

[65] See L. H. Walter, "On a Method of Obtaining Continuous Currents from a Magnetic Detector of the Self-Restoring Type," *Proc. Roy. Soc. Lond.*, vol. 77, A, p. 538, 1906.

a manner that the changes of magnetism produced by the oscillations create alternating induced currents in embracing coils of wire, which are rectified by a commutator in the usual dynamo machine manner. The inventor has described his apparatus as follows:—

Two ebonite bobbins, B, B (see Fig. 25), mounted on the same spindle, are rotated in the field of two horseshoe permanent magnets, NS, NS, these bobbins being wound, in a similar manner to those illustrated in connection with the pivoted bobbin detector previously referred to, with some feet of steel wire of suitable resistance. A winding of two coils, W, W′, at right angles to one another, of a hundred turns, is placed on each bobbin at right angles to the plane of the steel wire winding, as in a drum armature, corresponding coils, *i.e.*, W and W, W′ and W′, being connected in such a way that the E.M.Fs. generated are equal and opposite. The ends of the windings are connected to the segments of a four-part commutator, C. (For the sake of clearness, only one pair of corresponding windings, of one turn each, is shown connected in Fig. 25.) The steel wire windings of the two bobbins are exactly alike, the ends of one winding being insulated, while those of the other are connected to a pair of slip-rings, *r*, *r*, and brushes, by means of which the oscillations can be passed through the winding.

On testing this apparatus, with no oscillations acting, there was no potential difference at the brushes. On waves arriving, a steady deflection of the galvanometer was obtained in a direction corresponding to an increase of E.M.F. generated by the armature acted upon by the oscillations. By suitably proportioning the turns in the winding the sensibility was considerably increased. The usual speed employed is about five to eight revolutions per second. Higher speeds have been tried, and give a larger effect, but the zero is not so steady. Telephonic signals can, of course, be received simultaneously by connecting to the winding at some point before the E.M.F. is commutated. When a relay alone has to be actuated, however, it may be advantageous to so arrange matters that the generated E.M.Fs. do not exactly balance, and a small initial current, insufficient to actuate the relay, passes all the time through it. The change can be rapidly effected by a very slight shift of the brushes.

A further discussion of the action of electric oscillations upon magnetized iron has been given in another paper by J. Russell, in which he analyses and compares the observations of W. H. Eccles and C. Maurain with his own, for which the reader must be referred to the original papers (see J. Russell, "The Shift of the Neutral Points due to Vibration in the Intensity of Mechanical Vibrations or Electric Oscillations Superposed upon Cyclic Magnetization in Iron," *Proc. Roy. Soc. Edin.*, vol. 29, p. 1, 1908. Also J. Russell, "The Superposition of Mechanical Vibrations or Electric Oscillations upon Magnetization, and Conversely in Iron, Steel, and Nickel," *Trans. Roy. Soc. Edin.*, vol. 45, p. 491, 1907).

From the statements made in the above paragraphs it will be seen that investigators differ as to the precise explanation of the mode of action of certain forms of magnetic detector, but that in all cases the influence of a high frequency oscillating magnetic force upon iron or steel, whether merely retaining magnetization or else subject to a certain magnetic force, is to change suddenly its magnetic condition or magnetic properties. Broadly, it may be said that the influence of electric oscillations upon magnetized iron is similar in effect to that due to mechanical vibration.

The author has devised an instrument called by him a Campograph (from καμπή, a curve), which enables the hysteresis loop of a cyclically magnetized iron rod to be photographed (see *Proc. Phys. Soc. Lond.*, vol. 27, p. 316, 1915). By means of this an investigation was made of the effect of electric oscillations on the magnetic hysteresis and permeability of iron (see *Proc. Phys. Soc. Lond.*, vol. 28, p. 35, 1915, Dr. J. A. Fleming and Mr. P. R. Coursey, on "The Effect of Electric Oscillations on the Magnetic Properties of Iron"). In this paper numerous photographs are given confirming statements made above as to the effect of electric oscillations on the hysteresis and magnetic permeability of iron which is being taken round a cycle of magnetic force, whilst at the same time it is subjected to electric oscillations either circulating round the iron or else through it.

Speaking generally, the effect of applying continuous electric oscillations to an

iron wire being taken through a slowly periodic magnetic cycle is to increase the hysteresis and area of the B—H loop.

11. Electrolytic Oscillation Detectors.—A third class of detectors depend for their operation upon the power of electric oscillations to affect electrolytic conduction or the passage of currents through electrolytes when created by some independent and unidirectional electromotive force.

A discovery of considerable interest in connection with this subject was due independently to R. A. Fessenden, Commandant Ferrié, and W. Schloemilch,[66] who found that electric oscillations had a marked effect on the voltaic polarization of carbon or metallic electrodes when in an electrolyte. A very short, fine carbon filament, or very fine platinum wire, 0·001 mm. in diameter and 0·01 mm. long, is made the anode A in an electrolytic cell containing, say, dilute acid, the cathode K being a larger lead or platinum plate (see Fig. 26). This cell is placed in series with a shunted voltaic cell of slightly higher E.M.F. than the polarization cell, and a telephone is included in the circuit. The electromotive force of the shunted cell "polarizes" the electrodes of the electrolytic cell, and, in consequence of the deposit of oxygen gas upon the small carbon anode, the resistance of the cell increases so much that current through it is reduced nearly or quite to zero. If, then, the terminals of the electrolytic cell or detector are connected to the two

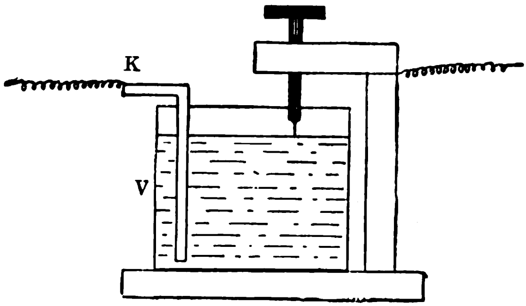


FIG. 26.—Ferrié, Fessenden, or Schloemilch Electrolytic Detector.

plates of a condenser inserted between two aerial wires, or between one aerial wire and the earth, and electric waves allowed to fall on these collecting wires, the electric oscillations depolarize the surface of the carbon or platinum anode and suddenly reduce the resistance of the cell. If, therefore, a telephone is placed in series with the shunted battery and cell, the sudden increase in the current through it causes a sound to be heard in the telephone, and by the impact of a greater or less number of trains of waves, sound signals on the Morse code can be heard in the telephone.

Observations have been made on this cell by M. Reich, who substituted for the carbon a fine platinum wire made by the Wollaston process, the end of this wire just protruding from a glass tube into which it was sealed.[67] One theory concerning this action is that the cause of the phenomena is the annulment of the anodic polarization by the electric oscillations. Another view advanced by

[66] See Commandant Ferrié, *Proceedings of the International Electrical Congress* (Paris, 1900), vol. ii. p. 289; see also W. Schloemilch, "A New Wave Detector for Wireless Telegraphy," *Elektrotechnische Zeitschrift*, 1903, vol. 24, p. 959; or *The Electrician*, 1903, vol. 52, p. 250. Fessenden described a detector in a United States Patent Specification, No. 12,115 of May 26, 1903, the original of which was No. 727,331 of May 5, 1903, consisting of a Wollaston wire or exceedingly fine platinum wire dipping a small depth into nitric acid, the vessel also containing another wire or electrode. He subsequently named this a liquid barretter. This patent application preceded the paper of Schloemilch.

[67] See "Observations on the Schloemilch Wave Detector," by M. Reich, *Phys. Zeitschrift*, 1904, vol. 5, p. 238.

Fessenden is, that the action is thermal and due to a change in the resistance of that portion of the electrolyte near to the fine platinum wire (see § 11).

V. Rothmund and A. Lessing have conducted experiments with this electrolytic wave detector, using a platinum point electrode of 0·025 mm. in diameter and dilute sulphuric acid at its maximum conductivity as the electrolyte.[68] Their conclusions are that the effect is a depolarization action caused by the high frequency currents. The small size of the anode is no doubt a necessity owing to the small quantity of electricity which is conveyed by the oscillations.

The electrolytic detector resembles in its general construction a Wehnelt interrupter. In the former case, however, the operating current is a high frequency alternating current, and in the latter a continuous one. In both cases, however, we have two electrodes of very unequal surface, one a platinum point of very small surface, and the other a larger one of any other metal.

J. E. Ives has also investigated the electrolytic detector, and given some good reasons supporting the view that the action is due to electrolytic polarization (see *Electrical World and Engineer*, New York, December 10, 1904). He employed an electrolyte having a zero temperature variation of resistance at 60° C., namely, a 2·5 per cent. solution of hypo-phosphorous acid. Below 60° C. the temperature coefficient is negative, and above it is positive. He found that the cell worked with this electrolyte. Also he deposited on the fine platinum anode platinum black. This deposit, as well known, reduces the polarization effect on platinum, and when the platinum wire was so treated the electrolytic cell became inoperative. As the platinum black deposit could not interfere with any heating effect, this experiment strongly supports the view that the action is electrolytic.

The above-described electrolytic cell has also been claimed as the invention of F. K. Vreeland (see Poincaré and Vreeland, "Maxwell's Theory and Wireless Telegraphy," p. 188, 1904), who states that, independently of Schloemilch, he found that a very minute anode of platinum wire, 0·0001 inch in diameter, placed in a cell containing nitric acid, together with a platinum cathode of larger surface, formed a very sensitive cymoscope far before an ordinary coherer.[69] It is to be noted that this last type of electrolytic detector will not operate unless the small surface is the anode, and that the resistance of the cell falls when electric oscillations act upon it, whereas the form of electrolytic detector described by De Forest increases in resistance by the action of electric waves.

Fessenden describes the production of the extremely fine platinum wire required as follows: A silver wire 0·1 inch in diameter has a core of platinum 0·002 inch in diameter. This silver wire is then drawn down to a diameter of 0·002 of an inch, and a short length of this wire is attached to the end of the screw A' (see Fig. 27), which is capable of being screwed down into a vessel of nitric acid. If the silver wire is immersed for a small fraction of a millimetre, then the acid dissolves off the silver and leaves an extremely fine platinum electrode immersed. If this becomes destroyed, then all that is necessary is to screw down the silver wire a further length into the liquid, and thus prepare another fine electrode. The necessary electromotive force for the polarization of the fine electrode is applied by a dry Leclanché cell shunted with a high resistance and a sliding contact on this resistance for taking off a fraction of a volt as in Fig. 28.

12. Thermal and Thermoelectric Detectors.—Since electric waves give rise to electric oscillations when they fall in the right manner upon open wire circuits, and these oscillations are high frequency electric currents, we can employ them to heat some very fine high resistance conductors and detect the wave by the heat it produces. In this case, however, we are measuring the *integral effect*, as certain

68 *Annalen der Physik*, 1904, vol. 15, p. 193; or *Science Abstracts*, 1904, vol. 7, A, p. 896.

69 It appears, however, from the judgment of Judge Wheeler in the United States Circuit Court given in the case of the National Electric Signalling Co. v. De Forest Wireless Telegraph Co., on October 16, 1905, that Vreeland was at the time of the invention an assistant to Fessenden and carrying out his directions. Priority in the invention of the electrolytic detector made as above described was therefore by this judgment awarded to Fessenden. In place of a silver-coated platinum wire immersed in nitric acid, it is also possible to use an iron-coated platinum wire produced by the Wollaston process which is immersed in dilute sulphuric acid, and in this case a silver cathode can be employed as in Fig. 28.

writers call it. The heat produced in a conductor by a train of decadent oscillations is proportional to the time-integral of the square of the instantaneous current value, and if we are employing an intermittent series of trains of oscillations it is proportional also to the number of trains of oscillations per second.

Hence thermal cymoscopes differ in this respect from coherers or magnetic detectors, in the operation of which the amplitude of the maximum voltage or current has an influence. No thermal wave detector has yet been invented which approaches in sensitiveness the best coherers, far less the magnetic or electrolytic detectors. An instrument in which heat is measured by the change in the resistance of a conductor produced by it is called a *bolometer*. The measurement of electric oscillations by the heat produced by them in a very fine wire is often called the *bolometric method* of detection. In this case some very fine high resistance wire, say, a wire of platinum, is made one arm of a Wheatstone's bridge, and its resistance is balanced against other conductors. In order to avoid the difficulties which arise from the heating of the bolometer wire by the bridge current, two similar wires must be placed in two arms of the bridge and a bifurcated arrangement employed, as shown in Fig. 41 of Chap. II. We can then

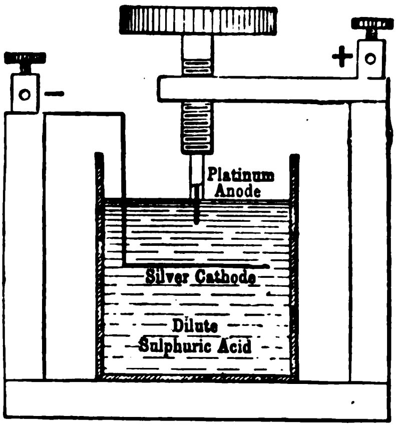


FIG. 27.

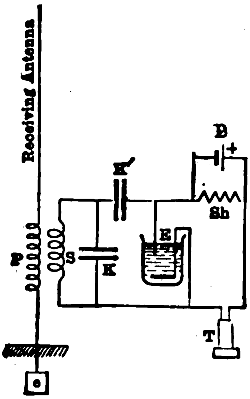


FIG. 28.

obtain a steady balance in the usual manner and bring the bridge galvanometer to zero. If then electric oscillations are passed through one of the fine wires, it is still more heated, and its resistance increased, and the bridge balance is upset. Hence the bridge galvanometer deflects. In place of a Wheatstone's bridge a sensitive differential galvanometer may be employed, and a double fine wire. One wire is placed in circuit with each coil of the differential galvanometer and a balance obtained. If then electric oscillations are passed through one of the wires, its resistance is increased, and the needle of the differential galvanometer deflects. In place of the differential galvanometer we may employ a differential telephone, and thus make the arrangement more sensitive.

As far back as 1889 experiments were made to employ the heating power of oscillations set up by electric waves as a means of detecting them.

W. G. Gregory described a radiation meter to the Physical Society of London, in which the elongation of a wire on which electric waves impinged was rendered visible by the use of an Ayrton and Perry twisted strip and mirror.[70]

H. Rubens and R. Ritter in 1890 employed a bolometric instrument in researches on electric gratings (see *Wied. Annalen*, vol. 40, p. 56, "Ueber das Verhalten von Drahtgittern gegen Electrische Schwingungen"). The details of their bolometer were as follows: Two rectangles, R and S, of fine iron wire 0·07

[70] *Proc. Phys. Soc. Lond.*, 1889, vol. x. p. 290.

mm. in diameter were employed (see Fig. 29). These were made the arms of a Wheatstone's bridge arrangement of conductors. One of these rectangles was connected with a linear oscillator, or antenna, A, which acted as a receiving wire, and when electric oscillations were set up in A by the impact on it of electric waves, these caused the circuits of the rectangle R to become heated, and so upset the balance of the Wheatstone's bridge. The deflection of the galvanometer G served then to detect and measure the electric radiation falling on the receiving wires.

C. V. Boys and W. Watson also gave an account, in 1890, of experiments made by them to measure electromagnetic radiation by means of the heat created by electric oscillations set up by it in linear conductors.[71]

C. Tissot has particularly studied the use of a bolometer for detecting electric waves at great distances from the source.[72] He employs an exceedingly fine platinum wire of great purity, the diameter of which is not more than 10 or 20 microns (1 micron = 0·001 millimetre). This wire is used in the arms of a bridge

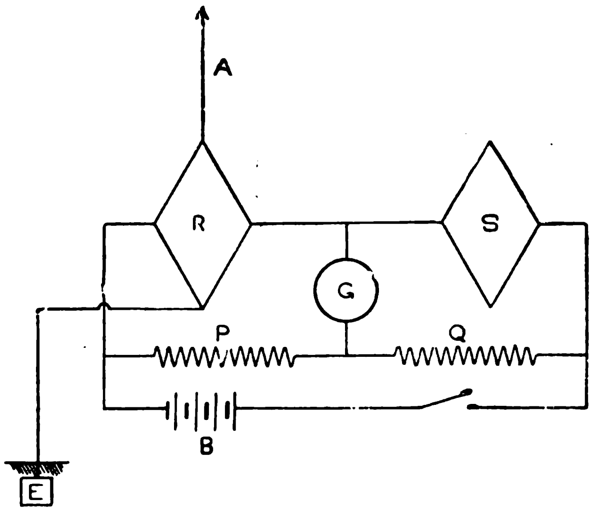


FIG. 29.—Bolometer Cymoscope. R, S, rectangles of fine wire forming the arms of a Wheatstone's bridge, with resistances, P and Q.

arrangement similar to that of Rubens and Ritter. With such a bolometer wire, he states that he has detected electric waves at a distance of 50 kilometres from the radiator when using the arrangements required for electric wave wireless telegraphy.[73]

W. Duddell devised in 1904 a thermal instrument of great sensibility for detecting electric oscillations.[74] He employs a form of Boys' microradiometer, in which a delicate thermocouple is suspended by a quartz fibre in a strong magnetic field. An attached mirror enables deflections to be estimated (see Fig. 30). Underneath this thermocouple he places a very thin and narrow strip of metal (gold leaf), through which the electric oscillations are passed. These oscillations heat the strip feebly. One junction of the small suspended thermocouple rests

[71] See *Proc. Phys. Soc. Lond.*, 1890, vol. xi. p. 20.

[72] C. Tissot, "Bolometers as Detectors of Electric Waves," *Journal de Physique*, 1904, vol. 3, p. 324; also *Science Abstracts*, 1904, vol. 7, A, p. 700.

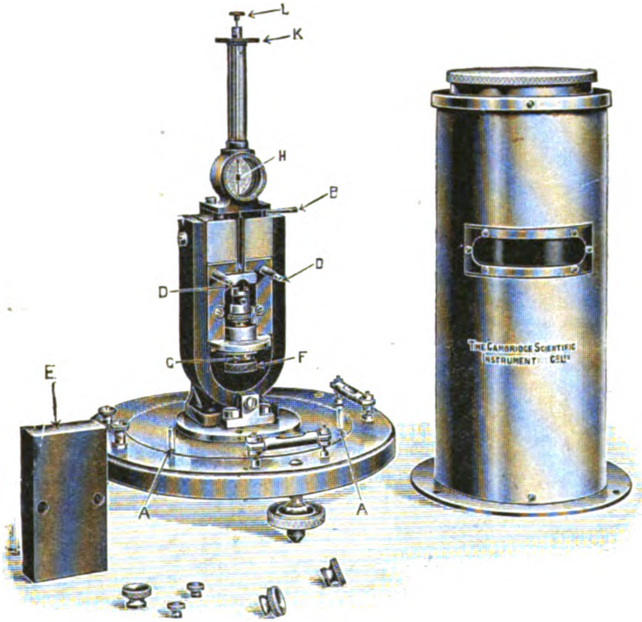
[73] See C. Tissot, *Comptes Rendus*, 1904, vol. 137, p. 846; or *Science Abstracts*, 1904, vol. 7, A, p. 100.

[74] See W. Duddell, "Instruments for measuring Alternating Currents," *Phil. Mag.*, 1904, vol. 8, p. 91. See also Chap. II. p. 234 of this book.

just above the strip but not quite touching it, and is therefore heated by radiation and convection. The couple is therefore traversed by a current, and is deflected in the magnetic field. If an ordinary Bell telephone is connected in series with the strip and a sound uttered to it, the alternating current so produced heats the strip sufficiently to make a large deflection of the ray of light reflected from the mirror attached to the thermocouple.

If the thin strip is placed in series with a pair of long rods or between an aerial wire and the earth, and if electric waves fall on these wires, then the electric oscillations set up heat the strip, and the instrument becomes a thermal cymoscope.

Instead of making the change in resistance of a fine wire detect the oscillations, we may detect a very small rise in temperature in it by placing in contact with the



[*From the Cambridge Scientific Instrument Company.*

FIG. 30.—Duddell Thermo-Galvanometer for Measuring very small Alternating Electric Currents.

wire a thermojunction. Such an arrangement was first employed by Klemencic in 1891.[75] The oscillations are sent through a fine constantan wire, and against this rests a thermoelectric couple of iron and constantan or other suitable metals. The ends of the couple are connected to a low resistance galvanometer. When a train of oscillations are passed through the fine wire they heat it, and the galvanometer connected to the thermocouple indicates the rise of temperature.

An improved arrangement of this kind was devised by the author in 1906, taking advantage of the position of tellurium and bismuth in the thermoelectric series, and also of the fact that such a fine wire rises to a higher temperature by the passage of a given current when placed in a high vacuum. A double glass test-tube, similar to a Dewar vacuum vessel, was constructed (see Fig. 31), the space between the two tubes being subsequently exhausted. Through the bottom

[75] See J. Klemencic, *Wied. Annalen*, vol. 42, p. 417, 1891.

of the inner test-tube were sealed four wires; two of these, *a*, *b*, were connected to a fine constantan wire, and the other two, *c*, *d*, were connected to a tellurium-bismuth thermojunction, T, formed of very fine wires of bismuth and tellurium, the junction being soldered by a special solder to the centre of the constantan wire. A high vacuum was then made in the interior space. When oscillations are passed through the constantan wire, a suitable low resistance galvanometer, G, being connected to the leads from the thermojunction, the galvanometer deflects, the deflection being proportional to the square of the integral value of the oscillations.

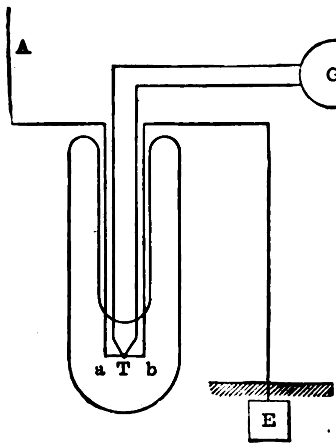


FIG. 31.—Fleming Thermoelectric Hot-Wire Microammeter.

The inclusion of the thermocouple and heater wire in a very high vacuum has a great effect in increasing the sensibility. It has been noted, both by the author and by P. Lebedew,[76] that for couples and wires of bright or polished metal the sensitiveness may be increased by so doing as much as 25 times. This great increase, however, takes place chiefly between a reduction of air pressure from 0·1 mm. to 0·0001 mm., and for much lower pressures there is hardly any change. A pressure of 0·01 mm. is sufficient to give almost the best effects (see Fig. 32).

The advantage of such a detector is the ease with which it can be calibrated

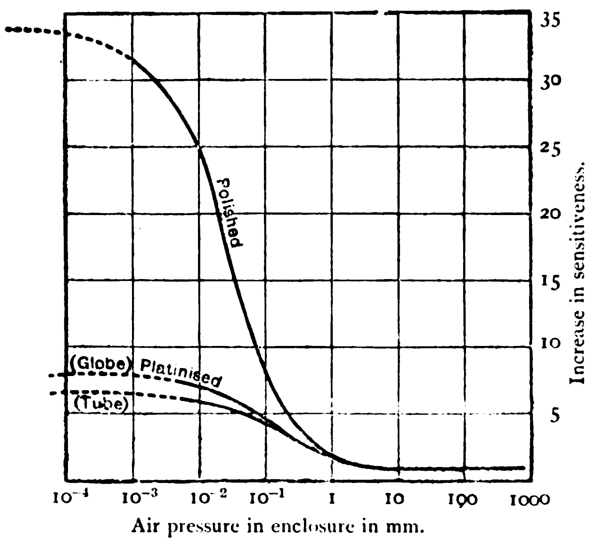


FIG. 32.—Curves showing the Increase in Sensitiveness of a Thermal Ammeter by enclosure in a space with Reduced Air Pressure.

by means of a known continuous current passed through the fine wire. For if we pass any train of oscillations which gives on the galvanometer the same deflection

[76] See P. Lebedew, *Ann. de Physik*, vol. 9, 1902; or *The Electrician*, October 3, 1902, p. 952.

as a certain direct current, we know that the mean square value of those oscillations must have the same ampere value as the unidirectional current which is thermally equivalent to them. It has been found, however, that it is not necessary to pass the oscillations through a wire. If pieces of two metals selected at opposite ends of the thermoelectric series are pressed in contact at one point, and if stout connecting wires make good contact with these pieces of metal at some other point, then, when oscillations are passed through the contact point, they produce heat there, and therefore excite a thermoelectromotive force. If, then, such a junction is inserted in an oscillatory circuit, and also has the terminals of a telephone connected to its leading-in wires, the telephone will respond by a sound to each passage of a train of oscillations. It is necessary, however, that the materials selected for the couple should have poor thermal conductivity, so that the heat generated at the small surface contact shall be localized there and not be conducted to the other junctions. L. W. Austin has described such thermoelectric detectors made with tellurium and aluminium and tellurium and silicon.[77] In constructing it, a bead of tellurium fused on the end of a springy brass wire is adjusted so as to press more or less tightly against an aluminium wire or disc. The couple, having a high resistance, is inserted as a shunt across the plates of a condenser in the receiving circuit, and a high resistance telephone is connected to the elements of the thermocouple. The question has been raised whether the action of this detector does depend upon a true thermoelectromotive force created by the heat produced at the small surface contact. The action as an oscillation detector may equally well be accounted for by the power which certain junctions, certain metals and non-metals, possess of rectifying a high frequency current. In other words, such junctions possess a unilateral conductivity.

The thermoelectric detectors are therefore very closely connected, as regards action, with another type of detector, called "rectifying detectors," which we proceed to consider.

13. **Rectifying Detectors.**—It has been found that a contact of small surface between certain dissimilar conductors, as, for instance, between tellurium and aluminium, also between silicon and copper, and carbon and steel, has a unilateral conductivity for electricity, and hence possesses the power of rectifying high frequency alternating currents. This was also observed by F. Braun in 1874 for some metallic sulphides or oxides and metals, and the same effect has been found in the case of a large number of other pairs of substances.[78]

L. W. Austin investigated the behaviour of a silicon-steel junction of this kind, and found that an electromotive force of 2·5 volts could pass a current of $25{,}000 \times 10^{-6}$ amps. from steel to silicon across the junction, but only a current of 6000×10^{-6} amps. in the opposite direction.[79] It follows, therefore, that such a junction rectifies an alternating current, and Austin has shown that when a steel-silicon junction is placed in series with an ordinary direct current galvanometer, electric oscillations acting on that circuit are rectified, and deflect the galvanometer. For small alternating voltages below about 0·2 of a volt, the rectified currents are proportional to the square of the alternating voltage. That this effect is not a pure thermoelectric effect is shown by the fact that the rectified current flows in the opposite direction to the true thermoelectric current produced by heating the junction.

This is the case for a carbon-steel and for a tellurium-aluminium junction, except for the last at low voltages.

A very large number of these combinations of non-metals and metals have been found which possess this rectifying power on trains of oscillations.

Thus G. W. Pickard has found that a rough surface of a mass of fused zinc oxide or of native red oxide of zinc in contact with a brass point rectifies oscillations, and may be used as a detector in association with a telephone as already

[77] See L. W. Austin, "On a High Resistance Contact Thermoelectric Detector for Electric Waves," *Physical Review*, vol. 24, p. 508, 1907.

[78] See Ferdinand Braun, *Pogg. Ann.*, vol. 153, p. 556, 1874.

[79] L. W. Austin, *Bulletin of the Bureau of Standards*, Washington, U.S.A., vol. 5 No. 1, 1908.

described.[80] The name "Perikon" was applied by G. W. Pickard, in U.S.A. Patent Specification, No. 886,154, to the contact rectifying detector made with fused oxide of zinc and a brass point, but it is now applied also to the rectifying contact detector consisting of chalcopyrite or copper pyrites in contact with zincite or native oxide of zinc, see U.S.A. Patent Specification, No. 912,726.

Many forms of clamp or holder for gripping the crystals or rectifying substances have been devised. The author has found the following a convenient arrangement for using and testing contact cymoscopes or wave detectors depending upon the rectifying power of a contact of two substances. One of these bodies, M (see Fig. 33), is held in a grip, and the other in the form of a point, C, is pressed against it by a screw, S, with divided head; the whole being enclosed under a glass shade to exclude dust. Terminals T and T′ are connected respectively to the two substances in contact.

The property of rectifying alternating currents and electric oscillations in virtue of a unilateral conducting power for electricity is also found in certain crystals. It was discovered in 1906 by General H. H. C. Dunwoody, of the United States army, that a mass of crystals of carborundum, which is an artificial silicide of carbon prepared in electric furnaces (see U.S.A. Patent of Acheson, No. 492,767 of 1893), can be used both with and without a local electromotive force as a detector of electric waves in radiotelegraphy.[81] If a single crystal of this substance is examined it will be found to have a hexagonal form and purple, green, or grey in colour, and to be somewhat translucent. The commercial carborundum as usually obtained consists of a mass of such crystals arranged and compacted in an irregular manner. Each individual crystal is a rather poor conductor, but certain picked specimens possess two marked characteristics, as shown by the measurements of G. W. Pickard[82] and G. W. Pierce.[83] In the first place, the crystal as a conductor possesses a unilateral conductivity, and in the next place, this conductivity does not obey Ohm's law, for the current-voltage curve is non-linear. It is not every crystal, however, of carborundum which exhibits in marked degree these characteristics. The most effective crystals are generally those of a silver grey lustre cut from the amorphous or underside of a mass of carborundum. For specific instructions as to the mode of selecting and testing them the reader is referred to an article by Mr. H. T. Worrall in the *Wireless World*, vol. iii. p. 434. Under an impressed E.M.F. of two volts a good crystal should pass a current 40 times greater in one direction than in the opposite. The curve in Fig. 34 shows the result of one set of observations by G. W. Pierce on the current in micro-amperes which passes through a crystal under certain applied voltages. If a gradually increasing electromotive force is applied to the crystal the current through it increases, but more rapidly than the voltage, as shown by the right-hand branch of the back line curve. If the E.M.F. is reversed then the current is reversed, but for the same value of the voltage is not so large. The left-hand branch of the curve should therefore be drawn with negative ordinates, but for the sake of economizing space the ordinates are drawn positively. Thus the curve in Fig. 34 shows a different conductivity of the same specimen under direct and reversed electromotive force, as observed by Professor Pierce. In some cases there is no sensible current until the applied voltage + or − exceeds a certain

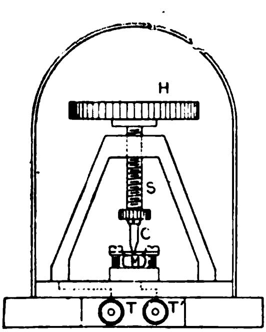


FIG. 33.—Contact Cymoscope. (Fleming.)

[80] See G. W. Pickard, U.S.A. Patent Specifications, No. 886,154, also No. 912,613 of September 3, 1907, and No. 912,726.

[81] See U.S.A. Patent Specification of H. H. C. Dunwoody, No. 837,616 of 1906; also British Patent Specification, No. 5332 of 1907.

[82] See G. W. Pickard, *Electrical World of New York*, vol. 48, p. 994, 1906.

[83] Also G. W. Pierce on "Crystal Rectifiers," *The Physical Review*, vol. 25, p. 31, July 1907. Also vol. 28, p. 153, 1909.

value which may be about 0·5 or 1 volt. This unilateral conductivity had been previously noted in the case of other minerals. Thus F. Braun, in 1874, found in it copper pyrites, iron pyrites, galena, and copper antimony sulphide, also in marked degree in psilomelan, a native oxide of manganese.

Braun could not find any evidence of electrolytic conduction or of thermo-electric effect in these cases to account for this curious asymmetry of conductance.

Professor Pierce has also discovered that the mineral hessite, which is as a telluride of silver or gold, and also anatase, which is an oxide of titanium, also possess

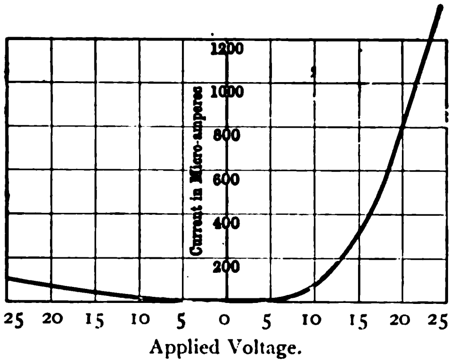


FIG. 34.—Characteristic Curves of a Carborundum Crystal for + and − E.M.F.

similar properties, and will therefore rectify alternating currents, since it passes a larger current when the voltage is in one direction than when it is reversed.

In crystals of carborundum the unilateral conductivity is a function of the voltage applied, as shown by the figures in the Table below, which gives the current in microamperes in one direction and the reverse through a crystal tested by Professor Pierce with various voltages.

RELATION OF CURRENT TO VOLTAGE, SHOWING UNILATERAL CONDUCTIVITY OF CARBORUNDUM.

	Current in microamperes.		
Volts.	C Commutator left.	C′ Commutator right.	$\frac{C}{C'}$
10·1	100	1	100
12·1	150	...	...
12·8	200	...	...
14·5	300	5	60
16·0	400	...	...
16·8	500	10	50
17·7	600	...	...
19·4	700	...	...
20·0	800	20	40
21·0	900	...	...
21·9	1000	30	33
23·2	1200	50	24
25·0	1500	...	...
27·5	2000	120	17

It will be seen that for this crystal under an impressed electromotive force of 10 volts the current in one direction is 100 times greater than in the opposite. Pierce found that in one specimen of carborundum, platinized on parts of its surface to make an improved contact, the current under an electromotive force of 34·5 volts was 527 times as great in one direction as in the opposite, and in another case under an electromotive force of 30 volts it was 3000 to 4000 times larger.

The rectifying power and the resistance of carborundum crystals have been investigated by Bertram Hoyle at various temperatures. He has found (see *Wireless World*, Sept. 1915, vol. iii. p. 356) that at the temperature of liquid air (−190° C.) the resistance of carborundum is greatly increased and its rectifying power reduced so that it is perfectly useless as a detector. On the other hand, by heating it it becomes more efficient as a rectifier, and at a certain temperature near to 400° or 500° C. it reaches a maximum sensitivity as a detector on account of its increased rectifying power and reduced resistance. Hence, as he points out, it would be an advantage to work with crystals heated to this temperature. Hoyle also confirms the author's prior statement that when an auxiliary voltage is applied to the crystal it should be such as to work at a point at which there is a change of curvature in the characteristic curve.

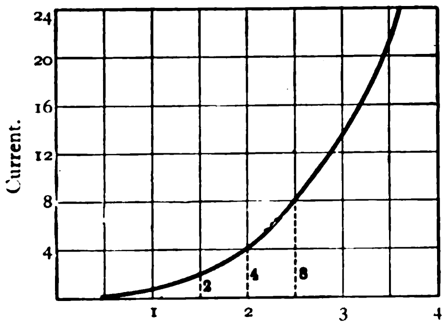


FIG. 35.—Characteristic Curve of Carborundum Crystal.

Besides using the crystals above named simply as rectifiers of an electric oscillation, we may use them in another way as oscillation detectors in virtue of the change of curvature at certain points on the characteristic curve. To do this we shunt the crystal by a telephone which has in series with it also a shunted voltaic cell so as to insert a fraction of a volt in the telephone circuit. There is, then, normally a certain current flowing through the crystal in one direction, and the voltage must be applied in the direction in which the crystal conducts best.

If, then, electric oscillations are superimposed on this unidirectional current by electric waves falling on the antenna, the steady E.M.F. acting on the crystal is periodically increased and decreased. Since, however, the current voltage curve is non-linear, the mean value of this pulsating current may be greater than the current due to the steady E.M.F., and hence the addition of the oscillations causes an increase in the current through the telephone and it therefore emits a sound. From the curve in Fig. 35 it will be seen that the current through a certain crystal corresponding to 2 volts steady E.M.F. is 4 microamperes. If, however, we add and subtract periodically 0·5 volt, then the currents due to 2·5 and 1·5 volts are respectively 8 and 2 microamperes, the mean of which is 5 microamperes. Hence the addition of the alternating E.M.F. of ± 0·5 volt to the steady E.M.F. of 2 volts increases the current through the crystal.

In addition to these cases of unilaterally conducting crystals there are other instances in which the contact between two different substances has a similar unsymmetrical conductivity, such, for instance, as a plumbago point in light

contact with a piece of galena or native sulphide of lead. Professor Pierce has shown that an excellent rectifying contact is obtained by pressing a copper point upon a crystal or flat piece of molybdenite, and the contact of a gold point with iron pyrites makes also an effective rectifier.

It would appear, therefore, that in some cases the rectification depends on a contact of small area between two materials, one of which is generally a pure metal, and the other an oxide or sulphide. In other instances efficient rectification is secured when the contact is one of large surface and low resistance. It seems, therefore, as if the rectification depends in some cases upon a surface action and in others upon the internal structure of one of the materials.

It has therefore been suggested that we should distinguish between "surface" and "body" rectification.

Various theories have been advanced to explain these effects.

R. H. Goddard (see *Physical Review*, vol. 34, June 1912) has examined the phenomena involved in the conduction of electricity at contacts of dissimilar substances, both in air and other gases, and also *in vacuo*, and drew from his experiments the following conclusions as to surface rectification :—

1. Pure elements give little or no rectification at a contact with pure metals, unless oxygen or air is present round the junction.

2. The effect of rectification or unilateral conductivity is due primarily to the presence of oxygen, and therefore of an oxide film (or else a sulphide film) at the junction.

3. Goddard considers, therefore, that these solid rectifiers operate in an analogous manner to the electrolytic rectifiers, such as the aluminium valve. In the case of the last mentioned arrangement a plate of aluminium is placed in apposition to a plate of graphitic carbon or some other metal, both being immersed in an electrolyte yielding hydroxyl as one ion. When the current flows in the electrolyte so that the aluminium is the negative electrode or cathode the current is much larger than when the aluminium is the positive or anode. It has been shown that this difference depends on the production of a film of aluminic hydroxide which is formed on the aluminium, and is not easily penetrated by the relatively large negative ions. If the current is reversed, then this hydroxide film is reduced and removed by the deposit on it of hydrogen ions.

The theory advanced by Goddard, therefore, is that the solid oxide or sulphide film at the bounding surface behaves in a similar manner. His view is that in a rectifying contact a solid film is present at the contact, and that large ions are packed against it giving a back E.M.F. when the current flows in one direction, whereas on reversal of the E.M.F. small negative ions pass freely through the film.

On the other hand, W. H. Eccles has worked out a theory of rectifying detectors based upon electrothermal considerations (see *Proc. Phys. Soc. Lond.*, vol. 25, p. 273, June 1913, W. H. Eccles "On Electrothermal Phenomena at the Contact of Two Conductors"). He points out that in the case of these rectifying detectors, by the aid of which the large part of the wireless telegraphy of the world is now conducted, we have two materials in contact, one or both of which are bad conductors of heat. Hence the temperature gradients in them, due to heat generated at the contact, will be steep. The heat is then created or absorbed at the junction and in proximity thereto. Again, these materials have in general large thermo-electric power and marked Thomson effects. Thus he points out that a pyrites-lead couple yields a thermo-electromotive force some 200 per cent. greater than a bismuth-lead couple. Also the temperature resistance coefficient of these oxides and sulphides is large and often negative. Eccles starts by considering mathematically the case of a material having a Peltier effect P, a Thomson effect or specific heat of electricity σ and a thermoelectric power p, which is placed between two pieces of identical metal having different values, P', σ', p'. The metal on one side making a good broad contact, and on the other side a small or imperfect contact with the material. He then shows that the heat generated at this contact by the resistance will be localized and produce a steep temperature gradient in at least one of the materials in contact. There will also be a Peltier effect at the junction, and a Thomson effect in the materials causing heat to be absorbed or generated at the contact and in the body of the materials along the temperature

gradient. The Joulean effect, depending as it does on the square of the current strength, will be independent of current direction, but the electromotive forces created by the Peltier and Thomson effects are dependent upon the direction of the current. Hence there arises an asymmetry in the counterelectromotive force produced, which is equivalent to a difference in electric conductivity in the two directions. An analytical discussion of it led Eccles to express the relation between the current i across the junction and the potential difference e between the conductors in contact at points beyond the temperature gradient in the form—

$$ae^2i^2 + bei + cei^2 - e + \rho i = 0$$

where ρ is the equivalent true resistance of the contact and a, b, and c are coefficients depending on the Thomson and Peltier effects and resistance change with temperature of the materials in contact. This expression is the equation of the steady current characteristic curve of the contact. Eccles then discusses several cases and shows that for certain values of a, b, and c we have either a curve symmetrical or non-symmetrical in the first and third quadrants. Also that this characteristic curve has changes of curvature at certain points.

Thus, for a metallic conductor the characteristic is, in accordance with Ohm's law, a straight line through the origin. For certain conductors, as, for instance, a D.C. carbon electric arc, the characteristic is a descending curve with concavity upwards, and hence the P.D. of the electrodes decreases with increasing current. In the case of contacts under consideration the characteristic may be either symmetrical on the two sides of the origin or not. If it is not symmetrical, then the reversal of the direction of the applied E.M.F. causes a change in the strength as well as direction of the current. In all cases the characteristic is non-linear. In this case the contact can act as a rectifier for alternating currents, and hence can be used alone without any auxiliary E.M.F. to rectify trains of oscillations into gushes of electricity in one direction, and therefore these trains, so rectified, will affect a telephone. Accordingly, a simple rectifier of this kind, placed in series with a telephone, rectifies trains of oscillations, and if these trains follow at regular and frequent intervals, a musical sound is produced in the telephone. On the other hand, if the characteristic curve is symmetrical, but has points of inflexion or change of curvature on it, then an auxiliary E.M.F. must be applied to bring the current through the contact to the point on the characteristic corresponding to that point of inflexion. The two cases are illustrated in the curves in Figs. 34 and 35, and the explanation of the last case has been given in connection with the latter diagram. As far back as 1906 it was pointed out by H. Brandes (see *Elektrotechnische Zeitung*, vol. 27, p. 1015, 1906) that any conductor which departs from Ohm's law can be used as a detector for electric oscillations. If the characteristic is non-symmetrical, then simply as a rectifier; but if it is symmetrical, although with changes of curvature in it, then it can be used as a detector by the aid of an added unidirectional electromotive force.

The author of this treatise also pointed out that in the case of his oscillation value described in § 15, the point of maximum sensitiveness is at or near a point of inflexion on the characteristic curve (see British Patent Specification, No. 13,518 of 1908; also Royal Institution Friday Evening Discourse, June 4, 1909).

A number of rectifying detectors were examined, at the author's suggestion, by P. R. Coursey,[84] who found that there was a good general agreement between the sensitivity curve plotted in terms of voltage and the curve representing the second differential of the characteristic curve for points corresponding to identical voltages, when these contact detectors were employed with an auxiliary voltage in series as above described. The author attempted to put the thermoelectric theory of their action to the test of experiment. For this purpose some rods of the most-used rectifying materials, *e.g.*, chalcopyrite, zincite, graphite, etc., were prepared by crushing the minerals and then compressing the powder into rods in a mould under great pressure. It was then found that these compressed materials had lost all rectifying power. Neither was it restored by intensely heating the rods in an

[84] See P. R. Coursey, "Some Characteristic Curves and Sensitiveness Tests of Crystal and other Detectors," *Proc. Phys. Soc. Lond.*, vol. 26, p. 97, 1914.

electric arc. It appears, therefore, as if the rectifying power depends upon a certain crystalline structure, which is destroyed by crushing to fine powder.

Mr. A. F. Hallimond has found that a number of these crystalline bodies can be arranged in order like a volta series of metals, such that, for any pair taken from the series, the maximum positive current flows under given E.M.F. from the mineral lowest on the list to the one highest. Zincite stands at the head of this list, and chalcopyrite near the bottom. The author has also noticed that there is a great difference in photoelectric power between the materials at opposite ends of this series.

In spite of all the research on this subject, we do not, however, seem to have arrived at a complete explanation of the phenomena.

The following references to United States Patent Specifications will afford the reader the means of gathering more information on the subject of contact or rectifying detectors.

G. W. Pickard, No. 836,531, applied for August 30, 1906, contains a claim for the use of silicon with copper as a thermoelectric detector.

G. W. Pickard, No. 888,191, applied for March 9, 1907, describes the construction of a rectifying detector of a brass point and a mass of silicon having a polished surface.

G. W. Pickard, No. 886,154, September 30, 1907, describes a rectifying detector consisting of a metal point and a mass of fused oxide of zinc.

G. W. Pickard, No. 912,726, of October 15, 1908. A detector composed of a mass of chalcopyrites (copper pyrites) and native red oxide of zinc. This is now called the "Perikon" detector.

Also reference may be made to the following United States Patent Specifications of G. W. Pierce :—

No. 879,061, January 11, 1907, covers the rectifying power of molybdenite.

No. 879,062, April 5, 1907, describes a rectifier made with oxide of titanium (anatase).

No. 879,117, April 5, 1907, describes the use of telluride of silver (hessite) as a rectifier.

No. 923,700, February 20, 1907, is an application of the rectifying power of molybdenite to control an alternating current so as to rectify it for the purposes of charging storage cells.

Also the reader will find some interesting information in the following papers :—

A. E. H. Tutton, *Wireless World*, vol. 1, p. 232, 1913, "Crystals as Rectifiers and Detectors."

H. Sutton, *The Electrician*, vol. 69, p. 66, 1912, "The Production of Rectifying Detectors by Mechanical Means."

14. Electrodynamic Detectors.—Since high frequency alternating currents or electrical oscillations create magnetic fields varying in a similar manner round the conductor through which they pass, and since these fluctuating fields can induce other currents in closed metallic circuits, we may construct electric wave detectors which depend for their operation upon electrodynamic forces of attraction and repulsion.

One such form of detector has been employed in researches by Professor G. W. Pierce.[85] It is a form of alternating current ammeter devised in 1887 by the author.[86] The writer showed then that if a silver or copper disc is suspended by a fine wire within a circular coil, so placed that the plane of the disc makes an angle of 45° with the axis of the coil, when an alternating current flows through the coil it will induce secondary currents in the disc, and the electromagnetic repulsion between the primary and induced currents will cause the disc to move so that its plane lies more nearly at right angles to the plane of the coil[87] (see Fig. 39, Chap. II.). The theory of the instrument has already been given in Chap. II. § 13.

[85] See G. W. Pierce, "Experiments on Resonance in Wireless Telegraphy Circuits," *The Physical Review*, September 1904, vol. 19, p. 201.

[86] See *The Electrician*, May 6, 1887.

[87] For an explanation of this fact, the reader is referred to the author's treatise on the "Alternate Current Transformer," vol. i, 3rd ed. § 12, p. 307.

This copper-disc alternating current galvanometer was employed by the author to measure telephone currents and other feeble alternating currents in 1887. More recently Professor Pierce increased the delicacy of the instrument by employing a disc of silver paper suspended by a quartz fibre, the disc being hung at an angle of 45° inside an ebonite tube, on the outside of which was wound a coil of insulated wire. A small fragment of silvered glass attached to the disc served to reflect a ray of light upon a scale, and indicated any movement of the disc. With this instrument quantitative measurements can be made of electric oscillations taking place in a circuit. Very much the same device has been employed by Fessenden,[88] who used a suspended silver ring and two fixed coils on either side of it, through which the oscillations passed.

This form of cymoscope, like the thermal instruments, measures the root-mean-square or integral value of the oscillations. The mechanical forces, however, are small, and these electrodynamic instruments are not as sensitive as even the best forms of thermal detector.

15. Ionized Gas and Thermionic Detectors.—The special qualities of gaseous conductors, especially rarefied gases in so-called vacuum tubes, have been utilized for the detection and measurement of electric oscillations, and therefore of electric waves.

Professor Righi availed himself of one striking peculiarity of rarefied gases as conductors, as follows: It is well known, as first shown by Varley, that if a glass tube having platinum electrodes sealed into it, and a vacuum of about one-thousandth of an atmosphere made in it, is subjected to electromotive force, no current will flow through it until a certain voltage, say of 300 volts or so, is exceeded. Beyond that limit the current which flows is almost exactly proportional to the excess of the voltage above this critical value. Hence, if a small vacuum tube is connected in series with a battery of voltaic cells giving some voltage a little less than the critical value, no glow will take place in the vacuum tube, because no current passes. If, however, the same circuit includes a coil in which electric oscillations are excited, then the electromotive force of these induced oscillations will, in one direction, be added to the electromotive force of the battery, and will send a current through the gas and cause it to glow. Righi employed a vacuum tube in which a very small space intervened between the electrodes, and employed a battery of 300 or more simple form of primary voltaic cells to produce the required "boosting" or auxiliary electromotive force.

The vacuum tube then glowed when electric oscillations were set up in the coil in series with it.

L. Zehnder employed a vacuum tube in a slightly different manner as a detector of Hertz oscillations.[89] He took advantage of another well-known fact connected with electrical discharge through rarefied gases.

A vacuum tube of the ordinary kind has, in addition to the usual platinum electrodes, another pair of electrodes at right angles, placed with their ends very close (see Fig. 36). If, then, a high potential battery, say of 300 or 400 cells, is applied in series with a high resistance, and the tube used in the ordinary way, we may adjust the number of cells until the electromotive force is just not sufficient to cause a glow discharge in the tube. Then if a very small discharge is sent between the transverse electrodes, this glow discharge causes the general mass of the rarefied gas to become a conductor for the steady battery electromotive force, and the vacuum tube bursts into glow. This arrangement is sometimes called a Zehnder "trigger tube," because the small transverse discharge, so to speak, sets off the longitudinal discharge in the tube. The transverse electrodes which convey the oscillatory discharge through the gas are placed quite close to the cathode of the continuous current electrodes, since it is known that at the cathode the great resistance to discharge is situated. In this manner a Hertzian spark too feeble to be visible at a distance can be rendered manifest by its power

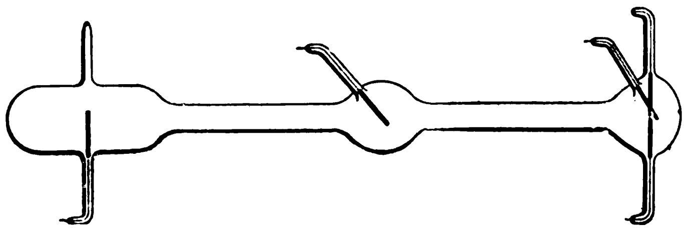
[88] See U.S.A. Patent Specification, No. 706,735, application of December 15, 1899.

[89] See "The Objective Representation of Hertz's Researches in Electrical Radiation," by L. Zehnder, *Wied. Annalen der Physik*, 1892, vol. 47, p. 82; also *The Electrician*, 1892, vol. 30, p. 253.

to start off another discharge from a powerful battery acting on the same mass of rarefied gas.

We have then to consider next a class of oscillation detector based on the emission by incandescent bodies of electrons, or thermions as they are now called. The first of these was devised by the author in 1904, called by him an *oscillation valve.*

If we seal into a highly exhausted glass bulb two carbon filaments as used in incandescent lamps (see Fig. 37*b*), we find that when both these filaments are



[*From "The Electrician."*

FIG. 36.—Zehnder's Trigger Vacuum Cymoscope.

cold, the vacuum or highly rarefied air left in the bulb is a very perfect non-conductor of electricity. Even an induction coil will not send a discharge through the bulb from one filament to the other if the exhaustion has been pushed far enough.

If, however, the carbon filaments are made incandescent by insulated batteries, then it is found that the electromotive force of a single cell is sufficient to send a current across the interspace between the filaments.[90] This can be proved if we

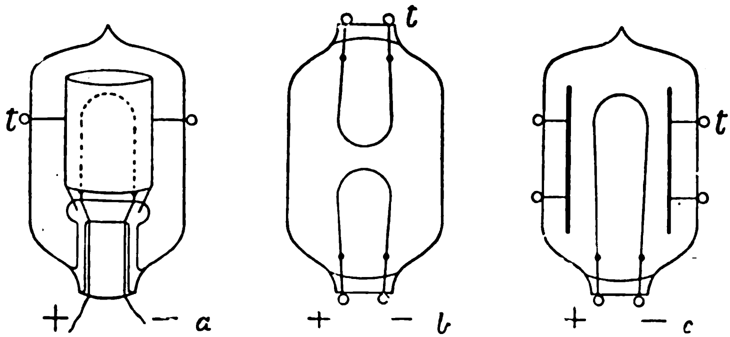


FIG. 37.—Fleming Oscillation Valves.

connect a galvanometer and single cell in series between the two ends of the carbon filaments, which are in connection with the negative poles of the respective insulated batteries. We may employ a single carbon filament and a metal plate or cylinder surrounding it (see Fig. 37*a* or 37*c*), and if we then render the carbon filament incandescent by a local battery, it is found that a single cell will pass a current through the vacuous space between the cylinder and the hot filament, provided that this single cell has its negative pole in connection with that end of

[90] See J. A. Fleming, "On Electric Discharge between Electrodes at Different Temperatures in Air and High Vacua," *Proc. Roy. Soc. Lond.*, 1890, vol. 47, p. 122; also *Proc. Roy. Institution*, vol. 13, p. 45, Friday Evening Discourse, Feb. 14, 1890.

the filament which is itself in connection with the negative end of the heating battery. If the connections of the single cell are reversed, then little or no current passes.

The space between the cold cylinder and the hot carbon filament possesses, therefore, a unilateral conductivity. Negative electricity can pass from the hot filament to the cold metal cylinder through the highly rarefied gas, but not in the opposite direction. The arrangement acts as an *electrical valve* for electric currents. The author furthermore discovered that this device could be used to separate out the two constituent currents of an electrical oscillation,[91] and so render it possible for an electrical oscillation to affect an ordinary galvanometer or a train of oscillations affect a telephone. To do this the valve, now called an oscillation valve, is used as follows:—

One of the above-described bulbs, O, has its insulated plate and filament connected respectively with the secondary coil, S, of an oscillation transformer and a galvanometer or current detector, G, joined in between one terminal of S and the

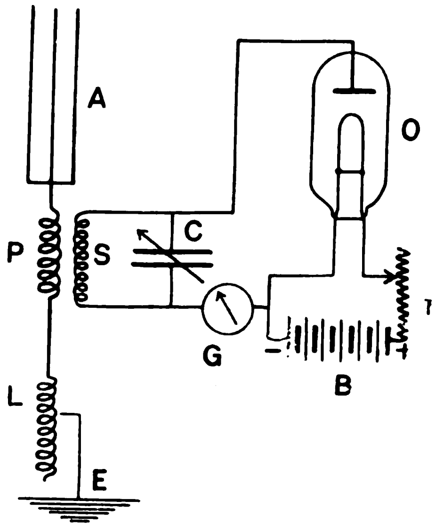


FIG. 38.—Mode of using a Fleming Oscillation Valve to rectify Electric Oscillations and render them detectable by an Ordinary Galvanometer, G.

negative terminal of the carbon filament (see Fig. 38). If electric oscillations are induced in this secondary circuit, S, by a primary coil, P, then when the carbon filament is made incandescent by an insulated battery, B, only one of the currents forming the oscillation is allowed to pass, viz. that in which the movement of negative electricity is from the carbon filament to the metal cylinder through the vacuous space. The galvanometer, therefore, is affected only by the flow of electricity in one direction, and its needle or coil is therefore deflected. In each train of oscillation the positive currents are, so to speak, sifted out from the negative, and only one set allowed to pass. We are therefore able to employ a sensitive mirror galvanometer of the ordinary type to detect the existence of electric oscillations in a circuit.

When the rectifying quality of the valve is to be used for detecting electric waves, or as a receiver in wireless telegraphy, the oscillation transformer, P, S, associated with the antenna, A, has its primary circuit included between the aerial wire, A, and the earth, E (see Fig. 39).

[91] See J. A. Fleming, "On the Conversion of Electric Oscillations into Continuous Currents by Means of a Vacuum Valve," *Proc, Roy. Soc. Lond.*, 1905, vol. 74, p. 476.

The secondary circuit of the transformer is closed by a condenser, C, and one terminal of this condenser is connected to the cylinder or plate of the valve, V, and the other to the negative terminal of the filament. A local battery, B, is employed to incandesce this filament. A telephone, T, is inserted in the circuit.

When electric waves fall on the antenna they excite oscillations in the condenser, C, and these are rectified by the unilateral conductivity of the rarefied gas between the filament and the cylinder, into an intermittent but unidirectional current in the circuit containing the telephone.[92] At each train of oscillation the telephone is traversed by a gush or flow of electricity in one direction giving rise to a sound.

Mr. Marconi modified this arrangement of the author's in 1907 by inserting an additional transformer, J, and condenser, C_1, as shown in Fig. 40.[93]

In both Mr. Marconi's and the author's above-described methods of employing the valve, we are utilizing the unilateral conductivity of the ionized gas which it possesses in virtue of the electrons discharged from the incandescent filament.

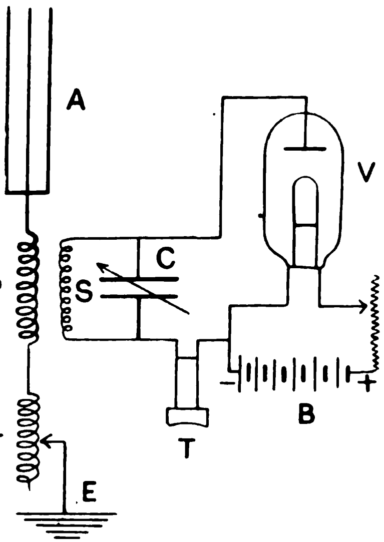


FIG. 39.—Fleming Oscillation Valve used as a Cymoscope or Electric Wave Detector in Wireless Telegraphy. A, antenna; V, valve; B, heating battery; T, telephone; E, earth plate.

On the other hand, we may employ this detector in another manner, depending on the fact that such ionized gas as a conductor does not obey Ohm's law.

It is well known that the conductivity of rarefied gases differs in nature from that of metals or electrolytes. If we apply a steadily increasing electromotive force to a mass of rarefied gas by means of two electrodes, the negative one being incandescent, then the current through the gas does not increase proportionately to the electromotive force. The current rises up to a maximum value, at which it is said to be saturated.[94] Hence the gas, as a conductor, does not obey Ohm's law. Also the conductivity, which is the ratio of current to voltage, rises to a maximum and then falls off. The curves in Fig. 41 show how the current and conductivity

[92] See British Patent Specification, No. 24,850, November 16, of 1904, J. A. Fleming; also U.S.A. Patent Specification, No. 803,684 of April 19, 1905; and German Patent, No. 186,034, issued May 6, 1907.

[93] See British Patent Specification, No. 887 of 1907, G. Marconi.

[94] See *Proc. Roy. Soc. Lond.*, 1905, vol. 74, p. 483; also Prof. Sir J. J. Thomson on "Conduction of Electricity through Gases," chap. viii.

of the vacuous space vary in one of the above-described oscillation valves when increasing voltages are applied between the metal cylinder and the carbon filament, the latter heated to various temperatures.

The resistance of the vacuous space may therefore vary from millions of ohms to a few ohms, according to the voltage applied and the temperature of the filament. The valve rectifies the oscillations or becomes more completely unilateral in conductivity the colder the metal cylinder is kept. If we allow the cylinder to become warmed by radiation from the filament, then the flow of electricity between the carbon filament and cylinder is not altogether in one direction. When made as shown in Fig. 37 and used with a carbon filament at that temperature at which it is working at about 3 watts per candle, the rectification is from 80 to 85 per cent.

If we consider a portion of the characteristic or current voltage curve which lies between the origin and the knee of the curve (which is shown on an enlarged scale in Fig. 42), we shall find that its curvature is not constant, but that

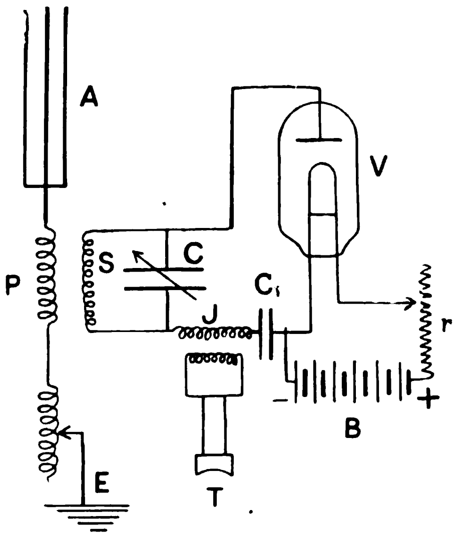


Fig. 40.—Marconi's Arrangement of Circuits for using the Fleming Oscillation Valve as a Radiotelegraphic Receiver.

corresponding to certain abscissæ representing a steady voltage applied to the ionized gas there is a marked change in curvature, so that the second differential of the curve has a large value. If, then, corresponding to this point we increase and diminish the voltage by small amounts, as by superposing an alternating voltage on the steady voltage, the corresponding current will have a mean value which will be greater than the current corresponding to the steady voltage, as already explained in connection with the carborundum crystal (see Fig. 35).

Hence if we apply this critical steady voltage in series with the ionized gas and with a telephone, we shall find that the addition in the circuit of an alternating voltage creates a sound in the telephone.

Accordingly, we can make use of this property of the ionized gas, viz. that its current-voltage curve is not linear, in other words, that it does not obey Ohm's law, in the following manner: An oscillation circuit (see Fig. 43), consisting of a condenser, C, and one coil of an oscillation transformer, P, S, is coupled to an antenna, A, which is in series with the other coil of the oscillation transformer. To one terminal of the condenser in the oscillation circuit is attached the cylinder of the vacuum valve, V. The filament of the valve is rendered incandescent by a

local battery, B, and regulated by a variable rheostat in series with it. This battery is also shunted by a high resistance, *r*, having a sliding contact upon it, and this contact is connected with the other terminal of the condenser in the oscillation circuit through a telephone, T. It will be seen, therefore, that this arrangement is equivalent to putting a certain exactly adjustable steady voltage

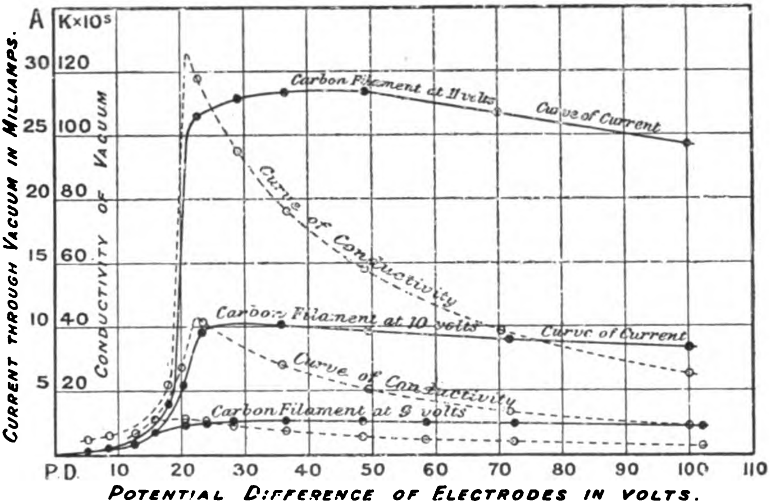


FIG. 41.—Curve showing the Variation of Current through a Fleming Oscillation Valve with Increasing Potential Difference between the Negative Carbon Filament Terminal and Insulated Plate.

in series with the telephone and the ionized gas of the valve, and also making arrangements for superimposing upon this critical steady voltage an oscillatory voltage derived from oscillations set up in the condenser circuit. If, then, the slider on the rheostat, *r*, is adjusted to a position such that the voltage applied

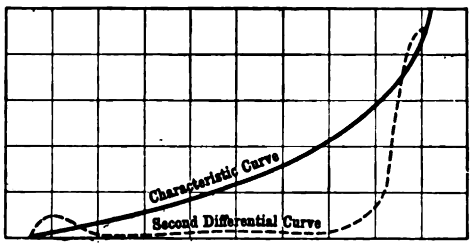


FIG. 42.—Lower part of the Characteristic Curve of Highly Rarefied Gas between Heated Electrodes.

to the ionized gas is exactly sufficient to bring it to that point on the characteristic curve at which a sudden change of curvature takes place, or change in conductivity, then it will be found that the setting-up oscillations in the condenser circuit will superpose upon this steady voltage an alternating voltage, and in accordance with the principles just above explained, this will cause a greater current to flow through the telephone, and a sound will therefore be produced in the telephone if electric waves continue to fall upon the antenna. The author found later on that greatly improved results can be obtained by employing a particular type of glow lamp

with a tungsten filament and an insulated cylinder of copper surrounding it. The electronic emission from the tungsten is greater than from carbon, probably because it is a better conductor, and can be raised without volatilization to a much higher temperature than carbon.[95] The author now uses this tungsten glow-lamp detector as above described. It will therefore be seen that both with the crystal and ionized gas detectors there are two methods of procedure in using them as radiotelegraphic receivers. We may either make use of the property of unilateral conductivity possessed by these substances and rectify trains of oscillations by means of them, or we may make use of the non-linear character of the characteristic curve. As already mentioned, any form of electric conductor in which the characteristic volt-ampere curve is not a straight line is capable of acting as a detector for electric oscillations owing to the fact that their characteristic curves exhibit changes of curvature at certain points, and that therefore the superposition

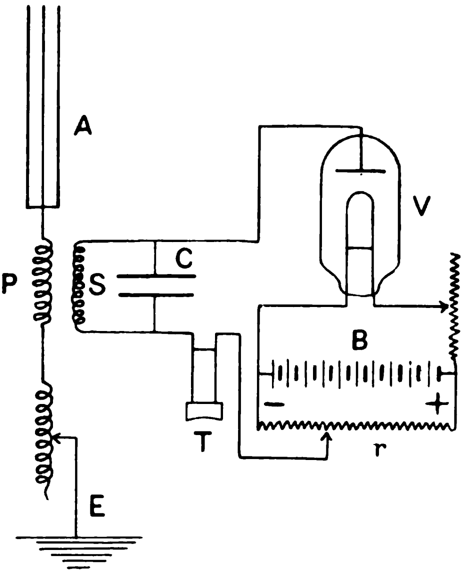


FIG. 43.—Diagram of Circuits when using the Fleming Oscillation Valve as a Cymoscope based on the Non-linear form of the Characteristic Curve of Rarefied Gas.

of an alternating voltage upon a steady voltage may be caused to create an increase of current through such a conductor.

In connection with this tungsten and carbon glow-lamp detector the question has been raised as to the origin of the ions which proceed from the filament. Some experimentalists have contended that they originate in gas occluded in the carbon or tungsten, and that if the filament is very carefully freed from this gas the ionic emission ceases.

Professor O. W. Richardson [96] has described careful experiments to show that this is not the case, and that the negative ions are electrons which are emitted from the incandescent body. These electrons are not created out of the tungsten or the surrounding gas; they must therefore be the free electrons which exist in the

[95] See British Patent Specification of J. A. Fleming, No. 13,518 of 1908.

[96] See *The Electrician*, vol. 73, p. 22, April 10, 1914, "On the Emission of Electrons from Tungsten at High Temperatures." See also *Phil. Mag.*, vol. 26, p. 345, August 1913. See also O. W. Richardson, *Phil. Trans. Roy. Soc.*, vol. 201, A, p. 497, 1903, "On the Electrical Conductivity Imparted to a Vacuum by Hot Conductors."

metal and bestow upon it electric conductivity according to the electronic theory. This emission of electrons is called a thermionic current. At high temperatures it may amount to several milliamperes from a filament 3 or 4 inches long and of the diameter used in tungsten lamps. Professor O. W. Richardson showed that the number of electrons, N, given off per square centimetre per second from an incandescent filament of carbon or metal at an absolute temperature θ, when in a good vacuum corresponding to the saturation current, is represented by the formula—

$$N = A\sqrt{\theta}\,\epsilon^{-b/\theta}$$

Where for carbon the constant, A, has some such value as 10^{34} and b some value between $7{\cdot}8 \times 10^4$ and $11{\cdot}9 \times 10^4$.

If the bulb including the conductor to be rendered incandescent is not exhausted to an extremely high vacuum and if the glass surfaces and the conductor itself are not perfectly freed from adhering or occluded gas by prolonged heating, then secondary effects arise which are due to the ionization of the residual gas by the electrons emitted by the incandescent body. The result is the production of positive ions from the gas molecules, and these being drawn in towards the filament bombard it and produce disintegration of the cathode surface. If, however, an extremely high vacuum is made in the bulb, and all occluded gases removed by heating, the phenomena of pure thermionic emission are obtained. By such precautions Dr. I. Langmuir has made very effective forms of the author's oscillation valve which for some reason he has found necessary to christen by the new name of Kenotron. The author, however, states distinctly in his British Patent Specification of November 1904, that the vacuum in the bulb should be as high as possible.

In April 1904 Dr. A. Wehnelt[97] described a discovery connected with the issue of ions from glowing solids, of considerable interest. There are some bodies, such as the oxides of barium and calcium, which exhibit very considerable thermionic emission, far greater than carbon and metals. He found that if a metal strip of platinum is covered with a thin layer of oxides of calcium, barium, or strontium, and if the platinum is rendered incandescent by an electric current, such a strip, if used as the cathode in a vacuum tube, gives an abnormally large discharge of negative ions. This cathode is now called a Wehnelt cathode. Such cathode can be made by placing a fragment of lime on a platinum strip which is electrically heated. Dr. Wehnelt applied his discovery in the construction of a form of vacuum valve by sealing into a glass bulb an anode terminal and also a cathode consisting of an oxide-covered platinum strip, the strip being heated to a bright red heat by an electric current, and the glass bulb exhausted of its air.[98] He described this device in a German patent in 1904, as an electric valve, but did not claim its use in connection with radiotelegraphy. As already mentioned, the author had shown in 1890 the use of a vacuum tube with incandescent cathode as a means of rectifying alternating currents; and in 1904 used it for rectifying electric oscillations and as a radiotelegraphic receiver.

The Wehnelt oxide cathode has been applied by R. von Lieben, E. Reisz, and S. Strauss in the construction of a sensitive relay which can also be used as a radiotelegraphic receiver.[99]

The construction of the appliance is as follows: A glass bulb like that of an incandescent lamp has in it a strip of platinum, K, coated with barium and calcium oxides as its filament (see Fig. 44). This is rendered incandescent by a local battery. Above the glower is a metal grid, H, which has an external terminal

[97] See A. Wehnelt, "Ueber den austritt Negativer Ionen aus glühenden Metallverbindungen und damit zusammenhängende Erscheinungen," *Annalen der Physik*, vol. 14, p. 425, 1904. Also A. Wehnelt, "On the Discharge of Negative Ions by Glowing Metallic Oxides," *Phil. Mag.*, July 1905, p. 80.

[98] See Dr. A. Wehnelt, "Ein Elektrisches Ventilrohr," *Ber. der Physikalisch Medizinischen Soc. in Erlangen*, vol. 37, 1905.

[99] See E. Reisz, "A New Method of Magnifying Electric Currents," *The Electrician*, vol. 72, p. 726, February 6, 1914. See also German Patent Specification of E. von Lieben, No. 179,807, and British Patent Specification, No. 1482 of 1911.

sealed through the glass. Above this is another platinum wire, A, sealed through the glass.

When used as a radiotelegraphic receiver the Wehnelt cathode is made incandescent by a battery of 30 volts. Across the terminals of this battery is a resistance, and a sliding contact on this resistance is connected to one terminal of the receiving circuit condenser. The other condenser terminal is connected to the grid. The anode, A, is connected through an auxiliary or boosting battery, and through a telephone receiver with the positive terminal of the heating battery.

A similar or very similar arrangement had previously been described by Lee de Forest in a United States Specification, No. 879,532, applied for January 29, 1907, one diagram of which is reproduced in Fig. 45. It is this particular form of ionized gas detector to which de Forest now applies the name *Audion* (see also British Patent Specification, No. 1427 of 1908).

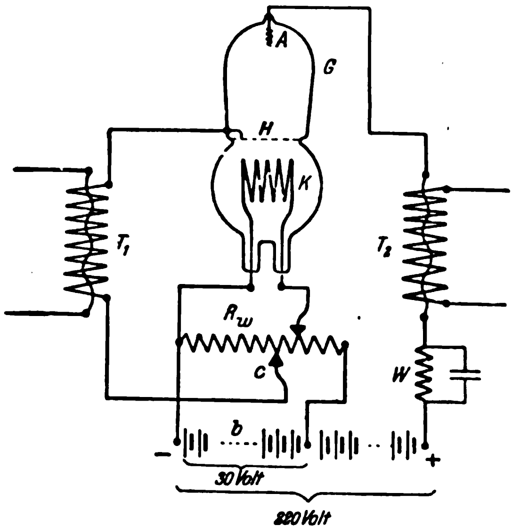


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FIG. 44.—Thermionic Relay of Lieben, Reisz, and Strauss.

This appliance consists of a glass bulb, D (see Fig. 45), having sealed into it a carbon or metal filament, F, like an incandescent lamp. The bulb is highly exhausted. Also a metal plate, *b*, carried on a wire sealed through the bulb, is fixed, as in the case of the author's oscillation valve. In addition to this another zigzag or grid of wire, *a*, is sealed into the bulb, which is placed between the filament and the above-named insulated plate. In operation the filament is rendered incandescent by a local battery, A, and the negative leg of the filament and the grid are connected respectively to the terminals of the condenser C in the closed receiving circuit. A separate condenser, C′, is interposed between the grid and the receiving condenser. The insulated plate *b* is connected with the positive terminal of the filament through a circuit which contains a telephone receiver, T, and a separate boosting battery, B, the negative pole of which must be joined to the positive leg of the filament. The *audion* differs, therefore, from the author's oscillation valve in having a grid and a plate sealed into the bulb in place of the single metal cylinder surrounding the filament. A very similar appliance has been developed by Dr. Langmuir and called by him a *Pliotron*. The distinguishing characteristic of this last-named form is the employment of tungsten wire for the incandescent filament, and grids of the same material for the two anodes. The

tungsten wire is most thoroughly freed from occluded gases and an extremely high vacuum made in the bulb. Under these conditions the physical phenomena in the bulb are almost wholly determined by the emission of electrons from the incandescent filament, and secondary effects due to the ionization of residual gases in the bulb are absent. In the case of the highly vacuous bulb containing one cold insulated plate in addition to the incandescent filament, which by whatever name it is rechristened is only the author's oscillation valve, we can employ it as already shown in two ways as a detector of oscillations. We can either make use of the simple rectifying power of the vacuous space to rectify trains of oscillations into single gushes of electricity, or we may make use of the fact that the characteristic curve has changes of curvature in it. Hence if a steady boosting voltage is employed to impress on the space between the filament and plate the particular voltage corresponding to this change of curvature, it will be found that the addition of a small alternating voltage increases the current through the vacuous space as already explained in connection with crystal detectors. But the vacuous bulb with hot electrode and grid and plate in it, as in Fig. 45, has an additional property. In this case a characteristic curve may be plotted, the abscissæ of which

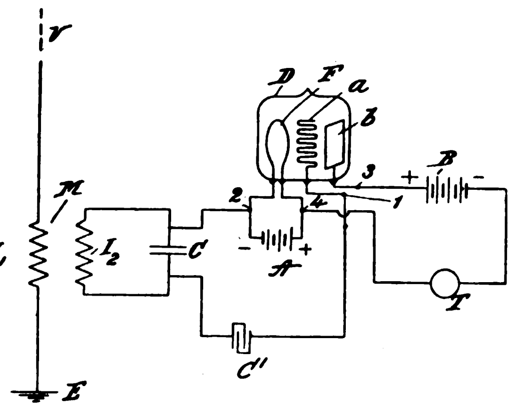

FIG. 45.—Double Anode Thermionic Detector or Audion.

are potential differences between the filament and the grid and the ordinates, the current flowing under the applied boosting voltage from the filament to the anode plate. It is found that this curve has marked changes of curvature, and that at one point it trends very rapidly upwards. This shows that at a certain point a small increase in the positive potential of the grid will make a very great increase in the current flowing to the plate. When the grid is charged negatively it repels the electrons reaching it from the filament and prevents them from reaching the plate. When the grid is positive it attracts the electrons and more pass through it and reach the plate.

The function of the grid is to catch or deflect the ions emitted from the incandescent body, and hence to alter the conductivity of the gaseous medium between the grid, *a*, and the plate, *b*, or second anode. This varies the current through the telephone, and by the use of a high-boosting voltage the arrangement acts as an amplifier as well.

In a high vacuum three-electrode valve the central portion of the characteristic curve is nearly a straight line. Hence any variation of grid potential, plus or minus, varies also in linear manner the current flowing from filament to plate. The amplifying power is measured by the ratio of increment of plate current to increment of grid potential. In a good instrument a variation of grid potential from -2 to $+2$ volts should increase the current by 1 milliampere.

The action of this last form of thermionic receiver is dentical with that of the

author's oscillation valve as far as concerns the physical operations taking place between the incandescent filament and the grid. The unilateral conductivity of this space causes the grid and any condenser in series with it to become charged negatively or to be robbed of a positive charge. Hence the occurrence of a train of oscillations in the aerial wire and closed condenser circuit in connection with it causes a pulsatory negative potential to appear on the grid. This reduces the current flowing from the battery through the telephone which has its negative terminal attached to the incandescent filament and its positive end to the anode or plate on the other side of the grid.

This arrangement has been called an amplifier because the variation in the current through the telephone is greater than would be the case if the telephone were inserted in the external circuit connecting the grid to the filament as in Fig. 39.

The fundamental principle of the appliance is, however, the same as in the case of the author's oscillation valve, viz. the thermionic emission of negative ions from the incandescent filament.

The reader may be referred for further information to a Paper by Dr. E. H. Armstrong, in the *Proceedings of the Institute of Radio Engineers* of New York, vol. iii. p. 215, September 1915, on "The Audion as Detector and Amplifier." Also to a Paper by Dr. Irving Langmuir in the same volume.

An important quality of this double anode thermionic detector is that two or more of these appliances can be connected in series, so that variations in the potential of the grid of the first creates greater variations in the anode plate, and this again is made to vary the potential of the grid of the second amplifier, and hence to create a still greater variation in its plate potential.

The arrangement is shown in Fig. 18 of Chap. X. It is called a chain of thermionic amplifiers. It will be seen by examining the diagram referred to that the oscillations in an antenna are rectified by the unilateral conductivity of the first thermionic bulb, and that its grid will have a negative charge created in it which in turn affects the potential of the anode plate and raises it. Hence there is a reduced current through the first plate circuit. This is transformed by the transformer coupling the two bulbs, so that variations of the negative potential of the grid of the second bulb take place. These in turn alter the current in the plate circuit of the second bulb, so that an increased variation takes place in the current passing through the telephone in that circuit. We have, therefore, a multiplying or amplifying action which enormously increases the effect produced by a feeble train of oscillations in the antenna.

One single thermionic detector with grid and plate can, however, be made to effect an amplifying action by coupling inductively the grid and plate circuits as shown in Fig. 7 of Chap. X. On reference to that diagram it will be seen that the oscillations in the antenna are caused to create oscillations in a coupled condenser circuit, which also includes the primary coil of an oscillation transformer, the secondary circuit of which is included in the plate circuit. Hence, any variations in this latter current react back on the grid circuit, and by winding the two transformer circuits in the right directions these currents in the plate and grid circuits may be made to amplify each other by a reaction which resembles that of the self-exciting dynamo.

Moreover, this reaction may be increased to a point at which the oscillations become self-sustaining, the energy being drawn from the battery in series with the anode plate. The appliance then becomes a means of producing oscillations as explained in Chap. X.

A large number of ways have been devised for using this three-electrode thermionic valve detector in wireless telegraphy as a receiving and amplifying detector. For details the reader must be referred to a special book on the subject by the author entitled "The Thermionic Valve in Radiotelegraphy and Telephony" (The Wireless Press, Ltd., Marconi House, Strand, London).

16. Physiological Oscillation Detectors.—The nervous system of all animals is affected by electric discharges and currents. The invention of the Leyden jar obtained notoriety chiefly in consequence of the "shocks" it could administer. In fact the effects were described by the early observers with an exuberance of

language scarcely justified by our experiences. One experimentalist on taking the discharge of a small Leyden phial declared that not for the whole kingdom of France would he endure another. Amongst other vertebrates the frog, that old martyr of science, seems to exhibit considerable sensibility when a discharge is passed down his sciatic nerve. Hence from the time of Galvani a frog's leg has been used as an indicator of electric currents. Accordingly, Dr. Lefeuvre of the University of Rennes, in France, has made use of it as a radiotelegraphic receiver. (See *The Electrician*, vol. 71, p. 93, 1913.)

The frog's leg is mounted so that contraction of the muscle moves a lever which makes a mark upon a revolving drum covered with blackened paper. The current received from an antenna is passed through the nerve (see Fig. 46), and the resulting contraction of the leg when a radiotelegraphic signal is received by

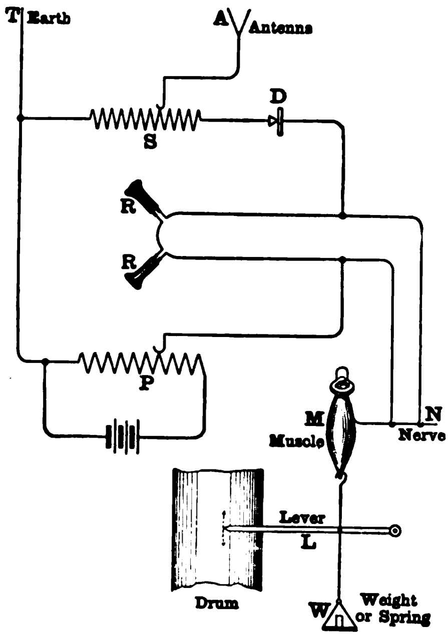

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Fig. 46.—Method of using a Frog's Leg Muscle for the Detection of Electric Waves.

the antenna makes a mark on the drum. In this manner Professor Lefeuvre has recorded signals coming from the Eiffel Tower station in Paris received at Rennes.

Fig. 47 shows the frog arranged as a receiver, and Fig. 48 the dot and dash time signals recorded.

17. Wave Measuring Instruments or Cymometers.—In addition to detecting the existence of electric waves passing through space by the oscillations which they can create in a linear conductor incident upon it, we often desire to measure the wave-length of these waves, either those sent out from a radiator or those received by a detector. In those cases in which the radiation is taking place, from a rod or wire in which high frequency oscillations are set up, the wave-length λ of the radiation is connected with the frequency n of the oscillations in the linear oscillator by the formula $V = n\lambda$, where $V = 3 \times 10^{10}$ cms. per second or is the velocity of radiation. Hence to determine the wave-length it suffices to determine the frequency of the oscillations in the antenna. Suppose that the

antenna is inductively associated with another closed circuit, and the radiator is a Marconi aerial wire. There are several methods by which we can ascertain the frequency of the oscillations taking place in this aerial wire. The following instruments have been invented for this purpose, and those devised by the author have been called by him *cymometers*. One form of instrument depends upon the establishment of stationary waves upon an associated helix.

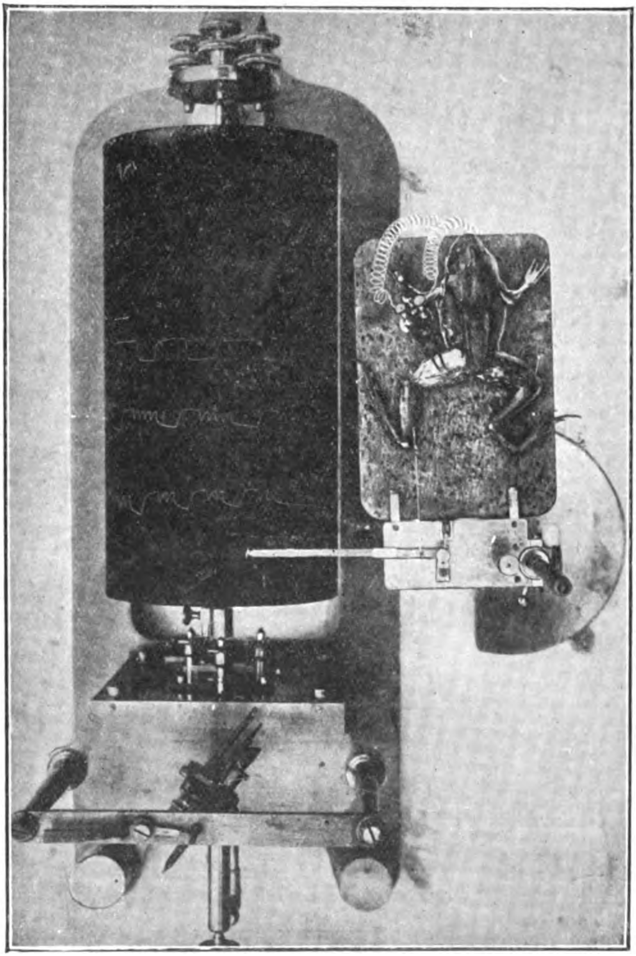

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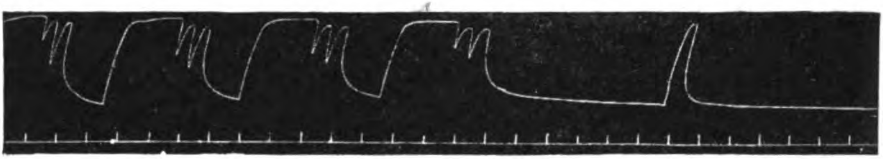
FIG. 47.—Frog's Leg used as a Radiotelegraphic Receiver.

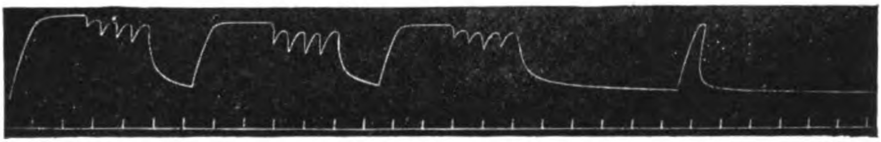
We have already explained, in Chap. IV., the conditions under which stationary electric oscillations of potential and current can be established upon an insulated helix of wire. Suppose that we have a helix consisting of a long ebonite rod, say 2 metres long and 5 cms. in diameter, wound over uniformly with a long spiral of fine silk-covered copper wire. This wire may suitably be of the size known as No. 32 S.W.G., and on such an ebonite core it will be possible then to wind in one layer of closely adjacent turns a helix of 5000 turns. Let such a helix,

K_1K_2, be supported on insulating stands (see Fig. 49) a couple of feet above a table in contiguity to an antenna, A, in which it is desired to measure the frequency. To some point near to the base of the aerial is attached a small insulated metal plate, which acts as one plate of a small air condenser, C_1, the other plate being connected to one end of the above-described insulated helix. On this helix slides a curved metal saddle, D, which is packed with tinfoil to make it fit the helix closely, and this saddle is connected by a wire with an earth plate, E_2. We provide also a Neon, or other vacuum tube, V. Let us assume, then, that the oscillations are excited in the aerial wire; we can move the saddle along the helix until such a position is found that one complete stationary wave of potential on the helix is included between the saddle and the end attached to the small air condenser. When this is the case, if we explore the space round the helix neewteb

Signal at 10.45—Series of dashes; one dot at 10.45.



Signal at 10.47—Dash and two dots; one dot at 10.47.



Signal at 10.49—Dash and four dots; one dot at 10.49.

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FIG. 48.—Physiological Records of Paris Time Signals received at Rennes, France, by Dr. Lefeuvre, by Frog's Leg-receiver.

these points with a Neon vacuum tube, we shall find that just over the end of the saddle the Neon tube does not glow, also it does not glow at a point half-way between the saddle and the condenser end, also it does not glow just at the end next the condenser. In order that this may be the case, it is necessary to shield the helix from indirect action of the oscillation in the aerial or the spark of the transmitter; it is necessary to place a metal plate (not shown in the diagram) close to the end of the helix which is in connection with the aerial. This plate must be perforated by a hole large enough to allow the end of the helix just to pass through, the air gap being large enough to prevent sparking from the end of the helix to this plate. This guard plate must also be connected to the earth by a wire. By moving the saddle about, it is then possible to find a position in which there is a node of potential over the saddle near the earth plate, and also half-way between, whilst at intermediate positions, $\frac{1}{4}$ of the way and $\frac{3}{4}$ of the way, there is an antinode and loop of potential, as indicated by a dotted line in the diagram in Fig. 49.

We have already explained, in Chap. IV., that the velocity with which the potential wave moves along the helix is inversely proportional to the square root of the capacity and inductance per unit of length of the helix, and we have shown how these quantities can be accurately measured. Hence this velocity can be determined for the helix in use in centimetres per second. By means of a scale, E, we can also measure the observed wave-length on the helix as indicated by the distance of the saddle from the earth plate, when the potential distribution is as above described. The quotient of this velocity along the helix by the observed wave-length gives us the frequency of the stationary oscillations

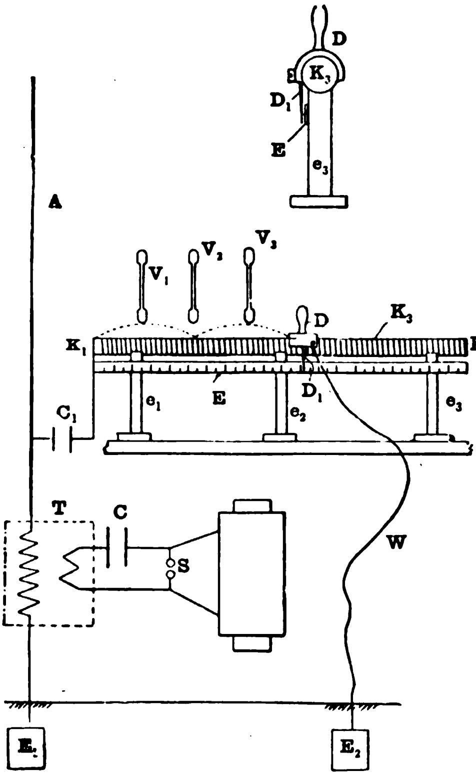


FIG. 49.—Helix Cymometer. (Fleming.) K_1K_2, helix of wire; D, sliding saddle; C_1, coupling condenser; A, antenna.

on the helix. This must also be the frequency of the oscillations in the aerial wire. Hence, if v stands for the velocity of propagation of the oscillations along the helix, and λ for the wave-length on the helix, or the length K_1D, then, if Λ stands for wave-length of the waves sent out into space from the aerial, we have the following formula for this wave-length :—

$$\Lambda = \lambda \frac{3 \times 10^{10}}{v}$$

In the above formula v must be measured in centimetres per second, and λ and Λ in the same linear units.

In this manner, given a helix sufficiently long, we can determine the frequency

of the oscillations in any aerial wire, and therefore the wave-length of the waves sent out into space from it.

The author has also devised a form of direct-reading portable cymometer, by which not only can the wave-lengths of waves used in wireless telegraphy be immediately ascertained, but also by its aid numerous measurements, such as the measurement of small capacities, inductances, and coefficients of coupling of oscillation transformers, can be made with great ease.[100] This instrument is constructed as follows :—

It consists of a condenser of variable capacity constructed of a tube of brass covered with ebonite, on the outside of which another concentric tube fits closely, but not so tightly as to prevent easy movement. If the tubes lie over one another, such a double brass tube with interposed tube of ebonite constitutes a tubular condenser, but if the outer tube is more or less slid off the inner brass tube the capacity is reduced almost proportionately to the displacement of the outer tube. Again, if we have a wire wound in the form of a helix round an ebonite tube, the turns being close together, but not touching, and if we have some form of clip which can be slid along the helix so as to make use of more or less of the spiral, we have a variable inductance.

These two appliances are combined together in the cymometer in such a way as to form a complete oscillatory circuit; the inner end of the tubular condenser (see Fig. 51) is connected to one end of the helix of wire by a copper bar, and the outer condenser tube, O, is connected to the helix by an embracing clip, K, so that as the outer condenser tube is displaced from the inner tube to reduce the capacity, the effective inductance in the circuit due to the spiral is reduced in the same proportion. The helix and the tubular condenser, which may be formed of two or more tubes, are mounted on a board, and by means of a handle the condenser tube can be moved and the inductance and capacity simultaneously altered, and in the same proportion. The appearance of the instrument is as shown in Fig. 50, and the scheme of connections as in Fig. 51. If, then, we place the long copper bar, BC, connecting the helix and condenser near but not very close to any other circuit, XY, in which oscillations are taking place, we can tune the cymometer circuit to the other circuit by moving the handle so as to vary the inductance and capacity of the cymometer. We must also have some means of determining when the current in the cymometer, or the potential difference of the tubes forming the condenser, is a maximum.

The author discovered that the most convenient way of doing this was by the use of a vacuum tube of the spectrum type, filled with Neon. Neon is a rare gas contained in the atmosphere, about 80,000th part by volume, and it is remarkable for its small dielectric strength and for the great brilliancy of the glow produced in it when placed in an alternating current field. If such a Neon tube is connected to the terminals of the tubular condenser, then, when the capacity and inductance are altered and the oscillation in the cymometer circuit thereby increased up to a maximum, it is easy to determine the position when this maximum takes place by the Neon tube beginning to glow, or glowing most brilliantly. The most recent form of the cymometer is shown in Fig. 52, in which a screw motion is provided for varying the capacity and inductance very slowly.

Another method of discovering when the current is a maximum in the cymo meter circuit is by inserting in the circuit of the copper connecting bar a fine wire of high resistance about a centimetre in length, having in contact with it a very sensitive thermo-junction of bismuth and iron. This thermo-junction is connected to a sensitive galvanometer, preferably a Paul single-pivot, low-resistance galvanometer (see Fig. 53). If, then, by the movement of the handle of the cymometer it is gradually tuned with any adjacent circuit in which oscillations are taking place, the increase in the current up to a maximum will be indicated by a gradual increasing deflection of the galvanometer, and it is quite easy to determine that adjustment of the cymometer in which the current is a maximum.

The cymometer has a graduated scale with a pointer moving over it, and the

[100] See J. A. Fleming, "The Application of the Cymometer to the Determination of the Coefficient of Coupling of Oscillation Transformers," *Phil. Mag.*, 1905, ser. 6, vol. 9, p. 758; also *Proc. Phys. Soc. Lond.*, 1905, vol. xix. p. 603.

instrument is calibrated by the manufacturer so as to show at a glance the frequency corresponding to any particular adjustment of the tubular condenser. The author has designed such instruments for reading frequencies from 50,000

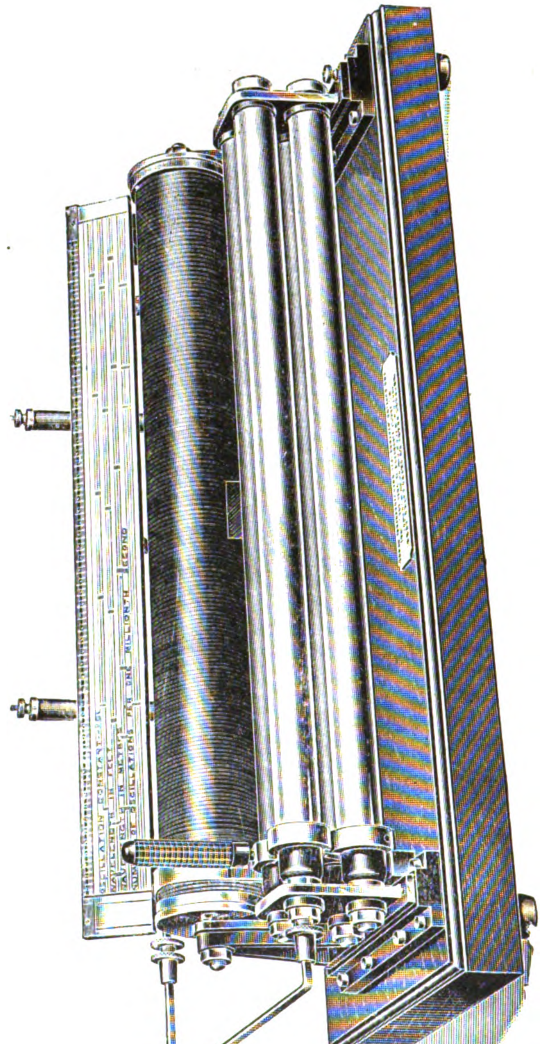


FIG. 50.—The Fleming Cymometer.

up to 5,000,000, and the appearance of the complete instrument is as shown in Figs. 50 and 53.

From the above descriptions it is evident that cymometers or wavemeters may be divided into the two broad classes of *open* and *closed* circuit instruments.

The general principles which must guide us in the construction or selection of a wavemeter were well stated by Professor A. Slaby.[101]

In the first place, it is obvious that the measuring instrument must not disturb

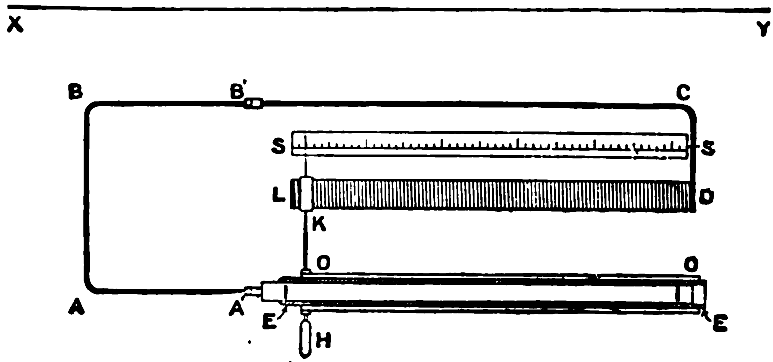


FIG. 51.—Connections of the Fleming Cymometer.

the natural frequency of the oscillation it is desired to measure. If we wish to measure the pressure of gas in a vessel or the electrical potential difference between two points, it is clear that our pressure gauge or voltmeter must not alter

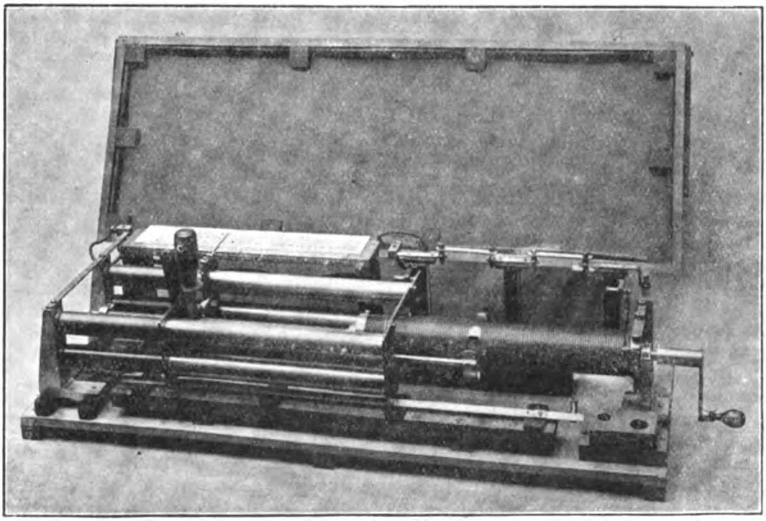


FIG. 52.—Recent Type of Fleming Cymometer for Measuring the Wave-Lengths of Radiotelegraphic Waves.

the value of the quantity we desire to measure by the very act of connecting the instrument used to take a measurement. If it does, although we may obtain a reading, we do not obtain the real value of the quantity we are seeking.

[101] See *Elektrotechnische Zeitschrift*, December 10, 1903, vol. 24, p. 1007.

In the same way, if we wish to know the frequency of the oscillations in any circuit, then it is clear that the instrument we employ must not alter the capacity or inductance of the circuit tested in the act of making a measurement. As the capacities and inductances with which we are concerned are generally very small, it is quite easy to be misled on this matter. Hence, if we are testing the frequency of the oscillations in an aerial wire, as used in wireless telegraphy, it is of the utmost importance not to disturb the small capacity of the aerial by connecting it to any object which will sensibly increase it. Also we must not increase its inductance by making loops or coils in it.

In all forms of open circuit or helix cymometer there is a great loss of energy by radiation. Hence these generally require more applied energy to work them than do the best forms of closed-circuit cymometer.

We may also distinguish cymometers by the mode in which they are coupled to the circuit, open or closed, in which the oscillations exist, the frequency of which we desire to know. This coupling may be electrostatic or electromagnetic, and involve either a capacity or a mutual inductance. In either case this coupling capacity or mutual inductance must be very small, for the reasons already given. It is easier to render the mutual inductance small, and better, therefore, to employ a closed-circuit cymometer. Hence the author has given preference to the closed-circuit form in the instrument designed by him.

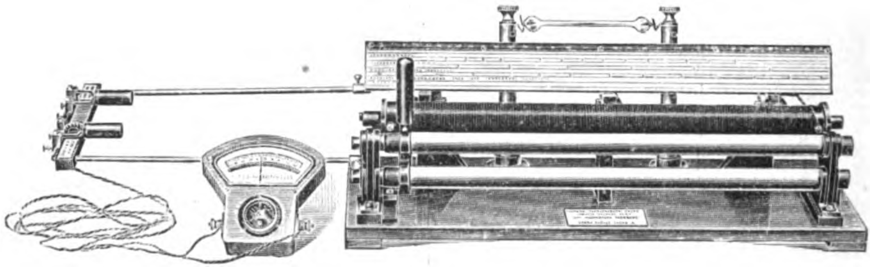
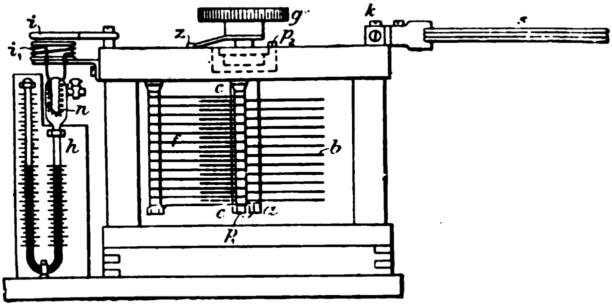


FIG. 53.

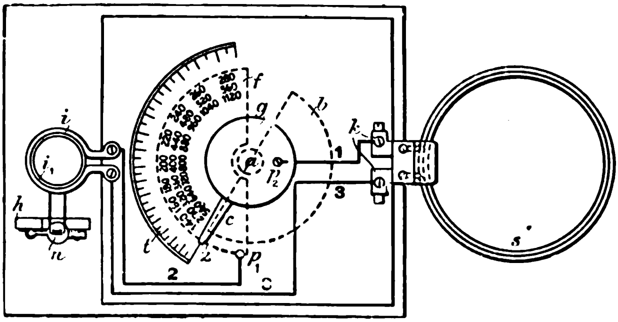
Another form of closed-circuit wavemeter has been designed by J. Donitz.[102] He employs an arrangement consisting of a circular coil having a definite inductance, in series with a condenser made of series of semicircular discs, the capacity of which can be varied within limits by the revolution of these discs on an axis, the arrangement of the condenser plates somewhat resembling that of a Kelvin multicellular voltmeter (see Figs. 54 and 55). These plates may be immersed in insulating oil. In inductive connection with part of the circuit is another small circuit, including a fine-wire platinum coil sealed up in the bulb of an air-thermometer. Hence the production of the maximum current in the inductance coil and condenser is estimated by the reading of the air-thermometer becoming a maximum. The instrument is used as follows: If it is desired to measure the frequency, and therefore the wave-length, of the oscillations in any circuit open or closed, a loop is formed on that circuit, which is placed parallel to and at some little distance from the circular coil of the wavemeter. The oscillations in the first circuit are then permitted to induce others in the wavemeter circuit, and the capacity of this last is altered by varying the condenser until the air-thermometer gives its maximum reading. When this is the case, it is assumed that the time period of the two oscillations is the same, that of the wavemeter being, of course, known from the known inductance and capacity of the circuit. Various coils are provided with the instrument to give it a suitable range of measuring power.

For portable and commercial purposes, the Marconi Company make up

[102] See *Elektrotechnische Zeitschrift*, 1903, vol. 24, pp. 920-925, No. 5; also *The Electrician*, January 1, 1904, vol. 52, p. 407; also German Patent, No. 149,350, Class 21 G.



(*Elevation.*)



(*Plan.*)

FIG. 54.—Dönitz Wavemeter.

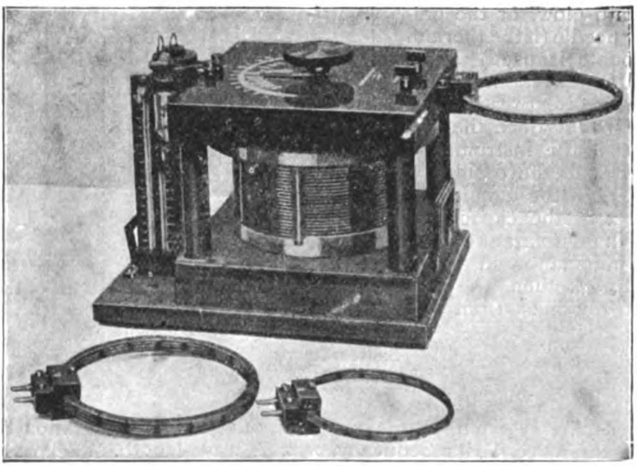


FIG. 55.—Dönitz Wavemeter. (*Perspective view.*)

a compact form of wavemeter shown in Fig. 56. It consists of a variable capacity condenser made with sheet ebonite as a dielectric, the capacity of which is changed by rotating a handle; and in series with this condenser is a fixed inductance in the lid of the box, consisting of a square coil of wire. This coil also serves for coupling with the circuit under test. The detector is a carborundum crystal, *Cr*, in series with a telephone, T, and the two are joined across the terminals of the condenser C, as in Fig. 57. The range of the instrument can be varied over wide limits by the use of interchangeable coils, L, having different inductances.

[*By permission of The Wireless Press, Ltd.*

FIG. 56.—The Marconi Company's Wavemeter.

18. Measurements made with the Cymometer—Frequency and Wave-Length.—The following measurements can then be made with the cymometer or equivalent forms of wavemeter.

1. *Frequency.*—It has already been shown that the frequency of the oscillations in a circuit having a capacity of C microfarads and the inductance L microhenrys, is given by the formula—

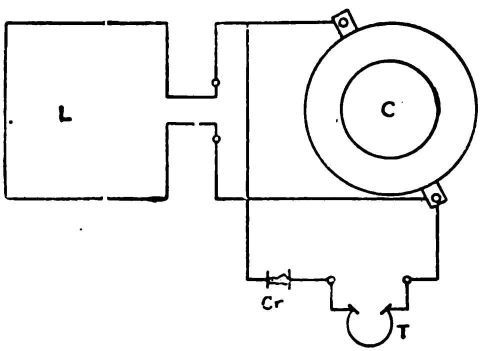
$$n = \frac{10^6}{2\pi\sqrt{C_{\mathrm{mfds.}} L_{\mathrm{mhys.}}}}$$

or if the inductance is measured in absolute electromagnetic units or centimetres, then the frequency is given by the formula—

$\frac{\sqrt{1000}}{2\pi}$

$$n = \frac{5{\cdot}033 \times 10^6}{\sqrt{C_{\mathrm{mfds.}} L_{\mathrm{cms.}}}}$$

If, then, we desire to determine the frequency of the oscillations in an antenna or other circuit, we place the bar of the cymometer in contiguity to that circuit, and move the handle of the cymometer along slowly until the cymometer circuit is in resonance with the other. This will be indicated by the Neon tube bursting into glow, or the galvanometer needle (if the thermo-electric couple is used) taking its maximum deflection. If the cymometer is direct reading, we can then see at once the frequency, or if the instrumental reading gives us the oscillation constant (O) of the circuit, viz. the product of the capacity of the circuit in microfarads ($C_{\mathrm{mfds.}}$) and the inductance in centimetres (L_{cms}), then the frequency is obtained from the formula—



[*By permission of The Wireless Press, Ltd.*

FIG. 57.—Scheme of Connections of the Marconi Company's Wavemeter.

$$n = \frac{5{\cdot}033 \times 10^6}{\sqrt{C_{\mathrm{mfds.}} \times L_{\mathrm{cms.}}}} = \frac{5{\cdot}003 \times 10^6}{O}$$

2. *Wave-Length.*—In all cases of wave motiont here is a relation between the eelocity of the wave, V, its frequency, n, and wave-length, λ, expressed by the qvuation—

$$V = n\lambda$$

The velocity of the electromagnetic waves being 300 million metres per second, or very nearly one thousand million feet per second, it follows that the wave-length is at once obtained by dividing this last number by the frequency. Hence, if the frequency of the oscillations in an antenna is determined, we have the wave-length of the emitted waves. If, then, we can determine the oscillation constant of the antenna, or of the circuit which is radiating, we have at once the following rules :—

Wave-length in feet $= 195{\cdot}56 \times$ oscillation constant.

Wave-length in metres $= 59{\cdot}6 \times$ oscillation constant.

Frequency in millionths of a second is $5{\cdot}033 \div$ oscillation constant.

In order to determine the wave-length, therefore, all that is necessary is to place the bar of the cymometer parallel, but not very near to a portion of the lower part of the antenna. For this purpose, a yard or two of the antenna may be laid in a horizontal position, if necessary. On exciting the oscillations in the antenna and moving the handle of the cymometer, we shall find a position in which the Neon tube glows most brilliantly if the cymometer has a suitable range. In the case of inductively coupled antennæ, it will generally be found that there are two wave-lengths being emitted, and therefore two positions in which the Neon tube has a maximum glow. In so using the cymometer, it is desirable to put the bar as far as possible from the antenna after having roughly discovered the approximate wave-length, and then to take a fresh reading, so adjusting the distance of the cymometer bar from the antenna, that the Neon tube only just glows on passing through to a position of resonance. With a little care it is possible to determine the wave-lengths of the order of 1000 or 1500 feet within 10 feet.

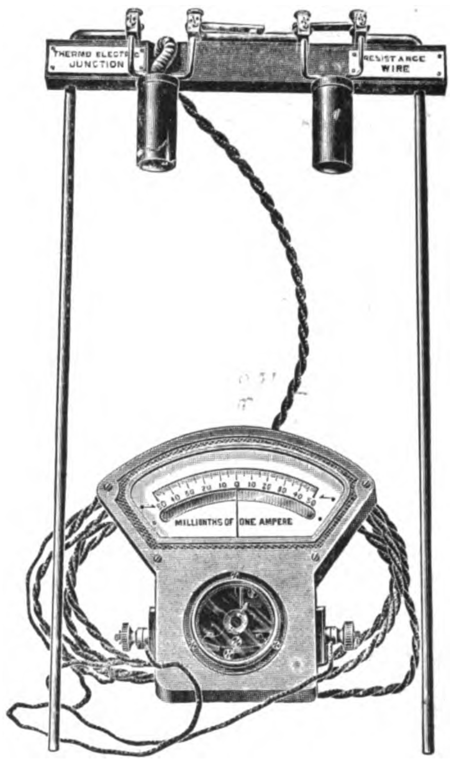


FIG. 58.

Four types of cymometers are now made, one suitable for measuring from about 30 metres to 1000 metres, another up to 1500 metres, a third up to 2000 metres, and a fourth up to 3000 metres, the lowest possible reading being generally about one-twelfth part of the highest possible reading for any one instrument, but with special cases greater ranges can be obtained. Hence a suitable cymometer must be employed for the particular measurements being made, the oscillation constants of the above four types ranging from about 1 to 12, 2 to 25, 3 to 37, and 4 to 50. For measurements in which greater accuracy of reading is required, it is better to employ, instead of the Neon tube, the thermoelectric detector, which is placed in the circuit of the cymometer. The circuit of the cymometer is cut in two places, or the simple double copper bend with which it is usually provided for completing the circuit can be replaced by a special double bend (see Fig. 58) containing two cuts in it, in one of which is inserted a fine resistance wire, and in the other a fine resistance wire having a thermoelectric junction in contact with it. These resistances and thermoelectric junction are contained in two ebonite boxes attached to the special bend, and a length of flexible connecting wire is provided,

by which the thermoelectric junction is connected to a special low-resistance, single-pivot sensitive galvanometer, that usually employed being made by Paul. There are short-circuiting straps for cutting out the thermoelectric junction, resistance, or the plain resistance. If we insert in the circuit only the resistance with the thermo-junction, and then employ the cymometer as above described, in proximity to any circuit in which oscillations are taking place, we shall find that as a handle is moved, tuning the cymometer more and more in circuit with the circuit under test, the ammeter exhibits a gradually increasing deflection, and at a certain position of the cymometer a maximum deflection is reached. In this position, therefore, the cymometer circuit is traversed by the maximum current, and, therefore, is in resonance with the circuit under test.

19. Measurement of Small Capacities and Inductances by the Cymometer.—The cymometer may be employed for the measurement of small capacities and inductances in the following manner:—

Each instrument is, or can be, supplied with a standard inductance consisting of one or more turns of insulated wire arranged round a rectangular frame. These inductances vary from about 4000 cms., or four microhenrys, up to 75,000 cms., or 75 microhenrys, depending on the pattern of cymometer in use. If, then, a certain small capacity, say that of a Leyden jar, has to be determined, it is done in the following manner. The jar is placed upon a sheet of ebonite, and one coating is

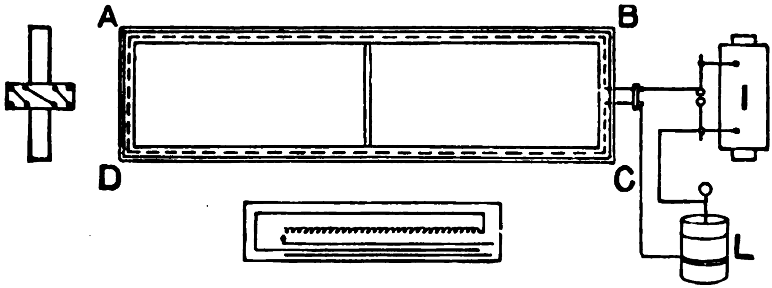


FIG. 59.

connected to one secondary spark ball of an induction coil, the other coating or terminal of the condenser being connected to one end of the above-mentioned standard inductance, whilst a second end of the standard inductance is connected to the other secondary spark ball (see Fig. 59). The spark gap, condenser, and inductance are all connected in series. The cymometer is then placed with its copper bar parallel, not very near to one side of the standard inductance. On working the coil, oscillations are set up in the circuit of the jar and inductance, and the handle of the cymometer is moved until the Neon tube glows most brightly. The scale reading of the cymometer then shows the oscillation constant of the cymometer in that position, that is to say, the value of the square root of the product of its capacity in microfarads, and its inductance in centimetres in its then position. The value of this quantity is called the *oscillation constant*, and is marked on the scale. It then follows that the oscillation constant for the circuit containing the unknown capacity must be the same. Hence, if we square the value of the oscillation constant and divide by the value of the standard inductance in centimetres, we have the value of the unknown capacity in microfarads. Thus, for example, suppose that the standard inductance is 5000 cms., and that the maximum glow in the Neon tube occurs when the cymometer pointer indicates an oscillation constant 10 on the scale, then the square of 10 being 100, and the quotient of $100 \div 5000$ being $\frac{1}{50}$, we know that the capacity of the condenser in question must be $\frac{1}{50}$ of a microfarad. The rule, therefore, is as follows: Square the oscillation constant and divide by the value of the standard inductance in centimetres, and the resultant quotient is the capacity of the jar or condenser in fractions of a microfarad.

In the same way the cymometer can be used with a standard condenser to determine the value of an unknown inductance, for if we determine as above described the capacity of a condenser by the aid of the cymometer, then join up this capacity with the unknown inductance and the spark gap, to form an oscillation circuit, putting in, if necessary, a yard of straight wire to lie parallel with the bar of the cymometer, and if we then determine the oscillation constant of this circuit, and find it to be O, then the inductance in the circuit must be equal to $\frac{O^2}{C}$, where C is the capacity of the condenser in microfarads, and this quotient gives the inductance in centimetres.

In those cases where a small inductance is measured, it can be determined as the difference between two inductances, viz. by joining up with the condenser of known capacity a standard inductance of known value, determining the oscillation constant as above, and then increasing the inductance of that oscillation circuit by adding in the small unknown inductance, and making a redetermination of the oscillation constant. Supposing, for instance, that the oscillation constant in the first instance is O_1, and in the second O_2, and that the standard inductance was, say, 5000 cms., and the value of the unknown and small inductance L, then we have the following equations :—

$$\frac{O_1^2}{C} = 5000$$

$$\frac{O_2^2}{C} = 5000 + L$$

$$L = \frac{O_2^2 - O_1^2}{C}$$

from which we can at once determine the value of L.

A large variety of such tests can be made with a cymometer, provided it is remembered that the oscillation constant marked on the scale of the cymometer is the square root of the product of its capacity reckoned in microfarads and its inductance in centimetres, corresponding to the position in which the handle of the cymometer is then placed.

The calculated value of the inductance L of a rectangular-shaped circuit made of round-sectioned copper wire may be obtained by the formula already given.[103]

The expression for L is as follows :—

$$L = 4\left\{(S+S')\log_e \frac{4SS'}{d} - S\log_e\left(S + \sqrt{S^2+S'^2}\right) - S'\log_e\left(S' + \sqrt{S^2+S'^2}\right) + 2\sqrt{S^2+S'^2} - 2(S+S')\right\} \quad . \quad . \quad . \quad (1)$$

where S and S′ are the lengths of the two sides of the rectangle, and d is the diameter of the round copper wire of which it is made.

The above formula is a strictly accurate one, for infinite frequency, and can easily be applied to any case of a real rectangular circuit. The logarithms are, of course, Napierian.

We can therefore construct a rectangular circuit of wire attached to the lid of the box of the cymometer, which has a known predetermined inductance of, say, 5000 cms. Strictly speaking, there is a small correction for the tails of parallel wire which connect the rectangle to the jar at one end and to the coil at the other. If considered necessary, this may be taken into account by employing a reduced case of the above formula for the inductance of the rectangle.

If there be a pair of circular-sectioned wires of diameter d placed at a distance D apart, the inductance for a length l of the parallel wires is given by the formula

$$L' = 4l\left(\log_e \frac{2D}{d}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, if the length of the tails of wire at each end of the rectangle is the same,

[103] See Chap. II. p. 155.

and equal to l, the inductance of the whole circuit is equal to $L_1 + 2L'$, where L_1 and L' have the values given by the formulæ above.

We can always check the result by using as a loop some form of circuit of which the inductance can be calculated.

Thus, if we bend a bare round-sectioned copper wire into a square, with the ends brought quite near together, we can predetermine its inductance.

We have here a reduced case of the general formula for a rectangular circuit. In expression above put $S = S'$, and put $4S = l$, then the formula reduces to—

$$L = 2l\left(\log\frac{4l}{d} - 2{\cdot}853\right) \quad . \quad . \quad . \qquad (3)$$

Strictly speaking, we should add to the value of the expressions (1) and (3) for the inductance of a rectangle and a square a term equal to $\frac{R'}{2\pi n}$, where R' is the high frequency resistance corresponding to a frequency n. The formulæ (1) and (3), as they stand, give the inductance for infinite frequency. The value of $\frac{R'}{2\pi n}$ is, however, generally negligible compared with the other term, and the expressions given may be taken to be the inductances for any frequency of the order of 10^6.

In the next place, we may employ the same instrument to determine the coefficient of coupling of the circuits of an air core transformer, such as an oscillation transformer used in wireless telegraphy. Suppose the inductance of the primary circuit to be denoted by L, that of the secondary by N, and the mutual inductance by M. Then $\frac{M}{\sqrt{LN}}$ is called the *coefficient of coupling*, and a quantity of importance in the theory of high frequency transformers.

We may join the two circuits of the oscillation transformer into one circuit, so that they assist or oppose each other in creating co-linked flux. In one case the effective inductance is equal to $L + 2M + N$, and in the other case it is $L - 2M + N$.

Hence if we treat the oscillation transformer, so joined up in the two ways, and measure as above its effective inductances, and call them L_1 and L_2, we have—

$$L_1 = L + 2M + N$$
$$L_2 = L - 2M + N$$

$$\text{Hence} \qquad M = \frac{L_1 - L_2}{4}$$

$$\text{and} \qquad L + N = \frac{L_1 + L_2}{2}$$

We can then determine directly and independently the larger of the two inductances L or N, and hence we can calculate the value of $\frac{M}{\sqrt{LN}}$, or the coefficient of coupling of the circuits. As an instance of such a determination, we may give the measurements made with a form of oscillation transformer used in wireless telegraphy. The primary circuit consisted of one single turn composed of eight turns of 7/22 insulated copper wire in parallel wound round a square wooden frame. The secondary circuit consisted of nine turns of the same standard wire wound over the primary circuit. The resultant inductances were measured by the cymometer with the circuits joined up to add and oppose each other.

The measured values were as follows :—

$$L_1 = L + 2M + N = 62{,}576 \text{ cms.}$$
$$L_2 = L - 2M + N = 49{,}621 \quad ,,$$
$$N = 55{,}445 \quad ,,$$
$$\text{whence we have} \quad M = 3239 \quad ,,$$
$$\text{and} \quad L + N = 56{,}098 \quad ,,$$
$$\text{therefore} \quad L = 653$$
$$\text{and} \quad N = 55{,}445 \quad ,,$$
$$\text{therefore} \quad \frac{M}{\sqrt{LN}} = 0{\cdot}54 \quad ,,$$

The coupling would therefore be called "close," as it is usual to call the coupling "close" or "tight" when the coefficient exceeds 0·1, and "loose" when it is smaller. The theory of the above-described instrument is involved in that of oscillation transformers generally, which has already been discussed.[104]

If there be two circuits each having inductance and capacity adjusted so that when separate and far apart each has the same oscillation constant and the same natural frequency n_0, then when these circuits are coupled together inductively with a coefficient of coupling $k=\frac{M}{\sqrt{L_1L_2}}$, where M is the mutual inductance and L_1 and L_2 the inductances of each circuit separately, it has been shown that we have created in the secondary circuit not one but two oscillations of different frequencies, n_1 and n_2, such that—

$$\left.\begin{aligned} n_1 &= n_0\frac{1}{\sqrt{1-k}} \\ n_2 &= n_0\frac{1}{\sqrt{1+k}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The condition that n_1 and n_2 should be equal, and equal to n_0, is that $k=0$. If the coefficient of coupling is not small, that is, if the mutual inductance of the two circuits is not small, we cannot employ a resonant or adjustable secondary circuit to ascertain the natural frequency of the oscillations in a primary circuit when the secondary circuit is not present. If the adjustable secondary circuit is the circuit as described of a cymometer, then in order that its indications may be correct there must be a very small mutual inductance between the cymometer circuit and the circuit we are testing.

It is evident, therefore, that if a circuit has in it oscillations of a certain frequency n_0, and we couple it inductively with another circuit which can be adjusted to have the same oscillation constant $\sqrt{CL}$, in order that oscillations of only one single frequency equal to n_0 should be induced in this adjustable circuit, it is essential that the coefficient of coupling k of the two circuits should be very small. Otherwise two oscillations of different frequency are excited, the frequency of one being greater and that of the other less than that of the free independent original frequency n_0 it is desired to determine.

In the form of cymometer here described, this necessary condition is fulfilled by making the mutual inductance between the cymometer and the circuit being tested very small. We have then to employ a sensitive detector for the condition of resonance, viz. a Neon vacuum tube. One characteristic of the author's form of cymometer is that only a small portion of the whole inductance of the cymometer takes part in creating mutual inductance. Another is that one single movement of a handle varies simultaneously and in the same proportion both the capacity and inductance of the instrument. In using a cymometer for measuring the frequency of the oscillations in any circuit, we have to be on our guard against disturbing the very quantity we wish to measure, or setting up in the cymometer circuit some oscillation of a different frequency. It is an obvious deduction from the above investigation, that in using the cymometer we should place the bar of the cymometer as far away as possible from the circuits being tested. We can make use of the cymometer itself to demonstrate the fact that in the close coupling of isochronous circuits we have oscillations of two periodicities set up. Thus suppose we have two circuits of the same time period when separated and we couple them together inductively. Then, if we investigate with the cymometer the oscillations set up in the secondary circuit, we find it to be a complex oscillation resolvable into components of different periods. The cymometer, therefore, acts just like an electrical spectroscope. It resolves the complex variations in a circuit into their simple components and shows us what they are. This effect is very marked in the case of inductively coupled aerials in wireless telegraphy. If we have a nearly closed condenser circuit with spark gap in which oscillations are set up, which is inductively coupled to an aerial or antenna, then, even if the two

[104] See Chap. III. p. 298, § 11.

circuits are, in common language, "tuned" to each other, so that they have the same independent time period, yet when coupled, if coupled at all tightly, there are two oscillations set up in the aerial of different frequencies, and two waves radiated of different wave-length, which may differ in length by 15 or 20 per cent.

20. The Measurement of the Logarithmic Decrement of Oscillations by the Cymometer.—The cymometer, or other direct reading wavemeter, affords also a ready means of obtaining the decrement of the oscillations in any circuit.

For this purpose it is employed to delineate a resonance curve, and from this curve the sum of the decrements of the cymometer and of the circuit under test can be obtained as explained in Chap. III. § 14. For this purpose we must provide the cymometer circuit with a hot-wire ammeter to read the mean-square value of the oscillations set up in its circuit. In the author's cymometer the current in the oscillatory circuit of the instrument is measured by inserting in it a short length of very fine wire against which is soldered a thermojunction consisting of a fine bismuth and fine iron wire. The high resistance wire consists of a length of about 3-5 cms. of constantan wire 0·05 mm. in diameter, having a resistance of about 6 or 7 ohms. If oscillations are passed through the constantan wire it is heated, and an E.M.F. is created in the thermojunction. This last is connected to a low-resistance single pivot Paul galvanometer as already described, so that this instrument is deflected by an amount depending on the mean-square value of the oscillations heating the thermojunction. If we pass through the fine wire various measured continuous currents, and note the steady deflection of the galvanometer, we know that any oscillations, damped or undamped, which subsequently produce the same deflection of the galvanometer, must be producing heat at the same rate, and therefore must have the same mean-square value as the direct current. In addition to this thermojunction wire, which serves the purpose of a hot-wire ammeter for small currents, we provide also another similar wire, but without a thermojunction, which can be inserted in the cymometer circuit at pleasure.

Let us suppose, then, that it is desired to examine the oscillations in any condenser circuit traversed by damped oscillations. A part of this last circuit must be brought near to a part of the testing or cymometer circuit so that it acts inductively on it but feebly. The cymometer circuit must then have its capacity and inductance, or both, varied slowly and at each setting; the natural frequency, n, of its circuit must be known, and also the root-mean-square value, a, of the current induced in its circuit. If there is only one single oscillation in the circuit under test having a frequency, N, then when the cymometer circuit is set to have this frequency the current in it will have a maximum value, A. If we insert in the cymometer circuit the known small resistance, R, then this will increase its damping by a known amount and the maximum current will be reduced to A′. If we call δ_1 the decrement of the primary circuit, δ_2 that of the cymometer, and δ_2' that due to the added resistance, R, then we have—

$$\delta_2' = \frac{R}{2NL_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where N and L_2 are the natural frequency and inductance of the cymometer corresponding to the position of resonance. Then by the formulæ already given (see Chap. III. § 14) we have—.

$$\delta_1 + \delta_2 - 2\pi\left(1 - \frac{n}{N}\right)\sqrt{\frac{a^2}{A^2 - a^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The observations are plotted as follows:—

Having observed the values of a and n over a sufficient range, we plot a curve having abscissæ $\frac{n}{N}$ and ordinates $\frac{a}{A}$, and the result is a resonance curve for the case in question, as shown in Fig. 60.

Since the frequency is connected with the oscillation constant by the formula $n = \frac{5{\cdot}033 \times 10^6}{O}$, we can also write the above formula of Drude and Bjerknes in the following form:—

$$\delta_1 + \delta_2 = 6{\cdot}2832 \frac{O_2 - O_1}{O^2} \sqrt{\frac{a^2}{A^2 - a^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In plotting out the resonance curve as above described, it is best to take the mean-square value of the maximum current as unity, and to correct the other currents in the corresponding ratio; and the same way for the frequencies, viz., the resonance frequency and any other frequency. If, then, we put x for $\left(1 - \frac{n}{N}\right)$ and y for $\frac{a}{A}$, we can write the above formula for the sum of the decrements finally in the form—

$$\delta_1 + \delta_2 = 6{\cdot}2832\, x \frac{y}{\sqrt{1 - y^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Since the resonance curve is not quite symmetrical with respect to its maximum ordinate, it is best to determine from the resonance curve the values of the frequency n lying on either side of the maximum current, which correspond to any given value of the cymometer current, and to take the mean of these values as the value to be put into the above formula.

It will be seen, then, that from such a resonance curve we can determine the sum of the decrements of the circuits under test, and that of the cymometer. This last has, however, been increased by the resistance of the fine wire inserted in its circuit, by means of which we determine the sum of the decrements. We have, therefore, to eliminate the latter quantity as follows: Suppose we insert in the cymometer circuit the small resistance, R, and that we again take a resonance curve. The decrement of the cymometer has been increased from δ_2 to $\delta_2 + \delta_2'$ where $\delta_2' = \frac{R}{2n_1 L_2}$, L_2 being the inductance of the cymometer circuit corresponding to the resonance frequency n_1. Now it has been shown in Chap. III. § 14, equation 166, that the mean-square value of the resonance current is inversely as the quantity $\delta_1\delta_2(\delta_1 + \delta_2)$. It follows, therefore, that if δ_2 is changed to $\delta_2 + \delta_2'$, and A^2 to A'^2, we must have the following relation, viz.—

$$A^2\delta_2(\delta_1 + \delta_2) = A'^2(\delta_2 + \delta_2')(\delta_1 + \delta_2 + \delta_2') \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or if we put X for $\delta_1 + \delta_2$ and X′ for $\delta_1 + \delta_2 + \delta_2'$, we may write it in the form—

$$\delta_2 = \frac{X'\delta_2'}{\left(\frac{A}{A'}\right)^2 X - X'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

But $$X = \delta_1 + \delta_2$$

therefore $$\delta_1 = X - \frac{X'\delta_2'}{\left(\frac{A}{A'}\right)^2 X - X'}$$

where $$X = 6{\cdot}2832\, x \frac{y}{\sqrt{1 - y^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

To determine δ_1 we have therefore to take two resonance curves, one as above described, and another in which the circuit of the cymometer has its decrement increased by a known amount, by the insertion of a second fine wire resistance in the gap provided for it.

The details of the measurements will perhaps best be understood by going through the calculations in a particular case. The reader should note, however, that in this particular numerical example the decrement used is that per half-period, and to obtain the usual value the figures given must be doubled.

A certain oscillation circuit was set up, and by means of the cymometer a pair of resonance curves drawn, one without and one with an added small resistance in the cymometer circuit. These curves were as shown in Fig. 60. From these curves measurements were made giving the R.M.S. values of the currents a, A, a', A′,

and at the same time of the quantities $x = 1 - \frac{n_2}{n_1}$ and $y = \frac{a}{A}$. A number of values of y were taken off the curve corresponding to various values of x not exceeding 0·05, and tabulated as under, and the value of $\delta_1 + \delta_2$ calculated by the formula above given.

$\frac{a}{A} = y$.	$\left(1 - \frac{n_2}{n_1}\right) = x$.	$X = \delta_1 + \delta_2$ per semi-period.
0·95	0·0120	0·115
0·90	0·0165	0·112
0·85	0·0205	0·104
0·80	0·0255	0·107
0·75	0·0293	0·105
0·70	0·0335	0·103

The mean value of X is then 0·108.

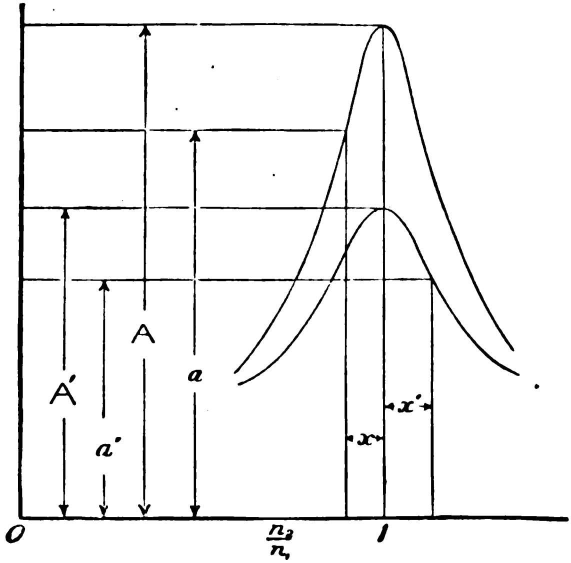


FIG. 60.—A Resonance Curve.

In the same manner, after increasing the resistance of the cymometer circuit, a second set of values was obtained as follows :—

$\frac{a'}{A} = y$.	$\left(1 - \frac{n_2}{n_1}\right) = x$.	$X' = \delta_1 + \delta_2 + \delta_2'$ per semi-period.
0·95	0·0125	0·120
0·90	0·0210	0·138
0·85	0·0255	0·130
0·80	0·0300	0·125
0·75	0·0345	0·124
0·70	0·0385	0·119

Hence the mean value of X′ is 0·126.

Accordingly we have from the curves and formulæ above given

$$\left(\frac{A}{A'}\right)=2{\cdot}34$$

$$\text{Hence}\qquad \left.\begin{aligned}\delta_1+\delta_2&=0{\cdot}108\\ \delta_1+\delta_2+\delta_2'&=0{\cdot}126\\ \delta_2'&=0{\cdot}018\\ \delta_2&=0{\cdot}0178\\ \delta_1&=0{\cdot}090\end{aligned}\right\}\text{All values per semi-period.}$$

The decrement of the primary circuit is, therefore, about 6 times that of the cymometer circuit. In this case the oscillation circuit being tested comprised a condenser or Leyden jar and an inductance of 5000 cms. and a spark gap of 2 or 3 mms. in length. The high frequency resistance of the inductance was calculated from the dimensions of the wire and found to be 0·23 ohm. As this circuit was a nearly closed circuit, the decrement was all due to resistance, partly of the metallic wire R and partly of the spark r, and this can be shown to be equal to $4n_1L\delta_1$, where L is the inductance of the circuit, and n_1 the frequency corresponding to resonance. Hence, if R and r are measured in ohms and L in centimetres, we have

$$R+r=\frac{4n_1L\delta_1}{10^9}$$

But $R=0{\cdot}23$, $L=5000$, $\delta_1=0{\cdot}090$, and $n_1=0{\cdot}95\times10^6$. Hence $r=1{\cdot}23$ ohms.

Also from the formula $m=\frac{4{\cdot}605+\delta_1}{\delta_1}$ we can show that each train of oscillations comprised about 52 semi-oscillations, or 26 periods.

Accordingly, the measurement of the decrement gives us all information about the nature of the oscillations taking place and the resistance of the spark.

If we had been testing the decrement of a radiotelegraphic antenna, we should probably have found a much larger decrement than 0·090, because then there would have been radiation to increase the damping, and therefore the decrement.

It will be seen, therefore, that by the use of the cymometer and the necessary adjuncts to it, we are enabled to obtain all the required information concerning the oscillations in the antenna of a radiotelegraphic transmitter employing the spark method of producing damped oscillations. When operating as above upon an antenna which is inductively coupled to the condenser circuit, the resonance curves will be found to be curves with double humps, as in Fig. 33, Chap. III.; and if these humps are not too close to one another, we may apply the above process to each humph separately, and obtain the decrement of each of the two coexisting oscillations in the antenna.

In making these measurements, the cymometer must of course stand on a table, and a certain length of the antenna must be bent round so as to be parallel with, but not too near, the bar of the cymometer. It will also be found necessary that the outer tube of the condenser should be connected to the earth by means of a terminal provided for that purpose.

We can make use of the measurements of decrement also to determine the high frequency resistance of any circuit. Thus, for instance, in the case of a particular primary circuit having an inductance of 5012 cms., and a capacity of 0·002645 mfd., and a spark gap of 1 mm., the resistance, R, added to the cymometer was 7·1 ohms, and the inductance L of the cymometer in that setting of the instrument corresponding to resonance was L=55,000 cms., and whilst the corresponding frequency=N was $1{\cdot}25\times10^6$.

$$\text{Hence}\qquad \delta'_2=\frac{R}{4NL}=\frac{7{\cdot}1\times10^9}{4\times1{\cdot}25\times10^6\times55000}=0{\cdot}0258$$

Also it was found that the resonance current A was 0·1195 amp. and that when the resistance of 7·1 ohms was inserted the resonance current A_1 was 0·0635 amp.

The observed corresponding values of $\frac{a}{A}$ and $\frac{n}{N}$ were then as recorded in the following Table :—

OBSERVATIONS FOR THE DELINEATION OF A RESONANCE CURVE

Cymometer currents as percentage of maximum current $=100\frac{A}{a}$.	Calculated value of $\sqrt{\frac{a^2}{A^2-a^2}}$	Measured value of $1-\frac{N}{n}$	Calculated value of $\delta_1+\delta_2$ per semi-period.
95	9·58	0·0067	0·0643
90	6·47	0·0098	0·0635
85	5·09	0·0126	0·0642
80	4·18	0·0152	0·0636
75	3·58	0·0177	0·0635
70	3·08	0·0205	0·0632
The mean value of $\delta_1+\delta_2=0·0637$			

Hence from the other observed quantities we have, by equation (9)—

$$A^2 \times 0·0637 \times \delta_2 = A_1{}^2 \times (0·0637+0·0258) \times (\delta_2+00258)$$

Therefore, inserting the above given values of A and A_1, we have $\delta_2=0·017$, and therefore $\delta_1=0·0467$.

The high frequency resistance R_1 of the primary circuit must therefore be such that $\delta_1=\frac{R_1}{4NL_1}$, where $N=1·25\times10^6$ and $L_1=5012$ cms. Hence—

$$R_1=4\times1·25\times10^6\times5012\times0·0467\times10^{-9} \text{ ohms, or } 1·17 \text{ ohms,}$$

and since the high frequency resistance of the inductance itself was found to be 0·31 ohm for that frequency, we find $1·17-0·31=0·86$ ohm as the resistance of the spark.

In taking these resonance curves it is a very great assistance to allow a steady blast of air under a pressure of about 16-20 inches of water to impinge on the gap between the spark balls. It not only keeps the balls cool, but blows away the arc which tends to form at each discharge, which would otherwise keep down the condenser terminal voltage.

21. Measurement of the Wave-Length and Decrement of Incident Waves.—It is quite easy in the above-described manner to measure the wave-length and damping of the waves sent out from a transmitting antenna. It is rather more difficult when we are concerned with the waves arriving on an antenna, since the oscillations set up in it are then much more feeble. The same principles, however, apply. It is merely a question of a more sensitive oscillation detecting instrument wherewith to measure the mean-square value of the oscillations in the receiving antenna.

The author has found that the most convenient detector for this purpose is the molybdenite-copper point rectifier of Professor G. W. Pierce. A small mass of molybdenite is held in a clip, and a copper point adjusted in light contact therewith. The contact has very effective unilateral conductivity, and rectifies the trains of oscillations so that they affect a high resistance telephone (1000 ohms) placed as a shunt across the condenser terminals of the cymometer.

The wave-lengths of the arriving waves can then be read off on the scale by adjusting the wavemeter circuits to give maximum sound in the telephone.

The cymometer or other wavemeter can then be employed to measure the decrement of the oscillations in the receiving antenna as follows :—

The first step is to plot a resonance curve, as already shown, by setting out as

ordinates the value of the mean-square current (J^2) in the cymometer circuit corresponding to various values of the natural frequency (n) of that circuit for various settings of the capacity and inductance within such limits that n does not differ from the resonance frequency N by more than 5 per cent. Then if J_r^2 is the mean-square value of the maximum or resonance current in the cymometer circuit, and δ_2 is the decrement of that circuit, and if δ_1 is the decrement of the oscillations in the antenna, we have by the usual Bjerknes formula—

$$\delta_1+\delta_2=2\pi\left(1-\frac{n}{N}\right)\sqrt{\frac{J^2}{J_r^2-J^2}} \qquad (12)$$

Assuming, then, that the resonance curve is drawn with J^2 as ordinates and n as abscissæ, we may select various values of n and J^2, and substitute them in the above formula, provided that n is within 5 per cent. of N. We may shorten the calculation by taking $n=\frac{95}{100}N$ and $1-\frac{n}{N}=\frac{1}{20}$, and determine the value of J^2, corresponding to this value of n.

Again, since the decrement of the cymometer circuit is given by the expression $\delta_2=\frac{R'}{2nL'}$, where R' is the high frequency resistance of that circuit and L' is its inductance corresponding to the frequency n, then for those types of wavemeter in which L' is either constant or varies proportionately to R', we have $\delta_2=\frac{C}{n}$, where C is some constant for the instrument which can be determined by experiment.

Accordingly, the semi-period decrement of the oscillations in the sending antenna is given by the expression—

$$\delta_1=\frac{2\pi}{20}\sqrt{\frac{1}{A^2-1}}-\frac{C}{n} \qquad (13)$$

where A^2 is the ratio of the mean-square values of the resonance current and the current corresponding to a frequency n which exist in the wavemeter circuit, n differing by 5 per cent. from the resonance frequency, and C being an instrumental constant of the wavemeter, viz. the value of $\frac{R'}{2L'}$ for that setting of the instrument corresponding to the frequency n.

The receiving appliance must consist of an inductance coil of adjustable inductance, and a coil in series with it which forms the primary of an oscillation transformer, the secondary circuit of which is movable so as to vary over wide limits the coupling of the two circuits. The secondary circuit is completed by a condenser of variable capacity, and the terminals of this capacity are connected also through a rectifying contact, such as the molybdenite, copper point and a high-resistance telephone in series with the latter.

The first step is to determine the ratio of the mean-square values of the resonance currents in the secondary circuit of the transformer when the secondary is set with various couplings, or at various distances from the primary. This can be done by sending constant oscillations through the primary circuit, and employing a low resistance hot-wire ammeter in the secondary circuit, or else a high resistance galvanometer, in place of the telephone, in series with the rectifying contact, the two being placed as a shunt across the condenser. In the experiment we are really obtaining the value of the square of the coupling $\left(\frac{M^2}{LN}\right)$ for the two circuits of the transformer.

Having done this we replace the telephone on the detector circuit and adjust the receiver to pick up the impinging waves of which it is desired to measure the decrement. The coupling of the two circuits of the transformer must then be reduced until the sound in the telephone just ceases to be heard. Let the mean-square current in the secondary circuit be then denoted by J_r^2.

The tuning is then altered by changing the capacity in the secondary circuit sufficiently to make its natural frequency differ by 5 per cent. from the resonance

frequency. The coupling is then strengthened until the sound in the telephone is again just audible. Let the mean-square current in the secondary circuit be then denoted by J^2, and the resonance mean-square current for exact tuning at frequency N corresponding to that particular coupling be denoted by ${}_1J_r^2$. We may then assume that $J_r^2=J^2$; and we have previously determined the ratio ${}_1J_r^2$ to J_r^2. Call this ratio a for the couplings in question. Then it follows that—

$$\frac{{}_1J_r^2}{J_r^2}=a=\frac{{}_1J_r^2}{J^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

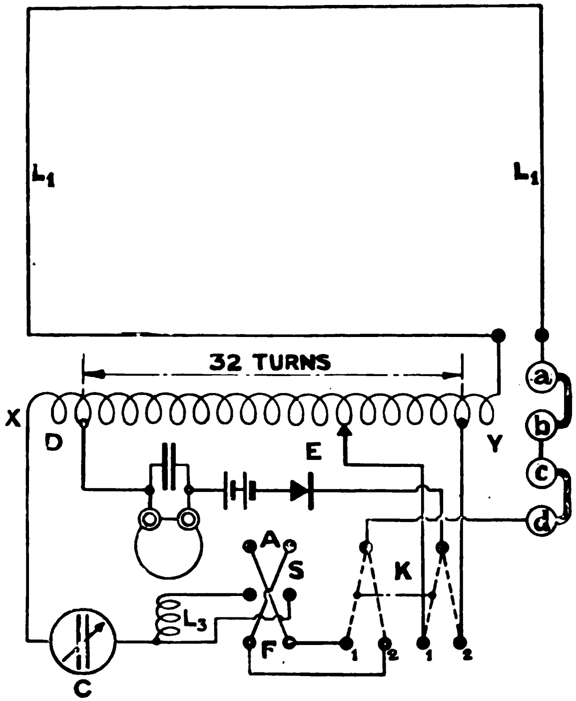


FIG. 61.—Scheme of Connections of Marconi Company's Decremeter.

and the decrement of the oscillation in the receiving antenna will be given by the same formula as before, viz.—

$$\delta_1=\frac{2\pi}{20}\sqrt{\frac{1}{a-1}}-\delta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

To determine δ_2, or the decrement of the secondary circuit of the oscillation transformer, a second set of readings must be taken in the same manner in which the decrement of the secondary circuit is artificially increased by the insertion in it of a fine wire of high resistance, r', the added decrement due to this is $\delta_2=\frac{r'}{2n'L_2}$, where n' is the frequency and L_2 the inductance of the secondary circuit at resonance. If then ${}_2J_r^2$ is the mean-square secondary current with this added resistance, and ${}_3J_r^2$ is the current for the same coupling with the resistance r' cut out, then by equation (9) we have—

$${}_3J_r^2\delta_2(\delta_1+\delta_2)={}_2J_r^2(\delta_2+\delta_2')(\delta_1+\delta_2+\delta_2') \quad . \quad . \quad . \quad . \quad (16)$$

If the coupling in this last experiment is so arranged that ${}_2J_r^2$ is a current just audible in the telephone, we can say that ${}_2J_r^2 = J_r^2$, and the ratio of ${}_3J_r^2$ to $J_r^2 = \beta$ is known from the previous calibration.

Hence from equations (15) and (16) we can determine δ_1.

The process is rendered much more simple in those cases in which the current in the receiving antenna is large enough to affect directly a sensitive thermal-micro ammeter such as Mr. Duddell's instrument, for we have then no difficulty in determining the ratio $\frac{J_r^2}{J^2}$ for the two frequency settings of the receiver.

For the rapid measurement of the decrement of damped oscillations in a circuit

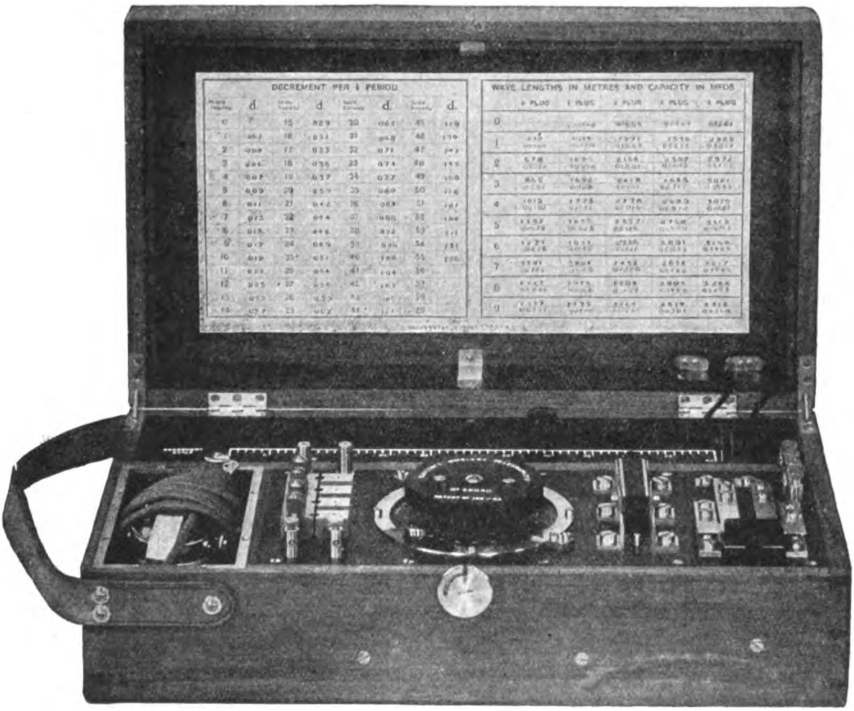


FIG. 62.—The Marconi Company's Decremeter.

the Marconi Company make up an instrument called a "decremeter," in which the detecting instrument is a telephone in series with a rectifying carborundum crystal. The principle of the decremeter will be understood from the diagram in Fig. 61, and the external appearance is shown in Fig. 62. It consists of a spiral inductance, XY, in series with a variable condenser, C, forming a closed oscillation circuit. A telephone and crystal detector are connected in shunt to a part of the inductance. The decremeter can therefore be used as a wavemeter. A portion of this inductance is in the form of a rectangle of wire, L_1 (see Fig. 61), which is contained in the lid of the instrument as in the latest form of Fleming Cymometer (see Fig. 52). This serves to couple the instrument circuit to the circuit under test. By varying the capacity of the condenser C the instrument circuit can be tuned to the tested circuit, and this tuning is recognized by the sounds in the telephone being at their loudest when the contact point E is moved up as near the end of the spiral as possible. The setting of the condenser is then left unaltered during

the decrement measurements. The inductance coil consists of wire in the form of a long spiral of small diameter, and hence, except near the ends, the potential fall down it is regular. For this reason, one end of the telephone is connected not quite at the end, and a sliding contact at E is provided on the spiral. A part of the inductance is in the form of a rectangle of wire, L_1, in the lid of the box, and this is held near the circuit under test. A resonance curve can thus be plotted by noting the position of the slider E for different values of the condenser capacity, E being adjusted to the point at which the sound in the telephone is just audible. Since the strength of the signal is inversely proportional to the scale reading DE, and the wave-length is proportional to the square root of the capacity, a resonance curve can be obtained by plotting the reciprocals of the scale readings DE against the square roots of the condenser capacities.

The decrement is obtained by applying the Bjerknes formula. For this purpose a throw-over switch, F, is provided, which throws in extra inductance L_3, so as to alter the frequency of the circuit by about 4 per cent.

The observation then consists in tapping the detector circuit off a fixed inductance of thirty-two turns of the spiral when the oscillation circuit is put out of tune by throwing the above-mentioned extra inductance into the condenser circuit, and then to obtain an equally strong signal by adjusting the slider again at another position with the extra inductance cut out and the circuit in tune with the circuit being tested.

A table is supplied with the instrument, which allows the decrement of the circuit under observation to be obtained from the above observations.

For a description of another form of direct-reading decremeter, by F. A. Kolster, the reader is referred to the *Proceedings of the Institute of Radio-Engineers*, vol. 3, p. 29, March 1915.

The circuits with which these oscillation detectors are combined to construct radiotelegraphic receivers are described in the next chapter.

PART III.—ELECTRIC WAVE OR RADIO-TELEGRAPHY

CHAPTER VII

THE APPARATUS OF RADIOTELEGRAPHY

1. Early Ideas and Experiments.—The reader who desires to study the history of all the various methods suggested for conducting wireless telegraphy must consult books more especially devoted to the historical side of the subject.[1]

From the earliest days of electric telegraphy, inventors had their attention directed to the problem of dispensing in part or entirely with continuous interconnecting wires. In 1838, Steinheil of Munich, acting on a suggestion made by Gauss, demonstrated that the earth could perform the function of a return for a telegraphic circuit, and thus made one of the most important contributions to practical telegraphy.

He seems, moreover, to have anticipated that in time improvements might be effected by which the necessity for any metallic circuit at all would be removed.[2] From the date of that suggestion the notion of telegraphy without wires may be said to have been ever present to the minds of telegraphic engineers.

The necessity for finding some solution of the problem of wireless telegraphy increased as the art of electric telegraphy itself extended, even if it were only to enable telegraphists to bridge over some short break or interval in a metallic circuit. Suffice it here to say that if we exclude the method depending on the employment of electromagnetic waves, the processes which had been previously found feasible or had been suggested were based upon—

(i.) The conduction of electric currents through the moist earth or the waters of rivers, lakes, or seas. This method particularly engaged the attention of Morse, Lindsay, Trowbridge, Preece, Rathenau, Strecker, Wilkins, and Melhuish.

(ii.) Electromagnetic induction between parallel metallic conductors, either complete circuits or circuits including earth returns. Suggested and studied by Trowbridge, Preece, Stevenson, and Lodge.

(iii.) A combination of methods (i.) and (ii.). Made into a practical system chiefly by the labours of Sir William Preece, aided by the British Postal Telegraph Engineers.

(iv.) Electrostatic induction between conductors separated by a greater or less distance. Brought to a working success by Edison, Gilliland, Phelps, and W. Smith, as a means of communication with moving railway trains.

The reader wishing to have some information with regard to the earlier researches of the above-named inventors may be referred to the following original

[1] We may particularly refer the reader to the excellent work by Mr. J. J. Fahie, "A History of Wireless Telegraphy" (Blackwood & Sons, London and Edinburgh).

[2] See Fahie's "History of Wireless Telegraphy, 1838-1899," 1899, p. 4 (Blackwood & Co.); also Fahie's "History of Electric Telegraphy to the Year 1837," pp. 343-348, for the history of the *earth return* in telegraphy.

Although Steinheil was not the first to employ or suggest the use of an earth return for completing an electric circuit, he was the first to apply it in practical telegraphy, and to realize its importance.

See also Steinheil, "Ueber Telegraphie, insbesondere durch galvanische Kräfte." Munich, 1838.

Interesting quotations from Steinheil's writings are given in Mr. Fahie's book on wireless telegraphy.

papers, as well as to the "History of Wireless Telegraphy," by Mr. J. J. Fahie above mentioned.

J. Trowbridge, "The Earth as a Conductor of Electricity," *American Acad. Arts and Sciences*, 1880.

W. H. Preece, "Recent Progress in Telephony," *British Association Report*, 1882.

W. H. Preece, "Electric Induction between Wires and Wires," *British Association Reports*, 1886 and 1887.

W. H. Preece, "Electric Signalling without Wires," *Journal Soc. of Arts*, February 23, 1894.

W. H. Preece, "Signalling through Space without Wires," *Proc. Roy. Inst. Lond.*, 1897, vol. xv. p. 467.

W. H. Preece, "Ætheric Wireless Telegraphy," *Proc. Inst. Elec. Eng. Lond.*, 1898, vol. xxvii. p. 869.

O. J. Lodge, "Magnetic Space Telegraphy," *Proc. Inst. Elec. Eng.*, 1899, vol. xxvii. p. 799.

In many cases suggestions were put forward which were based upon obviously erroneous ideas, and even embodied in patent specifications without being subjected to critical trial. Nevertheless, the best of the methods above classified had only enabled comparatively short distances to be covered. Even the most effective of them, viz. the method involving both conduction through the soil or water and electromagnetic induction between parallel wires, was extremely limited in its applicability by reason of the necessity for employing two parallel metallic wire circuits almost as long as the distance to be bridged.

A new era dawned when the scientific investigations commenced which finally placed us in possession of the principal facts connected with the generation and detection of electromagnetic waves, or as they are more shortly called, electric waves.

Maxwell's mathematical researches resulted in the enunciation in 1865 of his famous electromagnetic theory of light, but the difficulty of visualizing the nature of a luminous vibration on this theory retarded its appreciation. Hertz's discoveries and investigations, published in 1888, cast a flood of light upon its meaning, and whilst opening up a wide and promising field for experimental investigation, gave such enforcement to Maxwell's theory that it at once commanded general attention.

The matters, however, which chiefly interested physicists were the properties of the long waves generated in the æther by Hertzian methods, and the similarity between the effects connected with them and familiar optical phenomena. Hence a rapidly accumulated mass of experimental evidence was obtained, tending to show that luminous radiation may be electromagnetic in nature. Electro-optic phenomena were sedulously studied, and physical optics became, as it were, a department of electromagnetism.

In 1892 Nikola Tesla captured the attention of the whole scientific world by his fascinating experiments on high frequency electric currents. He stimulated the scientific imagination of others as well as displayed his own, and created a widespread interest in his brilliant demonstrations.

Amongst those who witnessed these things no one was more able to appreciate their inner meaning than Sir William Crookes. More than twenty years previously he had explored with wonderful skill and insight the phenomena of electrical discharge in high vacua, and had produced the instrument which subsequently produced the Röntgen rays. He allowed a trained scientific imagination to busy itself with the recent discoveries, and he wrote a now well-known article "On Some Possibilities of Electricity" in the *Fortnightly Review* for February 1892 (p. 173), in which he endeavoured to forecast some of the applications of high frequency electric currents and of Hertzian waves.

In this outlook into the future he clearly discerned the coming of a new form of wireless telegraphy based on an application of Hertz's discoveries to the communication of intelligence from place to place. In the course of the paper, Sir William Crookes made a cryptic reference to experiments in this direction he had witnessed "some years ago," which were subsequently explained to refer to un-

published investigations by the late Professor D. E. Hughes, in which signals were sent "a few hundred yards," without connecting wires, by the aid of a telephone. No details of the experiments were given, or any hint of how the result was obtained.

In the article to which reference is made we find much remarkable prognostication, but not a description of actual inventions.

It emphasized, in fact, how much at that date (1892) yet remained to be done. Speaking of electromagnetic waves and their properties, Sir William Crookes says (*loc. cit.*):—

"Here is unfolded to us a new and astonishing world, one which it is hard to conceive should contain no possibilities of transmitting and receiving intelligence.

"Rays of light will not pierce through a wall, nor, as we know only too well, through a London fog. But the electrical vibrations of a yard or more in wave-length of which I have spoken will easily pierce such mediums, which to them will be transparent. Here, then, is revealed the bewildering possibility of telegraphy without wires, posts, cables, or any of our present costly appliances. Granted a few reasonable postulates, the whole thing comes well within the realms of possible fulfilment. At the present time experimentalists are able to generate electrical waves of any desired wave-length from a few feet upwards, and to keep up a succession of such waves radiating into space in all directions. It is possible, too, with some of these rays, if not with all, to refract them through suitably shaped bodies acting as lenses, and so direct a sheaf of rays in any given direction; enormous lens-shaped masses of pitch and similar bodies have been used for this purpose. Also an experimentalist at a distance can receive some, if not all, of these rays on a properly constituted instrument, and by concerted signals messages in the Morse code can thus pass from one operator to another. What, therefore, remains to be discovered is—firstly, simpler and more certain means of generating electrical rays of any desired wave-length, from the shortest, say of a few feet in length, which will easily pass through buildings and fogs, to those long waves whose lengths are measured by tens, hundreds, and thousands of miles; secondly, more delicate receivers which will respond to wave-lengths between certain defined limits and be silent to all others; thirdly, means of darting the sheaf of rays in any desired direction, whether by lenses or reflectors, by the help of which the sensitiveness of the receiver (apparently the most difficult of the problems to be solved) would not need to be so delicate as when the rays to be picked up are simply radiating into space in all directions, and fading away according to the law of inverse squares.

"I assume here that the progress of discovery would give instruments capable of adjustment by turning a screw or altering the length of a wire, so as to become receptive of wave-length of any preconcerted length. Thus, when adjusted to 50 yards, the transmitter might emit, and the receiver respond to, rays varying between 45 and 55 yards, and be silent to all others. Considering that there would be the whole range of waves to choose from, varying from a few feet to several thousand miles, there would be sufficient secrecy, for curiosity the most inveterate would surely recoil from the task of passing in review all the millions of possible wave-lengths on the remote chance of ultimately hitting on the particular wave-length employed by his friends whose correspondence he wished to tap. By 'coding' the message even this remote chance of surreptitious straying could be obviated.

"This is no mere dream of a visionary philosopher. All the requisites needed to bring it within the grasp of daily life are well within the possibilities of discovery, and are so reasonable and so clearly in the path of researches which are now being actively prosecuted in every capital of Europe that we may any day expect to hear that they have emerged from the realms of speculation into those of sober fact. Even now, indeed, telegraphing without wires is possible within a restricted radius of a few hundred yards, and some years ago I assisted at experiments where messages were transmitted from one part of a house to another without an intervening wire by almost the identical means here described."

The above vague reference to experiments on telegraphy without wires over a short distance was at a later date illuminated by the account given by Professor D. E. Hughes himself, of the precise nature of these hitherto undescribed experiments.[3] In the course of his work on the microphone, Professor D. E. Hughes had occasion to notice the wonderful sensitiveness of a "microphonic" or loose joint between conductors, and its variation of resistance under impacts, such as those of sound waves. He included such an "imperfect contact" in series with a voltaic cell and a telephone, and found that the resistance of certain kinds of contact was effected by electric sparks at a distance. Using a contact between

[3] See a letter by Prof. D. E. Hughes in *The Electrician*, May 5, 1899, vol. 43, p. 40.

carbon and steel, he no doubt constructed some form of self-restoring coherer, and made the important discovery that the discharge of a Leyden jar at a distance caused a sudden variation in its electrical resistance, and hence a sound in the telephone included in its circuit.

Professor D. E. Hughes stated in a letter addressed to Mr. Fahie, on April 29, 1899 (*loc. cit.*), that he showed these experiments in December 1879 to Sir W. H. Preece, Sir William Crookes, Sir W. Roberts-Austen, Professor W. G. Adams, and Mr. W. Grove; also in February 1880 to Mr. Spottiswoode, then President of the Royal Society, and to Professor Huxley and Sir George Gabriel Stokes, the secretaries. In addition, he exhibited them to Sir James Dewar and Mr. Lennox. He was apparently discouraged from publishing the results at the time by finding that Sir George Stokes considered they were due to ordinary electromagnetic induction. It is, however, clear from the statements of Professor Hughes himself in 1899 that he had discovered (but not announced) in 1879 a number of facts afterwards rediscovered by Professor E. Branly in Paris in 1891, and he had, in fact, been using a self-restoring carbon-iron coherer in series with a telephone which was affected up to a distance of a few hundred yards by the electromagnetic waves created by an electric spark. If at the time he had publicly placed these observations on record, he would undoubtedly have anticipated some at least of Branly's work, but much remained to be done, which was subsequently done by Hertz and by Marconi, before electric wave wireless telegraphy, in any true sense of the word, could be translated from dream to fact.

Four years passed by, however, without any fulfilment of Crookes's scientific prophecy, although the most eminent physicists continued to work at the subject.

On January 1, 1894, the scientific world heard with profound regret of the death of Hertz.

On Friday, June 1, 1894, Sir Oliver Lodge delivered a memorial lecture on "The Work by Hertz," in the Royal Institution, London.

This lecture was remarkable in many ways. It gave many persons the opportunity of seeing, for the first time, striking experiments performed with Hertzian waves. The lecturer made use of a modified Branly's metallic filings tube, and also of a loose or imperfect metallic contact of his own invention, as a means of detecting the electric waves, and he gave to these devices the name *coherer*, by which they have since been known.

The tube was a glass tube loosely filled with iron borings and closed at the ends with metal plugs or caps. It is represented about one-third of full size in Fig. 3 of Chap. VI. The other form of coherer was a loose or microphonic contact between two pieces of metal, the pressure of which could be adjusted so that the junction offered too great a resistance to pass the current from a single cell, but *cohered* when electric waves fell upon it. In both cases the tapping back or decoherence was effected by hand after each experiment.

Experiments on the reflection, refraction, and polarization of these electric waves were shown, and their passage through stone walls from room to room. Yet, although replete with interest, the lecture, as originally delivered, contained not even a hint of a possible application of these electromagnetic waves to telegraphy. The lecturer throughout fixed the attention of the audience on the similarity between the effects obtainable with these waves and those better known effects produced by rays of light.

It was, in fact, an experimental demonstration of the undulatory character of the electromagnetic radiation from an oscillator, and of the electromagnetic nature of ordinary light.

Subsequently the lecture was published as a book, the first edition of which bore the title, "The Work of Hertz and some of his Successors." [4]

These experiments and some variations of them were repeated at the meeting of the British Association at Oxford in the following autumn, but here again no mention of the application of these waves to telegraphy was made, the object of

[4] In later editions issued after 1896 the title was changed to "Signalling across Space without Wires."

the experiments being to illustrate an electrical theory of vision, and to expound the properties of the electric waves.[5]

It is highly probable that these articles and lectures, bringing home so forcibly the power of an electric spark to affect or make a deflection of a galvanometer at a distant place, must have turned the thoughts of many ingenious persons to its utilization as a means of sending telegraphic signals. Subsequently we were informed that the matter had begun to occupy the minds of Dr. A. Muirhead, Admiral Sir H. B. Jackson (then Captain in the Royal Navy), and Professor R. Threlfall, and perhaps many more.

Amongst others, Professor A. S. Popoff, Professor in the Imperial Torpedo School in Cronstadt, Russia, directed his attention to the subject, attracted to it by Lodge's lecture, and desirous, as he says, of repeating the experiments both for lecture purposes, and for registering electrical perturbations taking place in the atmosphere. His apparatus and wave detector have already been described (see Chap. VI. § 3), as well as the publication of his description of them, and experiments conducted with them in January 1896, in the *Journal of the Physico-Chemical Society of Petrograd.*

It is beyond question, however, that the use he made of his apparatus was not the communication of intelligence to a distance, but for studying atmospheric electricity. The observations were made at the Institute of Forestry, Petrograd. Popoff says—

"Upon the building of the Institute, amongst other arrangements made for observing the direction and force of the wind, there was a small wooden mast about 4 sajen (28 feet) higher than the rods carrying the anemometers and weather-cocks, and which was furnished at the top with an ordinary lightning point and rod. This lightning rod, by means of a wire carried first on the wood of the mast, and further stretched across the yard on insulators into the meteorological observatory, was connected with the apparatus at the point A (Fig. 2), whilst the point B was connected to a wire which served as an earth conductor or connection for the other meteorological apparatus, and was connected to the water-supply pipes. The registering arrangements consisted of an electromagnet, to the armature of which there was attached a Richard pen writing on a Richard recording cylinder, making one revolution per week. It was found that the apparatus responded by a ring of the bell to every closing of an electric circuit which was recording observations of the direction and force of the wind, since electric oscillations were then set up in the conductors connected with the apparatus by the common conductor leading to the earth plate. In order to distinguish these marks from the others made by atmospheric electricity, the observers, who produced the ringing, made a note each time on the cylinder. This action upon the apparatus was, however, useful for the purpose of being sure that it continued in good order."

That this primary object was not telegraphy is shown by the paragraph with which he concludes his paper (*loc. cit.*). He says—

"In conclusion, I may express the hope that my apparatus, with further improvements, may be adapted to the transmission of signals to a distance by the aid of quick electric vibrations as soon as a means for producing such vibrations possessing sufficient energy is found."

We are left, then, with this unquestionable fact, that at the beginning of 1896, although the most eminent physicists had been occupied for nine years in labouring in the field of discovery laid open by Hertz, and although the notion of using these Hertzian waves for telegraphy had been clearly suggested, no one had overcome the practical difficulties, or actually given any exhibition in public of the transmission of intelligence by alphabetic or telegraphic signals by this means. The appliances in a certain elementary form existed, the advantages and possibilities of electric wave telegraphy had been pointed out, but no one had yet conquered the real practical difficulties, and exhibited the process in actual operation.

2. Marconi's Work, 1895-1898.—Meanwhile, a young investigator had been busy in Italy. Guglielmo Marconi was born at Bologna on April 25, 1874, and very early displayed an original and inventive mind. He studied physics under

[5] See *The Electrician*, August 17, 1894, vol. 33, p. 458. For pictures of the Lodge apparatus exhibited at Oxford, see *The Electrician*, vol. 39, p. 687.

Professor Rosa of the Leghorn Technical School, and made himself acquainted with the published writings of Professor Righi of the University of Bologna, whose valuable work on electromagnetic radiation was well known.

When little more than twenty years of age, Marconi had not only acquired much knowledge of Hertzian wave research, but he had clearly formed the intention of devoting himself to its utilization for effecting wireless telegraphy.

On his father's estate at the Villa Griffone, near Bologna, he began experimenting in June 1895 with Hertzian waves, using an ordinary spark induction coil, and making for himself experimental coherers or various forms of the Branly tube. Before long he originated an important improvement. Instead of employing the Hertzian form of radiator, he connected one terminal of the secondary circuit of his induction coil to a metal plate or net laid on the ground, and the other by a wire to a metal can or cylinder, placed on the summit of a pole. The spark balls were kept at such a distance that on closing the primary circuit of the coil an oscillatory spark passed between them. At the receiving end he similarly connected a metallic filings sensitive tube between an earth plate and an insulated conductor or capacity. He then began systematically to examine the relation between the distance at which the spark could affect his coherer and the elevation of his cans or cylinders above the ground. This brought him speedily to the discovery that the higher the cans the greater the distance over which he could work.

Thus in 1895 he was using cubes of tin about 1 foot in the side as elevated conductors or capacities, and found that when placed on the tops of poles 2 ms. high he could receive signals at 30 ms. distance, and when placed on poles 4 ms. high at 100 ms., and at 8 ms. high at 400 ms. With larger cubes of 100 cms. side fixed at a height of 8 ms. Morse signals could be transmitted 2400 metres, or 1½ miles all round.

Before this time, however, he had improved the Branly metallic filings tube, and produced his own nickel-filings sensitive tube already described (see Chap. VI. Fig. 4). He had combined this sensitive and regularly acting improved coherer with an electric tapping arrangement, but with more careful insight into the conditions to be fulfilled and a greater range of adjustment than previous workers.

He added also to the filings tube a pair of inductances or choking coils, intended to prevent the electric oscillations passing through the circuit in parallel with the tube, and compel them to expend their energy on the tube itself. He placed in series with the tube a single voltaic cell and a sensitive relay, and employed the relay to actuate a Morse printing instrument worked by a separate set of cells. In addition, he placed shunt circuits across the tapper break contacts and relay contacts to prevent sparking, and therefore disturbances of the sensitive tube by local effects.

Finally, he mounted the whole receiving arrangement on a board and enclosed the tube, tapper, and relay in a metallic box to shield them from the direct action of electric sparks made in their vicinity.

In the primary circuit of the induction coil at the transmitting end he placed a Morse sending key, and he connected the secondary terminals to the earth and to an elevated conductor as described. At the receiving end he connected, in the early experiments, one end of the coherer tube to an earth plate, and the opposite terminal to an elevated capacity. Lastly, he made such adjustments of the tapping arrangements that when a short series of oscillatory sparks were made at the induction coil by just depressing the Morse key in its primary circuit for one moment, the combination at the receiving end printed a *dot* on the Morse tap, and when the key was depressed for a longer time it printed a *dash*. In this manner the two signals required for forming an alphabet on the Morse code were obtained, and letters and words could be printed on the tape at the receiving end by properly handling the key at the transmitting end.

He employed at first the ball discharger of Professor Righi, which consisted of four solid brass balls, the two larger central ones being separated by a certain small interval, and the space between filled with vaseline oil kept in position by a non-conducting jacket or membrane.

In some experiments Marconi placed the discharge balls in the focal line of

a cylindrical parabolic mirror, and the receiver in the focus of another similar mirror, using, for the purpose of collecting the wave energy, two metal strips or rods, attached to the extremities of the coherer tube.

In 1896 he came to England with this apparatus, and on June 2, 1896, he applied for a British patent, No. 12,039, for the invention, which was duly granted. The complete specification was filed March 2, 1897.[6]

In July 1896 he introduced his invention and new method of telegraphy to the notice of Sir William Preece, then engineer-in-chief to the British Government Telegraph Service, who had for the previous twelve years interested himself in the development of wireless telegraphy by the inductive-conductive method.

On June 4, 1897, Sir W. H. Preece gave a lecture to a large audience at the Royal Institution in London on "Signalling through Space without Wires."[7] In this lecture, after expounding older and other methods, he devoted considerable time to exhibiting and explaining the Marconi apparatus, and spoke of it in the following terms :—

> "In July last Mr. Marconi brought to England a new plan. Mr. Marconi utilizes electric or Hertzian waves of very high frequency. He has invented a new relay which for sensitiveness and delicacy exceeds all known electrical apparatus. The peculiarity of Mr. Marconi's system is that, apart from the ordinary connecting wire of the apparatus, conductors of very moderate length only are needed, and even these can be dispensed with if reflectors are used."

Testifying to its practicability as a telegraphic method, Sir William Preece said—

> "Excellent signals have been transmitted between Penarth and Brean Down, near Weston-super-Mare, across the Bristol Channel, a distance of nearly nine miles. On Salisbury Plain Mr. Marconi covered a distance of four miles."

As regards the means used, it was stated that up to a distance of four miles a 6-inch spark coil sufficed, but for greater distances a 20-inch spark coil had been employed. In these experiments the method with reflecting mirrors was tried, but the chief part was carried out by connecting one terminal of the coherer and the spark coil secondary circuit respectively to earth, and each of the other terminals to nearly vertical wires upheld by masts, these wires terminating sometimes in metal plates or cylinders, or else the wires were upheld by balloons or kites covered with tinfoil in the manner shown in diagram in Fig. 1.

This testimony on the part of Sir William Preece at once attracted great attention. It was seen that Marconi had given a new form and very exalted powers to the Hertzian radiator. His long elevated wire upheld by the kite or mast corresponded to one wing of Hertz's oscillator, whilst the earth plate was the other. The induction coil charged the elevated wire, and when the spark jumped across the balls, the sudden discharge set up oscillations in that wire and radiated from it an electric wave. At the receiving station this wave fell upon the aerial wire, and by cutting across it created in the wire high frequency alternating electromotive force and electric oscillations. These oscillations passing through the coherer changed its conductivity and enabled the local cell to send a small current through the relay and operate the Morse printer. The ingenious tapping arrangements kept the coherer constantly in a receptive condition, so that if the key at the transmitting station was operated in Morse fashion the shorter or longer group of sparks made at the spark balls was indicated by a short or long mark on the Morse inker.

The evidence at this date all goes to show that the highest authorities on the subject admitted the novelty of Marconi's telegraphic method and appliances.

Sir William Preece, at the conclusion of his lecture, combated the contention

[6] The United States of America equivalent patent was numbered originally No. 586,193, applied for December 7, 1896, and issued July 13, 1897. After amendment it was reissued as No. 11,913, granted June 4, 1901.

[7] See *The Electrician*, June 11, 1897 vol. 39, p. 216; also *Proc. Roy. Inst. Lond.*, 1897, vol. xv. p. 467.

which appears to have been raised, that Mr. Marconi had done nothing new, and said (*loc. cit.*)—

"He has not discovered any new rays; his receiver is based on Branly's coherer. Columbus did not invent the egg, but he showed how to make it stand on its end, and Marconi has produced from known means a new electric eye more delicate than any known electrical instrument, and a new system of telegraphy that will reach places hitherto inaccessible. . . . Enough has been done to prove and show that for shipping and lighthouse purposes it will be a great and valuable acquisition."

The news of these successful demonstrations spread abroad and excited great interest. Amongst those who had been giving attention to the utilization of Hertzian waves was the late Dr. Slaby, at that time a Professor in the Technical High School at Charlottenburg, Berlin, and he at once hurried to England to

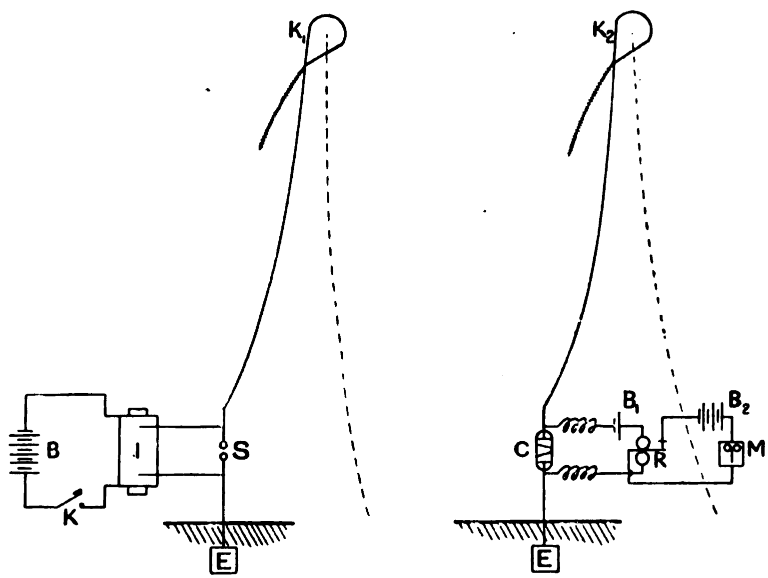


FIG. 1.—Marconi's Apparatus for Wireless Telegraphy in 1896. B, battery; I, induction coil; S, spark balls; K, sending key; E, earth plate; K_1, K_2, kites upholding aerial wires; C, coherer; R, relay; M, Morse printing instrument; B_1, B_2, batteries.

discover how Marconi had solved the problem that had hitherto baffled him (Professor Slaby).

After seeing and assisting in the experiments across the Bristol Channel, Professor Slaby wrote a magazine article on "The New Telegraphy,"[8] and made the following remarks:—

"In January 1897, when the news of Marconi's first successes ran through the newspapers, I myself was earnestly occupied with similar problems. I had not been able to telegraph more than one hundred metres through the air. It was at once clear to me that Marconi must have added something else—something new—to what was already known, whereby he had been able to attain to lengths measured by kilometres. Quickly making up my mind, I travelled to England, where the Bureau of Telegraphs was undertaking experiments on a large scale. Mr. Preece, the celebrated engineer-in-chief of the General Post Office,

[8] See Dr. A. Slaby on "The New Telegraphy," *The Century Magazine*, April 1898, vol. 55, p. 867.

in the most courteous and hospitable way, permitted me to take part in these; and in truth what I there saw was something quite new. Marconi had made a discovery. He was working with means the entire meaning of which no one before him had recognized. Only in that way can we explain the secret of his success. In the English professional journals an attempt has been made to deny novelty to the method of Marconi. It was urged that the production of Hertz rays, their radiation through space, the construction of his electrical eye —all this was known before. True; all this had been known to me also, and yet I was never able to exceed one hundred metres.

"In the first place, Marconi has worked out a clever arrangement for the apparatus which by the use of the simplest means produces a sure technical result. Then he has shown that such telegraphy (writing from afar) was to be made possible only through, on the one hand, earth connection between the apparatus and, on the other, the use of long extended upright wires. By this simple but extraordinarily effective method he raised the power of radiation in the electric forces a hundredfold."

The two and a half years between June 1896 and December 1898 were occupied by Marconi with numerous public demonstrations of the utility of his system of wireless telegraphy. Space cannot be afforded for a detailed history, but the general facts are as follows:—

The autumn of 1896 was occupied with experiments carried out before representatives of the British Government Postal Telegraph Department, and communication was established over a distance of 2 miles. Tests were also carried out in the presence of the Navy and Army representatives (Captain Jackson, R.N., and Major Carr, R.E.) on Salisbury Plain, during the month of March 1897, when transmission over a distance of 8 miles was demonstrated. In May 1897 the experiments already described, between Penarth and Weston-super-Mare, were made across the Bristol Channel, a distance of 9 miles. In July 1897 Marconi undertook demonstrations for the Italian Government at Spezzia, in Italy, and covered a distance of 12 miles between warships. Communication was then set up by him between Alum Bay, in the Isle of Wight, and Bournemouth, England, a distance of about 14 miles over sea, and the working of the system was inspected by the author in April 1898. Marconi was at that time using as the transmitter a 10-inch spark induction coil, and a discharger consisting of four balls of brass, each about 2 inches in diameter, spaced slightly apart in an ebonite frame.

One of the outer balls was connected by a thick wire to an earth plate, and the other outer ball by a wire to an insulated strip of wire netting about 120 feet in length, which was upheld by an ebonite insulator attached to a sprit hauled up to the top of a 120-foot mast (see Fig. 2). These balls were also in connection with the secondary terminals of the induction coil, and the four brass discharge balls were set with air gaps about $\frac{1}{4}$ inch, or 5 to 6 mms., long between the balls. In the primary circuit of the induction coil was placed a massive Morse key with heavy platinum contacts. Marconi had at that time abandoned the use of the Righi discharger with balls in oil. The receiver used was exactly as already described. With this apparatus telegraphic messages were sent in Morse code at about a rate of 12 to 15 five-letter words per minute. The working of this Isle of Wight to Bournemouth plant was inspected by many notable men, *e.g.*, Lord Tennyson, Lord Kelvin, and others; and Lord Kelvin gave practical expression to his opinion that it was already in a commercial condition by paying for a message sent by him to Sir William Preece at the General Post Office, London, on June 3, 1898.

In May 1898 communication was established for the Corporation of Lloyds between Ballycastle and the Lighthouse on Rathlin Island in the North of Ireland, the distance being 7·5 miles.

In July 1898 the Marconi telegraphy was employed to report the results of yacht races at the Kingstown Regatta for the *Dublin Express* newspaper. A set of instruments were fitted up in a room at Kingstown, and another on board a steamer, the *Flying Huntress*. The aerial conductor on shore was a strip of wire netting attached to a mast 40 feet high, and several hundred messages were sent and correctly received during the progress of the races. The distances were from 5 to 20 miles.

At that time His Royal Highness, the then Prince of Wales, had the misfortune

to injure his knee, and was confined on board the royal yacht *Osborne* in Cowes Bay. Mr. Marconi fitted up his apparatus on board the royal yacht by reqnest, and also at Osborne House, Isle of Wight, and kept up wireless communication for three weeks between these stations. The shore mast was 105 feet high, and the wire on board the yacht 83 feet high. The distances covered were small; but as the yacht moved about, on some occasions high hills were interposed, so that the aerial wires were overtopped by hundreds of feet, yet this was found to be no obstacle to communication.

The success of these demonstrations led the Corporation of Trinity House to afford an opportunity for testing the system in actual practice between the South Foreland Lighthouse, near Dover, and the East Goodwin Lightship, on the Goodwin Sands. This installation was set in operation on December 24, 1898, and proved to be not only most successful, but of the greatest practical value. It was shown that when once the apparatus was set up it could be worked by ordinary seamen with very little training.

At the end of 1898 electric wave telegraphy had thus been established by Marconi on a practical basis. He had demonstrated its utility, especially for communication between ship and ship and ship and shore. A work which could not be accomplished by any other system.[9]

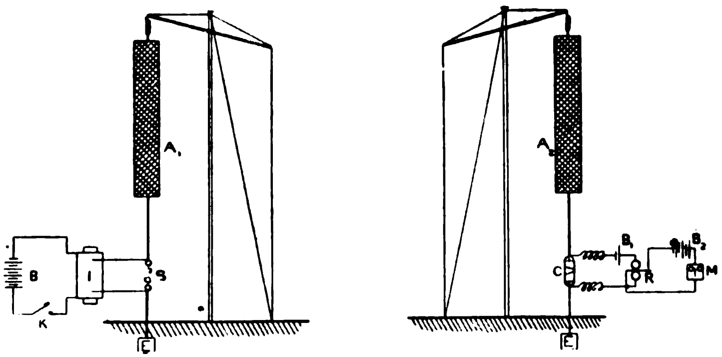


FIG. 2.—Marconi's Apparatus for Wireless Telegraphy as used in 1896-98. A_1, A_2, strips of wire netting constituting the antennæ upheld by insulators at the top of masts. The remaining letters refer to apparatus as mentioned under Fig. 1.

It had been shown that the advantages were as follows:—

(i.) It worked as well by night as by day, and in bad weather, fogs, or storms, as well as in fair weather; provided that the proper insulation of the aerial wire or elevated conductor was maintained.

(ii.) In certain electrical conditions of the atmosphere, and during thunder-storms, some difficulty was usually found in working, owing to the atmospheric discharges affecting the sensitive tube, and therefore making stray marks on the Morse tape of the printer, but seldom sufficient to interrupt communication altogether.

(iii.) The interposition of high hills, trees, or the curvature of the earth did not prevent communication, though slightly affecting the power required. It worked particularly well over sea surface, and between ships and shore stations.

(iv.) The apparatus could be set up and handled by any ordinary telegraphist, and the record was made on paper strip in the usual Morse code.

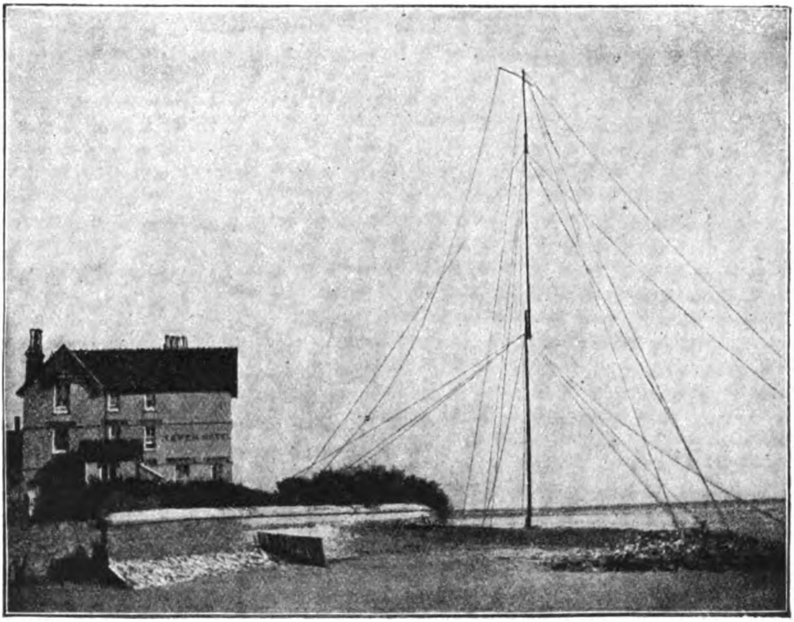
(v.) It easily covered distances far beyond those feasible or attained by other systems of wireless telegraphy.

[9] A summary of his work on wireless telegraphy up to the beginning of 1899 is given in a paper read by Mr. Marconi to the Institution of Electrical Engineers on March 2, 1899. See *Journal of the Inst. Elec. Eng.*, 1899, vol. 28, p. 273.

(vi.) Lastly, the apparatus required was by no means costly, and, with the exception of the mast required for upholding the aerial wire, it occupied but little space, and was particularly adapted for use on board ship.

The general appearance of the collected sending and receiving apparatus required inside the station or cabin is shown in Fig. 7.

3. Marconi's Improvements in 1898 and 1899.—Marconi was desirous of working over still greater distances than those already covered, but the difficulties of erecting masts for elevating the aerial conductor beyond a certain height were considerable. A mast 100 or 120 feet high is a comparatively simple thing to set up. It can be erected in three sections, and the aerial wire can be supported by insulators from a cross sprit at the top (see Fig. 3).



[*By kind permission of the Marconi Wireless Telegraph Company.*

FIG. 3.—The Haven Hotel, Sandbanks, Poole, and the Wireless Telegraph Mast. At this station much of Mr. Marconi's research work on wireless telegraphy was carried out between 1898 and 1908.

At the beginning of 1899, masts 120 to 140 feet high were employed, and an aerial wire consisting of a stranded copper wire 7/20 or 7/22 (generally an india-rubber insulated wire) was used. Very often a cylinder of wire netting was attached at the top or insulated end, and sometimes two or more aerial wires in parallel were used. The insulators were round rods of ebonite, about 24 inches long and 1 inch in diameter. When using simple wires and the receiving and transmitting apparatus of the 1896-1898 type, Marconi had found that the maximum distance which could be covered seemed to increase in proportion to the square of the height of the aerial wire, so that with aerials 100 feet high at each end the maximum working distance was four times that obtained with aerials 50 feet high.

He introduced, however, at this date an improvement into his receiving arrangements which had the result of increasing its sensitiveness. Instead of inserting the sensitive metallic filings tube or cymoscope directly between the

earth plate and the bottom of the receiving aerial wire, an oscillation transformer of a particular form was interposed (see Fig. 4).

In considering the production of stationary electric oscillations in wires in Chap. IV., it has been explained that if a vertical wire is set up with its lower end in connection with an earth plate, then when the fundamental oscillation is set up in the wire we have a node of potential, and an antinode or maximum of current at the lower or earthed end of the aerial.

If, then, we cut this aerial wire near the earth and insert a coherer between the earth plate and the aerial wire, the production of oscillations in the wire only results in establishing a relatively small difference of potential between the terminals of the coherer. This instrument is, in fact, being employed in a very inefficient manner, and inserted in the wrong place. This was clearly pointed out by A. Slaby, who also suggested a way of overcoming the difficulty. Marconi

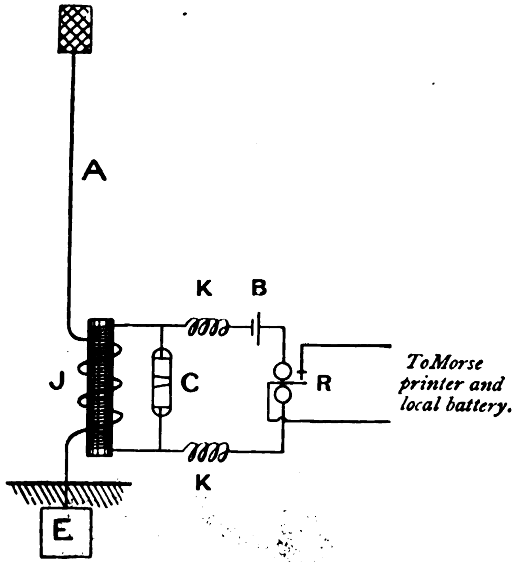


FIG. 4.—A, receiving antenna, or aerial wire, ~~with capacity plate at summit ;~~ J, jigger, or oscillation transformer; C, coherer, or sensitive tube; E, earth plate; R, relay; B, relay cell; K, K, choking coils.

adopted the better plan of inserting in the base of the aerial wire near the earth the primary coil of a small air core transformer, J, of a peculiar kind, the secondary terminals of which were connected to the sensitive tube, C (see Fig. 4). In this manner the large current existing near the base of the aerial was, so to speak, transformed into high voltage for use at the terminals of the coherer. To do this effectively, however, requires a special form of transformer. Lodge had previously suggested in a British patent specification the employment of a transformed oscillation for affecting the coherer, so that it was operated by secondary oscillations, and not directly by those in the receiving rods.[10] Lodge, however, gave no details of construction, and from the diagram in his specification it is impossible to determine the dimensions and the nature of the circuits suitable for making a transformer which will be operative in any given case. Marconi discovered, after innumerable experiments, the proper form to give to such an oscillation transformer, and particularly described it in patent specifications and lectures.

[10] See Lodge's British Patent, No. 11,575 of 897

Marconi's oscillation transformer, or "jigger," in one form consists of a glass tube about 1 cm. in diameter and 4 to 8 cms. in length. On this is wound a primary circuit consisting of a length of silk-covered copper wire, which may vary in diameter, according to circumstances, from No. 26 to No. 40 S.W.G. This coil is put on in one layer, or in two or more layers, which may be joined in parallel or in series.

The inventor points out that the best results are obtained when the secondary circuit of the oscillation transformer has a total length equal to that of the transmitting aerial. This, however, must be understood to apply to the form of simple transmitting aerial up to that time used. We shall consider more particularly in another chapter the general physical theory of these oscillation transformers.

A large variety of forms of receiving oscillation transformer have at various times been employed by Marconi, and it forms a very important element in his system. In order to secure good results, or, in fact, any result at all, the length of the secondary circuit of this receiving oscillation transformer must bear a certain relation to the length of the wave used.

4. Marconi's English Channel Experiments in 1899.—Just before Easter, 1899, Marconi obtained from the French Government permission to erect a mast for wireless telegraph experiments at Wimereux, near Boulogne, in France, and a corresponding mast was erected at the South Foreland Lighthouse, near Dover, on the coast of England. The distance of these stations from one another was 32 miles (50 kilometres).

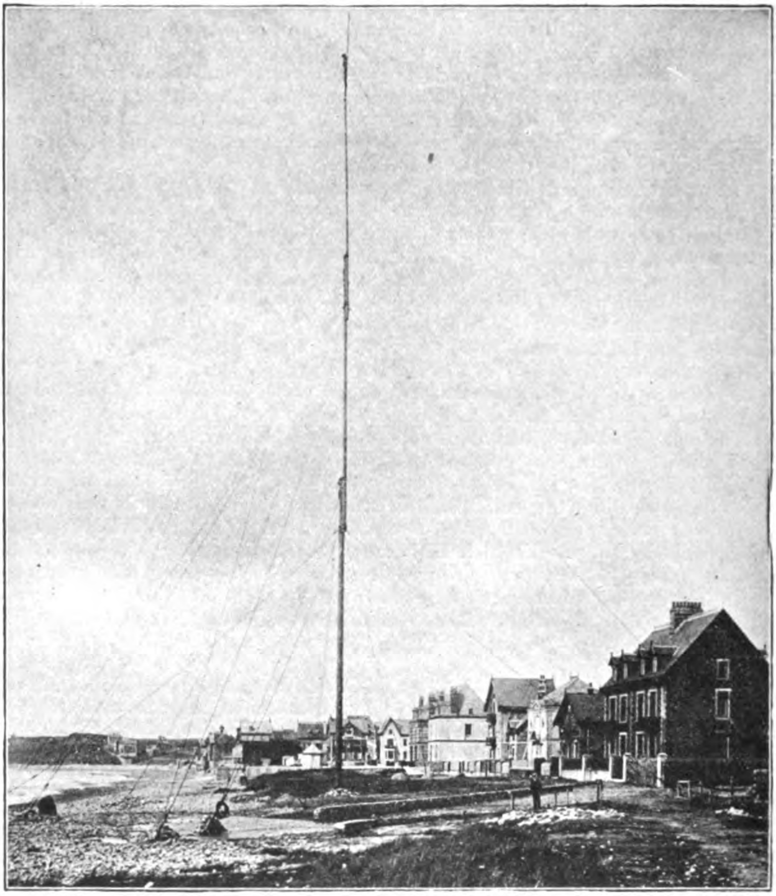
The apparatus for sending and receiving was erected in a small room in the South Foreland Lighthouse on the English side of the Channel, and on the French side in the Chalet d'Artois, at Wimereux (see Fig. 5).

The aerial wires were single-stranded copper wires 150 feet long, insulated with india-rubber, and upheld at the top by ebonite rods as insulators. As soon as the plant was complete Marconi transmitted messages, on March 27, 1899, across the English Channel, and sent communications in this manner from Wimereux to numerous scientific friends in England. The result was to create an immense public interest in the achievement all over the world. Up to that moment wireless telegraphy by electric waves had attracted only a very limited general attention; but the bridging of the English Channel by electric waves was one of those sensational feats which at once aroused the daily press to lively comment on the matter. The author, after spending some time in examining the appliances and working, wrote a letter to the *Times*, published on April 3, 1899, part of which was as follows:—

"During the last few days I have been permitted to make a close examination of the apparatus and methods being employed by Signor Marconi in his remarkable telegraphic experiments between South Foreland and Boulogne, and at the South Foreland Lighthouse have been allowed by the inventor to make experiments and transmit messages from the station there established both to France and to the lightship on the Goodwin Sands, which is equipped for sending and receiving ether wave signals. Throughout the period of my visit, messages, signals, congratulations, and jokes were freely exchanged between the operators sitting on either side of the Channel, and automatically printed down in telegraphic code signals on the ordinary paper slip at the rate of twelve to eighteen words a minute. Not once was there the slightest difficulty or delay in obtaining an instant reply to a signal sent. No familiarity with the subject removes the feeling of vague wonder with which one sees a telegraphic instrument merely connected with a length of 150 feet of copper wire run up the side of a flagstaff begin to draw its message out of space and print down in dot and dash on the paper tape the intelligence ferried across 30 miles of water by the mysterious ether.

"The apparatus, moreover, is ridiculously simple and not costly. With the exception of the flagstaff and 150 feet of vertical wire at each end, he can place on a small kitchen table the appliances, costing not more than £100 in all, for communicating across 30 or even 100 miles of channel. With the same simple means he has placed a lightship on the Goodwins in instant communication, day and night, with the South Foreland Lighthouse. A touch on a key on board the lightship suffices to ring an electric bell in the room at South Foreland, 12 miles away, with the same ease and certainty with which one can summon the servant to one's bedroom at an hotel. An attendant now sleeps hard by the instruments at South Foreland. If at any moment he is awakened by the bell rung from the lightship, he is able

to ring up in return the Ramsgate lifeboat, and, if need be, direct it to the spot where its services are required, within a few seconds of the arrival of the call for help. In the presence of the enormous practical importance of this feat alone, and of the certainty with which communication can now be established between ship and shore without costly cable or wire, the scientific criticisms which have been launched by other inventors against Signor Marconi's methods have failed altogether in their appreciation of the practical significance of the results he has brought about.



[*By kind permission of the Marconi Wireless Telegraph Company, Ltd.*

FIG. 5.—Mast and Antenna Wire at the Chalet d'Artois, Wimereux, Boulogne, whence the first wireless messages were sent across the English Channel in March 1899 by Mr. Marconi.

"Up to the present time none of the other systems of wireless telegraphy employing electric or magnetic agencies has been able to accomplish the same results over equal distances. Without denying that much remains yet to be attained, or that the same may not be effected in other ways, it is impossible for anyone to witness the South Foreland and Boulogne experiments without coming to the conclusion that neither captious criticism nor official lethargy should stand in the way of additional opportunities being afforded for a further extension of practical experiments. Wireless telegraphy will not take the place of

telegraphy with wires. Each has a special field of operations of its own, but the public have a right to ask that the fullest advantage shall be taken of that particular service which ether wave telegraphy can now render in promoting the greater safety of those at sea, and that, in view of our enormous maritime interests, this country shall not permit itself to be outraced by others in the peaceful contest to apply the outcome of scientific investigations and discoveries in every possible direction to the service of those who are obliged to face the perils of the sea. If scientific research has forged a fresh weapon with which in turn to fight nature, 'red in tooth and claw,' all other questions fade into insignificance in comparison with the inquiry how we can take the utmost advantage of this addition to our resources."

Although many scientific men at that time refused to admit that these cross-Channel experiments were indications of the utility of the Marconi telegraphy, some of the remarks in the author's letter to the *Times* just quoted received singular confirmation a few days later. During a dense fog on the Channel on April 28, 1899, a steamer, the *R. F. Matthews*, outward bound, ran into the East Goodwin Lightship and inflicted serious damage. The lightship, however, being provided with the Marconi apparatus, was able to communicate at once with the station at South Foreland Lighthouse, and tugs and a lifeboat were sent out immediately from Ramsgate to the assistance of the lightship. But for this timely aid the lightship would most probably have sunk. These demonstrations were continued uninterruptedly during the year 1899.

In the autumn of that year the British Association held its annual assembly at Dover. This meeting, taking place just a hundred years after the date of Volta's epoch-making invention of the Voltaic pile, was made the occasion of certain celebrations. The author, by request, delivered an evening discourse on "The Centenary of the Electric Current" before the British Association in the Town Hall, Dover.[11] At his suggestion a mast had been erected on the tower for the purposes of wireless telegraphy (see Fig. 6). The Marconi apparatus was set up on the lecture table and placed in direct communication with the South Foreland Lighthouse (4 miles), with Wimereux, in France (33 miles), and with the East Goodwin Lightship (12 miles) (see Fig. 7).

During the lecture messages were sent to the President of the French Association for the Advancement of Science (M. Brouardel), then meeting at Boulogne, and numerous messages exchanged with the South Foreland station and the East Goodwin Lightship. Subsequently messages were sent from Wimereux, in France, and received directly at a Marconi station established at Chelmsford, in England, a distance of 85 miles, of which 30 miles were over sea and 55 miles over land. The height of aerials at both stations was 150 feet.

In the same year, the interest of the public being greatly aroused over the races for the International Cup between British and American yachts, Mr. Marconi went over to the United States and employed his apparatus and system of telegraphy between a ship and the shore, for reporting the results of the races during their progress, for the *New York Herald* newspaper. Over four thousand words were transmitted in less than a total of five hours' work done on different days.

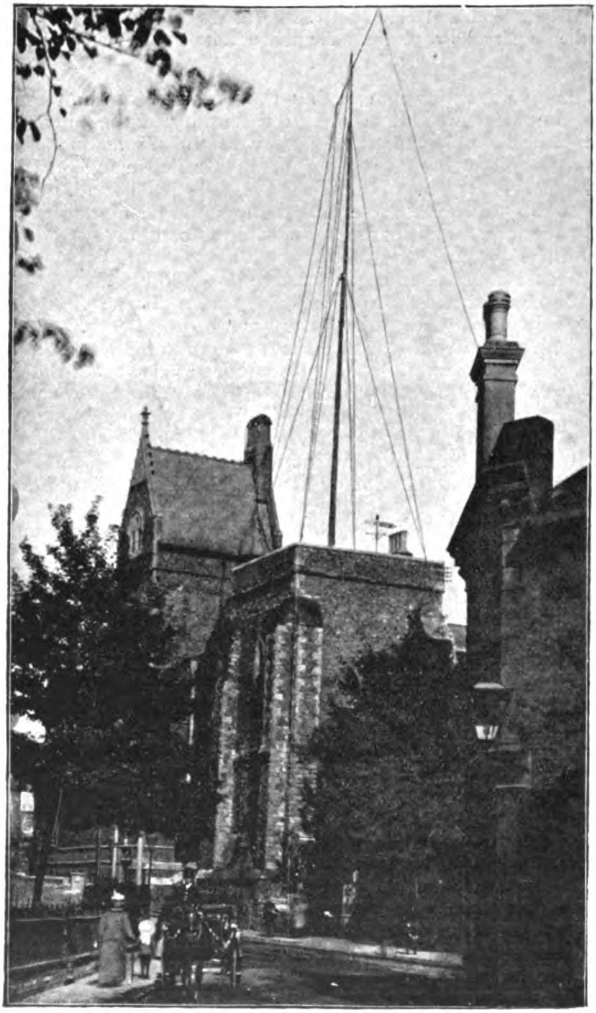
A more important application was, however, made in July and August 1899, during the naval manœuvres of the British Navy. Three vessels of the Reserve Squadron were fitted with the apparatus, and most important evolutions were carried out by orders given by Marconi wireless telegraphy. Two cruisers (*Juno* and *Europa*) were equipped, and in some cases important orders and information were transmitted instantly 85 miles. A full account of the result obtained was published by Commander S. Statham, R.N.[12] In this work the value of the oscillation transformer in the receiving aerial was fully demonstrated, and also the fact that the curvature of the earth seemed in no way to interfere with the transmission of the electromagnetic waves radiated from the aerials even over great distances. These demonstrations assisted to establish electric wave wireless telegraphy both for naval and mercantile marine purposes on a firm basis.

[11] See *The Electrician*, vol. 43, p. 764, 1899.

[12] See article in the *Army and Navy Illustrated*, August 1900; also a Friday Evening Discourse at the Royal Institution by G. Marconi, February 2, 1900, *Proc. Roy. Inst.*, vol. xvi. No. 94, p. 251.

By the end of 1900 the new supermarine wireless telegraphy had taken an unassailable position as an essential aid to navigation, commerce, and naval operations.

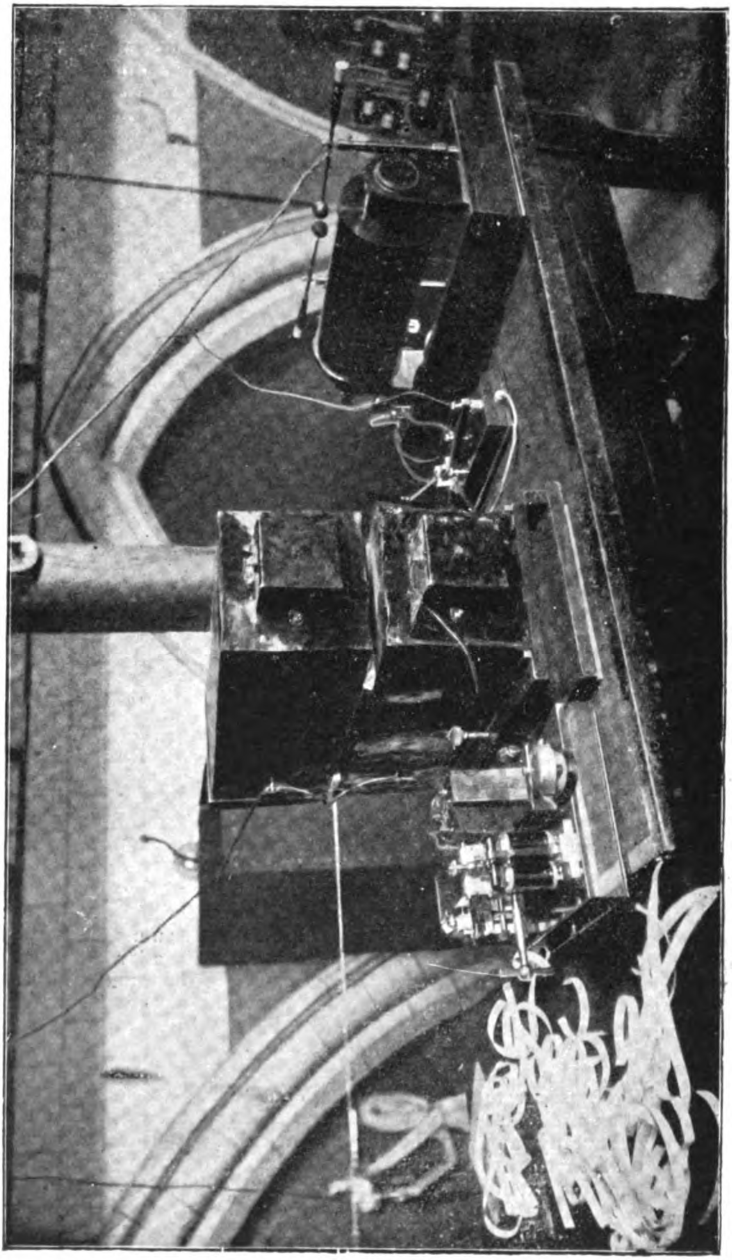
5. The Evolution of Syntonic Wireless Telegraphy, 1897-1901.—From the very commencement of practical electric wave telegraphy it was recognized that



[*From "The Electrician."*

FIG. 6.—Mast and Marconi Aerial Wire erected on the Tower of Town Hall, Dover, August 1899, for Reception of Messages from France during the Meeting of the British Association in September.

some means must be found for limiting the receptivity of wireless telegraph stations. The simple form of wave-detecting arrangement, first used by Marconi before he introduced the peculiar oscillation transformer just described, is sensitive to electric waves varying very considerably in wave-length; in fact, a single



[From "The Electrician."

FIG. 7.—Marconi Wireless Telegraph Apparatus arranged on the Lecture Table in the Town Hall, Dover, for a Lecture by Dr. J. A. Fleming, F.R.S., on "The Centenary of the Electric Current," before the British Association, September 1899.

electromagnetic impulse, or so-called solitary wave, if strong enough, will affect it. Hence atmospheric electrical discharges and stray or vagrant waves sent out by any source are readily picked up by it.

Several distinct problems here present themselves. In the first place, we may desire to make any given receiving station normally responsive only to electromagnetic waves of one particular wave-length. In the next place, we may wish to render that station proof against deliberate attempts to hinder communication by throwing on to it violent vagrant or disturbing waves. Thirdly, we may want to prevent foreign stations from picking up messages not intended for them which are being sent out from some transmitter, and intended only for some particular receiving station.

The first problem is an easier one to solve than the second and third. We shall defer to a later section the consideration of the different practical solutions which have been offered of these problems, and confine ourselves here to a brief mention of the work done between 1897 and 1900 on this subject.

Almost immediately after the application by Marconi for his first patent for electric wave telegraphy, Sir Oliver Lodge, who as we have seen had devoted himself strenuously to Hertzian wave research, applied for and obtained a British patent (No. 11,575 of 1897) for "Improvements in Syntonized Telegraphy without Line Wires." This patent proved subsequently to be a fundamental one in con-

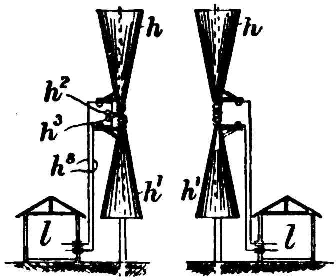


FIG. 8.—Lodge's Wing-shaped Antennæ for Electric Wave Telegraphy.

nection with this subject. Lodge clearly recognized that to place electric wave or radiotelegraphy on a practical basis, the transmitter and receiver must be syntonized to each other or made to have the same natural time-period of free electric oscillations. Under these conditions oscillations would be set up in the receiving circuits by the impact of the waves sent out from the corresponding transmitting station, but would not be set up by those of different and non-identical frequency. He also saw that the oscillations in the transmitter must therefore be feebly damped so that each train of oscillations must comprise a large number of oscillations. Hence each train of waves falling on the receiver would be able by the cumulative effect of its many impulses to set up oscillations in the receiving circuits having considerable amplitude. He therefore devised apparatus for conducting syntonic wireless telegraphy as follows:—

His radiator consisted of a pair of "capacity areas," or triangular-shaped metal plates, h, h' (see Fig. 8), separated by a spark gap, but having an inductance coil, generally shown as a spiral of a few turns, interposed. In some cases this radiator was to be used horizontally, and in other cases vertically. In this last case the lower metal wing or area might be connected to the earth, or partly buried in the earth, and the upper wing extended by connection to an insulated plate.

Lodge asserted that this form of radiator was capable of persistent or long-sustained oscillations, suitable, therefore, for effecting syntonic telegraphy. He was well aware, and states (*loc. cit.*, p. 2, line 53) that unless the radiator provides these sustained trains of waves, no true syntonic action is possible. A part of the specification is taken up with descriptions of methods of charging electrically these oscillators. The receiving arrangement was to consist of a pair of capacity

areas (one of which might be the earth) similar to the transmitter, but containing in its circuit a Branly coherer, consisting of a tube of metallic filings with a "clock, or a tuning fork, or a cog wheel, or other device" mounted on the stand of the coherer to cause a tremor of sufficient intensity. This vibrator or decoherer was evidently to be maintained continuously in action. In some cases the coherer was inserted in the secondary circuit of "a species of transformer," the primary of which was in the circuit of the collecting wings, but no detailed instructions are given for making this oscillation transformer or properly relating the lengths of its circuit and its turns to the capacity and inductances of the collecting circuit. Without this adjustment the oscillation transformer is a detriment rather than an advantage.

Early in 1900 Marconi applied for a British patent (No. 7777 of April 26, 1900), in which appliances were described for conducting syntonic telegraphy as well as simultaneous multiplex telegraphy with single aerials.

Some mention of these advances was made by the author in a letter published in the *Times* of October 4, 1900, in which the results of certain remarkable demonstrations given in the previous month were described. Reference was also made to them in Cantor lectures on "Electric Oscillations and Electric Waves," given by the author to the Society of Arts in November and December 1900; and they were subsequently more fully discussed in a paper read to the Society of Arts by Mr. Marconi on May 15, 1901, entitled "Syntonic Wireless Telegraphy." [13]

The particulars of the apparatus described in the above-mentioned specification of Marconi are as follows :—

At the transmitting end the original arrangement of an aerial wire connected to one spark ball of the induction coil, the other being earthed (now called a plain aerial), was exchanged for an aerial consisting of a pair of inductively coupled circuits. A condenser, usually taking the form of a battery of Leyden jars, had one terminal connected to one spark ball of an induction coil, and the other to the primary circuit of an oscillation transformer. The opposite terminal of this transformer circuit was joined to the second spark ball. These spark balls were placed, as usual, in connection with the secondary terminals of an induction coil. The secondary circuit of this oscillation transformer was inserted between the aerial wire and the earth plate, and an adjustable inductance coil included in the circuit (see Fig. 9).

The oscillation transformer is constructed as follows: It consists of a square wooden frame, wound over with a number of lengths of highly insulated, thick-stranded copper cable joined in parallel, so as to make a primary circuit of one turn of extremely low resistance. In some cases two or more turns may be employed. Over this is wound a secondary circuit of 5 to 10 turns, and the oscillation transformer is usually immersed in a vessel of highly insulating oil. This secondary circuit is joined in between the aerial and the earth, a variable inductance being interposed. When in position the oscillation transformer forms an inductive coupling between two circuits—one a nearly closed oscillation circuit of large capacity and small inductance, and the other an open oscillation of much smaller capacity and greater inductance.

These circuits are more or less closely "coupled" by varying the distance between the primary and secondary. By the adjustment of the variable inductance inserted between the earth plate and the secondary circuit of the oscillation transformer, and by variation of the capacity of the condenser in the primary circuit, the two circuits are brought into resonance with each other. When oscillations are set up in the closed circuit by the discharge of the condenser, the energy stored up in the Leyden jars is gradually drawn off and radiated by the open circuit. The closed circuit thus forms a reservoir of energy, and it is in itself a slightly damped circuit or persistent oscillator. The open circuit is a good radiator, and is kept supplied with energy by the reservoir. Hence we have a much more persistent train of oscillations set up in the aerial at each discharge than would be the case if the only storage of energy were that due to the small capacity of the aerial. The important matter, however, is the proper "tuning" of the two coupled circuits. This can be effected in several ways :—

[13] See *Journal of the Society of Arts*, issue for May 17, 1901, vol. 49, p. 505.

One plan is to employ a hot-wire voltmeter which is connected to two points on the circuit of the earth wire leading from the secondary circuit of the oscillation transformer to the earth plate. When oscillations are set up in the aerial, there is a difference of potential between these points, and the needle of the hot wire voltmeter makes a more or less steady deflection. This reading depends not only upon the maximum value of the oscillatory current during each train of oscillations, but upon the logarithmic decrement, and upon the number of groups of oscillations which take place per second. If the spark gap remains the same length, and the number of spark discharges per second is kept constant, then any change in the capacity of the condenser in the primary circuit or in the inductance of the aerial circuit will make this voltmeter reading either greater or less. We then make some small change in one of these factors, say the condenser capacity, such that

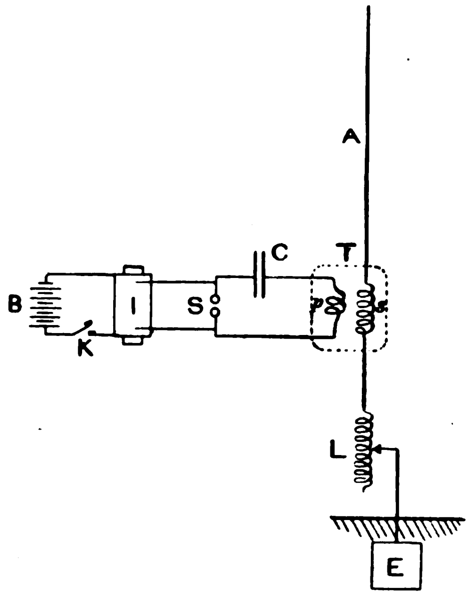


FIG. 9.—Arrangement of Transmitting Apparatus in Marconi System of Syntonic Wireless Telegraphy. A, antenna; L, tuning inductance; E, earth plate; *p*, *s*, oscillation transformer or jigger; C, condenser; S, spark balls; I, induction coil; B, battery; K, sending key.

the voltmeter reading is slightly increased. We then continue in the same direction until the voltmeter reading begins to decrease again. In this manner we can tell approximately when we have given such value to the capacity that the current in the aerial is a maximum for a given spark length and spark frequency. This indicates that the two coupled oscillation circuits are approximately in syntony. Another method is to alter the inductance in series with the aerial and secondary circuit of the oscillation transformer until the maximum potential difference between terminals of this secondary circuit is reached, as evidenced by the spark discharge between them being of the greatest possible length.

For the purposes of this test, a sliding ball discharger, highly insulated, with means for easily altering the distance of the balls, is joined across the secondary terminals of the transformer.

A third method is to hold a rectangle of wire near the lower part of the aerial, the rectangle having inserted in it a vacuum tube, preferably one containing

rarefied neon.[14] If the rectangle is placed with one side parallel to and near the aerial, the oscillatory currents induced in it will cause the vacuum tube to glow. We now alter either the inductance or capacity in either of the circuits, and notice whether the tube glows at a greater or less distance from the aerial, and so proceed to make small changes until we have succeeded in making the tube glow at the greatest possible distance from the aerial. This indicates that we have produced the maximum oscillation of current in the aerial. The spark length and spark frequency must, of course, remain unchanged during the test.[15]

Turning next to the receiver, the diagram of connections of Marconi's syntonic receiver is shown in Fig. 10.

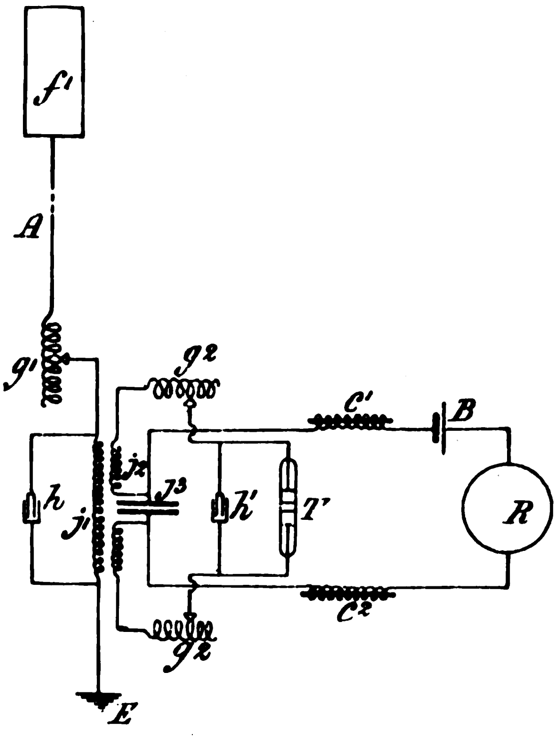


FIG. 10.—Arrangement of Receiving Apparatus in the early Marconi System of Syntonic Wireless Telegraphy. A, antenna; E, earth plate; g^1, g^2, tuning inductance; j^1, j^2, jigger; j^3, jigger condenser; c^1, c^2, choking coils; T, sensitive tube, or coherer; R, relay; B, battery.

A is the aerial wire, which may or may not be terminated in a plate or cylinder, *f*. At the foot of this aerial is an adjustable inductance, *g*, and this is connected to an earth plate, E, through the primary circuit, j^1, of an oscillation transformer. The terminals of this transformer are connected by a small sliding condenser, *h*. The secondary circuit, j^2, of this transformer is cut in the middle, and a condenser, j^3, inserted. The outer terminals of the secondary circuit are

[14] The advantages of using rarefied neon in a vacuum tube as a means of detecting electrical oscillations were first pointed out by the author in a paper read to the British Association in 1904. See *Phil. Mag.*, October 1904, p. 419.

[15] Another and more effective means is to employ the author's cymometer to make a measurement of the oscillation constant of the open and closed circuit respectively, and then to adjust the circuits so that they have the same oscillation constant. See Chap. VI. § 16.

connected through two small variable inductances, g^1 and g^2, with the terminals of the sensitive tube, T, and are also connected by an adjustable condenser, h'. From the terminals of the middle condenser, j^3, proceed two wires, which pass through choking coils, C^1 and C^2, and include the relay, R, and local cell, B, for working the relay. The Morse inker or other telegraphic instrument and associated battery connected to the relay are omitted from the diagram. The oscillation transformer, or jigger, placed in this receiving arrangement had its secondary circuit wound as already described in Marconi's three British Specifications, No. 12,326 of 1898, and Nos. 6982 and 25,186 of 1899 (see § 3 of this chapter).

To syntonize the receiver with itself and with the transmitter, the two circuits, viz. the open circuit, comprising the receiving aerial, and the closed circuit, comprising the secondary circuit of the oscillation transformer, and the inductances g^1 and g^2, and the condensers j^3 and h' in series with it, must be adjusted so that this open and closed circuit are in resonance with each other, and have the same natural time period as the transmitter circuits intended to correspond with them. These different frequencies are technically termed the various *tunes*, and the operation of putting the circuits into syntony or resonance is called *tuning* the receiver and transmitter to themselves and to each other.

In his British Patent Specification, No. 7777 of 1900, Marconi gives the details for nine *tunes*. For example, one tune he calls No. 7, and he gives the particulars of the transmitter and receiver as follows:—

The transmitting aerial consists of four vertical stranded 7/22 copper wires, each 48·6 ms. long, connected together at the top or insulated end, but kept apart throughout their length by being suspended from the arms of a wooden cross, each arm of which is 4 ms. long.

The capacity in the primary of the oscillation circuit consists of a number of Leyden jars in parallel, having a total capacity of 0·016 mfd.

The oscillation transformer consists of a square wooden frame, the side of which is 30·48 cms., or 12 inches, in length, wound over with a primary circuit of one turn, the total length of the primary being 150 cms. The secondary circuit consists of six turns of insulated wire wound on the same frame, three turns on each side of the primary. These two circuits are made of highly insulated india-rubber covered stranded copper cable, and the transformer, when made, is immersed in a vessel of highly insulating oil.

The oscillation transformer in the receiver has a secondary circuit consisting of 73·15 ms. of single silk-covered copper wire, No. 40 S.W.G., wound in one layer on a glass tube 5 cms. in diameter. The secondary is divided at its middle point. There are two primary circuits, each consisting of 2·75 ms. of copper wire 0·7 mm. in diameter, wound on tubes 6·5 cms. in diameter. The two primaries are placed over the two sections of the secondary circuit, and are joined in parallel. Another tune he calls No. 8, and gives particulars as follows:—

The transmitting aerial consists of a single stranded 7/22 copper wire 48 ms. long.

The condenser in the primary circuit of the transmitter consists of one or more Leyden jars having a total capacity of 0·007 mfd. The oscillation transformer in the transmitter has a primary circuit consisting of ten insulated wires, each 1·5 ms. in length, wound once round a square frame, the size of which is 30 cms., the ten wires being joined in parallel. The secondary circuit consists of 48·64 ms. of insulated wire, wound over the primary in 16 layers, the first or inner layer having nine turns, the second eight turns, the remainder seven, six, five, and two turns respectively.

The oscillation transformer in the corresponding receiver has a secondary circuit consisting of 48·64 ms. of single silk-covered copper wire 0·37 mm. in diameter, wound on a tube 9·6 cms. in diameter in one layer and cut at its middle point, to insert the condenser. The primary is 3·64 ms. long, made of wire 0·7 mm. in diameter, wound symmetrically over the middle portion of the secondary circuit in one layer.

In both cases the receiving aerial is identical with the transmitting aerial. Marconi states that these two tunes give good signals over distances of 190 miles.

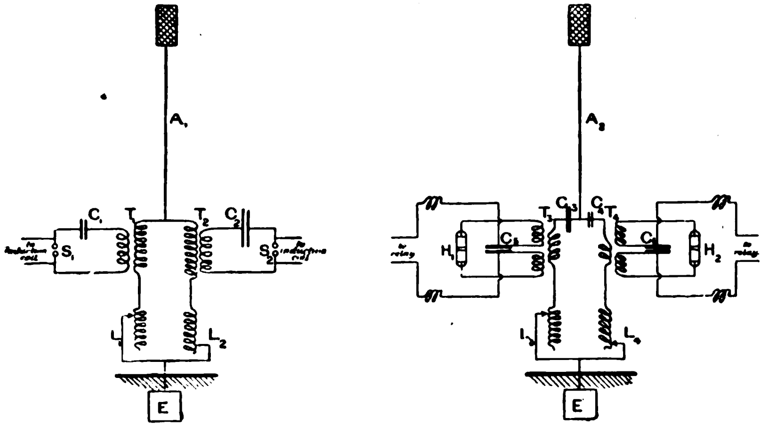
The invention of the above-described apparatus enabled Marconi, in the

summer of 1900, to conduct and exhibit duplex wireless telegraphy by sending and receiving simultaneous messages from one and the same aerial.

The arrangements at the sending and receiving ends were as shown in Fig. 11. At the transmitting end the two transmitters are connected to the same aerial wire, and each transmitter is operated independently by its own key. Two sets of waves are, therefore, radiated from the aerial, one which we may call the A wave, and the other the B wave.

At the receiving end two receiving sets are also connected, as shown, to one and the same aerial, and when the adjustments are properly made, one of these receivers responds only to the A wave, and the other only to the B wave. Hence the transmitters may be set to work simultaneously, and simultaneous but different messages received on the two transmitters.

6. General Principles of Electric Wave or Radiotelegraphy.—Having sketched in bare outline as above described the initial inventions in connection with radiotelegraphy it will be to the advantage of the student to depart from the strict historical development of the subject and to group the information to be



Transmitting station. *Receiving station.*

FIG. 11.—Arrangement of Transmitting and Receiving Apparatus in Marconi System of Multiple Syntonic Wireless Telegraphy.

given under headings determined by the function and nature of the appliances used in radiotelegraphy rather than to attempt to place inventions in chronological order.

The remarkable achievements of Marconi in connection with the invention of a method of wireless telegraphy, far exceeding in utility and promise anything previously accomplished, provided a powerful stimulus to innumerable inventors to follow in his steps. Hence in all countries an army of workers threw themselves eagerly into this field. Much of what was done was, in course of time, rendered antiquated by fresh inventions, and in the struggle for existence the non-useful or less useful inventions fell into the background. It is, therefore, of no utility at the present time to occupy the pages of a new edition of this treatise with descriptions of apparatus which has ceased to be used more than is necessary to explain the development of the art.

Modern radiotelegraphy is based upon the production and emission of a particular type of electromagnetic waves of great wave-length which are created by high frequency electric oscillations set up in an arrangement of wires called an *aerial* or *antenna*. These waves are either radiated in groups called *wave trains*, each train consisting of a series of waves of the same wave strength but gradually

decreasing amplitude, or else are undamped waves of constant amplitude and wave-length.

In the first case we are concerned not only with the *oscillation frequency*, or the reciprocal of the periodic time of each separate oscillation, but with the *group frequency* or number of wave trains emitted per second. To create intelligible signals these groups of waves, or the continuous stream of waves, are cut up into other larger groups or sections of two magnitudes, one containing more groups or waves than the other. These waves are absorbed by the receiving antenna, and caused to re-create similar but feeble electric oscillations in a circuit called the receiving circuit, and their energy is in turn utilized to generate some form of visible or audible or printed signal in a receiving instrument.

The signals are made on the International Morse or other code in which the alphabet, phrases, or numerals are indicated by collections of long and short signals, one called a *dot* and the other a *dash*, grouped in various ways to form letters, words, and numbers. The telegraphic alphabet as employed in the British Postal Telegraph Service and also in wireless telegraphy is the International Morse Code as follows. The unit is the *dot signal*, which may either consist in a short mark made on a strip of paper, Morse telegraphic tape, or a short sound, tick, or buzz made by a telephone or a brief deflection of a galvanometer needle or spot of light. A *dash signal* is a longer mark, sound, or deflection, equal in magnitude or duration to three dots. A *dot space* is a blank equal in duration to a dot, left between the signs forming a letter, and a *dash space* is a blank equal in duration to a dash left between letters forming a word. A space equal to five dots is allowed between words, and a longer space between sentences.

The following signs form the code :—

THE INTERNATIONAL MORSE ALPHABET.

A	• —	M	— —
Ä	• — • —	N	— •
B	— • • •	O	— — —
C	— • — •	P	• — — •
CH	— — — —	Q	— — • —
D	— • •	R	• — •
E	•	S	• • •
F	• • — •	T	—
G	— — •	U	• • —
H	• • • •	V	• • • —
I	• •	W	• — —
J	• — — —	X	— • • —
K	— • —	Y	— • — —
L	• — • •	Z	— — • •

THE NUMERALS.

1	• — — — —	6	— • • • •
2	• • — — —	7	— — • • •
3	• • • — —	8	— — — • •
4	• • • • —	9	— — — — •
5	• • • • •	0	— — — — —

Bar for fractions — • • — •

SIGNS.

Understand • • • — •

Repeat • • — — • •

Go on — — •

Wait • — • • •

Call signal • • • —

A full stop • • • • • •

A hyphen — • • • • —

An apostrophe • — — — — •

A semicolon — • — • — •

A note of interrogation or request for repetition of anything not understood } • • — — • •

A note of exclamation — — • • — —

Error • • • • • • • • •

End of transmission • — • — •

End of message • • • — • —

Call for help
Ship in great danger } • • • — — — • • •

In addition to the International Morse Code another alphabet called the American Morse Code is in use in the United States. It is as follows :—

THE AMERICAN MORSE ALPHABET.

A	• —	N	— •
B	— • • •	O	• •
C	• • •	P	• • • • •
D	— • •	Q	• • — •
E	•	R	• • •
F	• — •	S	• • •
G	— — •	T	—
H	• • • •	U	• • —
I	• •	V	• • • —
J	— • — •	W	• — —
K	— • —	X	• — • •
L	——	Y	• • • •
M	— —	Z	• • • •

Full stop • • — — • •

Interrogation — • • — •

Exclamation — — — •

If, therefore, the stream of continuous waves or the trains of damped waves sent out from the transmitting station can be intermitted, that is started and stopped at pleasure for a greater or lesser interval of time, and if these can be made to influence some recipient device giving visible or audible signals of corresponding duration, we have the means of transmitting the words of any language which is alphabetic.

In the case of Chinese, Japanese, and other non-alphabetic languages, the ideographs can be numbered, and the numbers transmitted and then translated.

Generally speaking, if the signal made at the receiving end is visual, the receiver is called telegraphic, but if it is audible and heard in a telephone, the reception is called telephonic. There is no true scientific distinction between the two methods; the telephone when used in this matter is a telegraphic instrument of a particular kind, making an audible signal, just as a buzzer or Bright's bell are other forms.

The signal can, of course, be sent in ordinary conversational words or in code words, and by far the larger portion of all commercial and naval intelligence is transmitted by special code. There are considerable advantages in the reception of a message by some form of receiver which prints it down on paper tape, or records it by photography, as we then possess a record which subsequently can be critically considered, and any errors or obscurities in it perhaps rectified by experienced guessing, whereas the failure to receive rightly a single letter or word on the telephonic method cannot be overcome in the same manner. On the other hand, the telephonic methods generally admit of greater speed, as the appeal is made directly to the ear, and the essential inertia or delay in the recording device is absent. In the same manner, automatic sending by means of punched tape has great advantages over ordinary hand sending, in speed spacing, and accuracy. The most usual obstacle to intelligibility is bad spacing in letters and words. Great precision is necessary to secure the perfect exactness in the time duration of the signs, sounds, or deflections which constitutes good sending on the Morse code. Moreover, each hand sender has his own peculiarities, and can be recognized by these, just as each individual has his own caligraphy. Hence the advantages of automatic sending by means of punched tape are considerable.

The transmitting apparatus involves the following elements:—

(i.) The radiator, aerial wire or antenna, including the earth plate or else the balancing capacity.

(ii.) The arrangements for producing the electric oscillations in the antenna, either intermittent or else persistent or undamped.

(iii.) The source of electromotive force or charging voltage.

(iv.) The key or controller for starting and stopping the oscillations.

At the receiving station the apparatus may be analysed into—

(i.) The receiving antenna and earth plate or balancing capacity.

(ii.) The cymoscope or wave detector and associated oscillatory circuits.

(iii.) The recording or signal producing instrument.

Each wireless telegraph station is equipped with sending and receiving apparatus, whilst the antenna is usually common to both, and used for sending and receiving alternately, being switched over from the transmitter to the receiver as required, or it can be simultaneously used with the aid of special appliances.

We shall proceed to discuss some of the scientific questions involved in the working and construction of each part of this apparatus.

7. The Aerial Wire or Antenna, its Construction and Support.—The simplest form of radiator or antenna is a single metallic wire upheld in a nearly vertical position by an insulator from a mast, tower, or chimney, the said wire being either bare or insulated. As regards material, tinned hard-drawn copper, phosphor-bronze, or aluminium wire, bare or else insulated with india-rubber, is generally employed. The wire may be solid or stranded, a stranded wire of tinned copper 7/20 or 7/22 S.W.G. being a convenient size. Bare steel wire cannot be used, as its magnetic qualities would damp out the oscillations too quickly. If, however, the steel is thickly galvanized, it may be employed on an emergency. The author has for a long time past made use of aluminium wire for antennæ. As the high frequency oscillations are entirely on the surface of the

wire, specific resistance does not come in question. The only qualities of the aerial wire with which we are concerned, other than magnetic permeability, are tensile strength, durability, weight, and cost per pound or kilogramme. Since the specific gravity of aluminium is 2·7, and that of copper 8·9, a given size and length of copper wire will weigh rather more than three times that of a similar aluminium wire.

The tensile strength of pure aluminium is 5·27 tons per square inch, and that of hard copper is about 15 tons per square inch. Alloys of aluminium are, however, now prepared having nearly the same density as pure aluminium, but a tensile strength equal to that of hard-drawn copper. The prices of copper and aluminium per ton vary from year to year, but are approximately £80 to £90 per ton for copper and about £200 for aluminium at the present date (1916). Hence an aerial wire of any given diameter and length in aluminium will weigh about one-third of an equal-sized wire in copper. Experience has shown that if galvanic action is avoided by not making contact between it and other metals, aluminium will stand very well the action of town or sea air.

In the case of large aerials the element of weight is important, and the use of aluminium or alloys of aluminium having a density not exceeding 3 and tensile strength not less than 20 tons per square inch, as material for the aerial wire, will be found advantageous.

The aerial wire has to be upheld in a vertical position, and the usual method is to suspend it from a mast or tower. In Marconi's earliest experiments he employed metal cans placed hat-wise upon wooden poles and connected by a wire with the oscillation producer. In his fundamental patent he describes metal plates suspended from wooden frames, and in some of his earliest demonstrations, as in his first transatlantic achievement, he employed kites and balloons to sustain the wire (see Fig. 1).

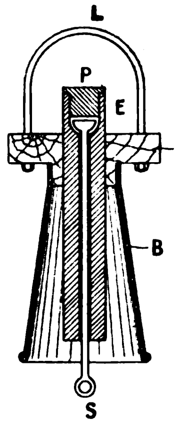


FIG. 12.—Antenna insulator. (Fleming.) E, ebonite tube; P, ebonite plug; S, brass pin; W, oak cross bar; B, bell-shaped metal protector; L, suspension loop.

The next improvement was to erect a ship mast on shore and suspend the wire from a sprit or gaff at the top. Marconi employed in his cross-Channel work between England and France, in 1899, 150-feet masts in three sections, strongly stayed with hemp ropes. For his earliest work at Poldhu and Nova Scotia, 200-feet masts were erected, each put up in four sections, and a ring of such masts was used. The masts were supported by steel wire stays divided into sections by dead-eyes.

Subsequently wooden lattice towers were erected strongly stayed, but these have since been replaced by the Marconi Company by steel tubular masts. In some stations metal lattice towers are used insulated at the lower end, and in other places, as at the Marconi station at Clifden, in Ireland, wooden masts are still employed.

In the case of ships or lightships, it is usual to add a gaff to the existing masts to gain greater height. In some cases two gaffs are used, and from a stay between the aerial wires suspended. This was first done in the case of the Italian warship *Carlo Alberto*, placed at Mr. Marconi's disposal by the Italian Government for experimental purposes. In the case of battleships it is usual to add a special gaff or gaffs to the masts to secure greater height for the antenna.

The sufficient insulation of the aerial at the top is an important matter. It was at first customary to employ a simple ebonite rod attached to the gaff, from the lower end of which the wire was suspended. Later flanged and shielded ebonite rods have been employed. The author has designed and used for some time an effective form of insulator made as follows: A thick ebonite tube, E, has a recess turned out at the top (see Fig. 12). In the interior of the tube a brass rod, S, fits tightly, the upper end of which is formed like the head of a pin. This head lies in the cupped recess of the ebonite, and an ebonite plug, P, is then

tightly fitted in with Chatterton's compound. The ebonite tube is gripped outside by a cross bar of oak, W, which carries also a brass suspension loop, L. The result of this construction is that the ebonite is under compressional and not

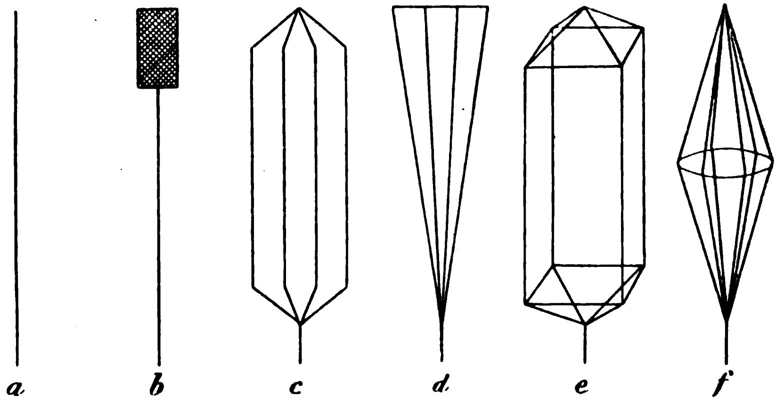


FIG. 13.—Various forms of Antenna used in Electric Wave Telegraphy. *a*, plain aerial wire; *b*, wire with capacity plate at top; *c*, multiple wire antenna; *d*, fan-shaped antenna; *e*, cage or quadruple wire antenna; *f*, double cone antenna.

tensional strain, and is able to stand a greater pull between the loop and eye of the pin.

In addition, a wider metal tube, B, is fitted to a ring embracing the ebonite rod, and this tube is made water-tight at the upper end so that the metal tube acts as a petticoat to keep the ebonite rod dry. The lower end of the brass rod is formed into a loop to which the aerial wire is attached, and the ebonite insulator is suspended from the gaff of the mast. In some cases a chain of two or three such insulators can be employed. The aerial wire on board ship sometimes requires tying back to keep it clear of rigging, and in this case the tie-backs or stays must be connected to insulators of the above kind.

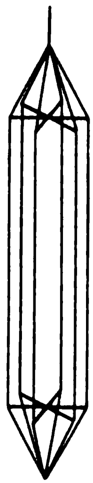


FIG. 14. Star or Six-fold Antenna as used on British Battleships.

In place of a single wire antenna two or four wires may be employed, arranged in parallel and connected at the ends to the arms of a wooden cross.

Such multiple wire aerials may take several forms. They may take the form of parallel wire or cage-shaped or else fan-shaped conical or double-cone multiple wire aerials (see Fig. 13).

On battleships and large liners the antenna now consists of a series of six or eight parallel wires, which are kept apart by star-shaped wooden separators. This six-fold antenna is stretched from mast to mast on insulators, and brought down also sometimes fore and aft. A vertical wire or wires connects it with the signalling cabin (see Fig. 14).

A cone-shaped antenna, consisting of a very large number of wires, was originally employed in the first large radio-telegraphic stations of the Marconi Company at Cape Breton and at Cape Cod; but after the invention by Mr. Marconi of the bent or *directive antenna*, partly vertical and partly horizontal, these cone antennæ have been replaced by antennæ of the type shown in Fig. 15, which have the peculiar property of projecting radiation more strongly in the direction away from which the free end points. The theory and action of these forms of directive antennæ will be considered in a subsequent section (see § 8).

Lodge suggested a form of insulated roof antenna, in which a metal surface

is carried on insulators placed on metal or wooden supports. An effective and much used form of antenna is the umbrella antenna, in which a number of nearly vertical wires have radial extensions from the top dipping downwards (see Fig. 16). The advantage of a multiple antenna is that it has a larger capacity, and therefore yields a longer wave-length, than a single wire of the same length. In addition to this the subdivision and separation of the mass of the antenna into separated portions greatly reduces the effective inductance, and so aids the increase of the antenna current.

In all cases the mast or tower has to be supported by stays, and if these are made of steel rope, as is generally the case, they must be cut up into sections by means of insulators, so that they are not of nearly the same length as the antenna wires themselves, or, in other words, are much out of tune with the antenna wires considered as electrical oscillators. If this is not done and if the stay wires happen to have a natural period of electrical oscillation near to that of the waves for which that particular station is tuned, they would absorb a good deal of the

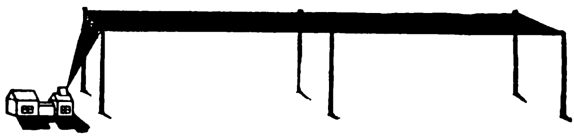


FIG. 15.—Marconi's Bent or Directive Antenna.

energy of these waves and so diminish the amount available for absorption by the true antenna.

In many cases forms of antenna are employed, consisting of wires or groups of wires arranged in the shape of the letter T or of the letter L turned on its side so: ┌. In which the horizontal part is carried between two masts and the vertical part leads to the signalling house.

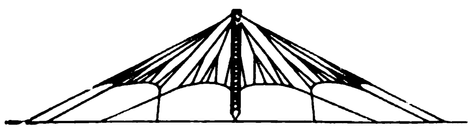


FIG. 16.—Umbrella Antenna.

8. Classification of Antennæ. Open and Closed Circuit Antennæ.—The various forms of antennæ used may be classified according to certain definite principles. We have first the general but ill-defined distinction between open and closed antennæ already mentioned.

The typical open circuit antenna is a straight wire insulated at the upper end and upheld so as to stand nearly vertically to the earth's surface. In it electric oscillations can be set up either by charging the wire and suddenly discharging it to earth, as in the original Marconi method, or by connecting it directly or inductively to a syntonic nearly closed energizing circuit in which oscillations are established by the arc or spark method.

This open oscillator is characterized by a perfect spacial symmetry. The magnetic field round it is distributed in circles with their centres on the wire (see Fig. 18), and the lines of electric force lie in radial planes intersecting on the wire as shown in Chap. V. § 7. Hence it radiates strongly and equally in all directions. It has therefore a large radiation decrement, and its oscillations will be damped out quickly unless energy is continually supplied to it from an energizing circuit. Such an oscillator is frequently called an *electric oscillator*. In comparison with this, at the other extreme we have the closed or *magnetic oscillator*, in which a nearly closed conductive circuit is interrupted by a condenser with plates near

together and at most by a short spark gap (see Figs. 17 and 18). In this case the magnetic field is only symmetrical with regard to the plane of the oscillator but not with regard to a vertical line. If we suppose such a closed oscillator placed with its plane vertical and to be traversed by a current, then if at any instant the current is flowing round it counter-clockwise the internal magnetic field will be

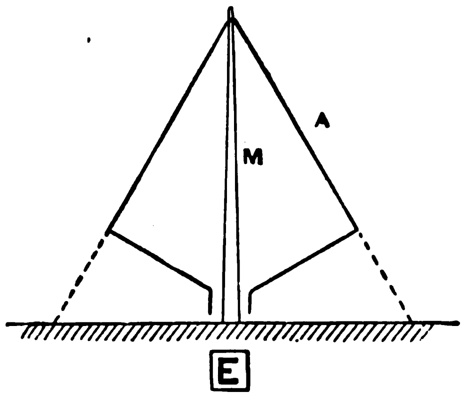


FIG. 17.—A Closed Circuit Antenna.

directed towards the spectator and the lines will double back and outside the circuit will be directed away from the spectator, thus completing their circuit (see Fig. 20). If we represent the section of a magnetic line of force by a small circle, we can put a dot in the circle to indicate that the direction of the force is towards the reader and a cross if it is away from the reader, and in the diagram in

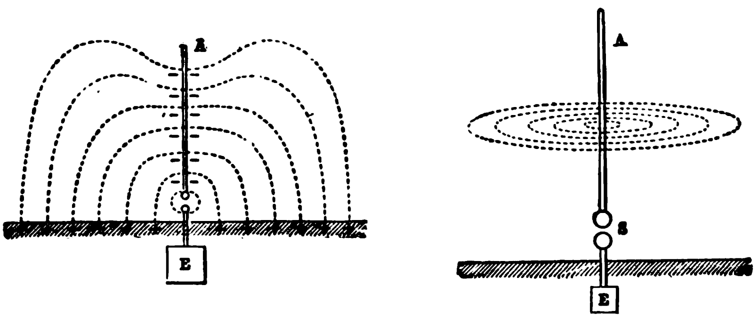


FIG. 18.

Lines of Electric Force of Linear Antenna.

Lines of Magnetic Force of Linear Antenna.

Fig. 20 the direction is so indicated for the instant when the current in the antenna is in the counter-clockwise direction. A closed circuit when traversed by a high frequency current or oscillation gives rise not only to magnetic but to electric force in the external space and radiates electromagnetic waves. We have already obtained (see § 13, Chap. V.) expressions for the electric and magnetic forces of the closed or magnetic oscillator and for the radiation from it, hence these need not be repeated. But from the remarks there made as to the form of the

electric and magnetic field of the closed oscillator it will be evident that its radiation is a maximum in the plane of the oscillator.

To sum up, we may say that although the closed circuit antenna has generally less radiative power and less radiation decrement than the open circuit antenna, yet it has a certain asymmetry of radiation which gives it importance and utility in radiotelegraphy. Again, we have many different types of antenna intermediate between the straight, vertical open circuit antenna and the completely closed antenna, which possesses in intermediate degree the useful qualities of both open

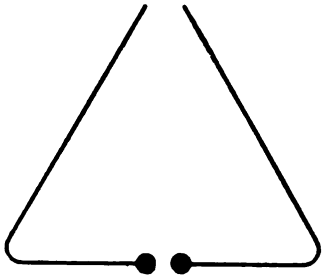


FIG. 19.—Nearly Closed Antenna.

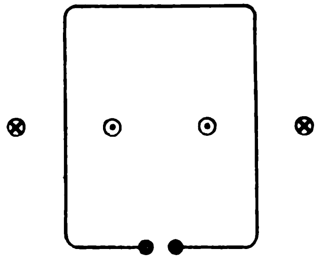


FIG. 20.—Diagram illustrating the Disposition of the Magnetic Field of a Closed Antenna.

and closed antenna. For instance, a straight wire antenna inclined to the surface of the earth or a wire partly vertical and partly horizontal has a non-symmetrical radiative power.

9. Bent or Inclined Antennæ.—A theory of the operation of bent or inclined antennæ may be based upon the assumption that they may be regarded as

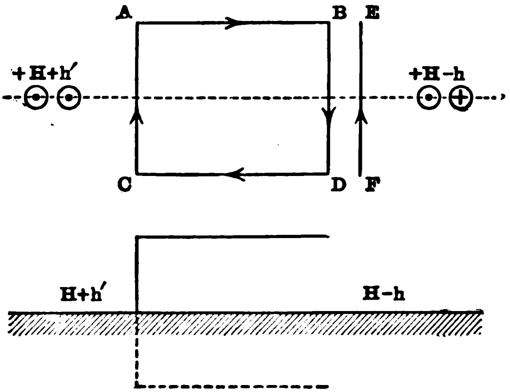


FIG. 21.

equivalent to the conjunction of an open or electric oscillator and a completely closed or magnetic oscillator. Consider, for example, a square or rectangular circuit ABCD (see Fig. 21), in which electric oscillations are taking place. The magnetic field of such an oscillator consists of closed lines which embrace the circuit of the oscillator. These lines are all perpendicular to the plane ABCD in crossing that plane, and if the current is in any instant going round in the same direction to the hands of a watch, that is, in the direction ABDC, the lines of magnetic force in the included space are proceeding away from the reader, and returning on all sides outside the area towards the reader.

In Fig. 21 the small circles represent the section of a pair of such lines of magnetic force outside the oscillator returning back on both sides in the equatorial line by the two little circles marked +H, in which the magnetic flux is towards the reader. On the other hand, if we consider a simple open antenna EF of the same height as the side of the rectangle BD, and consider the nature of the magnetic force round it when a current is flowing upwards in it, it will be seen that these lines are circles lying in planes at right angles to the antenna, and that the sections of these lines in that plane may be represented by the little circles $+h'$ and $-h$, marked respectively with a dot and a cross. If, then, we suppose the open and closed circuits to be placed so that the open one is in close contiguity to one side of the closed one (see upper diagram of Fig. 21), and that the oscillations in these parts of the two circuits in contiguity are always in opposite directions, then it is quite easy to see that the field due to the open circuit antenna will assist the field due to the closed circuit antenna on the left-hand side, but tend to weaken it on the right-hand side. So that if we call the field due to the open antenna on the one side h, and on the other side h', the resultant field due to the combined open and closed antennæ will be $H+h'$ on the left-hand side, $H-h$ on the right-hand side.

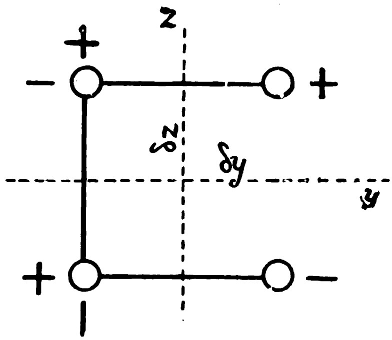


FIG. 22.

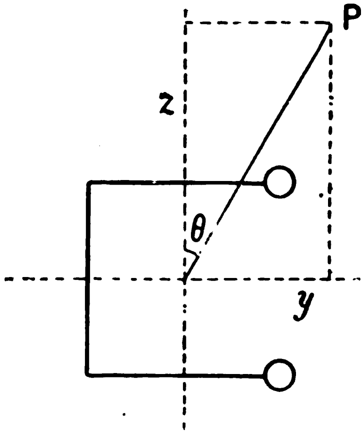


FIG. 23.

We can now imagine the two oscillations in the continuous wires BD, EF which are opposed in direction to annihilate each other, and the result is that we are left with a bent antenna as in the lower diagram of Fig. 21, in which if oscillations are set up we are able to produce a field which is non-symmetrical, being greater on the side away from which the open ends point. Such an antenna is called a bent antenna, and if we imagine it half buried in the earth, the surface of the earth being a plane of zero potential, it produces the same effect above the earth's surface as one-half of a complete double bent antenna. It follows, then, that an earthed antenna partly vertical and partly horizontal must produce a non-symmetrical radiation.

The author has given an analytical discussion of the theory of a bent antenna, based on the above view that it could be considered as composed of the superposed Hertzian oscillators. (See J. A. Fleming, *Proc. Roy. Soc. Lond.*, ser. A, vol. 78, p. 1, 1906. A note on the theory of Directive Antennæ, or Unsymmetrical Hertzian Oscillators.)

The analytical treatment of the subject presents, however, enormous difficulties unless we limit consideration to the case in which the current in the oscillator is assumed to have the same value at all points at the same time, and also that the dimensions of the oscillator are small compared with the distance from it of the points at which the field is considered. The first assumption is not strictly true

for any ordinary radiotelegraphic antenna, but is necessary to bring the case under mathematical treatment.

One form of bent oscillator of the above kind may be considered to be made up by the superposition of three Hertzian electric oscillators placed at right angles to each other, the poles being so arranged that at the two corners poles of opposite signs are superimposed, the oscillations in all being synchronous and similarly directed (see Fig. 22). Hence, to obtain the field of this bent oscillator, we need merely to calculate those of the components and add them together.

Let a single electric oscillator be placed with its centre at the origin and axis coinciding with the z axis. Let oscillations exist in it of period $2\pi/n$ and radiation be emitted of wave-length $2\pi/m$. Suppose the length of the oscillator to be denoted by δz, the electric charge at either pole at any instant by q, the uniform current in the axis by i, whilst Q and I are the maximum values of q and i which vary so that $q=Q \sin nt$ and $i=I \cos nt$. Also let $\phi=Q\delta z$ be the maximum electric moment of the oscillator. We have, therefore, $I=Qn$ or $\phi n=I\delta z$, and $n/m=v$, the velocity of propagation of the radiation through space. The scalar potential V at any point P whose distance from the origin is r, is given by

$$V=-\frac{\phi}{k}\frac{d}{dz}\left(\frac{\sin(mr-nt)}{r}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is the dielectric constant of the medium, and $r^2=x^2+y^2+z^2$.

Also, if F, G, and H are the components of vector potential at P, we have in this case $F=G=0$, and

$$H=-I\delta z\frac{\cos(mr-nt)}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If, as before, we employ the symbol Π to stand for

$$\frac{\sin(mr-nt)}{r}$$

we can write the above expressions (1) and (2) in the form

$$V=-\frac{\phi}{k}\frac{d\Pi}{dz}, \quad H=\phi\frac{d\Pi}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If we suppose this doublet to be moved parallel to itself in the negative direction so that its centre is displaced by a distance $-\frac{1}{2}\delta y$, the scalar and vector potentials at P become—

$$V=\left(-\frac{\phi}{k}\frac{d\Pi}{dz}\right)+\frac{1}{2}\frac{d}{dy}\left(\frac{\phi}{k}\frac{d\Pi}{dz}\right)\delta y \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$H=\phi\frac{d\Pi}{dt}-\frac{1}{2}\frac{d}{dy}\left(\phi\frac{d\Pi}{dt}\right)\delta y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Consider, then, two other similar doublets of length δy, and maximum moment ϕ', placed with poles pointing in opposite directions and axes parallel to the axis of y, the doublets having centres at distances $+\frac{1}{2}\delta z$ and $-\frac{1}{2}\delta z$ from the origin and poles arranged as in Fig. 22. The scalar and vector potentials at the point P of these last two doublets constituting together a double doublet are obviously given by

$$V=-\frac{\phi'}{k}\frac{d^2\Pi}{dz\,dy}\delta z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$G=\phi'\frac{d^2\Pi}{dz\,dt}\delta z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence, if three such short, straight oscillators, having equal currents and charges, are placed round the origin so as to create a doubly-bent oscillator, the

scalar and vector potentials of this oscillator at a point P (see Fig. 23), the distance of which from the origin is large compared with the linear dimensions of the oscillator, are given by—

$$V = -\frac{\phi}{k}\frac{d\Pi}{dz} + \frac{1}{2}\frac{\phi}{k}\frac{d^2\Pi}{dy\,dz}\delta y - \frac{\phi'}{k}\frac{d^2\Pi}{dz\,dy}\delta z \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

$$\left.\begin{aligned} F &= 0 \\ G &= \phi'\frac{d^2\Pi}{dz\,dt}\delta z \\ H &= \phi\frac{d\Pi}{dt} - \frac{1}{2}\phi\frac{d^2\Pi}{dy\,dt}\delta y \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

where $\phi'\delta z = \phi\delta y$.

The electric and magnetic forces at the point P, of which the axial components are X, Y, Z, and α, β, γ, can be obtained from equations (8) and (9) at once by the aid of the relations—

$$\left.\begin{aligned} X &= -\frac{dF}{dt} - \frac{dV}{dx} & \alpha &= \frac{dH}{dy} - \frac{dG}{dz} \\ Y &= -\frac{dG}{dt} - \frac{dV}{dy} & \beta &= \frac{dF}{dz} = \frac{dH}{dx} \\ Z &= -\frac{dH}{dt} - \frac{dV}{dz} & \gamma &= \frac{dG}{dx} - \frac{dF}{dy} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

For the present purposes we require only the electric and magnetic forces perpendicular to the radius vector r, taken at its extremity, when that radius is taken in the plane xy, which is normal to the plane yz in which the oscillator is situated. Hence we need only calculate the value of Z, α, and β for the case in question.

If we write M for $I\delta y\delta z$ and call this the magnetic moment of the bent oscillator, so that $\phi\delta y = \phi'\delta z = M/n$, we have the following equations for the potentials and forces in the field at points not very near the oscillator :—

$$\left.\begin{aligned} V &= -\frac{\phi}{k}\frac{d\Pi}{dz} + \frac{\phi}{2k}\frac{d^2\Pi}{dy\,dz}\delta y - \frac{\phi'}{k}\frac{d^2\Pi}{dz\,dy}\delta z = -\frac{\phi}{k}\frac{d\Pi}{dz} - \frac{M}{2kn}\frac{d\Pi}{dy\,dz} \\ G &= \phi'\frac{d^2\Pi}{dz\,dt}\delta z = \frac{M}{n}\frac{d^2\Pi}{dy\,dt} \\ H &= \phi\frac{d\Pi}{dt} - \frac{\phi}{2}\frac{d^2\Pi}{dy\,dt}\delta y = \phi\frac{d\Pi}{dt} - \frac{M}{2n}\frac{d^2\Pi}{dy\,dt} \\ Z &= -\phi\frac{d^2\Pi}{dt^2} + \frac{M}{2n}\frac{d^3\Pi}{dy\,dt^2} + \frac{\phi}{k}\frac{d^2\Pi}{dz^2} + \frac{M}{2kn}\frac{d^3\Pi}{dy\,dz^2} \\ \alpha &= \phi\frac{d^2\Pi}{dy\,dt} - \frac{M}{2n}\frac{d^3\Pi}{dy^2\,dt} - \frac{M}{n}\frac{d^3\Pi}{dz^2\,dt} \\ \beta &= -\phi\frac{d^2\Pi}{dx\,dt} + \frac{M}{2n}\frac{d^3\Pi}{dx\,dy\,dt} \end{aligned}\right\} \quad . \quad . \qquad (11)$$

Performing the necessary differentiations on the function—

$$\Pi = \frac{\sin(mr - nt)}{r}$$

and collecting terms in sin $(mr - nt)$ and cos $(mr - nt)$, which for shortness will be written sin χ and cos χ, also putting v for n/m or $1/\sqrt{\mu k}$ where μ and k are the permeability and dielectric constant of the medium, we have the following expressions for Z, α, and β :—

$$Z=\left\{\phi(m^2r^2-1)-\phi(m^2r^2-3)\left(\frac{z}{r}\right)^2+\frac{M}{2v}\frac{3}{mr}\left(\frac{y}{r}\right)\right.$$
$$\left.+\frac{M}{2v}\frac{(6m^2r^2-15)}{mr}\left(\frac{y}{r}\right)\left(\frac{z}{r}\right)^2\right\}\frac{\sin\chi}{kr^3}$$
$$+\left\{\phi mr-\phi 3mr\left(\frac{z}{r}\right)^2-\frac{M}{2v}(m^2r^2+3)\left(\frac{y}{r}\right)\right.$$
$$\left.+\frac{M}{2v}(15-m^2r^2)\left(\frac{y}{r}\right)\left(\frac{z}{r}\right)^2\right\}\frac{\cos\chi}{kr^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\alpha=\left\{\phi v m^2r^2\left(\frac{y}{r}\right)+\frac{M}{2}3mr\left(\frac{y}{r}\right)^2-\frac{M}{2}3mr+3Mmr\left(\frac{z}{r}\right)^2\right\}\frac{\sin\chi}{r^3}$$
$$+\left\{\phi v mr\left(\frac{y}{r}\right)-\frac{M}{2}(m^2r^2-3)\left(\frac{y}{r}\right)^2-\frac{3M}{2}\right.$$
$$\left.-M(m^2r^2-3)\left(\frac{z}{r}\right)^2\right\}\frac{\cos\chi}{r^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\beta=-\left\{\phi v m^2r^2\left(\frac{x}{r}\right)+\frac{M}{2}3mr\left(\frac{x}{r}\right)\left(\frac{y}{r}\right)\right\}\frac{\sin\chi}{r^3}$$
$$-\left\{\phi v mr\left(\frac{x}{r}\right)-\frac{M}{2}(m^2r^2-3)\left(\frac{x}{r}\right)\left(\frac{y}{r}\right)\right\}\frac{\cos\chi}{r^3} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Suppose we limit attention to the value of the electric force e and the magnetic force d at right angles to the extremity of the radius vector r, the former being parallel to the z-axis and the latter being drawn in the plane of xy. The magnetic force d in this direction is equal to $\alpha y/r-\beta x/r$. Hence we obtain its value by multiplication of the values of α and β by y/r and x/r and subtraction. Then putting $z=0$ in the above equations and writing $\cos\psi$ for y/r we have—

$$e=\left\{\phi(m^2r^2-1)+\frac{M}{2v}\frac{3}{mr}\cos\psi\right\}\frac{\sin\chi}{kr^3}$$
$$+\left\{\phi mr-\frac{M}{2v}(m^2r^2+3)\cos\psi\right\}\frac{\cos\chi}{kr^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$d=\{\phi v m^2r^2\}\frac{\sin\chi}{r^3}+\left\{\phi v mr-\frac{M}{2}m^2r^2\cos\psi\right\}\frac{\cos\chi}{r^3} \quad . \quad . \quad . \quad . \quad (16)$$

If we denote the amplitudes of e and d by E and D, we have finally—

$$E=\frac{1}{kr^3}\sqrt{\left\{\phi(m^2r^2-1)+\frac{3}{2}\frac{M}{v}\frac{\cos\psi}{mr}\right\}^2+\left\{\phi mr-\frac{M}{2v}(m^2r^2+3)\cos\psi\right\}^2} \quad . \quad (17)$$

$$D=\frac{1}{r^3}\sqrt{(\phi v m^2r^2)^2+\left(\phi v mr-\frac{M}{2}m^2r^2\cos\psi\right)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where E is the amplitude of the electric force perpendicular to the radius vector and to the equatorial plane, and D is the amplitude of the magnetic force perpendicular to the radius vector and in the equatorial plane.

If the expressions under the radicals in (17) and (18) are written out, it is easy to show that when ψ is 180° the values of E and D are both greater than when $\psi=0°$.

If we put $M=0$ in the above equations they reduce to the values given by Hertz for the electric and magnetic forces of the short straight oscillator or doublet taken in the equatorial plane, the electric force being parallel to the axis and magnetic force at right angles. When mr is large compared with unity we have $kE^2=\mu H^2$, showing that the energies of the magnetic and electric components of the wave then become equal. Also there is a minimum value of D and E corresponding to a value of ψ such that

$$\cos\psi=\frac{2\phi v}{M}\frac{1}{mr}, \text{ or } \cos\psi=\frac{2\phi v}{M}\frac{mr}{m^2r^2+3} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The above expressions are numerically small when r is large compared with the wave-length of the radiation. Hence a minimum value of the forces at the extremity of the radius vector is found, corresponding to some azimuthal angle ψ rather less than 90° reckoned from the direction in which the free ends of the bent oscillator point.

The degree of this fore-and-aft inequality in the plane of the oscillator will depend upon the ratio of the magnitude of the quantities $\frac{1}{2}Mmr$ and ϕv or upon the ratio of δy to $2/m^2r$, that is upon the ratio of

$$\frac{\delta y}{\frac{1}{2}\delta z+\delta y} \text{ to } \frac{4}{m^2rl}, \text{ i.e., to } \frac{\lambda}{\pi^2rl},$$

where $2l$ is the total length of the bent oscillator. The greatest inequality between the fore-and-aft radiation in the plane of the oscillator will exist when π^2 times the ratio of the sum of the lengths of the two horizontal parts of the oscillator to its total length is as nearly as possible equal to the product of the ratios of λ^2/l^2 and l/r. The ratio λ/l is fixed by the geometrical form of the oscillator, hence the inequality in radiative power in the fore-and-aft directions for a given oscillator essentially depends upon the ratio of wave-length to the distance of the point at which observations are made, and at large distances will only be sensible when long wave-lengths are employed.

The above theoretical examination of this operation of a bent oscillator shows clearly that its unsymmetrical radiation in the equatorial plane depends not upon absolute wave-length, but upon the ratio of wave-length to the distance of the receiving point, and upon the proportion between the length of the vertical and of the horizontal portions of the oscillator.

The above investigation shows that if an antenna is bent or inclined so that it is partly vertical and partly horizontal, the result is to produce a non-symmetrical radiation; in other words, to give the antenna a *directive* quality as regards radiation, and that this arises from the fact that such an antenna may be regarded as the result of a combination of the open and closed type. This property of the bent or inclined antenna was, however, not discovered by mathematical analysis, but experimentally in the endeavour to achieve the feat of directing radio-telegraphic waves.

Another and quite different theory of directive antennæ has been developed by H. v. Hoerschelmann (see *Jahrbuch der Drahtlosen Telegraphie*, vol. v. p. 15, and p. 188, 1911). This theory, based on a rather elaborate mathematical analysis, led to the conclusion that the directive properties of a Marconi bent antenna depend essentially upon the production in the earth's crust beneath the antenna of two vertical electric currents which act like vertical antennæ, the currents in these being in opposite phases. Hence the conclusion arrived at is that if the earth is a very good conductor there would be no directive effect, because no currents could penetrate deep into the earth. Also if the earth were a perfect non-conductor no such currents could be formed. Let μ be the magnetic permeability, κ the dielectric coefficient, and σ the conductivity of either the air or the soil. Let $p=2\pi n$ where n is the frequency, then let k^2 be written for $c^{-2}\{\mu\kappa p^2+jp4\pi\sigma\mu\}$ where c is the unitary ratio, or the velocity of light, and let suffixes 1 and 2 have reference respectively to the value of k for air and earth. Then Hoerschelmann shows that wave amplitudes along various radii defined by an azimuth angle ϕ will be given by an expression of the form

$$R = Ak_2 + Bk_1 \cos \phi$$

where A is the height of the vertical part of the antenna, and B that of the horizontal part.

Now the equation $r=a+b \cos \phi$ is the polar equation to a Limaçon which is the inverse of a conic section with respect to the focus. Although Hoerschelmann's expression for the polar curve defining the intensity of radiation in various azimuths is in accord to a considerable extent with experiment, yet his conclusion that the directivity of such a bent antenna depends upon the finite conductivity of

the soil or surface on which the antenna stands has not yet been confirmed by experiment.

The theory of the bent antenna has also been discussed by Prof. H. M. Macdonald (see *Proc. Roy. Soc. Lond.*, vol. 81, A, p. 394, 1908). He has shown that at a distance r from the radiator the square of the amplitude of the electric or magnetic force vertical to the earth at a point having an azimuth θ is given by an expression of the form—

$$\left(A - \frac{B}{r}\cos\theta\right)^2 \frac{k^2}{r^2} + A^2 \frac{k^2}{r^4}$$

The above expression is nearly of the same mathematical form as the expressions given in the equations (17) and (18), and denotes a figure of 8 curve having one loop larger than the other.

Macdonald remarks that the essential feature of all arrangements for giving direction to electric waves is the interference of two sets of waves differing in phase and proceeding from two sources at a distance apart.

Another theory of directive antennæ has been enunciated by J. Zenneck (see *Phys. Zeitschrift*, vol. 9, p. 50, 1908, or *Science Abstracts*, vol. 11, B, June 1908, abs. 705, based on the view that at great distances from a sloping sending antenna the electric field is an alternating field more or less inclined to the vertical and having a more or less considerable horizontal component. Hence Zenneck shows the result would be to project the radiation better in the direction away from which the antenna slopes.

10. Experimental Investigations with Bent or Directive Antennæ.—At a very early stage in connection with radiotelegraphy the problem of directing the radiation, or concentrating it in certain directions, presented itself to inventors. The earliest attempts to give direction to an electric beam involved the use of parabolic mirrors. Hertz showed that electric waves could be reflected according to the same laws as rays of light, and in some of his earliest experiments he employed a pair of cylindrical parabolic mirrors for this purpose. In the focal line of one mirror a linear oscillator was placed, and in the focal line of the other a linear resonator. When the mirrors were placed in apposition, a beam of electric radiation was transmitted from one to the other, the beam being mostly confined to the space between the mirrors. In order that this experiment may succeed, it is necessary, however, to use radiation of a wave-length which is small, or at least not large, compared with the dimensions of the mirror. Thus Hertz used cylindrical parabolic mirrors 12·5 cms. in focal length, which were about 2 metres high and 1 metre wide, and employed electric waves having a wave-length of about 66 cms, or about 2 feet in length.[16]

In some of his earliest experiments on electric wave telegraphy Marconi adopted the same plan, and used copper parabolic mirrors, by the aid of which he projected a beam of electric radiation in a certain direction for about 2 miles.[17]

In place of cylindrical parabolic mirrors, other inventors have proposed to employ vertical wires or rods arranged along a parabolic base line drawn on the ground, the radiator being placed in the focal line, *e.g.*, S. G. Brown (see British Patent Specification, No. 14,449 of 1899; also see U.S.A. Patent Specification of Lee de Forest, No. 748,597 of 1902).

The devices are, however, unavailable when very long electric waves are employed. It is perfectly impracticable to construct mirrors of dimensions comparable in size with the length of electric waves of 500 or 1000 feet in wave-length, and to employ smaller mirrors would be like attempting to conduct optical experiments with mirrors having dimensions of less than one hundred-thousandth part of an inch.

A new line of investigation was, however, opened up by the observation that the radiation of a pair of antennæ with currents in opposite directions, or of a sloping antenna, and particularly that of a vertical loop or nearly closed antenna, is non-symmetrical. In the British Patent Specification of S. G. Brown,

16 See Hertz, "Electric Waves," English translation by D. E. Jones, p. 175.

17 See G. Marconi, "On Wireless Telegraphy," *Journal Inst. Elec. Eng.*, 1899, vol. 28, p. 282.

No. 14,449 of 1899, a diagram is given showing a pair of vertical antennæ said to be separated by half a wave-length, each attached to one of the spark balls of an induction coil. This arrangement is correctly stated to radiate and absorb best in the plane of the wires. It was also found that in the case of a closed loop it is greater in the plane of the loop than in any other plane. Observations concerning this phenomenon were made in 1898 and 1899 by S. G. Brown, by K. Strecker, and by A. Slaby, and in 1899 by J. Zenneck, and later by H. von Sigsfeld and by F. Braun in Germany.[18] Arrangements were also described in patent specifications by M. R. Garcia,[19] L. de Forest,[20] and J. S. Stone[21] for locating the direction of the transmitting station.

This pioneer work, however, did not sufficiently lead to practical achievement, whilst in some cases results said to have been obtained are clearly in contradiction with well-ascertained facts. In other cases there was no doubt a correct observation, but it was not followed up. Thus, L. de Forest states that an antenna formed of vertical and horizontal insulated rods with a spark gap placed at their junction is directive, and radiates more towards the side on which the vertical branch of the antenna is placed and in the plane of the antenna (U.S.A. Patent Specification, No. 749,131, applied for March 6, 1901).

The problem which seems first to have attracted attention was that of determining the direction of the transmitting station at the receiving station.

F. Braun states he had employed in 1903, as a means of so doing, a receiving antenna not horizontal, but sloping upwards at a small angle towards the incoming wave (see *The Electrician*, vol. 57, p. 247, 1906, and *Phys. Zeitschrift*, iv. p. 363, 1903).

Again, L. de Forest described in a United States Patent Specification an arrangement consisting of a metal plate or grid, longer in a horizontal than in a vertical direction, which is swivelled round a vertical axis so as to be capable of being oriented.[22] In the vertical part he places an electrolytic receiver of some kind shunted by a telephone and local cell. He states that when the grid is placed broadside on to the incident waves it collects the largest amount of energy, and the oscillation detector is then most vigorously affected. Hence by rotating it round into this position the receiving operator can determine the direction of the radiant point. He states that with a collecting screen 15 feet by 6 feet in size, he has been able to locate within 10° the direction of a transmitting station 7 miles away. Another device by the same inventor consists in employing a horizontal antenna swivelled so as to rotate round a vertical portion, and in this is placed an electrolytic receiver of some kind shunted by a telephone and local cell. According to the specification, the horizontal portion may be extended in one or both directions, or may consist of a closed loop.

The inventor states that when turned round so that the direction of the horizontal part coincides with the direction of the incident waves, the oscillation detector in the vertical part gives its maximum indication and its minimum when the horizontal part lies transversely to the direction of motion of the signal waves.

J. S. Stone, following the prior suggestions of S. G. Brown (*loc. cit.*), proposed to place two vertical receiving antennæ at a distance apart equal to one-half of a wave-length of the waves employed, and to arrange these so as to be capable of rotating round an axis halfway between them. If, then, these two antennæ are placed in the line of direction in which the incident waves are travelling, the inventor states that they will be oppositely affected, and if the variations of potential or current at their respective bases are inductively combined so as to be added together, they will not affect a receiving instrument. On the other hand, if the line joining the two antennæ is perpendicular to the direction in which the

[18] For a brief account of this early work on directive radiotelegraphy, the reader may be referred to an article in *The Electrician*, vol. 57, p. 220, May 1906.

[19] See U.S.A. Patent Specifications of M. R. Garcia, No. 795,762, applied for January 10, 1901.

[20] See U.S.A. Patent Specifications of L. de Forest, No. 749,131, 1904, being a divided part of an original No. 720,568, March 6, 1901.

[21] See U.S.A. Patent Specifications of J. S. Stone, Nos. 716,134, 716,135 of 1902.

[22] See U.S.A. Patent Specifications of L. de Forest, Nos. 771,818, 771,819, applied for May 28, 1904.

waves are travelling, then the potential or current variations in them, being in the same phase, can be added together so as to affect a receiving instrument operated upon by the two antennæ jointly. Hence the direction in which the incident wave is travelling may be ascertained by finding the position in which the two receiving antennæ must be placed so that their joint effect on the detector may be *nil.* Apart, however, from the mechanical and almost insuperable difficulties of so dealing with antennæ, which must be hundreds of feet apart in the case of the

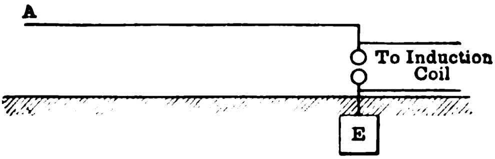


FIG. 24.—Marconi's Bent Transmitting Antenna for Directing Electric Radiation.

waves used in electric wave telegraphy, this proposal neglects altogether the effects which arise from the damping in the wave train. When a wave train of highly damped electric waves is travelling through space, although the electric and magnetic forces at places separated by half a wave-length are opposite in direction at any instant, they are not equal in magnitude. Accordingly, there would be a differential effect on the two antennæ placed half a wave-length apart in the line of motion, due to the difference in magnitude of the simultaneous forces in opposite directions. Hence this plan of S. G. Brown and Stone, though ingenious and re-described subsequently by many other patentees, does not seem to have reached practical realization.

The first really practical solution of the problem, however, was obtained when Marconi made systematic observations of the radiation intensity in various

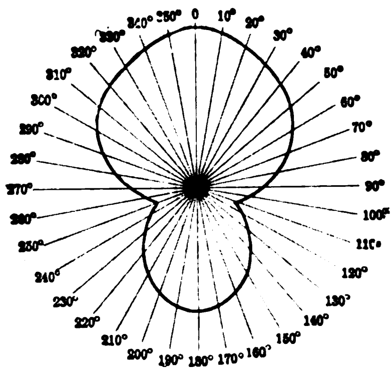


FIG. 25.—Polar Diagram showing the Energy Radiated by a Bent Antenna in Various Azimuths.

directions, but at equal distances, round an antenna having a short part of its length vertical and a longer part horizontal, combined with similar observations of the absorptive power of such an antenna in various directions.[23]

Setting up at one place such a bent transmitting antenna (see Fig. 24), Marconi took observations at equal distances around it, but in different azimuths, by means

[23] See G. Marconi, "On Methods whereby the Radiation of Electric Waves may be mainly confined to Certain Directions, and whereby the Receptivity of a Receiver may be restricted to Electric Waves emanating from Certain Directions," *Proc. Roy. Soc. Lond.*, ser. A, vol. 77, p. 413, 1906.

of a simple straight vertical receiving antenna, having in its circuit some form of quantitative oscillation detector, such as a Duddell thermal ammeter. The intensity of the radiation in any direction is then taken as proportional to the mean-square value of the currents read by the ammeter in the receiving circuit.

He set off these currents or intensities in a diagram as the radii of a polar curve (see Fig. 25), and obtained a closed curve something like a figure 8 with two unequal loops, the radii of this curve representing the intensity of the radiation for various angular directions round the bent transmitter. It was then seen that the radiation is greatest in one direction, and that is the direction away from which the free end of the bent radiator points. It is also a minimum in another direction approximately 110° from the maximum direction, and it has a secondary or intermediate minimum 180° in the opposite direction, that is, in the direction in which the free end of the bent antenna points. The shape of this curve can be fully accounted for theoretically, as shown in the previous section, by assuming that the bent antenna is a combination of a closed or magnetic oscillation and an open or electric oscillator.

A large number of observations were thus obtained by Marconi with bent transmitting antennæ and vertical or open receiving antennæ, and also with vertical or symmetrical radiating antennæ and bent receiving antennæ placed in various relative positions, and these observations proved that an antenna which radiates best in any one direction absorbs best, as a receiving antenna, waves which are coming from that direction, and also that when an antenna is constructed which is partly vertical and partly horizontal, the radiation is non-symmetrical, being greater in some directions than in others. Marconi's observations were made with radiating and receiving antennæ from 30 to 45 metres in length, separated by distances varying from about 250 metres to 600 or 700 metres, and he then found that for the same distance between the antennæ the intensity of the radiation, as measured by a thermal or magnetic oscillation detector, was sometimes as much as four times greater in the direction away from which the free end of the bent radiator pointed than in the same direction. The wave-length of the waves used in his experiments was about 150 metres, and hence the maximum distance at which experiments were carried out was only about four or five wave-lengths. Practical experience, however, shows that the same directive qualities exist at very much greater distances, but theory points to the fact that at extremely large distances the asymmetry tends to vanish, and that any bent oscillator, however arranged, has no asymmetry of radiation for very large distances. In one experiment he employed a horizontal wire 100 metres in length, placed at a slight distance above the earth's surface, and connected at one end through a spark gap with the earth. Such a transmitter sent out waves approximately 500 metres in length. The receiving antenna was a vertical wire 8 metres in length, tuned to the period of the transmitter by means of a syntonizing coil and connected to the earth through a magnetic oscillation detector. The signals were quite distinct at 16 kilometres when the horizontal part of the radiator pointed away from the receiver, but only very weak at 10 kilometres when the free end of the transmitter pointed towards the receiving wire, and quite undetectable at 6 kilometres when the free end of the transmitter pointed at right angles to the line joining the transmitter and receiver.

Again, at Clifden, Connemara, Ireland, by means of a horizontal conductor 230 metres in length as a receiving antenna, and connected to the earth through a magnetic oscillation detector, Marconi found it possible to receive with clearness all the signals transmitted from the Poldhu station at a distance of 500 kilometres, provided that the free end of the horizontal receiving antenna pointed directly away from the direction of Poldhu, whilst no signals at all could be received if the horizontal wire at Clifden made an angle of more than 35° with the line of direction of Poldhu. Furthermore, he found that he could receive signals from the Admiralty Station on the Scilly Isles at Mullion in Cornwall, a distance of 85 kilometres, by means of a horizontal receiving antenna 50 metres in length placed 2 metres above the ground, one end of the wire being connected to the earth through a magnetic oscillation detector, provided that the free end of the wire at Mullion pointed away from the Scilly Isles, but that no signals could

be received if the horizontal portion was swivelled round so as to make an angle of more than 20° with the line joining Mullion with Scilly.

It is an obvious consequence of the Law of Exchanges which holds good for electromagnetic radiation as well as for heat and light, that any form of antenna which radiates better in one direction than another must absorb best radiation arriving from the direction towards which it radiates best.

Hence, by means of a horizontal wire 60 metres in length, supported 2 metres above the ground and connected at one end to the earth through a magnetic oscillation detector, Marconi was able to locate the direction of an invisible ship 16 miles away, sending out electromagnetic waves, by noticing the direction in which the free end of the horizontal receiving antenna had to be placed in order to make the signals most strong. This direction was a direction opposite to that from which the waves were arriving.

Marconi employed, therefore, a pair of such bent antennæ (as in Fig. 26) as a means of achieving a practically useful directive telegraphy.[24]

He places a long horizontal insulated wire or wires parallel to the earth's surface, and at a distance from it small compared with the length of the wire. One end of this wire is insulated, and the other end is connected to an earth plate either through a pair of spark balls, if it is a transmitting antenna, or through an oscillation detector of some form if it is a receiving antenna. These antennæ at the two stations (transmitting and receiving) are arranged back to back—that is, with their free or insulated ends pointing away from each other, but with the

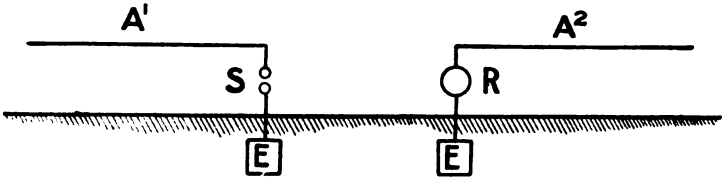


FIG. 26.—Marconi's Directive Antennæ for Electric Wave Direction Telegraphy. A^1, transmitting antenna ; S, spark balls ; A^2, receiving antenna ; R, oscillation detector ; E, earth plates.

horizontal wires in the same vertical plane (see Fig. 26). In the diagram A^1 is the horizontal transmitting antenna and A^2 is the receiving antenna. The spark balls S are placed in the short vertical branch of the antenna A^1, and a receiving instrument, such as a magnetic detector, R, in the same number of the receiving antenna. The two earth plates are denoted by E. Marconi then found that the receiving instrument R is most strongly affected when the antennæ are placed as shown in Fig. 26, and with horizontal parts in one vertical plane. If, however, the receiving or transmitting antenna is turned round into any other position, the indications in the receiving instrument rapidly fall off in intensity ; a variation of even 10° or 15° from alignment generally sufficing to render the signals insensible.

Some experiments of the same kind were made by the author in the same year (see *Phil. Mag.*, December 1906, "On the Electric Radiation from Bent Antennæ," a paper read before the Physical Society of London, November 23, 1906, by J. A. Fleming). A vertical radiating antenna was employed consisting of a single wire which could be bent over at various heights from the ground, so as to make a bent antenna partly vertical and partly horizontal, the ratio of the horizontal to the vertical lengths being varied at pleasure. A vertical receiving antenna was employed at distances varying between 80 to 150 feet, and in the receiving antenna a hot-wire oscillation detector of the thermoelectric type, devised by the author, was employed to measure the R.M.S. value of the current created in the receiving antenna. The transmitting antenna had its horizontal part swivelled round in various directions at intervals of 15°, in the several positions the current created

[24] See British Patent Specifications of G. Marconi, No. 14,788, of July 18, 1905.

in the receiving antenna was measured, the oscillations being excited in the transmitting antenna by means of a spark gap of constant spark length. The total length of the transmitting antenna was 20 feet, and the height of the receiving antenna was the same length.

The following Table shows the current in the receiving antenna in arbitrary units for each position of the horizontal part of the transmitting antenna.

Radiation from a Bent Earthed Transmitting Antenna 20 feet in total length. Receiving Antenna vertical and 20 feet high. Distance between receiver and transmitter 138 feet.

Length in feet of vertical part of Transmitter	5	4	3	2	1
Length in feet of horizontal part of Transmitter	15	16	17	18	19
Radiated wave-length in feet	100	100	105	106	110
Azimuth of horizontal part of Transmitter in degrees.	Current in the receiving Antenna in arbitrary units.				
0	100	100	100	100	100
15	98	97	94	92	93
30	92	85	96	83	75
45	82	79	79	77	67
60	78	74	70	71	58
75	77	67	59	56	45
90	72	66	57	52	48
105	71	65	57	46	41
120	70	66	62	53	49
135	72	64	60	54	48
150	73	80	58	67	59
165	70	74	56	69	60
180	82	69	64	63	68

These observations clearly confirm Marconi's observations that the radiation from a bent antenna is unsymmetrical, being greatest in a direction opposite to

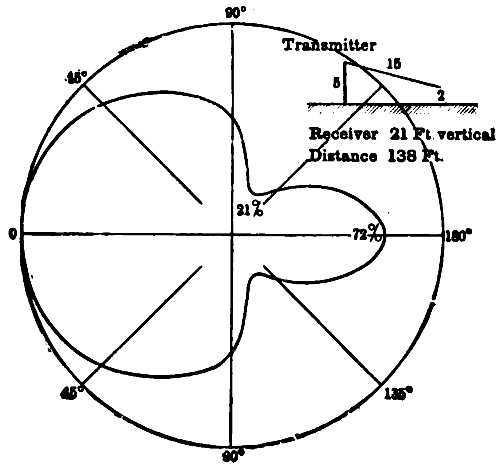


FIG. 27.—Polar Diagram, Radii of which denote Currents in the Receiving Antenna due to Radiation from a Bent Transmitter placed in Various Positions.

that towards which the free end of the antenna points. It was also found that by bending down the free end towards the earth, as in Fig. 27, the radiation became still more unsymmetrical, as shown by the polar curve in Fig. 27, in which the radii represent the strength of the currents in the receiving antenna corresponding

to various relative positions of the horizontal or inclined part of the transmitting antenna. It will be seen from the polar curve that the tipping down of the horizontal part causes nearly the whole of the radiation to be sent out towards that side opposite to which the free end points.

Marconi directed his attention also to the location of the radiant point, and discovered experimentally that the form of bent antenna which projects its radiation most intensely in a direction opposite to that in which the free end points receives or absorbs best radiation arriving on it from the same direction. Hence, he employed such a bent receiving antenna which could have its horizontal part swivelled round its vertical part to locate the direction of the sending antenna.

Marconi also invented a stellate receiving antenna consisting of a number of wires arranged in a radial manner from a centre to locate the direction of a sending station[25] (see Fig. 28). In this case a single oscillation detector or receiver has one terminal connected to an earth plate, and the other successively to the inner ends of the various radial antenna. Note is then taken of that position in which the signals are loudest, and the transmitter must then be in a direction opposite to that in which the free end of that particular radial receiving antenna, thus found to give the best signals, points.

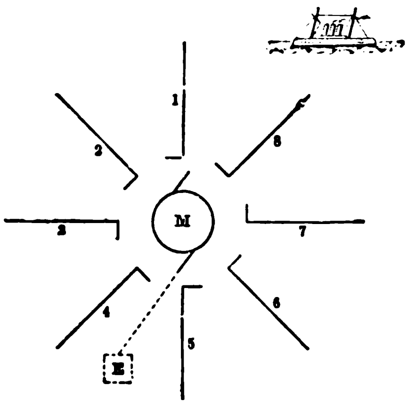


FIG. 28.

Marconi has given in his paper on directive antennæ (*loc. cit.*) a large number of observations, set out in the form of polar figure-of-eight curves, which delineate the intensity of the radiation in various azimuths from bent antennæ of various proportions by the length of their *radii vectores*, and also the current induced in such a receiving antenna, and its power of locating the direction of the radiant point. We shall refer again to his work in the chapter on Radiotelegraphic Stations.

The observations of Marconi, and of the author also, confirm the truth of the deductions which can be made from the theory given in § 9. If we plot out the values of E given by equation (17), § 9, for the electric force of a bent antenna perpendicular to and in the equatorial plane for some fixed value of the quantity mr, but for varying values of the azimuthal angle ψ, it will be found that they plot out into a figure-of-eight curve, such as that in Fig. 25. Moreover, the theory above given shows that the non-symmetricality depends not on the distance r alone, but on mr, or on the ratio of distance to wave-length, and upon the ratio of the lengths of the vertical and horizontal portions of the antenna.

This deduction also agrees with the observations of Mr. Marconi, who says[26]:—

"I have observed that, in order that the effects should be well marked, it is necessary that the length of the horizontal conductors should be great in proportion to their height

[25] See G. Marconi's British Patent Specification, No. 3127, of February 8, 1906.
[26] See *Proc. Roy. Soc. Lond.*, vol. 77, A, p. 415, 1906.

above the ground, and that the wave-lengths employed should be considerable, a condition which makes it difficult to carry out such experiments within the walls of a laboratory.

"I have found the results to be well marked for wave-lengths of 150 metres and over, but have not been able to obtain as well-defined results when employing much shorter waves, the effects following some law which I have not yet had time to investigate."

Another entirely different method of giving direction to electric waves has been devised by F. Braun, which depends upon the interference of electric waves travelling in the same direction but different in phase.[27] In Braun's method, three simple vertical wire antennæ are set up in positions corresponding to the angular points of an equilateral triangle, and oscillations are created in these antennæ which differ from one another in phase. These oscillations with definite phase differences were produced by a method devised by N. Papalexi and L. Mandelstam.[28] By these arrangements it is possible to cause the waves emitted by the three antennæ to combine together and assist one another in certain directions, but to neutralize one another in certain directions.

The experiments were carried out on a large open space near Strasburg. Wooden poles 20 metres high were planted at the corners of an equilateral triangle whose sides were 30 metres long. Antennæ wires, each approximately 33 metres long, terminated in wire netting stretched parallel to the ground and at a small distance above it. These constituted the balancing capacities. In the centre of the triangle an observation hut was constructed from which the wires ran out horizontally to the mast at a height of $2\frac{1}{2}$ metres above the ground. At a distance of 1300 metres a receiving station was constructed and a receiving wire erected attached to a pole 20 metres high. In the circuit of this receiving wire was placed a hot-wire oscillation detector, by means of which the current in the receiving wire could be measured.

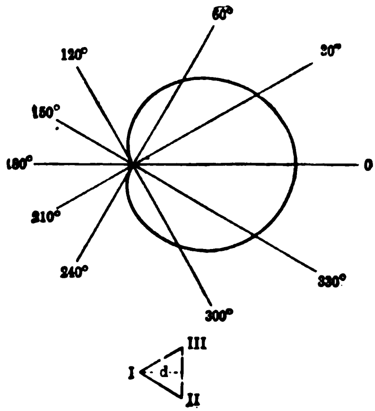


FIG. 29.

In a number of the experiments the oscillations in two of the transmitting antennæ were of the same phase, but differed from those in the third antenna by a definite amount, say, by 100°. The amplitude of the oscillations in the two antennæ in the same phase was half that in the third antenna. Under these conditions, if observations are taken of the current in the receiving antenna at equal distances, but in different azimuths round the triple transmitter, it is found that in one direction the radiation is a maximum, and in the opposite direction it is nearly zero, varying in accordance with the radii of a polar curve, as shown in Fig. 29.

The method, although ingenious, has not the simplicity and practicality of the bent receiving and transmitting antennæ employed by Marconi.

11. The Predetermination and Measurement of the Capacity of an Antenna.—An important measurement in connection with an antenna is its electrical capacity, as it is upon this that the current into it, and therefore the radiation from it, partly depends. There are certain cases in which we can predetermine with fair accuracy the electrostatic capacity, but then only by certain assumptions as to the distribution of the charge upon it.

Thus consider the case of a single circular-sectioned wire of which the length is large compared with the diameter. If an electrical charge is given to it, then

[27] See *The Electrician*, vol. 57, pp. 222, 244, May 25 and June 1, 1906. Prof. F. Braun, on "Directed Wireless Telegraphy."

[28] See *Science Abstracts*, vol. 9, A, *abs.* 1277, 1906; or *Phys. Zeitschr.*, vol. 7, p. 303, 1906. "A Method of obtaining Oscillations in Different Phases."

the electric density or charge per unit of length would be greater near the ends than in the middle. The true capacity of such a wire can only then be found if it is far removed from all other conductors by considering it to be an extreme case of an ellipsoid. If we consider the charge per unit length of the wire to be constant all along it, and call this density ρ, and the length of the wire l, and its diameter d, or radius of cross-section r, then we have seen (see § 7, Chap. II.) that an expression for the potential V near the centre of the wire is as follows :—

$$V = 2\pi d\rho \log_e \left(\frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}}\right)$$

By hyperbolic trigonometry we have

$$\operatorname{Sinh}^{-1} x = \log_e (x + \sqrt{1 + x^2})$$

Hence $$V = 2\pi d\rho \sinh^{-1}\frac{l}{d}$$

If, as usual, l/d is very large compared with unity then

$$V = 2\pi d\rho \log_e \frac{2l}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The whole charge on the wire Q is equal to $l\pi d\rho$, and hence the approximate capacity $C = Q/V$ is given by

$$C = \frac{l}{2 \log_e 2l/d} \text{(electrostatic units)} \quad . \quad . \quad . \quad . \quad (21)$$

or using ordinary logarithms and reckoning in micro-microfarads we have

$$C = \frac{l}{4 \cdot 6052 (\log_{10} 2l/d) \times 0 \cdot 9} \text{(m.-mfds.)} \quad . \quad . \quad . \quad . \quad (22)$$

We have already pointed out that these expressions will in general assign a capacity rather smaller than that given by actual measurement, since the formula is based on the supposition that the earth is at a considerable distance from the wire. Nevertheless, if we desire to predetermine approximately the capacity of a single *very thin* vertical aerial wire having one end near the earth, we can do so by increasing the value given by the above formulæ by about 10 per cent.

Thus a wire 0·1 inch in diameter, 7/22 S.W.G. in size, and 200 feet long, upheld in a nearly vertical position with base near the earth, was found to have a capacity of 0·00038 mfd., whereas the above formula would predetermine it to be 0·00033 mfd.

This matter has been experimentally examined by A. E. Kennelly and S. E. Whiting (see *Electrical World*, New York, vol. 48, p. 1239, 1906; or *Science Abstracts*, vol. x. A, 1907, *abs.* 290).

If a metal rod and plate are immersed in a conducting fluid, then the electrical resistance between the plate and metal in various positions is inversely proportional to the electrostatic capacity between them in the same positions, if we suppose the conducting fluid replaced by a dielectric medium. Hence, if a copper wire and large copper plate are immersed in a solution of sulphate of copper, the measurement of the resistance between them in various positions leads to a knowledge of the capacity of that wire with respect to the plate when both are in air. According to the measurements of Kennelly and Whiting, the capacity of a metal cylinder 80 diameters long is about 8·9 per cent. greater when the lower end of the cylinder is near the ground than it is when the cylinder is far removed and in free space.

It has already been pointed out that if a number of insulated wires or strips are placed parallel and near to each other, the actual measured capacity falls short of the sum of the capacities of each wire taken alone and separate in space, by an amount which is greater as the wires are nearer together. This is shown by the figures in Table I., obtained in the Pender Laboratory, University College, London.

A number (1 to 11) of flat iron strips about 1 inch wide, 15 feet long, and 0·05 inch thick, were hung up in a large room, and the capacity measured with the strips at different distances apart. The results in arbitrary units were as follows :—

TABLE I.

Number of strips.	Distance apart in inches.				Sum of separate capacities.
	12 inches.	6 inches.	3 inches.	In contact.	
1	1·00	1·00	1·00	1·00	1·0
2	1·74	1·45	1·34	1·19	2·0
3	2·31	1·80	1·61	1·27	3·0
4	2·79	2·10	1·85	1·44	4·0
5	3·28	2·42	2·03	1·46	5·0
6	3·75	2·70	2·21	1·54	6·0
7	4·18	2·98	2·36	1·59	7·0
8	4·61	3·25	2·52	1·72	8·0
9	5·03	3·51	2·68	1·81	9·0
10	5·46	3·77	2·82	1·96	10·0
11	5·90	4·00	2·97	1·99	11·0

This result shows that if a large number of wires are arranged in parallel to form a cage or conical aerial, the capacity of the whole is not nearly equal to that of the sum of the separate wires when very far apart.

From other experiments the writer has found that four equal and parallel wires, placed at a distance of about one-fiftieth of their length apart, have only twice the capacity of one wire, and twenty-five wires only about five times the capacity of one wire.

Hence, in the case of multiple wire aerials, the best way to determine the capacity is to measure it by means of the methods described in Chap. II., with the rotating commutator.[29] However complex in form, the aerial has a certain capacity with respect to the earth which is best expressed in micro-microfarads.

We give below, in Table II., the measured results obtained in certain definite cases, which will be a guide in estimating.

TABLE II.

CAPACITY OF AERIAL WIRES OR ANTENNÆ IN MICRO-MICROFARADS (MMFDS.).

1 mmfd. $= 10^{-6}$ of a microfarad.

A vertical wire 0·1 inch diameter and 110 feet long, with bottom end 5 feet from the earth suspended in the open air	= 205	mmfds.
A nearly vertical wire 0·1 inch diameter and 200 feet long, with end near the ground suspended in the open air.	= 308	,,
A single wire ship aerial, wire about 0·1 inch diameter and 150 feet long .	= 300	,,
A vertical wire 0·14 inch diameter and 12 feet long, hung up in a large room	= 32	,,
A single wire of 0·1 inch diameter and 14 feet long, suspended vertically in a large room	= 40	,,
Four vertical parallel wires 110 feet long and 0·1 inch diameter, spaced 6 feet apart at angles of a square	= 583	,,
Twenty-five vertical wires 0·1 inch diameter, 200 feet long, arranged fan-shape with top ends about 2 feet apart	= 1640	,,
One hundred and sixty wires, each 0·1 inch diameter and 100 feet long, arranged conically with bottom ends together 10 feet above ground and top ends 2 feet apart	= 2685	,,
Four vertical wires 0·1 inch diameter, each 45 feet long, placed fan-shape in front of a building 6 feet apart, bottom ends 10 inches apart connected to copper bus bar	= 485	,,

[29] See J. A. Fleming and W. C. Clinton, "On the Measurement of Small Capacities and Inductances," *Phil. Mag.*, May 1903, ser. 6, vol. 5, p. 505.

The inference to be drawn from the above figures is that, as regards mere capacity, a few wires spaced far apart are better than a great many close together. The capacity of an aerial may be increased, however, by adding metal cylinders or galvanized iron wire netting cylinders at the top to a considerable extent. Such *capacity areas*, as they are called, are electrically equivalent to an increase in length in the wire.

Otherwise a horizontal length of wire may be added in one or both directions at the top of a vertical wire, making what is called a T-shaped aerial wire.

In some cases a number of horizontal wires are stretched parallel to each other at a height above the ground, and a wire or wires brought down from them vertically at one end.

It is quite easy to measure the capacity of an aerial experimentally, and thus accumulate experience as to the capacity of given types of antenna. No method is so convenient as the rapid charge and discharge method involving the use of the rotating commutator described in Chap. II.

The antenna A is attached to the middle brush, and one terminal of a well-insulated battery, B, and of a movable coil galvanometer, G, to each of the outside brushes respectively of a rotating Fleming and Clinton commutator, C (see Fig. 30). The other terminals of battery and galvanometer must be put to a good earth, E. On setting the commutator in rotation, the antenna is alternately charged by the battery and discharged through the galvanometer. The antenna must, of course, be well insulated at the top. The process of calibration of the galvanometer and calculation of the capacity have already been described (see Chap. II. § 7).

Previously to taking a measurement of an antenna it is always necessary to determine its insulation. This can best be done by ascertaining what current as measured by a sensitive galvanometer can be detected when this galvanometer is inserted between the antenna and a battery having one terminal to earth and the other to the second terminal of the galvanometer.

For this purpose a battery of small secondary cells, say 50 or 100 cells, made up in test tubes, is very useful. The battery must in general have an electromotive force of 50 to 200 volts. A convenient substitute for a battery is the small self-exciting dynamo or magneto machine contained in the Evershed and Vignoles "Megger" for measuring insulation resistances. This little dynamo has an electromotive force of 500 volts or so. The first step is to calibrate a sensitive mirror galvanometer so as to know the current in microamperes corresponding to any observed deflection or scale displacement of the spot of light. This being done, one terminal of the galvanometer is connected to the insulated antenna, which must be previously disconnected from the earth plate, and the other terminal of the galvanometer is joined to the insulated terminal of the battery or dynamo, the second terminal of the latter being connected to the earth plate.

We then observe the deflection of the galvanometer, and determine the current flowing into the antenna in microamperes. If we then determine the electromotive force of the battery or dynamo in volts, the quotient of this voltage by the current in microamperes gives us the insulation resistance of the antenna in megohms. It is well to try the experiment first with a low voltage battery or single cell, and a coarse galvanometer or simple detector, lest there should be an accidental contact of the antenna with some non-insulated body. By this means the sensitive galvanometer will be preserved from destruction. As might be expected, the insulation of an ordinary antenna varies very much with the weather, being extremely high (several hundred megohms) in dry frosty weather and very low (perhaps a few hundred thousand ohms) in wet weather. The capacity measurement should always be made when the insulation is very high, as otherwise the charge put into the antenna by the battery partly leaks out before it can be discharged through the galvanometer by the rotatory commutator. For this reason in working the arrangement shown in Fig. 30, for measuring the antenna capacity by the commutator, it is always well to make two experiments, one with the galvanometer inserted in the charging circuit, or in series with the battery, and one with it in the discharge circuit or in parallel with the battery. The equality of the capacity measurements in the two cases is a proof of the good insulation of the

antenna. If the insulation of the antenna is very good, then its capacity may be determined by charging it from a battery, say, at 100 volts, and discharging this charge into a larger condenser, say, of ⅓ microfarad size. If this process is repeated fifty to a hundred times, we accumulate a large charge in the large condenser, and this can be measured in the usual way by the "throw" given on a ballistic galvanometer by comparing the "throw" given when the accumulated charge in the condenser is due to, say, 100 discharges into it of the antenna charged at 100 volts with the "throw" given by the discharge of the same condenser charged at 2 volts.

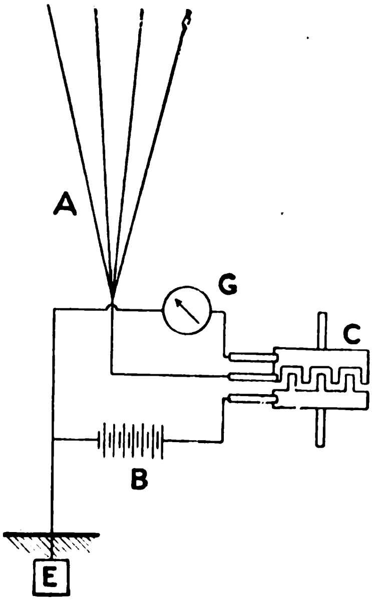


FIG. 30.—Mode of determining the Electrical Capacity of an Antenna, A, by means of a Rotating Commutator, C, Battery, B, and Galvanometer, G.

At all radiotelegraphic stations regular measurements should be made of the insulation and capacity of the antenna, the former being stated in megohms and the latter in microfarads or micro-microfarads, or in electrostatic units of which 9×10^5 equal 1 microfarad.

It is possible within limits to predetermine the capacity of an aerial under certain assumptions as to the distribution of the charge. Thus Prof. G. W. O. Howe (see *The Electrician*, vol. 73, pp. 829, 859, 906, 1914) has given formulæ for this purpose which can be applied to parallel wire antennæ or umbrella antennæ of certain forms.

If we consider a long straight wire to be made up of short sections of equal length insulated from each other, and if we imagine these sections each to receive an equal charge of electricity, then each of them would have a certain potential. If we suppose the units then all conductively connected together the charge would redistribute itself, and the potential would become the same for all sections. This uniform potential will be approximately the same as the average potential of the insulated sections. Hence the capacity of the antenna can be calculated when we know this average potential, for it is equal to the quotient of the total charge by the mean potential. Thus consider the case of a long straight wire of length l. Take any point P in it at a distance al from one end where a is some fraction less than unity. Then its distance from the other end is $(1-a)l$. The potential at P which may be denoted by V_P, is the sum of the potentials due to the two sections of the wire, or

$$V_P = 2\pi r\rho \int_0^{al} \frac{dx}{\sqrt{r^2+x^2}} + 2\pi r\rho \int_0^{(1-a)l} \frac{dx}{\sqrt{r^2+x^2}}$$

This is equal to

$$2\pi r\rho \left(\log_e \frac{2al}{r} + \log_e \frac{2l(1-a)}{r} \right)$$

or to

$$4\pi r\rho \left(\log_e \frac{l}{r} + \log_e 2\sqrt{a(1-a)} \right)$$

The mean value of $\log_e 2\sqrt{a(1-a)}$ between $a=0$ and $a=1$ is $-0{\cdot}309$. Hence the mean potential of the wire is—

$$V = 4\pi r\rho \left(\log_e \frac{l}{r} - 0{\cdot}309 \right)$$

This mean potential is a little less than the potential at the centre, and a little greater than the potential at the ends. The values of $\left(\log_e \frac{l}{r} - 0.309\right)$ for various values of $\frac{l}{r}$ have been calculated by Howe as follows :—

$\frac{l}{r}$	$\log_e \frac{l}{r}$	$\log_e \frac{l}{r} - 0.309$.	Percentage differences.
200	5·30	4·99	6·18
600	6·38	6·07	5·1
2000	7·59	7·28	4·25
6000	8·68	8·37	3·7
20000	9·90	9·59	3·2
40000	10·59	10·28	3·0

Hence for such a straight wire of length l cms. and diameter $2r$ cms. and uniform electric charge $2\pi r\rho$ units per centimetre of length the mean potential when far removed from the earth is therefore

$$V = 4\pi r\rho\left(\log_e \frac{l}{r} - 0.309\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and the capacity in electrostatic units is

$$C = \frac{l}{2\left(\log_e \frac{l}{r} - 0.309\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

If, however, such a wire is not far from the earth its potential will be reduced and capacity increased.

Suppose that the wire is a vertical wire of length l, with its bottom end at a distance h from the earth, and that the wire has a positive charge of $2\pi r\rho$ units per unit of length. Then there is a negative induced charge on the earth, and the effect of this in surrounding space is the same as that of an *electrical image* of the wire, or is the same as if there were a wire with a negative charge placed below the surface of the earth exactly in the position of the optical image of the first wire, reflected in the earth's surface ; the earth's induced surface charge being removed. To find the effect of this image we have to calculate the potential of a uniformly distributed negative charge on a wire of length l at a point on the axis of the wire produced, and at a distance a from the nearest end of the wire. It is easy to show that this potential is $-2\pi r\rho \log_e\left(\frac{a+l}{a}\right)$. We may then regard the potential of the actual vertical wire as being its own mean potential due to its charge *minus* a potential at its centre due to the charge on the image which is at a mean distance $2h + \frac{l}{2} = a$. Hence the mean potential of the wire is

$$V = 2\pi r\rho\left(2\log_e \frac{l}{r} - 0.618 - \log_e \frac{2h + \frac{3}{2}l}{2h + \frac{1}{2}l}\right)$$

If h is small compared with l, then the above approximates to

$$V = 2\pi r\rho\left(2\log_e \frac{l}{r} - 0.618 - \log_e 3\right) = 2\pi r\rho\left(2\log_e \frac{l}{r} - 1.72\right) \quad . \quad . \quad (25)$$

The capacity of such a vertical wire, with its lower end near the earth, is then

$$C = \frac{l}{2\log_e \frac{l}{r} - 1.72} \text{ (electrostatic units)} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Suppose that a wire 110 feet long and 0·1 inch diameter is placed vertically with its bottom end 5 feet from the ground, we have $\frac{l}{r}=26{,}200$, and $2\log_e\frac{l}{r}=20{\cdot}34$.

Hence, since 110 feet = 3352 cms. we have

$$C=\frac{3352}{20{\cdot}34-1{\cdot}72}=180 \text{ (electrostatic units)}$$

or 200 micro-microfarads.

On referring to Table II. on a previous page it will be seen that the actual measured capacity of such an antenna was found to be 203 mmfds., or a difference of scarcely 1 per cent. from the predicted value.

It is clear, therefore, that in this case the proximity to the earth increases the free capacity of the wire by about 8 per cent.

Suppose we then consider the case of a single horizontal wire of length l, and diameter d, placed parallel to the earth's surface, and at a height h above it, the length being great compared with the height h. It can be shown that if the earth were a perfect conductor the capacity of that wire would be

$$C=\frac{l}{2\log_e\frac{4h}{d}} \text{ (electrostatic units)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Since the earth is far from being a perfect conductor the capacity will not generally be as large as that given by this formula, but it will be greater than that of the same wire in free space.

An important case is that of several parallel wires, the length being great in comparison with the distance between them. This has been treated by Howe. Let there be n wires each of length l, and at distances d apart. We have then in the first place to find the potential at a point outside a charged wire, and at a distance d cms. from it.

Consider a wire of length l divided into two sections βl and $(1-\beta)l$, and take a point at P at a distance d from the junction of the two sections. Then the potential at P is equal to

$$V_P=2\pi r\rho\left(\int_0^{\beta l}\frac{dx}{\sqrt{d^2+x^2}}+\int_0^{(1-\beta)l}\frac{dn}{\sqrt{d^2+x^2}}\right)$$

$$=2\pi r\rho\left\{\left(\sinh^{-1}\frac{x}{d}\right)_0^{\beta l}+\left(\sinh^{-1}\frac{x}{d}\right)_0^{(1-\beta)l}\right\}$$

$$=2\pi r\rho\left(\sinh^{-1}\frac{\beta l}{d}+\sinh^{-1}\frac{(1-\beta)l}{d}\right) \text{ accurately}$$

$$=2\pi r\rho\left(\log_e\frac{2\beta l}{d}+\log_e\frac{2(1-\beta)l}{d}\right) \text{ nearly}$$

If we place n wires each of length l parallel to each other, and at distances d apart, Howe shows that the mean potential of the whole group is

$$V=4\pi r\rho\left(n\log_e\frac{l}{d}+\log_e\frac{d}{r}-B\right)$$

and the capacity in electrostatic units is

$$C=\frac{nl}{2\left(n\log_e\frac{l}{d}+\log_e\frac{d}{r}-B\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

where B is a function of n as follows :—

No. of wires n.	Value of B.	No. of wires n.	Value of B.
2	0	8	6·40
3	0·46	9	8·06
4	1·24	10	9·80
5	2·26	11	11·65
6	3·48	12	13·58
7	4·85		

In the case of an actual antenna we have generally certain wires which are horizontal, or nearly so, and certain leading down wires which are nearly vertical, as in the case of a simple T-aerial or Marconi directive aerial.

Take the case of the T-aerial consisting of two single wires. We have then several sources of potential for each wire. In the case of the horizontal wire we have (1) its own charge, (2) the charge on the vertical wire, (3) the charge on the image of the horizontal wire, and (4) the charge on the image of the vertical wire. Four similar effects contribute to create the potential of the vertical wire.

In making these calculations, it is most convenient to assume that the uniform charge on the antenna is everywhere one electrostatic unit per centimetre of length. Then for each wire we calculate the potential due to its own charge from the formula $V_1 = 2\left(\log_e \frac{l}{r} - 0{\cdot}31\right)$. If the wire is a horizontal wire at a height h above the earth, then there is a negatively charged image at a distance h below the surface, and the reduction of potential of the horizontal wire due to its own image is, according to Howe's calculations—

$$V_2 = 2\left(\sinh^{-1}\frac{l}{2h} + \frac{2h}{l} - \sqrt{1 + \frac{4h^2}{l^2}}\right)$$

If the wire is a vertical wire then the reduction of its potential due to the uniform charge on its own image is—

$$V_3 = \frac{1}{h-a}\log_e \frac{2^{2(a+h)}h^{2h}a^{2a}}{(a+h)^{2(a+h)}}$$

where h is the height of the top of the wire from the ground, and a is the distance of the bottom end from the ground.

Howe has also given formulæ for predetermining the mean potential produced all along a given wire by the uniformly distributed charge on a neighbouring wire placed at an angle to the first and in the same or different planes. These results have been embodied in curves which can be applied to the case of umbrella antennæ. But for these we must refer the reader to a paper on "The Capacity of Aerials of the Umbrella Type," by Professor G. W. O. Howe, in *The Wireless World* for October 1915, p. 426. As an illustration of his method, Howe gives the following calculations for a simple T-aerial.

The length of the vertical part is 100 feet $= h$, the length of the horizontal part is 200 feet $= l$. The semi-diameter of the wire is 0·048 inch $= r$, and the ratio $l/r = 50{,}000$. Also the charge per unit of length $= 2\pi r\rho = 1$. Then for the horizontal wire we have $2\left(\log_e \frac{l}{r} - 0{\cdot}31\right) = 20{\cdot}98$, and for the increase in its potential, due to the charge on the vertical wire, we have one 1·76, as given from Howe's curves (see *The Electrician*, vol. 73, p. 807). Also for the reduction in potential to the image of the horizontal wire we have $-0{\cdot}94$; and for the image of the vertical wire we have $-0{\cdot}63$.

Hence, adding up the potential of the horizontal wire, we have as follows :—

Potential due to its own charge	20·98
,, ,, charge on vertical wire	1·76
,, ,, charge on image of horizontal wire	−0·94
,, ,, charge on image of vertical wire	−0·63
Total potential	21·17

In the same way for the vertical wire we have—

Potential due to its own charge	19·58
,, ,, charge on horizontal wire	3·52
,, ,, charge on image of vertical wire . .	-1·09
,, ,, charge on image of horizontal wire . .	-1·27
Total potential . .	20·83

The total charge on the wire is—

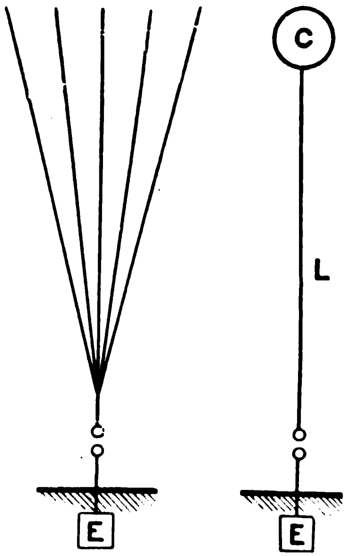
$$300 \times 30{\cdot}48 = 9150 \text{ units}$$

and the mean potential is—

$$\frac{200 \times 21{\cdot}7 + 100 \times 20{\cdot}83}{300} = 21{\cdot}06$$

Hence the capacity is 9150/21·06 = 435 E.S. units or 483 micro-microfarads.

In the same manner Howe shows that we can approximately predetermine the capacity of a 10-wire T-aerial of certain given dimensions; but for this we must refer the reader to his articles in *The Electrician*, vol. 73; see also vol. 77, p. 880.



(*a*) (*b*)

FIG. 31.—(*a*) Real Fan-Shaped Antenna with Distributed Capacity, and (*b*) Ideal Antenna with Capacity Localized at the Summit.

12. The Oscillation Constant of an Antenna.—Another important quantity connected with an antenna is its *oscillation constant* which determines the wave-length of the radiation emitted by it. Every antenna has its own natural period of electrical vibration, depending upon its capacity and inductance. We may compare it to a straight elastic strip of steel, gripped at one end in a vice. If we bend the strip and release it, it vibrates isochronously with a time period depending upon its flexural elasticity and its mass.

Consider the case of a fan-shaped antenna wire having a pair of spark balls near the base (see Fig. 31). Let the balls be connected with the secondary circuit of an induction coil, and electric oscillations set up in the wire. These are executed with a certain definite time period, depending upon the capacity and inductance of the wire. In the actual wire these two qualities are, so to speak, mixed up together, or there is so-called distributed capacity and inductance. We can, however, imagine an antenna in which the capacity is all collected at the top, and the inductance alone left in the wire. If the inductance of the wire without capacity be denoted by L, and the capacity at the top is denoted by C, then it is evidently always possible to so adjust the magnitude of C and L that the hypothetical simple antenna has the same electrical time period of oscillation as the real complex antenna. In this case the imaginary capacity concentrated at the top is called the *equivalent capacity*, and the inductance of the vertical wire without capacity is called the *equivalent inductance*. If the equivalent capacity C is measured in microfarads, and the equivalent inductance L in centimetres, then the quantity $\sqrt{CL}$ is called the *oscillation constant* (O) of the hypothetical antenna.

The time period T of this last antenna is connected with its oscillation constant by the relation $T = \frac{O}{5{\cdot}033 \times 10^6}$.

The real antenna, which has the same period T, will also have the same

oscillation constant. We can, therefore, define the oscillation constant of an antenna to be numerically equal to the product of its natural time period T, and the constant $5{\cdot}033 \times 10^6$.

The mathematical predetermination of the oscillation constant of an antenna, in any but the simplest cases, presents great difficulties. It can, however, be obtained for a simple rod antenna as follows :—

Let l be the length in centimetres of the antenna supposed to be a circular-sectioned wire of diameter d, and let λ be the length in centimetres of wave radiated from it when it is giving its fundamental electrical oscillation. Then, since the velocity of radiation is 3×10^{10} cms. per second, if T is the natural time period of oscillation, we must have—

$$T = \frac{\lambda}{3 \times 10^{10}}$$

Hence the oscillation constant O is nearly given by $O = \frac{5\lambda}{3 \times 10^4}$.

The relation between l and λ for a single thin wire antenna is expressed by $\lambda = 4l$, but for a fan or multiple antenna, such as that shown in Fig. 31, it is more nearly $\lambda = 5l$.[30, 31]

On the first supposition we have $O = \frac{l}{1500}$, and on the second $O = \frac{l}{1200}$.

For multiple antennæ, or antennæ with capacity plates at the top, we cannot predetermine the oscillation constant, but it can be found experimentally with great ease by means of the author's cymometer (see Chap. VI. § 15).

Suppose any aerial or antenna, A, to be set up, and that it is desired to ascertain its natural time period or to adjust it to have any required time period. We provide the aerial with a pair of spark balls, S, at the base inserted between the aerial and the earth plate, and place the Fleming cymometer, Cy, with its copper bar parallel to and near the base of the aerial (see Fig. 32). We connect the spark balls to an induction coil, I, and set up oscillations in the aerial. The handle of the cymometer is then moved until the neon vacuum tube of the cymometer glows most brightly, and the cymometer will then indicate the fundamental oscillation constant of the aerial by its scale reading, since when the cymometer is in tune with the aerial their oscillation constants must agree. In making this measurement the cymometer should be kept as far away from the aerial as possible so as to avoid increasing the capacity of the aerial. We can then insert inductance in the earth wire of the aerial or alter its capacity until we give it any required oscillation constant and natural time period.

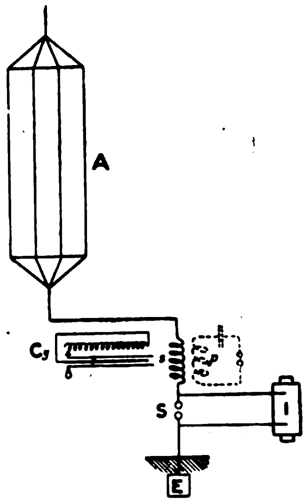


FIG. 32.—Mode of determining the Oscillation Constant of an Antenna by the Author's Cymometer. A, antenna under test ; Cy, cymometer : S, spark ball ; I, induction : ps, oscillation transformer with primary circuits ; p, out of operation ; E, earth plate.

Thus, for instance, we may vary an inductance inserted in series with an aerial until we give it an oscillation constant of 5 or 10, corresponding to a natural time period of one-millionth of a second or one half-millionth of a second.

The measurement of the oscillation constant of the aerial gives us at once the length of wave

[30] See Mr. H. M. Macdonald, Adam's Prize Essay, "Electric Waves," p. 111.

[31] See also J. A. Pollock (*Journal of Roy. Soc. of New South Wales*, 1903, vol. 37, p. 198), who found that for a Hertzian rod oscillator the ratio of wave-length to rod-length was 2·3 to 2·45, and therefore for an earthed rod oscillator $\frac{\lambda}{l} = 4{\cdot}6$ to $4{\cdot}9$, or nearly 5. See also last section of Chap. IV.

radiated from it when used as a simple plain aerial in the original Marconi manner. For the velocity of electromagnetic radiation being 3×10^{10} cms. per second, or 10^9 feet per second nearly, it follows that the length of wave radiated is nearly 200 times the oscillation constant, when wave-lengths are reckoned in feet, and 60 times when wave-lengths are reckoned in metres. More exactly, the rules are—

$$\text{Wave-length (in feet)} = 195{\cdot}56 \times \text{oscillation constant}$$
$$\text{Wave-length (in metres)} = 59{\cdot}6 \times \text{oscillation constant}$$

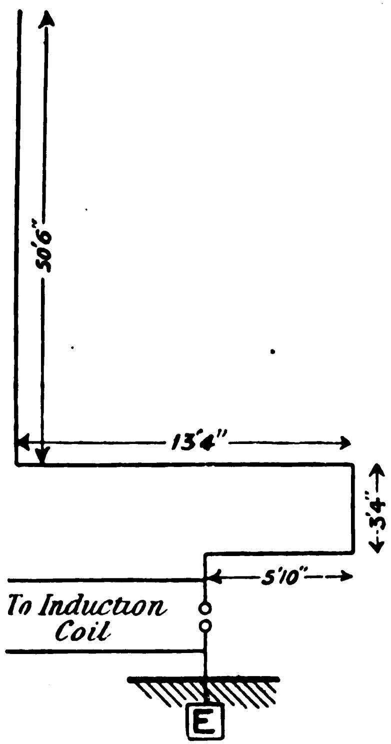


FIG. 33.—Dimensions of Antenna at University College, London, used for a Special Experiment as a Plain Aerial.

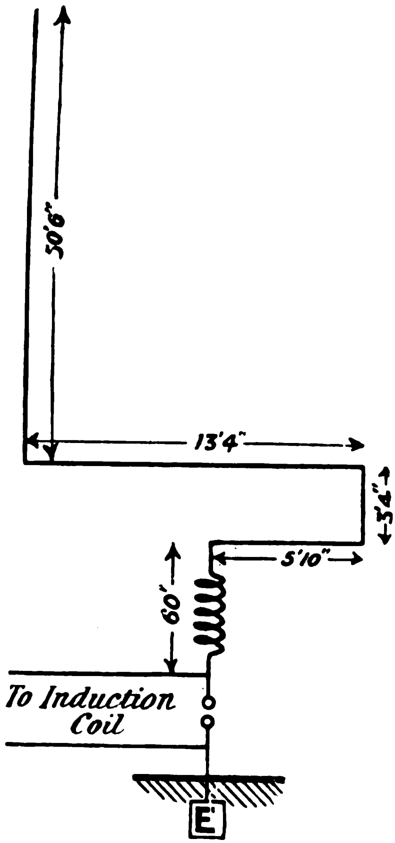


FIG. 34.—The same Antenna as in Fig. 33, but with an Inductance Coil inserted in Series with it above the Spark Balls.

the oscillation constant being the square root of the product of the capacity in microfarads and the inductance in centimetres.

Thus, for instance, an aerial set up at University College, London, had the form and dimensions shown in Fig. 33. The dimensions are given in feet and inches. It was arranged as a simple aerial 73 feet long. By means of the author's cymometer placed near to the lower horizontal bend, the oscillation constant was determined, and found to be 1·85. Hence the length of the fundamental wave radiated is 370 feet, and this is very nearly five times the total length of the aerial, since $5 \times 73 = 365$.

This measurement of the ratio of wave-length to antenna-length agrees with Professor H. M. Macdonald's theory, and with the confirmation of it just given.

The above-described aerial wire then had an inductance coil of wire 60 feet in length inserted between the aerial and spark balls, this coil being the secondary circuit of an oscillation transformer (see Fig. 34). The total length of the open oscillating circuit was now 73+60 feet, or 133 feet. The oscillation constant was then found to be 4·85, and hence the fundamental radiated wave-length was now 970 feet. This is nearly equal to five times (73+2×60), thus showing that the 60 feet of wire wound on a frame forming the secondary circuit of the oscillation transformer was equivalent to much more than 60 feet of additional length to the aerial, owing to its greater inductance per unit of length. Hence a great error may be committed in estimating the radiated wave-length even of a single wire aerial, if it is assumed (as some writers have done) that the wave-length of the radiator wave is four times the total length of wire composing the aerial, including that of any coiled wire forming an inductance in series with it.[32]

In a third case, a fan-shaped aerial of four wires, each 50 feet in length (see Fig. 35), and having the secondary circuit of an oscillation transformer in series with them, was tested in the same way. In this case the oscillation constant was found to be 6·9, and the radiated fundamental wave-length 1380 feet. The above examples show how much the form, capacity, and inductance of the aerial affect the wave-length of the radiated. Generally speaking, great errors have been made in guessing or assuming the radiated wave-lengths of antenna in the absence of careful measurements with the cymometer.

The correct formula for calculating the wave-length of the radiated waves from such an antenna with distributed inductance in series with a coil of localized inductance has already been given in § 7 of Chap. IV.; and it can be pre-determined by the equation (85) in that section when the total inductance of the antenna and that of the coil is known.

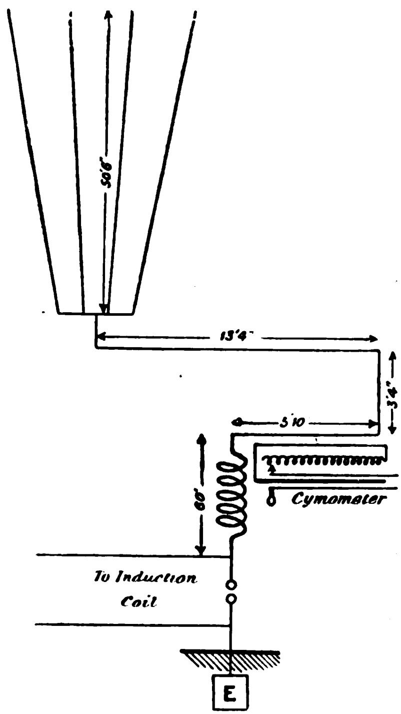


Fig. 35.—Determination of the Oscillation Constant of a Fan-Shaped Antenna by the Cymometer.

13. The Earth Plate or Balancing Capacity.—We have in the next place to consider that part of the radiator which is complementary to the antenna. The antenna itself is analogous to one-half of a Hertzian oscillator, and there must therefore be another conductor with respect to which the antenna has capacity. This may be either a plate of metal laid in or on the earth, or a conductor consisting of wires insulated from it. Mr. Marconi has always strongly held the view that the lower end of the antenna should be in good conductive connection with the earth. On the other hand, Sir Oliver Lodge has maintained that the complement to the antenna should be a conductor insulated from the earth. We shall consider in Chap. IX. the function of the earth in radiotelegraphy, and shall therefore not discuss here the reasons for and against a conductive earth.

[32] See Messrs. W. Duddell and J. E. Taylor, "Wireless Telegraph Measurements," *Journal Inst. Elec. Eng. Lond.*, 1905, vol. 35, p. 341.

If an earth plate is used it should consist of copper plates or wires well connected. They may lie on the ground or be buried in the ground as an earth plate. They should extend away from the foot of the antenna for a distance at least equal to its height and be of good conducting metal. Some addition to the damping is without doubt introduced by the earth connection, but there are ways of compensating for it; for although connection to earth may increase the decrement of an oscillator, the decrement can also be decreased by the addition of inductance and capacity to it. Hence we may compensate for one by the other. In numerous cases the use of a balancing capacity perfectly insulated from the earth is impracticable. When an earth plate is employed certain precautions should be taken in making it. It is desirable in the first place to have it in two separate portions, so that the resistance to earth can be measured by measuring the resistance between the two earth plates. This cannot be done when the plate is in one piece. Also, if possible, provision should be made for wetting the earth plate and for examining it periodically to see if corrosion has set in.

The form of this plate is important. It is found that long narrow strips give less earth-plate resistance than a single square or round strip.

The general theory of earth-plate resistance is as follows: Let a conductor of any form be supposed to be buried in an infinitely extended medium of resistivity, ρ. Then suppose the conductor buried in the medium to be charged to a potential V, and to have a charge Q. The quotient $\frac{Q}{V}$ is the capacity (C) of the body. Let I be the current proceeding normally from unit area of the conductor into the medium. Let E be the normal electric force, and dn an element of length of the normal, and dS an element of surface of the conductor. Then—

$$-\frac{dV}{dn}=E \text{ and } \int E dS = 4\pi Q$$

also $$\frac{dV}{dn}dn = \frac{\rho dn}{dS} I dS = E dn \text{ by Ohm's law.}$$

Hence $$\rho \int I dS = \int E dS = 4\pi Q$$

or $$\frac{4\pi Q}{\rho V} = \frac{\int I dS}{V} = K = \text{conductance of the dielectric.}$$

Therefore $$\frac{4\pi}{\rho} C = K$$

and $$\frac{1}{K} = R = \frac{\rho}{4\pi C} \qquad (29)$$

Hence the total resistance of the buried conductor is numerically equal to the quotient of the resistivity of the surrounding earth by the capacity of the body in homologous units. Hence for any given position that form of earth plate will give the least earth resistance which has the largest capacity.

In making an "earth" we are concerned with initial cost and durability. The cheapest form of earth plate consists of a number of stout, stranded, thickly galvanized iron wires, or, better, bare stranded copper wires spreading out radial-fashion like the roots of a tree underground. In that manner we obtain the greatest earth-plate capacity. Strips of zinc plate are also often used. In this case, however, care must be taken not to solder a copper wire to the zinc if the joint is buried underground, or else the plate at the joint will be destroyed by galvanic action.

In large stations the earth plate, when used, is very extensive. It consists of plates of copper laid in a circle round the base of the antenna and connected by thick and numerous copper wires with one common earth terminal. If the ground is rocky or dry the earth plate should have a very considerable area, and extend as far as possible in the direction in which the transmission of waves mostly takes place.

If a balancing capacity is used it may consist of wires radiating from the base

of the mast or tower which supports the antenna, these wires being strained between insulators fixed to posts of such height that it is possible to walk under the wires easily.

In the case of portable or military stations the earth plate is formed of several large sheets of copper gauze which are spread on the ground and kept in contact with it by a good many large stones or masses of earth.

14. The Coupling and Excitation of the Oscillations in an Antenna.—In modern radiotelegraphy the original or plain self-excited Marconi antenna is now hardly ever used.

Owing to the relatively small capacity of a simple linear antenna wire with respect to the earth the possible energy storage is small, and in consequence of the large radiative power of such an oscillator this energy is thrown off almost entirely in the first oscillation or two that occur when the spark takes place at the spark balls. Hence the radiated wave train comprises at most one or two waves. Therefore, the effect on the receiving circuit is merely that due to a transitory impulse, and no true syntony or tuning is possible between transmitter and receiver. Hence in modern radiotelegraphy the antenna is connected either directly or inductively with an energizing circuit, in which a much larger store of energy can be accumulated and then given up to the antenna as required. When the inductive coupling is used a two-coil oscillation transformer is interposed, one circuit of which is inserted between the antenna and the earth and the other connected with the energizing circuit, and in the case of the so-called direct coupling a single coil or auto-transformer is similarly employed. In both cases the two coupled circuits are syntonized together.

The general theory of the operation of such transformers when both circuits have condensers attached to their terminals has been given in a previous chapter (see Chap. III. § 14).

In the case of an oscillation transformer used in the transmitting apparatus, the circuit which contains the spark gap and the smaller of the two inductances, and therefore the larger of the two capacities, is called the primary circuit, whilst the other is called the secondary circuit. When employed to create oscillations in an antenna, the secondary circuit of the transformer is connected between the antenna and the earth, or else a large capacity called the balancing capacity replaces the earth (see Fig. 8, § 5 of this chapter). If the antenna is insulated and symmetrical, the secondary circuit is inserted in the centre.

It is found that no advantage ensues from winding the primary circuit of an oscillation transformer used in connection with a transmitting antenna with more than one turn of wire. The inductive effect of the primary circuit depends on the magnetic field it creates. This, again, is the result of the ampere-turns of the primary current. The current in this circuit is chiefly determined by the inductance of the primary circuit, and the effect of increasing the turns on the primary circuit is, on the whole, to decrease the ampere-turns, since the inductance varies as some power of the number of turns lying between 1 and 2.

Accordingly, Mr. Marconi constructs his "transmitting jiggers" or oscillation transformers for use with sending aerial wires with one, or at most two, turns in the primary circuit. These may be made, however, of several single turns arranged in parallel.

The secondary circuit generally consists of more than one turn—say five to twenty turns—wound under or over the primary circuit. Very good insulation must be secured between the two circuits, and it is necessary to immerse the whole coil in a vessel of insulating oil.

A convenient mode of construction is to make two square frames of wood, each side about 50 cms. in length, and the width being 5 to 10 cms.

On one frame highly insulated and well-stranded copper wire is wound, a large number of strands being employed in parallel, each being taken once round the frame. The wire should be high conductivity copper wire, size No. 40 S.W.G., each wire slightly insulated with varnish, and a large number of these insulated wires, say 200, twisted together and insulated over all with india-rubber, and then again a sufficient number of these compound wires laid in parallel once round the frame and the ends soldered to copper strips.

In this manner we construct a circuit with low high frequency resistance and also small inductance. On the other frame a sufficient number, say 5 to 10 turns, of well insulated fine-stranded wire of the same kind are wound, and the two frames placed side by side in highly insulating transformer oil in a stoneware (not metal) vessel. When joined respectively in series with the condenser of the energizing circuit and with the antenna, the two circuits of the transformer can be tuned together by a supplementary inductance coil placed on one or other circuit.

In practice it is advisable to employ oscillation transformers both in transmitting and receiving apparatus, which permit the coupling to be varied over wide limits by altering the distance of the primary and secondary circuits. As regards the transmitter, this is best achieved by winding the primary and secondary circuits on separate square wooden frames, which are put together in one stoneware vessel of insulating oil if a closed coupling is required, or in separate vessels separated from each other by a certain distance if a weak coupling is required. In the case of the receiving circuit, where a high insulation is not required, the author has found that the best way of achieving it is to wind the two circuits in flat spirals on the surfaces of two hinged boards, so that they can be approximated or removed from each other by opening or closing the boards, more or less like a book. This plan is not practicable in the case of the transmitter, because very high insulation is necessary, and the two circuits must in general be immersed in transformer oil.

When using a two-coil oscillation transformer as a means of inducing oscillations in the antenna circuit, it is generally necessary to insert also in series with the antenna a variable inductance called a *variometer* or *tuning coil* by means of which we can alter the inductance of the antenna circuit. The antenna possesses capacity with respect to the earth and also has inductance. The secondary circuit of the oscillation transformer inserted in series with it has also inductance, but it is necessary to have the power of varying the total inductance so as to make the product of the antenna capacity and the total antenna inductance agree with the same product for the energizing circuit connected with it.

The usual form of this variable inductance is a bare copper wire spiral either flat or cylindrical with the turns not in contact. A shifting connection allows more or less turns of this spiral to be inserted between the earth plate or balancing capacity, and the lower or earth end of the oscillation transformer. The arrangement is shown diagrammatically in Fig. 42(*a*) in the next section of this chapter.

When an auto or single coil transformer is used it consists of one single layer turn of copper wire or tube wound into a spiral with turns not touching. On this spiral there are two shifting contacts or travelling clips and one fixed end terminal.

The coil is by its end terminal to the lower end of the antenna. The outermost travelling terminal is connected to the earth plate or balancing capacity, and the intermediate travelling terminal and the lower end of the antenna are connected to the energizing circuit. By shifting the travelling terminals the two coupled circuits can be tuned.

15. The Nature of the Oscillations set up in Antennæ of Various Types.—We have in the next place to consider the nature of the oscillations produced in various types of antennæ, and the distribution of potential and current along it. When oscillations are taking place in the aerial, whether created by direct charge, as in the original Marconi method, or by coupling it to another closed oscillating circuit directly or inductively, two conditions must always hold good.

(i.) There must be a current node at the upper or insulated end, and a potential antinode or loop at the same place.

(ii.) At the earth plate end there must be a node of potential and an antinode or loop of current.

On the wire there will be produced, provided it has a suitable length, stationary waves of potential and current.

We have already given (see Chap. IV. §§ 1 and 2) the theory of the production of such stationary waves on wires. A general confirmation of theory has

been obtained from experiments made with wire antennæ by Drude, Braun, Slaby, Chant, and Ives.

1. *The Oscillations in a Simple Antenna.*—Consider first the case of the original plain wire Marconi aerial. Theory shows that when oscillations are excited in it by disruptive discharge, the fundamental oscillation is such that the potential oscillation has no amplitude at the earthed end, and a maximum at the insulated or top end, and that odd harmonic oscillations can exist, viz. the 3rd, 5th, 7th, etc., in which there are 1, 2, 3, etc., nodes of potential in addition to the node at the base, forming 1½, 2½, 3½, etc., semi-waves of potential distribution on the wire, as indicated by the dotted lines in Fig. 19 of Chap. IV. In these diagrams the black lines indicate the antenna, and the distance of the dotted line from it the potential amplitude at that point. In the same manner, there will be loops and nodes of current with the condition that the free end is a node and the earthed end a loop of current.

Experience shows that it is not easy to excite the higher harmonics in a plain antenna. The most usual mode of oscillation is the fundamental. The explanation of this is probably to be found in the fact that owing to their greater frequency the higher harmonics are better radiated than the fundamental vibration. Hence the antenna tends to get rid of its harmonic oscillations, and to keep going its fundamental oscillation. Thus F. Braun explored the potential distribution in a horizontal free-ending wire attached at one end to one secondary

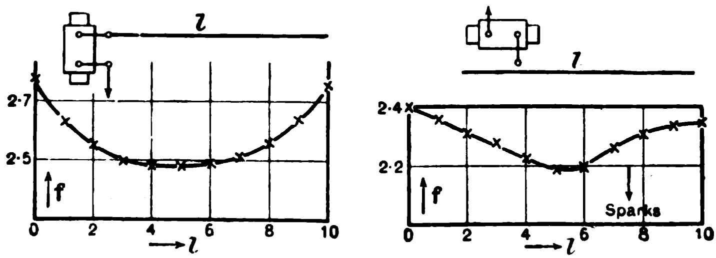


Fig. 36.—Diagrams illustrating the Results of Slaby's Experiments on the Distribution of Potential along a Linear Oscillator. The ordinates of the curved line denote spark potentials at the corresponding points in the wire.

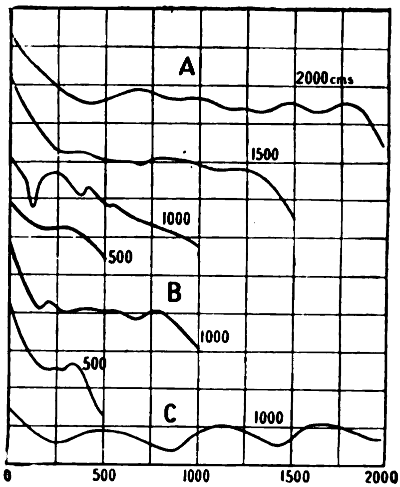
spark ball of an induction coil, the other ball being earthed or attached to an equal wire or capacity.[33] Braun hung on to the wire vacuum tubes at various places, each vacuum tube having a short tail of wire attached to its lower end, and he found no evidences of potential nodes, but a general increase in potential along the wire from the spark ball to the free end. Slaby made a more careful exploration, measuring the potential at each point in the antenna by means of a spark micrometer consisting of a blunt metal point opposed to a flat surface of carbon, the distance being capable of adjustment by a fine screw.[34] He earthed one of the secondary spark balls of an induction coil, and attached to the other a horizontal wire antenna 10 metres long and 1 mm. in diameter. He explored the distribution of potential along this wire. He found a distribution of potential as represented by the ordinates of the dotted line in Fig. 36, showing the existence of a stationary wave of potential in the wire with a minimum in the middle of its length. Higher harmonics were absent. If two equal insulated antennæ were attached to the two spark balls, then a regular distribution of increasing potential was found in each wire, as shown by the ordinates of the dotted line, where the abscissæ represent distance from the spark balls of the coil, and the ordinates the micrometer spark length or potential amplitude at that point in the antenna.

[33] F. Braun, *Phys. Zeitschrift*, 1900, vol. iii. p. 143.

[34] See A. Slaby, *Elektrotechnische Zeitschrift*, 1902, p. 168; or *The Electrician*, 1902, vol. 49, p. 6,

C. A. Chant has also made similar measurements, using a form of Rutherford magnetic detector attached to the horizontal antenna by which to measure the potential or current at that point in the wire.[35] He employed as antenna a bare copper wire 0·7 mm. in diameter, stretched horizontally, and attached one end to one secondary spark ball of an induction coil, the other ball being either (1) earthed, (2) attached to an equal antenna, or (3) left insulated. He varied the length of the antenna from 500 to 2000 cms., and delineated a series of curves, the ordinates of which represent the potential amplitude in the antenna and the abscissæ distances from the free end. In the case when one spark ball was earthed these curves show a general increase in potential along the wire from the spark balls to the free end, but the curve is somewhat irregular; and in the case of the antenna 1000 cms. long there is a decided minimum or node of potential at 150 cms. from the free end (see Fig. 37, curves A).

In the case when one spark ball of the coil was not earthed (curves C) there was no general rise of potential along the antenna attached to the other ball, but



DISTANCES MEASURED ALONG THE WIRE FROM THE FREE END.

FIG. 37.—Curves obtained by Chant, representing the Potential Distribution in a Linear Oscillator directly charged. Curves A, one spark ball earthed; Curves B, equal wires attached to spark balls; Curves C, one spark ball insulated.

a series of nodes and loops of potential, the nodes appearing at distances 130, 425, 715, 1000 cms. from the end of the antenna 1000 cms. long. These clearly correspond to a stationary harmonic oscillation of 3½ semi-waves of potential, each having a wave-length of 580 cms. For $3 \times 290 + 130 = 1000$, and as we always find that the final semi-loop is less than one-quarter of a wave-length, in fact, nearly one-fifth of a wave-length, this agrees with the above observations.

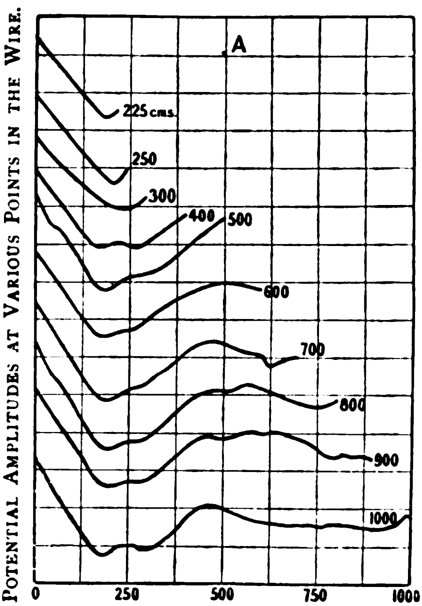
Hence we can say that experiment confirms the theory that the excitation of electric oscillations in a simple vertical antenna, earthed at its lower end through a spark gap, results generally in the production of the fundamental oscillation of the antenna with a potential amplitude, increasing all the way up the wire from the bottom or earthed end up to the top or free end. Also that the length of the wire comprises something rather less than one-quarter of a wave of potential.

There seems to be some difference of opinion as to the ratio between the wave-

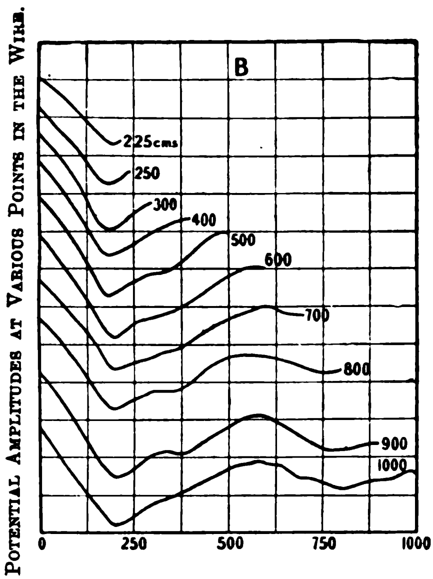
[35] See C. A. Chant, "On the Variation of Potential along Transmitting Antenna in Wireless Telegraphy," *The American Journal of Science*, January 1904, vol. 17.

length λ and actual length l of a simple linear Hertzian oscillator. If a pair of rods, very thin compared with their length, are placed in one line, the total length of the oscillator being l, then the wave-length of the waves emitted when they are used as a Hertzian oscillator is, according to the theory of M. Abraham (*Wied. Ann.*, 1898, vol. 66, p. 435), equal to $2l$, and Lord Rayleigh agrees (see *Phil. Mag.*, 1904, vol. 8, p. 105). According to the theory of Prof. Macdonald, the ratio is $2{\cdot}53l$.

The experimentally determined ratios lie between these two extremes. Thus P. Drude, for a rod oscillator of 1 mm. diameter and 4 metres long, found $\frac{\lambda}{l}=2{\cdot}1$ (see *Ann. der Physik*, 1903, vol. 11, p. 965), and this was confirmed by F. Conrat. A result in close agreement for thin-wire oscillators from 8 to 30 metres long has



Distances measured along the Wire from the Free End.



Distances measured along the Wire from the Free End.

Chant's Curves representing the Potential Distribution in an Inductively Coupled Antenna.

FIG. 38.—Curves A. Inductive Coupling of Antenna and Condenser Circuit with Balancing Capacity at one end of the Secondary Circuit of the Oscillation Transformer and Antenna attached to the other end.

FIG. 39.—Curves B. Inductive Coupling of Antenna and Condenser Circuit with one end of the Secondary Circuit of Oscillation Transformer earthed and Antenna attached to the other.

been obtained by Prof. G. W. Pierce (see *Proc. Amer. Acad. of Arts*, vol. 45, March 1910), who found a mean value of $2{\cdot}094$. On the other hand, A. D. Cole, for an oscillator 3 mms. in diameter and 80 mms. long, found $\frac{\lambda}{l}=2{\cdot}52$, which is in accordance with Macdonald's theory. (See also last section of Chap. IV.)

For a thin single-wire Marconi antenna of height h the ratio $\frac{\lambda}{h}$ may approach 4, but is generally a little larger, and for a branched antenna or thick rod experiment generally finds a ratio near to or a little below 5.

2. *The Oscillations in an Inductively Coupled Antenna.*—Mr. Chant also studied the distribution of potential in an antenna in which the oscillations were excited inductively or by contact with a closed oscillating circuit. He varied the length of the antenna (from 225 to 1000 cms.) attached to one terminal of the secondary circuit of the oscillation transformer, the other terminal of this transformer being attached (1) to earth, (2) to an antenna of equal length, (3) to a large capacity, and (4) insulated (see Figs. 38 and 39).

In each case he explored the potential distribution and found always a minimum of potential at some distance between 150 and 200 cms. from the free end of the antenna. In the case of antennæ longer than 500 cms., there seemed to be a secondary maximum of potential between 450 and 600 cms. from the free end, and a secondary minimum at about 750 to 800 cms. from the free end (see Fig. 39).

In the majority of instances there was no agreement between the natural time period of oscillation of the condenser circuit and that of the inductively or directly connected antenna. Hence the oscillation created on the latter was a forced

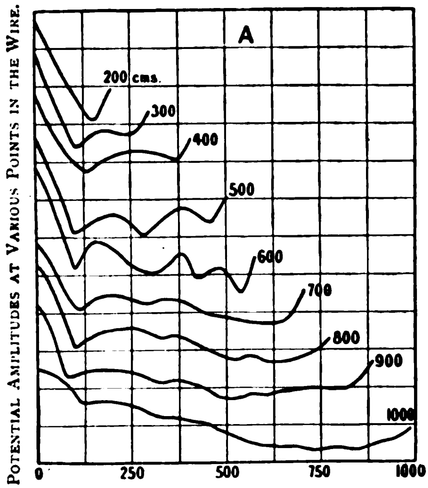


Fig. 40.—Chant's Curves representing the Potential Distribution in an Antenna Wire coupled directly to a Condenser Circuit, with a Balancing Capacity instead of Earth Connection to the latter.

oscillation, and the distance from the free end of the wire to the first minimum of potential may be taken as equal to rather less than one-quarter of the wave-length due to the closed circuit oscillator. Chant's conclusions are, that by the inductive method of connection between non-syntonized circuits, we excite in the antenna chiefly a forced oscillation due to the condenser circuit, and that the change in the length of the antenna makes but little difference in the position of the first node, provided the antenna is long enough to contain at least one-quarter of a stationary oscillation due to the condenser circuit. On the other hand, in the case of the simple Marconi aerial and the antenna direct-coupled to a condenser circuit, the principal oscillation is the fundamental vibration of the antenna itself; the most effective arrangement in the latter case being when the fundamental natural oscillation time period of the antenna agrees with that of the condenser circuit (see Fig. 40).

In connection with this part of the subject the reader may be referred to an interesting paper by Mr. J. E. Ives, on the "Wave Lengths and Overtones of a Linear Electrical Oscillator." See *Physical Review*, vol. 30, February 1910.

The case to which the chief technical interest attaches, however, is that in which the natural free time period of the condenser circuit agrees either with the fundamental natural free period of the coupled antenna or with a harmonic of the latter, the coupling not being very close, that is, the "coefficient of coupling," k, being not greater than 0·1.

We have seen in Chap. III. § 11 that when the two circuits of an oscillation transformer are syntonized, or have the same oscillation constant ($\sqrt{CL}$) when separate, then when associated together inductively the potential differences created at the terminals of the condensers on the primary and secondary circuits are in the ratio of these capacities, and not in the ratio of the number of turns on the two circuits of the oscillation transformer.

Also we have seen that oscillations of two frequencies are then set up in the circuits. We cannot only resolve the complex vibration which then occurs in each circuit mathematically into two constituent components, but we can by means of the oscillograph tube, and the cymometer, actually see the electrical beats, and detect or measure the wave-length of the two components.

If we connect a Gehrcke oscillograph vacuum tube (see Chap. I. § 6, Fig. 35) to the terminals of the condenser or across parts of the inductance of the secondary circuit, then, as explained already, the length of the glow light on the electrodes of the vacuum tube will vary proportionately to their potential difference. If, then, the tube is examined in a rapidly revolving mirror, making from 50 to 100 turns per second, the recurrent images of the glow light will be separated out, and we shall see a series of bright lines of greater or less lengths which denote the varying currents or potential differences in secondary circuit.

If we examine in this way the oscillations in a primary circuit with secondary circuit removed, we see an image consisting of a series of decreasing lines which is the single damped oscillation in the circuit (see Fig. 36, Chap. I.). If, however, the secondary circuit is closed, then the image is as represented in Fig. 21, Chap. III., which are from photographs taken by Hans Boas of Berlin.

This last photograph shows the electrical beats in the secondary oscillation, the gradually increasing and diminishing oscillations being the result of the superposition of two sets of oscillations of different frequencies.

The same electrical beats are well shown in the photographs of the oscillations in coupled circuits, taken by Professor Taylor Jones (see Fig. 19, Chap. III.), with a mirror oscillograph of his own invention (see *Phil. Mag.*, August 1907, p. 238), which represents the oscillation train in the form of a wavy line imprinted on a photographic plate, the ordinate of the line at any point representing the periodic potential difference between two points on the oscillatory circuit.

The effect of varying the closeness of the coupling (k) in an oscillation transformer is best shown by drawing a series of resonance curves with various couplings. When the two circuits are syntonized, but the coupling is very weak, we have seen that in the secondary circuit there is an oscillation of one single frequency of the common period. If then we make slight variations in the natural frequency of the secondary circuit by varying its inductance and observe the current (R.M.S. value) J_2 in that circuit, and compare it with the maximum or resonance value J_{2max}, we can plot a resonance curve of which the ordinates are the ratio $\left(\frac{J_2}{J_{2max}}\right)^2$, and the abscissæ are the ratio $\frac{n_2}{n_1}$ of the frequencies in the secondary and primary circuits. We shall then have a curve with a single maximum (see Fig. 60, Chap. VI.). If the closeness of coupling is increased, we shall find that we obtain a resonance curve having two maximum ordinates differing in absolute value, and the curve resulting from the plotting of $\left(\frac{J_2}{J_{2max}}\right)$ in terms of $\frac{n_2}{n_1}$ has a double hump (see Fig. 41).

Again, if we increase the value of k still more, we get a curve with two widely separated humps of different height, showing that there are two frequencies corresponding to two maximum values of the secondary current. Thus the more we increase k the wider apart and the higher do these two maxima of antenna

current lie. We have already shown (see Chap. III. § 14) that the ordinate y of the resonance curve, where $y=\left(\frac{J_2}{J_{2max}}\right)^2$, is connected with the sum of the decrements δ_1 and δ_2 of the two circuits separately by the equation—

$$\delta_1+\delta_2=2\pi\left(1-\frac{n_2}{n_1}\right)\sqrt{\frac{y}{1-y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and it follows that—

$$y=\left(\frac{J_2}{J_{2max}}\right)^2=\frac{\left(\frac{\delta_1+\delta_2}{2\pi}\right)^2}{\left(1-\frac{n_2}{n_1}\right)^2+\left(\frac{\delta_1+\delta_2}{2\pi}\right)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Since, then, there are two values of the frequency of the secondary circuit for which resonance occurs, which are more widely separated, the greater k (see Chap. III. § 15), it follows that there must be two maximum values of the ordinate of the resonance curve.

This matter has already been treated from the theoretical point of view in § 14 of Chap. III., and hence need not be discussed again here.

We have also already considered the theory of the oscillation transformer (see Chap. III. § 11) with capacity in each circuit, as given by Oberbeck,[36] and

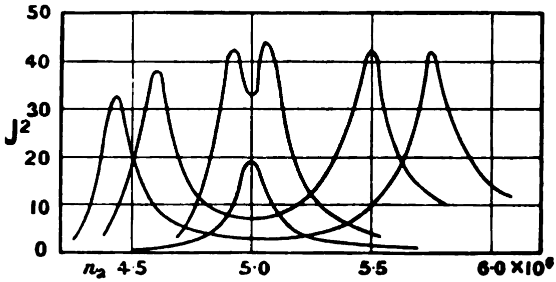


FIG. 41.—Resonance Curves for Various Degrees of Closeness of Coupling of Primary and Secondary Oscillation Circuits. The curve with single maximum corresponds to very loose coupling, and the curves with double maximum to various degrees of close coupling of the circuits.

the problem has been treated by G. Seibt[37] with special reference to electric wave telegraphy.

Let us consider first the case of an antenna earthed through the secondary circuit of an oscillation transformer having a certain coefficient of coupling k (see Fig. 42). Let the primary circuit contain a condenser of capacity C_1, and let the equivalent inductance of the primary circuit when in presence of the secondary circuit of the oscillation transformer, but with the antenna and earth removed, be denoted by L_1. Then let C_2 be the capacity of the antenna with respect to the earth and L_2 the equivalent inductance of the antenna and secondary circuit of the oscillation transformer. Let M be the mutual inductance of the two circuits of the transformer and V_1 and V_2 the maximum values of the potential differences of the primary and secondary circuits, and I_1 and I_2 the currents in them considered as vectors. If then we assume that the oscillations are undamped, that is, that the resistances of the two circuits are negligible, we may write the vector equations connecting potential and current for the two circuits as follows :—

[36] See A. Oberbeck, *Wied. Ann. der Physik*, 1895, vol. 55, p. 627.
[37] See G. Seibt, *Physikalische Zeitschrift*, August 1, 1904; or *L'Éclairage Électrique*, October 1904, vol. 41, p. 2.

$$V_1 + jpL_1I_1 + jpMI_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$
$$V_2 + jpL_2I_2 + jpMI_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

also
$$I_1 = jpC_1V_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$
$$I_2 = jpC_2V_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Then, eliminating from the above equations I_1, I_2, V_1 and V_2, we have—

$$p^4 - p^2\frac{C_1L_1 + C_2L_2}{C_1C_2(L_1L_2 - M^2)} + \frac{1}{C_1C_2(L_1L_2 - M^2)} = 0 \quad . \quad . \quad . \quad . \quad (36)$$

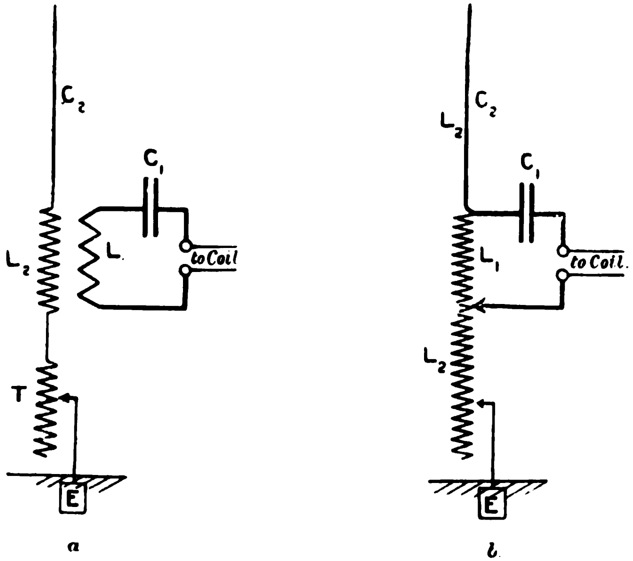


FIG. 42.

(*a*) Antenna and Condenser Circuit Inductively Coupled. (*b*) Antenna and Condenser Circuit Directly Coupled.

C_1, condenser ; L_1, inductance ; E, earth plate ; C_2, antenna.

Hence, solving this quadratic (36)—

$$p^2 = \frac{1}{2C_1C_2(L_1L_2 - M^2)}\left\{(C_1L_1 + C_2L_2) \pm \sqrt{(C_1L_1 - C_2L_2)^2 + 4C_1C_2M^2}\right\} \quad . \quad (37)$$

Suppose, then, that the oscillation constants of the antenna and condenser circuits are made equal by adjusting the capacity and inductance, so that $C_1L_1 = C_2L_2 = CL$. Then the above solution for p^2 reduces to—

$$p^2 = \frac{1}{CL} \cdot \frac{1 \pm k}{1 - k^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where $k = \dfrac{M}{\sqrt{L_1L_2}}$. Since $p = 2\pi n$, we may write the solution in the above critical case as follows :—

Let n_0 denote the natural frequency of each circuit alone, so that $n_0 = \dfrac{1}{2\pi\sqrt{CL}}$; also let p_1 and p_2 be the two roots of the equation (36), and let $p_1 = 2\pi n_1$, and $p_2 = 2\pi n_2$.

Then we have from (38)—

$$\left.\begin{aligned} n_1 &= n_0 \frac{1}{\sqrt{1-k}} \\ n_2 &= n_0 \frac{1}{\sqrt{1+k}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

It follows that—

$$\left.\begin{aligned} n_0^2 &= \frac{n_1^2 + n_2^2}{2} \\ \text{and} \qquad k &= \frac{n_1^2 - n_2^2}{n_1^2 + n_2^2} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

These equations show us that if k has any value less than unity, and if the open and closed circuits have the same oscillation constant when separate, then when coupled together the oscillation set up in the open circuit is a complex oscillation which is composed of two oscillations of different frequencies, n_1 and n_2. The more nearly k approaches to unity, the greater will be the difference between n_1 and n_2, and the difference of either of them from n_0.

Since the length λ of the wave radiated from the antenna is connected with the frequency n of the oscillations in the antenna by the equation $u = n\lambda$, where u is the velocity of radiation, viz. 3×10^{10} cms. per second, or 10^9 feet per second, it follows that there are two waves of wave-length λ_1 and λ_2 radiated from the tuned inductively coupled area, and these wave-lengths are connected with the natural fundamental wave-length λ_0 of the antenna and associated secondary transformer circuit, and the coefficient of coupling k by the equations—

$$\left.\begin{aligned} \lambda_1 &= \lambda_0 \sqrt{1-k} \\ \lambda_2 &= \lambda_0 \sqrt{1+k} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

$$2\lambda_0^2 = \lambda_1^2 + \lambda_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

$$\text{and} \qquad k = \frac{\lambda_2^2 - \lambda_1^2}{\lambda_2^2 + \lambda_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Again, since the wave-lengths of the radiated waves are proportional to the equivalent oscillation constants O, we may write the equations (42) and (43) in the form—

$$2O_0^2 = O_1^2 + O_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$$k = \frac{O_1^2 - O_2^2}{O_1^2 + O_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Also we can calculate the relative energy of the two secondary oscillations of frequency, n_1 and n_2. For the secondary current has a maximum value I_2 and therefore a maximum energy $\frac{1}{2}L_2I_2^2 = W$. But $I_2^2 = C_2^2V_2^2p^2$ by (35). Therefore $W = C_2L_2\left(\frac{C_2V_2^2}{2}\right)p^2$.

Since C_2L_2 is the square of the oscillation constant (O^2) of the antenna, we see that—

$$W = O^2p^2\left(\frac{C_2V_2^2}{2}\right)$$

Again, by (38) there are two values of the product O^2p^2, viz. $\frac{1}{1-k}$ and $\frac{1}{1+k}$.

If, then, W_1 and W_2 are taken to denote the maximum energies of the two resultant oscillations of frequency, n_1 and n_2, we have by (38) and (39)—

$$\frac{W_1}{W_2} = \frac{1+k}{1-k} = \frac{n_1^2}{n_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

or the oscillation which has the greatest frequency has the greatest energy.

The above deductions from theory can be experimentally confirmed by means of the author's cymometer. The following are the details of an experiment made with an inductively coupled antenna at the Pender Laboratory of University College, London, which will illustrate the facts.

The antenna consisted of four aluminium wires each 50 feet in length, arranged fan-shape, the wires being 5 feet apart at the top, and at the bottom joined to a copper bar from which proceeded a thick stranded copper cable bent rectangularly, and in all 23 feet long. The bottom of this antenna was attached to one terminal of the secondary circuit of an oscillation transformer, consisting of a length of 45 feet of 7/22 copper wire, wound in nine turns on a square wooden frame. The other end of this secondary circuit was earthed. The total length of antenna from earth to summit was 118 feet. The antenna was arranged as in Fig. 34. The total capacity was 0·000538 mfd. A cymometer was then placed with its copper bar parallel to a portion of the antenna, and a pair of spark balls inserted between the oscillation transformer and the earth, so as to form a simple Marconi aerial. Using the cymometer as described, the oscillation constant O_0 of this antenna was found to be 6·9. Hence the equivalent inductance L_2 of the antenna and associated transformer circuit was 88,500 cms., for $C = 0·000538$, and $(6·9)^2 = \frac{538}{10^6} \times 88,500$.

The primary circuit of the oscillation transformer consisted of one turn of about 6 feet in length of a massive conductor made of seven lengths of 19/22 copper cable, arranged in parallel, wound over the secondary circuit.

This primary circuit was connected in series with a spark-ball discharger and a glass-plate condenser, having a total capacity of 0·0357 mfd. This closed circuit had its capacity C_1 adjusted, so that in connection with the inductance L_1 of the above thick circuit and connectors the circuit had an oscillation constant of 6·9, equal to that of the antenna circuit. Hence we have $L_1 = 1330$ cms. When the oscillations were created by connecting the spark balls to an induction coil as usual, and the antenna oscillations tested by the cymometer, it was found that there were two oscillation periods in the antenna, showing that it was radiating waves of two wave-lengths. The cymometer gave readings for the two oscillation constants, viz. $O_1 = 8·5$ and $O_2 = 5$, corresponding to radiated æther waves of 1000 feet and 1700 feet in wave-length.

$$\text{Hence} \quad O_0 = 6·9, \quad O_1 = 8·5, \quad \text{and } O_2 = 5$$
$$\text{also} \quad O_0^2 = 47·6, \quad O_1^2 = 72·2, \quad \text{and } O_2^2 = 25$$

The condition to be fulfilled is—

$$2O_0^2 = O_1^2 + O_2^2 \quad \text{or} \quad \frac{(O_1^2 + O_2^2)}{O_0^2} = 2$$

and we see that—

$$\frac{72·2 + 25·0}{47·6} = 2·04$$

instead of being exactly equal to 2·0, as it should be. The difference from theory is only, however, 2 per cent. The correctness of the above formula connecting the values of O, O_1, and O_2 has been also confirmed by the experiments of C. Fischer (see *Annalen der Physik*, vol. 22, p. 265, 1907, or *Science Abstracts*, vol. x. A, 1907, *abs.* 702).

Furthermore, the coefficient of coupling k of the oscillation transformer should be given by—

$$k = \frac{O_1^2 - O_2^2}{O_1^2 + O_2^2} = \frac{72·2 - 25·0}{72·2 + 25·0} = 0·49$$

In Chap. II. § 4, and Chap. VI. § 16, we have already given direct measurements of the coefficients of coupling of similarly constructed oscillation transformers, and shown that k has a value not far from 0·5. Hence the value deducted from the above cymometer readings is likely to be right.

Again, since the lengths of the two waves of the radiated waves are $\lambda_1 = 1700$

feet, and $\lambda_2 = 1000$ feet, the natural wave-length λ_0 of the free independent antenna should be $\frac{(1700)^2 + (1000)^2}{2} = 1400$ feet, and this agrees with the measurement of the free oscillation constant 6·9, for the radiated wave-length is always nearly 200 times the oscillation constant of the antenna.

We see again, therefore, how erroneous it is to assume that for such a coupled aerial the radiated wave-length is four times the total height of the antenna *plus* the length of its associated inductance coil. For the antenna, if coupled so as to be in syntony with the condenser circuit, radiates two waves of different wave-lengths, neither of them related to the height of the aerial by the simple fourfold relation.

These wave trains possess different maximum amplitude and different damping or decrements, and to this matter we shall revert again presently.

3. *The Oscillations in a directly Coupled Antenna.*—A large number of electric wave telegraph stations are equipped with transmitters consisting of a condenser, spark gap, and inductance in series with each other, one end of the inductance coil being earthed and the other connected to an antenna. The condenser and spark gap are in series, and placed as a shunt across the inductance (see Fig. 42(*b*)).

Following an investigation of G. Seibt,[38] we shall first determine the relation between the constants of the circuit and the frequency. Let C_1 be the capacity of the primary condenser, and L_1 the inductance of the coil in series with it. Let L_2 be the inductance of the antenna, and C_2 its capacity with respect to the earth.

Let I_1 be the current through the condenser, I_2 the current into the antenna, and I_1' the current in the inductance coil. Let V be the potential difference of the terminals of the inductance, or of those of the condenser. Lastly, let $p = 2\pi n$, and $j = \sqrt{-1}$, as usual. We have, then, the following vector equations :—

$$\left.\begin{aligned} V &= jpL_1 I_1' \\ V &= -j\frac{I_1}{pC_1} \\ V &= jI_2\left(pL_2 - \frac{1}{pC_2}\right) \\ \text{also} \quad I_1 &+ I_1' + I_2 = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Eliminating the symbols for current and potential, we have a biquadratic in p, viz.—

$$p^4 - p^2\frac{C_2L_2 + C_2L_1 + C_1L_1}{C_1C_2L_1L_2} + \frac{1}{C_1C_2L_1L_2} = 0 \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Since the above expression has unequal roots, it indicates that oscillations of different frequencies are set up in the antenna.

Suppose that the length of the antenna is so adjusted that its own free oscillation constant is the same as that of the condenser circuit taken alone, then we shall have—

$$C_1L_1 = C_2(L_1 + L_2) = CL, \text{ say}$$

Under these conditions the solution of the biquadratic (48) becomes—

$$p^2 = \frac{1}{C_2L_2}\left(1 \pm \sqrt{\frac{L_1}{L_1 + L_2}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

$$\text{or} \quad p = \sqrt{\frac{1}{C_2L_2}\left(1 \pm \sqrt{1 - \frac{1}{1 + \frac{L_1}{L_2}}}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

$$\text{but} \quad \frac{1}{C_2L_2} = \frac{1}{CL}\left(1 + \frac{L_1}{L_2}\right)$$

$$\text{hence} \quad p = \sqrt{\frac{1}{CL}\left(1 + \frac{L_1}{L_2}\right)\left(1 \pm \sqrt{1 - \frac{1}{1 + \frac{L_1}{L_2}}}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

[38] See G. Seibt, *Physikalische Zeitschrift*, August 1, 1904; or *L'Éclairage Électrique*, October 1, 1904, p. 27, "A Comparison between the Direct and Inductive System of Coupling in Transmitters for Wireless Telegraphy."

Let us write in the above solution $\frac{1}{1-\rho^2}$ instead of $1+\frac{L_1}{L_2}$, and we then have as the solution in the syntonic case—

$$p=\sqrt{\frac{1}{CL}\cdot\frac{1\pm\rho}{1-\rho^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

or

$$n=\frac{1}{2\pi\sqrt{CL}}\cdot\frac{1}{\sqrt{1\pm\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Hence there are oscillations of two different frequencies excited in the antenna. Call these frequencies n_1 and n_2, and let $n_0=\frac{1}{2\pi\sqrt{CL}}$ be the frequency of the condenser circuit or antenna alone. Then we have—

$$\left.\begin{aligned} n &= n_0\frac{1}{\sqrt{1+\rho}} \\ n_2 &= n_0\frac{1}{\sqrt{1-\rho}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

as the values of these frequencies.

Also, since $u=n\lambda$, where u is the velocity of the waves radiated from the antenna, we have—

$$\left.\begin{aligned} \lambda_1 &= \lambda_0\sqrt{1+\rho} \\ \lambda_2 &= \lambda_0\sqrt{1-\rho} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

as equations giving us the wave-lengths of the waves radiated, where λ_0 is the natural free fundamental wave-length of the aerial.

On comparing the above equations (52) with the similar equations (38) for the inductively coupled antenna, we see that the quantity above called $\rho=\frac{\sqrt{C_2}}{\sqrt{C_1}}$ appears in the same place as the coefficient of inductive coupling k, and may hence be called the coefficient of direct coupling. Accordingly, if ρ is small, that is, if the antenna capacity is small compared with that of the condenser, only waves of one wave-length are emitted from the antenna, but if the capacity of the antenna is of the same order as that of the condenser in the closed circuit, then two waves of different wave-length will be emitted, as in the inductively coupled case.

The method of inductive coupling, however, gives a great range of adjustment, because we can without altering the capacity of the antenna or condenser vary k over wide limits by moving the two circuits of the oscillation transformer to or from each other.

As in the case of the inductive coupling, so in that of the direct coupling, when two different wave-lengths are emitted in virtue of syntonism between the open and closed oscillating circuits, these two waves have different maximum intensities and different damping.

It is, however, not quite so easy to produce in a direct-coupled antenna oscillations of two frequencies, as in the case of an inductively coupled antenna. The reaction of the antenna on the energizing or reservoir circuit, on which this duplexing of the frequency depends, is less well marked in the case of the direct-coupled antenna than in the case of the inductively coupled one.

As an illustration of the application of the foregoing formula, we may give the following measurements made with a transmitting plant set up in the Pender Laboratory at University College, London. The antenna was inductively connected, through an oscillation transformer having a coefficient of coupling 0·5, with a condenser circuit having a capacity of 0·025 mfd. and an inductance of 2000 cms. The antenna circuit was syntonized with the condenser circuit separately, so that each has the same oscillation constant when uncoupled. We have then

$k=0{\cdot}5$, $C_1=0{\cdot}025$ mfd., $L_1=2000$ cms. Hence the oscillation constant $\sqrt{C_1L_1}=7{\cdot}07$, and the frequency n_0 in the condenser circuit taken alone is—

$$n_0=\frac{5\times 10^8}{\sqrt{C_1L_1}}=\frac{5}{7}\times 10^8$$

Hence the corresponding wave-length λ_0 is 1400 feet, since the velocity of radiation is 10^9 feet per second nearly Accordingly, we have $\sqrt{1+k}=1{\cdot}224$ and $\sqrt{1-k}=0{\cdot}707$, and the two wave-lengths emitted by the coupled aerial have values, λ_1 and λ_2, such that—

$$\lambda_1=\lambda_0\sqrt{1-k}=1400\times 0{\cdot}7=980 \text{ feet}$$
$$\lambda_2=\lambda_0\sqrt{1+k}=1400\times 1{\cdot}124=1714 \text{ feet.}$$

To sum up, then, the effects taking place in the case of the inductive coupling of two oscillatory circuits both having capacity, inductance, and resistance, are as follows:—

If the circuits are nearly syntonized, that is, have nearly the same oscillation constant or time period of oscillation when free and separate, then when coupled inductively, and oscillations created by discharge in one circuit, we have in the other circuit a complex oscillation which may be analysed into the sum of a forced oscillation and a free oscillation. These combine to produce a resultant oscillation with periodic maxima resembling the effect of *beats* in music, and this, again, may be analysed into the sum of two oscillations of different frequencies. Nevertheless, the resultant oscillation has a single definite mean-square or effective value given by the expression—

$$J^2=\frac{E^2}{16L_2^2}\cdot\frac{a_1+a_2}{a_1a_2}\cdot\frac{1}{4\pi^2(n_1-n_2)^2+(a_1+a_2)^2}\quad . \quad . \quad . \quad . \quad (56)$$

where E is the maximum electromotive force acting in the secondary circuit of inductance L_2, and a_1 and a_2 are the damping factors, such that $a_1=n_1\delta_1$ and $a_2=n_2\delta_2$, δ_1 and δ_2 being the decrements of the two circuits when separate (see Chap. III. § 14).

If I_1 is the maximum value of the primary current in the condenser circuit, then $E=Mp_1I_1$ where M is the coefficient of mutual inductance. But $I_1=C_1V_1p_1$, and $M=k\sqrt{L_1L_2}$, also $p_1^2C_1L_1=1$; accordingly—

$$E^2=k^2L_1L_2C_1^2V_1^2p_1^4=k^2C_1V_1^2L_2p_1^2$$
$$\text{or}\quad \frac{E^2}{L_2^2}=\frac{k^2V_1^2}{L_1L_2}$$

Therefore we have—

$$J^2=\frac{k^2V_1^2}{16L_1L_2}\cdot\frac{a_1+a_2}{a_1a_2}\cdot\frac{1}{4\pi^2(n_1-n_2)^2+(a_1+a_2)^2}\quad . \quad . \quad . \quad . \quad (57)$$

The maximum value of J takes place when $n_1=n_2$, or when the circuits are in resonance. Denoting this resonance value of the mean-square current by J^2_{max}, we have—

$$J^2_{max}=\frac{k^2V_1^2}{16L_1L_2n^3}\cdot\frac{1}{\delta_1\delta_2(\delta_1+\delta_2)}\quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Since $C_1L_1n_1^2=\frac{1}{4\pi^2}$, the above equation may be written—

$$J^2_{max}=\frac{\pi^3k^2}{L_2p}\cdot\frac{C_1V_1^2}{2}\cdot\frac{1}{\delta_1\delta_2(\delta_1+\delta_2)}\quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

This shows us that the maximum or resonance value of the secondary or antenna mean-square current is proportional to the energy storage in the primary circuit, and inversely as the reactance of the secondary circuit, and determined also by a function of the decrements.

When the circuits are in resonance we have also $C_2L_2p^2=1$ and $n=\frac{5\times 10^6}{O}$, where

O is the common oscillation constant of the two circuits. If then we reckon the primary and secondary capacities C_1 and C_2 in microfarads, the primary voltage V_1 in volts, and the inductances in centimetres, and remember that π^4 is nearly 100, the expression for J^2_{max} may finally be put in the form—

$$J^2_{max} = \frac{k^2}{1000} \cdot \frac{C_2}{O} \cdot \frac{C_1V_1^2}{2} \cdot \frac{1}{\delta_1\delta_2(\delta_1+\delta_2)} \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Thus, suppose an inductively coupled antenna has a capacity $C_2 = 0{\cdot}0005$ mfd., and that the primary circuit contains a condenser of capacity $C_1 = 0{\cdot}025$ with a spark gap of 2 mms. Then $V_1 = 8000$ volts. Let the oscillation constant, when the circuits are tuned, be $O = 5$, corresponding to a frequency of $n = 10^6$. Further, let $k = 0{\cdot}5$, $\delta_1 = 0{\cdot}04$, $\delta_2 = 0{\cdot}1$. The mean-square value of the antenna current will then be—

$$J^2_{max} = \frac{1}{4 \cdot 10^3} \cdot \frac{1}{10^4} \cdot \frac{8 \cdot 10^5}{56} \cdot 10^5 = 36 \text{ nearly}$$

or the antenna current will be 6 amperes nearly.

To secure this high value, exact resonance is necessary. A very little want of tuning will reduce this antenna current considerably. If I_2 is the maximum value of the first oscillation of current in the antenna, and if there are N groups of oscillations per second, we have already shown (Chap. III. § 1) that—

$$J_2^2 = \frac{NI_2^2}{4n\delta}$$

Suppose in the above case that $N = 50$, or 50 condenser discharges are made per second, then $4n\delta_2 = 4 \times 10^5$, and I_2 has a value of nearly 540 amperes. This calculation shows us the enormous currents which may exist at certain instants in a perfectly tuned antenna, inductively coupled to a syntonic condenser circuit, and also by inference the extremely high potentials which may exist at the open or upper end. For an infinitesimal fraction of a second the aerial is carrying a current which would more than suffice to melt if it continued.

The value of the mean-square current in the antenna can always be ascertained by inserting in it a hot-wire ammeter, or else a bundle of a number of No. 36 platinoid wires, and ascertaining of how many strands this bundle must be composed, so that the wires may be melted or made red-hot.

The value of the maximum potential $V_{2(max)}$ at the upper end of the antenna can be obtained from that of the maximum value of the current in the following manner. Since the energy stored in the antenna is alternately electrostatic and electrokinetic, if L_2 is the antenna inductance and C_2 the capacity, we must have—

$$L_2I_2^2{}_{(max)} = C_2V_2^2{}_{(max)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Hence $$V_{2(max)} = I_{2(max)}\sqrt{\frac{L_2}{C_2}} = I_{2(max)} \cdot \frac{O}{C_2}$$

Thus, to take the above instance, we have found that $I_{2(max)} = 780$ amperes for the antenna with oscillation constant of 5 and capacity $C_2 = 0{\cdot}0005$ mfd.

Therefore $$V_{2(max)} = 540\frac{5 \times 2000}{\sqrt{1000}} = 175{,}000 \text{ volts (nearly).}$$

The $\sqrt{1000}$ is a factor to adjust the units, so that whilst potential and current are reckoned in volts and amperes, capacity is reckoned in microfarads, and inductance in centimetres.

Hence the voltage at the upper end of this antenna, when in resonance, would be equivalent to about a 20-cm. spark taken between spherical metal balls of very large size. Wireless telegraphists are all aware of the extremely long sparks which can sometimes be drawn from the upper end of the antennæ. This shows the necessity for high insulation at the supporting insulator.

16. Classification of Methods for the Practical Production of the Oscillations in the Antenna.—We have already shown in Chap. I. that the oscillations

which can be created in the antenna are either trains of intermittent damped oscillations or else continuous undamped oscillations. The former are generated by the free oscillations of a condenser which is charged and discharged through a low resistance circuit. The latter are created either by an electric arc of a particular kind or mechanically by a high frequency alternator.

The method employing the free oscillations of a condenser necessitates the use of a discharger of some kind which automatically permits some form of condenser to be charged and then discharged at regular intervals. These intervals may be as large as one-fiftieth of a second or more; or they may be as small as one-thousandth of a second or less. Low frequency discharges are, however, very little used at present, for with modern telephonic methods of reception the pitch of the sound heard in the telephone is that of the spark or discharge frequency. It is found that a rather shrill note having a frequency of about 500 per second is best heard over and above sounds of lower note due to atmospheric discharges.

Again, the character of this telephonic sound is greatly improved as far as hearing is concerned by a perfect regularity in the discharge period. Hence in most cases in which intermittent condenser discharges are employed to produce the oscillations we endeavour to give to these a perfectly regular frequency of about 300 to 500.

Methods for producing such regular high frequency discharges are generally called musical spark methods, because of the character of the telephonic sound produced by them. The types of transmitter employing such condenser discharges are called spark transmitters. Certain methods have been evolved as explained in Chap. I. for producing condenser discharges of such high frequency that there is practically no interval of time between the end of one train of oscillations and the beginning of the next. Such closely sequent trains differ only from continuous trains in that there is a periodic rise and fall in the amplitude of the oscillations.

Furthermore, in the case of intermittent discharges produced by sparks the oscillations set up in the primary or spark circuit may be feebly damped, and may be employed to produce simultaneous secondary oscillations in the antenna circuit by means of an oscillation transformer as already described. If, however, the primary spark endures for any finite time, there will be an inductive reaction between the secondary oscillations and the primary circuit which remains in effect closed as long as the spark endures, which results in the production of a complex oscillation in both circuits resolvable into oscillations of two periods as already explained in Chap. III. § 6.

On the other hand, if the primary circuit spark is very quickly extinguished, then the primary condenser discharge is nearly dead-beat. If this discharge takes place through one circuit of an oscillation transformer, the other circuit of which is in series with the antenna, then the antenna circuit is subjected to a brief electromotive impulse which sets it in free sustained oscillations. Such a type of primary spark is called a *quenched spark*, and this mode of exciting the oscillations in the secondary circuit is called the method of shock or impact as already explained in Chap. III. § 14. We need not, therefore, allude to it here at any greater length.

The particular forms of discharger used to produce this effect are described in the next section.

We can, therefore, classify the methods so far used for the production of oscillations in radiotelegraphic transmitters as follows:—

(A) Methods for the production of intermittent trains of damped oscillations;

(B) Methods for the production of continuous or undamped oscillations.

The damped trains may be produced either by (*a*) regular or musical spark discharges, (*b*) irregular condenser discharges.

The discharges or sparks may be (α) prolonged or (β) quenched sparks. Each of these types requires special appliances for its production.

17. Condensers for Radiotelegraphy.—We have already mentioned in Chap. I. the principal forms of condenser used in wireless telegraphy. We may add here a little more special information on the subject.

Condensers are used both in the transmitting and receiving circuits, but in the

former the dielectric has to stand very considerable voltages, whereas in the latter the electric pressure is very small. Also the capacities required in the transmitting part are much larger than those in the receiver. Hence, in the transmitting condensers we require dielectric strength and large capacity. In both cases we desire as little energy dissipation as possible, because this damps out free oscillations and causes rise in temperature.

In the receiving portion it is best to employ, if possible, air condensers. The choice, generally speaking, lies between air, oil, and ebonite as dielectrics. The condensers used are required to have adjustable capacity, and are generally made on the multiple sector plate type with capacity varied by turning a shaft to which one set of plates is fixed as shown in Fig. 63 of Chap. I.

[By permission of the Proprietors of "The Electrician."

FIG. 43.—Seibt Air Condenser of Variable Capacity.

Such a variable condenser is generally enclosed in a glass vessel so that it can, if need be, be filled with oil. The use of a mineral oil as dielectric would about double the capacity as compared with air, but the advantage is a doubtful one, as the oil has certainly greater dielectric energy loss than the air, and hence the result is to destroy the sharpness of the resonance curve, and therefore of the tuning. The same can be said of similar condensers with ebonite as dielectric. A very neat form of variable air condenser has been introduced by the Sterling Telephone Company made on a plan due to Seibt. The condenser is turned out of a solid block of aluminium, the plates or blades of the movable parts being slightly thinner than the recessed grooves in the fixed part. Hence a very thin air space is left, perhaps about 0·5 or 0·25 mm. The construction and external appearance is shown in Fig. 43. By cutting off the movable plates, so that the top

ones enter slightly before the lower plates, it is possible to make the scale absolutely equidivisional in terms of capacity. The condenser is so compact that one having a capacity of 2000 cms., or about $\frac{1}{450}$th of a microfarad, weighs only 600 grammes, and occupies a space about 10 cms. high and 10 cms. in diameter.

Such condensers are, therefore, very well adapted for receiving circuits, but not for circuits with any high voltage. On the other hand, for the transmitting condensers we are limited to air, glass, and oil as our dielectrics. Various forms of glass condensers have been described in Chap. I. § 11.

Mr. Marconi has used at some of his power stations very large air condensers. Although some advantages accrue, such a condenser is necessarily very bulky, and this requires long metal connections. Hence the saving in dielectric loss is balanced more or less by increase in resistance loss. It is most usual to employ metal sheets, zinc or tin plate in oil, either with or without glass plates put in between the metal electrodes. The whole condenser and oil is contained in a galvanized iron box with lid. The connections pass through insulators in the lid.

If glass is employed, condensers may be built up of sheets of good crown glass with metal plates interposed. The best method of construction is to make the metal plates of thin sheet zinc, which are cut out with a tool in the form shown in Fig. 44. These plates are built up with alternate sheets of glass, each cut an inch larger every way than the metal plates, the zinc plates being arranged with the lugs alternating, as shown in Fig. 60, Chap. I. The sheet zinc should not be too thin, so that when the whole mass of plates is immersed in oil in a stoneware or metal vessel, the oil penetrates in between the glass plates and excludes all air. The zinc plates on one side are connected together to a terminal and also those on the other. A sheet of crown glass 3 mms. or $\frac{1}{8}$ inch thick will bear safely an alternating voltage of 20,000 volts, equivalent to a 6 or 7 mm. spark. When higher voltages are employed such condenser boxes must be arranged in series. For any given dielectric there is a certain energy storage per cubic centimetre which cannot be exceeded, and in the case of glass this is equal to about 0·01 joule per cubic centimetre, or about 200 foot-pounds per cubic foot. Hence from this figure can be calculated the bulk of condenser glass required for a given energy storage or output of the condenser. The oil in which the condenser plates are immersed should be the highest insulation transformer oil and a thin paraffin oil or resinous oil is preferable to a thick vegetable oil such as linseed oil. It is found, however, that glass plates employed as the dielectric in a condenser age with use and after a time punctures of the glass become more frequent. In Germany it is usual to employ condensers of the Moscicki type, consisting of large glass tubes closed at the bottom like large test-tubes a couple of metres long and a decimetre in diameter, the glass being blown so as to be thicker at the top than at the bottom. These tubes are then coated within and without with copper-plated silver to such a depth that at the edge of the metal the glass is, say, half a centimetre in thickness, but below that only two or three millimetres. As already explained, this prevents puncture at the edges. Owing, however, to the ageing qualities of glass it has been found preferable in large stations to employ air condensers consisting of metal plates in air, either air at ordinary pressures with the plates separated to a considerable distance, or else placed in a vessel with compressed air, which, therefore, has a much higher dielectric strength.

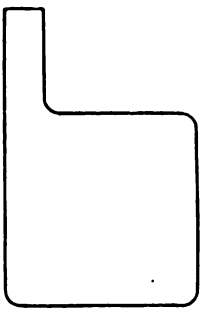


FIG. 44.

It remains to notice the manner in which these condensers should be connected to one another, and to the inductance and to the spark gap, in order to secure the best results. It is convenient to construct the primary condenser of a number of separate condensers, such as Leyden jars or glass-plate or micanite condensers, the elements being arranged either in series or in parallel, to give the capacity and dielectric strength required. If a number of condensers are to be arranged in parallel, all in series with the primary coil of the oscillation transformer and with

a spark gap, then it is desirable that the length of the oscillatory path through its separate condenser should be the same. For this purpose condensers should be arranged as shown in Fig. 45*a*, and not as shown in Fig. 45*b*. In the first case the length of the oscillatory circuit for each condenser is the same; in the second case it is different.

Where very high potentials are used, it is necessary to arrange condensers in series, or they may be partly arranged in series and partly in parallel, but the same conditions as to length of oscillatory path should be fulfilled. The connections between the condensers are best made with finely stranded cables made up of cotton-covered No. 36 S.W.G. copper wire twisted together. It is necessary to have sufficient surface for the discharge path and to avoid as much as possible introducing resistance into the primary circuit, so as to keep the decrement of the primary circuit as low as possible. In some cases it is necessary to charge condensers in series and discharge them in parallel, and this may be done in one of several ways shown in the appended diagrams (see Fig. 46).[39] In the appended

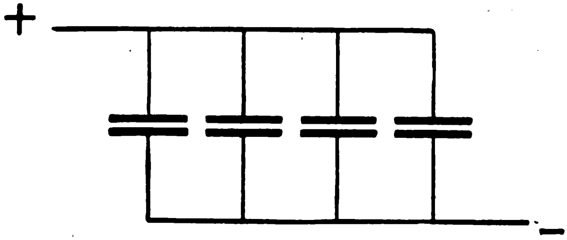


FIG. 45*a*.

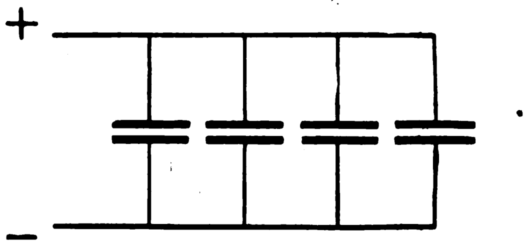


FIG. 45*b*.

Two Modes of arranging Condensers in Parallel in a Discharge Circuit.

diagrams, C_1, C_2 are the condensers, L_1 is the inductance coil, and S_1, S_2 are two spark gaps; I is an induction coil. It will be seen that the condensers are charged in series and discharged in parallel.

The best arrangement of condensers in the oscillation circuit of a wireless telegraph transmitter is determined by the following considerations. Let us assume that the antenna is inductively connected to the condenser or oscillation circuit. Then, in order that we may have syntony, we must make the oscillation constants of the two circuits the same. Let C_2 be the capacity of the antenna circuit and L_2 its inductance, so that $\sqrt{C_2L_2}$ is its oscillation constant. Let C_1 be the capacity of the condenser in the primary or nearly closed circuit and L_1 its inductance. Also let R_1 be the high frequency resistance of this circuit, including that of the spark gap. Then the oscillation constant is $\sqrt{C_1L_1}$ and the damping factor is $\frac{R_1}{2L}$. It has generally been the custom in spark telegraphy to make C_1 from ten to twenty times C_2, and hence L_2 must be from ten to twenty times L_1 in magnitude.

[39] See German patent granted to Fritz Lesemann, Class 21A, No. L. 18,521.

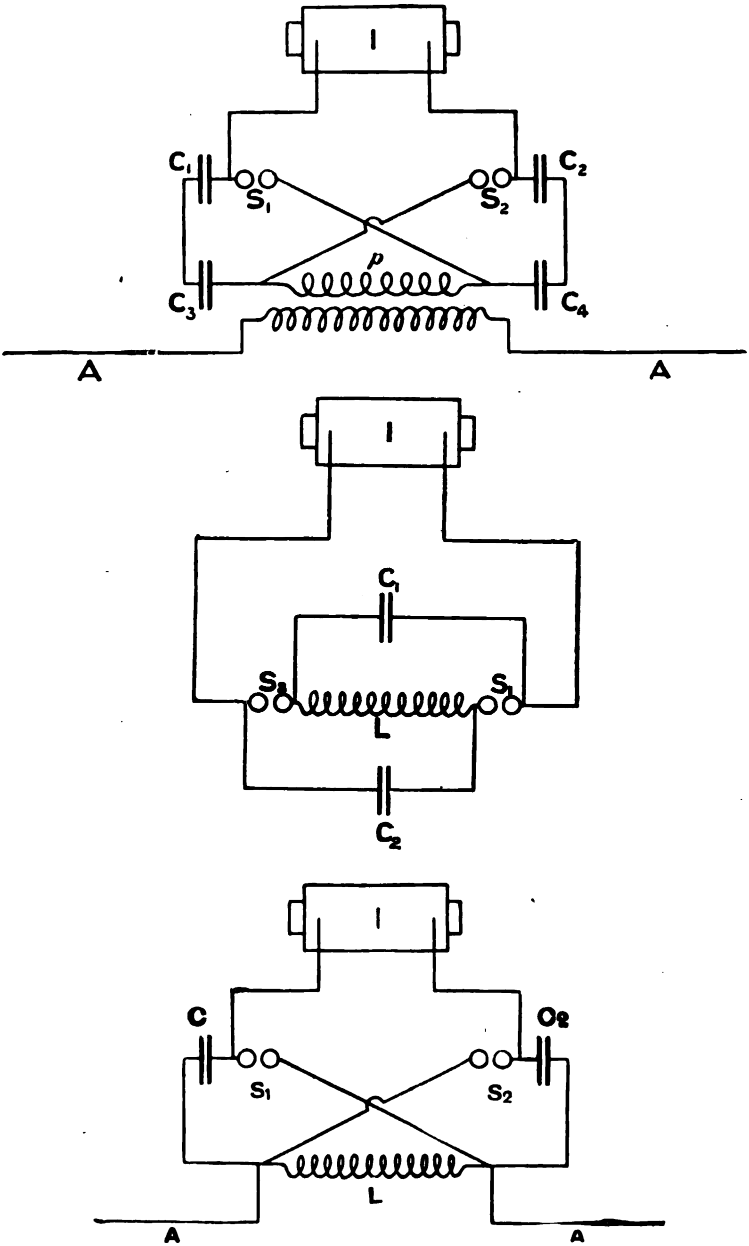


FIG. 46.—Lesemann's Method of Charging Condensers in Series and Discharging them in Parallel from an Induction Coil. I, induction coil; C_1, C_2, condensers; S_1, S_2, spark gaps; L, inductance coil; p, s, oscillation transformer; A, antennæ.

Accordingly, C_1 is a large capacity and L_1 is a small inductance. The resistance R_1 of the condenser circuit is largely made up of spark resistance. Hence for a given spark length the smaller we make L_1 the larger will become the damping and the greater the decrement of the oscillations.

It is advantageous, therefore, to keep the capacity C_1 as small as possible, so as to use as much inductance as possible in the condenser circuit. We can then only obtain the required energy storage by charging the condenser to a high voltage. This must, however, be achieved without increasing the length of spark gap, and can be accomplished by constructing the primary condenser of separate condensers which are charged in series, and each discharged across its own short

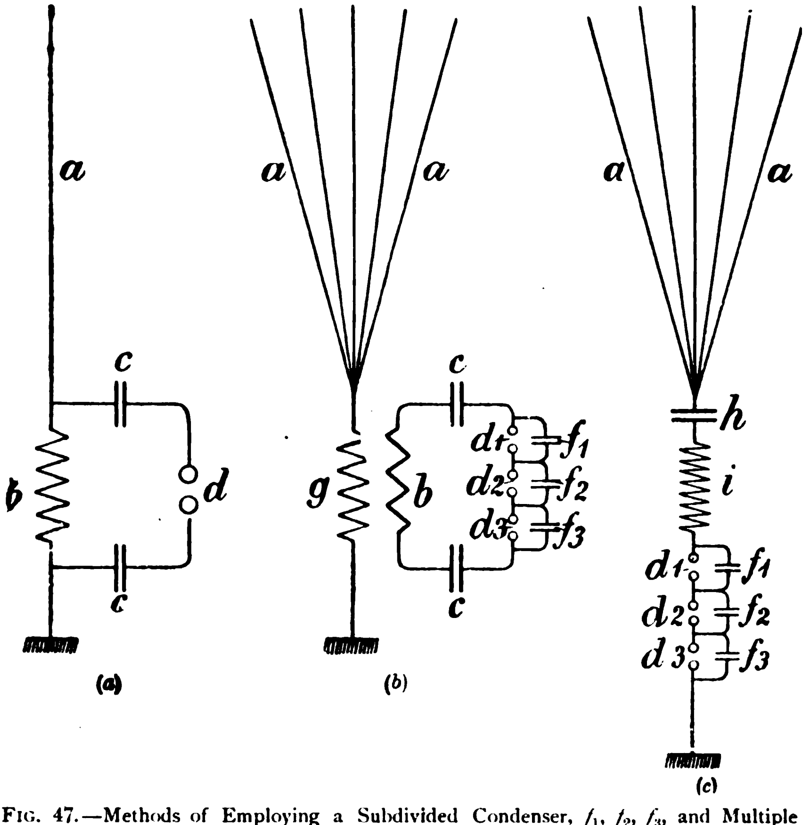


FIG. 47.—Methods of Employing a Subdivided Condenser, f_1, f_2, f_3, and Multiple Spark Gap, d_1, d_2, d_3, in Connection either with a Directly Coupled Antenna as in (*a*), or an Inductively Coupled Antenna, as in (*b*), or a Simple Antenna, as in (*c*), to obtain a Smaller Resistance Decrement, and therefore more Persistent Oscillations in the Condenser Circuit.

spark gap, the several spark gaps being connected in series. The arrangements for effecting this are shown in Fig. 47, and are taken from a British Patent Specification, No. 20,804 of 1904, granted to the Wireless Telegraph Company of Berlin.

These diagrams show the mode of connecting the condensers and spark gaps in the case of inductively and directly coupled antennæ. The advantage to be gained by the use of relatively small capacity in the primary oscillation circuit is

shown by the following example. Let us suppose that the primary capacity $C_1 = 0{\cdot}1$ mfd., and that the primary inductance $L_1 = 10^5$, and that the spark length used is 1 cm. corresponding to 30,000 volts. Then the frequency n is 5×10^4, and if the spark be assumed to have a resistance of 1 ohm the resistance decrement δ is 0·05.

If, then, we divide the condenser, say, into five parts, and arrange these in series, each with a spark gap of 1 cm. in length and a separate discharge path for each condenser, having an inductance, say, of 1250 cms., the frequency of the oscillations will rise to 10^6, and yet the whole stored energy will be the same. If the main discharge circuit has still an inductance of 10^5 cms., we shall then, by the mere fact of raising the frequency, find we have lowered the decrement to 0·0125, or to one quarter of its original value. Hence, without affecting the quantity of the stored energy, we have yet made the number of oscillations per train much greater.

The arrangement of the capacity of the primary condenser is not, therefore, a matter of indifference, and with a certain capacity at disposal we can make use of it by certain arrangements more advantageously than by others.

18. Dischargers for Spark Radiotelegraphy.—The physical process involved in the production of damped electric oscillations and corresponding electric wave generation consists in charging some form of condenser and then suddenly releasing the charge in the form of a spark, and permitting the free oscillations of the condenser to take place across the spark gap. When dealing with condensers of small capacity charged with voltages of a few thousand volts, *e.g.*, 20,000 or so, no particular difficulty occurs, all that is required is a spark gap, or series of spark gaps, consisting of a couple or more of metal balls of brass, zinc, or iron, separated by intervals of a few millimetres which are inserted in series with the condenser and its inductive discharge circuit. The charging voltage is supplied by an induction coil or small transformer, the secondary circuit of which is connected to the spark balls, or outer members of the series of balls.

As the potential difference of the balls increases the condenser becomes charged, and at a certain P.D. the limit of the dielectric strength of the air in the air gap for that particular gap length used is reached, and the air insulation breaks down. The spark gap at once becomes conductive and the circuit of the condenser is completed. It then oscillates until the energy of the charge is dissipated as heat and radiant wave energy.

The first point to notice in connection with this operation is that there may be a very large number of discharges of the condenser during one semi-period of the transformer or during an interruption of the primary circuit of the induction coil.

This will happen when the spark gap is short. As the charging voltage and P.D. of the balls increase, a point is reached at which the air gap insulation breaks down and a train of oscillations ensues. If, however, the duration of this train is very short compared with one cycle of the charging voltage, then after this train of oscillation has subsided the P.D. of the balls increases again. Another spark and another set of oscillations will then take place, and this may be repeated three or four, or even many dozen, times during one semi-period or cycle of the charging voltage. Hence it is quite erroneous to assume that there is only one spark per interruption of the primary of the coil or per semi-period of the alternating current feeding the transformer. There may be a great many sparks in this time. This is called the phenomenon of multiple sparks. Moreover, these sparks do not occur at equidistant periods of time. They are generally very irregular. This is partly because the dielectric strength of the air in the spark gap is determined by the state of ionization in which it is left after the last discharge.

Another reason is the production of an electric arc or direct discharge of the transformer or induction coil across the gap, and until this is extinguished the condenser cannot again become charged. The actual discharge which occurs across the gap may be partly a true oscillatory condenser discharge, which we shall call the spark discharge, and partly a discharge of the nature of an electric arc taking place between the balls, which is due to current coming directly out of the induction coil or transformer, and not at all to the oscillatory discharge of the condenser.

This arc discharge keeps down the potential difference of the balls, and it must be destroyed before the condenser can again become fully charged.

When employing large power transformers the production of an arc discharge has to be prevented by special means, and the object of design in large power dischargers is to prevent this arcing, and to cause the spark to be due entirely to electric energy coming out of the condenser and not at all to energy supplied directly by the transformer. In short distance radiotelegraphic apparatus such as is provided on ships and small coast stations the usual source of voltage is a 10- or 12-inch spark induction coil. In these cases the discharger often takes the simple form of a pair of brass balls connected to the secondary terminals of the coil. There are the following objections to this rudimentary form of spark gap. The opposed surfaces of the balls become rough or worn away in time, the exact inter-distance or length of spark gap is not readily adjustable, the ionized air between the balls is removed irregularly by draughts, and lastly, but not least, the noise made by the spark is annoying and enables the signals being sent to be read by ear a long way off. In all cases, therefore, the spark balls should be enclosed in a more or less sound-proof chamber. This may be of cast iron or very thick wood lined with asbestos. If the chamber contains air, then the spark will combine some of the nitrogen and oxygen into oxides of nitrogen, and ultimately with the aid of moisture these form nitric acid, which is deposited on the walls of the vessel. Hence the chamber must either contain some alkaline material, lime, potash, or soda, to absorb these vapours, or else fresh air must be continually passed through the chamber, or it must be made air-tight and filled with some gas, such as nitrogen, which is unaltered by the spark. Provision should also be made for easily altering the spark-gap length and for changing the opposed surfaces as they become eroded.

The author has found that this is most easily achieved by carrying the metal balls which form the discharge surfaces in a holder something like a ball castor used on the legs of tables. The ball is contained in a closely fitting tube, having a spiral spring behind it, and the tube has a screw-ring on the end keeping the ball in place. The ball can then be turned round into various positions so as to expose clean surfaces when required.

Two such balls may be carried on the end of screwed rods having divided heads passing through metal bosses let into ebonite insulators, and these may be mounted so as to contain the balls in a stout box.

Another convenient form of discharger devised by the author consists of a pair of thick brass discs with rounded edges. These are carried on brass spring pedestals attached with a screw and nut so that the discs can be turned round to bring fresh places into opposition. The discs and pedestals are mounted so that the discs are on the same plane and their edges a few millimetres apart. The distance between the sparking points is then varied by means of two screws, which pass through the sides of a stout wooden containing box and press on the spring pedestals so as to force the discs nearer together. In all cases where the spark gap is contained in a silencing chamber it is convenient to have a peephole glazed with dark glass through which to examine the spark. It is extremely injurious to the eyes to look for long at a condenser spark between metal terminals.

A convenient form of discharger for small transmitters of not more than 1½ kilowatt power as used by the Marconi Company is shown in Fig. 48. It consists of a box of very stout wood lined with asbestos. Ebonite insulators carry the spark balls which are supported on screwed shafts so as to vary the spark length. Below the balls will be seen a pair of needle points which are placed with points at a greater distance apart than the ball surfaces. These are called the guard points. If any sudden rise of pressure due to resonance effects takes place, the spark jumps between these points and saves the condenser plates from puncture.

In the case of dischargers for large power stations intended for conducting long distance radiotelegraphy, special difficulties have to be overcome in constructing a suitable discharger for heavy spark discharges. The chief difficulty encountered is the tendency of the transformer or transformers employed to charge the condensers to start an arc discharge at the same time, which, as already explained, reduces the potential difference of the balls, so that the condensers shunted across the discharger cannot again become charged until the arc is extinguished. Furthermore,

with large discharge currents the metallic surfaces between which the discharge takes place become worn away very rapidly. This last difficulty is to some extent mitigated by the employment of a rotating disc discharger. In this case continually new and cool surfaces are presented between which the discharge takes place. The most convenient form for these surfaces is in the shape of two large cast-iron wheels with rounded edges, which are caused to rotate rapidly by electric motors in the same direction. An air blast may also be employed to keep the surfaces between which a discharge takes place cool, so as to prevent volatilization of the metal and to some extent to limit the arc discharge.

Another invention of the author in connection with transmitting apparatus is for a discharger (see British specification, No. 25,383 of November 20, 1903, or United States patent, No. 792,014), consisting of balls which are set in revolution by electric motors or other means, and included in a chamber in which nitrogen or carbonic acid gas is compressed.

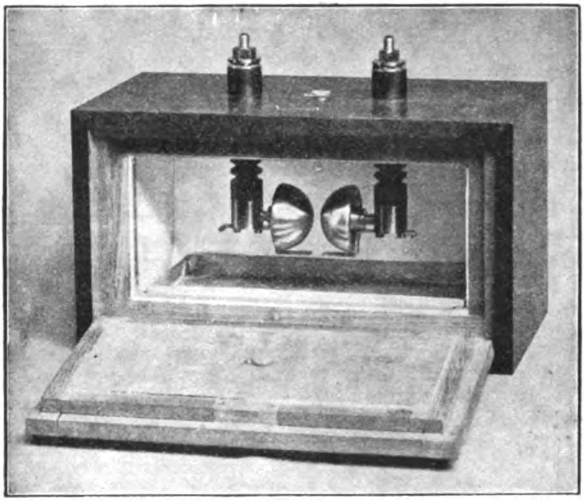


FIG. 48.—Enclosed Spark-Ball Discharger in Silencing Chamber (Marconi).

The balls or discs between which the discharge takes place are driven round at a slow pace by means of gearing, which in turn is driven by a small electric motor or clockwork. When electric motors are employed, each ball or disc is preferably driven by its own motor, and these motors are contained in a cast- or wrought-iron sound-proof chamber, which also contains the ball discharger. As the contact surfaces are continually being changed they wear more evenly, and the kind of spark, therefore, required for the performance of electric wave telegraphy is better preserved. If these balls or discs are hollow, water may be caused to circulate through them and so keep them cool. It is found that a very great advantage is secured by using a short spark taken in compressed nitrogen enclosed in a strong iron reservoir, as the electric discharge can be made perfectly noiseless, and the unpleasant effects arising from this sound are obviated. In order to make the contact between the revolving ball or disc and the external electrical generator (whether it be a transformer, induction coil, or any other means), mercury cup contacts are used. The shaft carrying the ball or disc has on it a copper cup containing mercury, and a stout copper pin connected with the external circuit dips into this mercury. The disc can, therefore, revolve, and yet a good connection is kept up with the external circuit.

Another effective method of quenching or preventing the formation of an electric arc is by the use of a powerful air blast. This has the property of extinguishing the true arc discharge, but does not interfere with the condenser spark discharge. A convenient method of applying the blast is as follows:—The discharge takes place between a metal ball carried on the end of a rod and the edge of a metal tube, the ball being placed near the open end of the tube so as more or less to block it up. A powerful air blast is supplied through the tube, and the air escapes between the ball and the edge of the tube and continually extinguishes the electric arc which tends to be formed.

Even in the case of small dischargers produced by an induction coil, it is an advantage to direct a jet of air supplied through a glass nozzle to act on the space between the discharge balls. This serves to destroy any true electric arc which may form, but does not hinder the condenser discharge. The particular conditions under which this air blast is of use are set forth in a paper by the author and Mr. Richardson, entitled "The Effect of an Air-Blast upon the Spark Discharge in Condenser Circuits" (see *Phil. Mag.*, May 1909), read before the Physical Society of London, March 26, 1909, in which it is shown that for short sparks, 1-3 mms. in

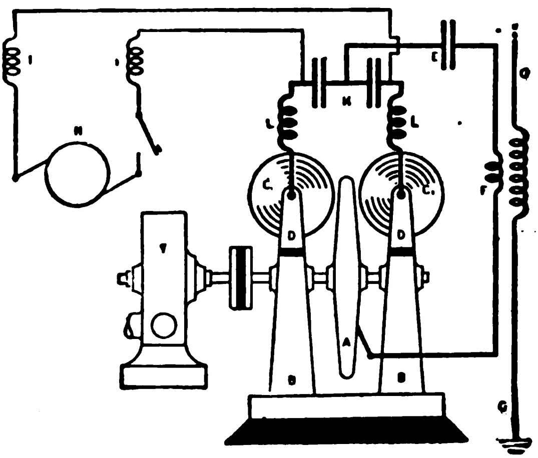


FIG. 49.—Marconi High-Speed Rotating Disc Discharger.

length, the air blast is an advantage, as it both increases the mean-square value of the oscillatory current and greatly steadies it. This air blast serves to make the discharge current more regular, and is of great use in certain measurements where steady high frequency currents have to be generated.

In the design of dischargers for long-distance radiotelegraphic stations, Mr. Marconi made a very great advance in the years 1906-7 by the invention of his high-speed rotating disc dischargers, which have greatly increased the speed of signalling possible in the case of large radiotelegraphic transmitters on the spark system, and also have provided a means for producing practically continuous oscillations, or at least trains of electric oscillations very rapidly succeeding each other without any silent interval between them. The construction of these dischargers was described by the inventor himself in a lecture at the Royal Institution on March 13, 1908.[40] One form of the Marconi high-speed disc discharger for the production of practically continuous oscillations is constructed as follows:—

A metal disc, A (see Fig. 49), insulated from the earth, is caused to rotate at

[40] G. Marconi, "On Transatlantic Wireless Telegraphy," *The Electrician*, vol. 60, p. 883, March 1908. Also British Patent Specifications, G. Marconi, Nos. 8462, 1907; 8463, 1907; 20,119, 1907; also 8581 and 8582, both of 1909: and United States Patent, No. 935,383, 1909.

very high speed, by means of a high-speed electric motor or steam turbine. The shaft which carries this disc passes through bearings in two pedestals, B, the upper parts of which, D, are insulated from the lower parts. These upper parts carry two other discs, C_1 and C_2, which may be called the polar discs, which can also be rotated at a very high speed. These polar discs have their edges placed very close to the surfaces or edges of the metal disc A. The two polar discs are connected respectively through suitable brushes or rubbing contacts to the outer ends or terminals of two condensers, K, joined in series, and these condensers are also connected through suitable inductive resistances, I, to the terminals of a high tension continuous current dynamo, H, although in some cases an alternator may be used instead. These condensers, K, will be referred to as the reservoir condensers. Against the central high-speed or metal disc a suitable rubbing contact is provided, and connected between this contact and the middle point of the two condensers K is inserted an oscillatory circuit, consisting of a smaller condenser, E, in series with an inductance, which last is connected inductively or directly to the antenna. The circuit containing the condenser E and the primary coil F of the oscillation transformer is suitably tuned to the period of the antenna G. If the necessary conditions are fulfilled, and a sufficient electromotive force is employed when the dynamo H is put in action, and the discs caused to rotate, a discharge will take place between the outer discs and middle disc, which discharge is neither an oscillatory spark nor an ordinary arc, and powerful oscillations will be created in the signalling condenser E and the oscillatory circuit F.

Mr. Marconi states that in order to obtain good effects a peripheral speed of over 100 metres a second is desirable, and, therefore, particular precautions have to be taken in the construction of the discs both as to material and balancing. The inventor gives the following explanation of the operation of the discharger. He says: "Let us imagine that the source of electricity is gradually charging the double condenser K, and increasing the potential at the discs, say C_1 positively and C_2 negatively; at a certain instant the voltage will cause the charge to jump across one of the gaps, say between C_2 and A. This will charge the condenser E, which will then commence to oscillate, and the charge in swinging back will jump from A to C_1, which is charged to the opposite potential. The charge of E will again reverse, picking up energy at each reversal from the condensers K. The same process will go on indefinitely, the losses which occur in the oscillatory circuit EF being made good by the energy supplied from the generator H. If the disc is not rotated, or rotated slowly, an ordinary arc is at once established across the small gaps, and no oscillations take place. The efficient cooling of the discharge by the rapidly revolving disc seems to be one of the conditions necessary for the production of the phenomena."

If, therefore, a continuous current dynamo is employed, such a discharger, when properly adjusted, enables undamped oscillations to be obtained, the arc discharge being entirely suppressed, and replaced by a regular high frequency alternating current supplied from the condenser E. The principle, therefore, which underlies the working of the discharger is that to obtain an arc discharge between metal surfaces one of these surfaces, namely, the negative, must be allowed to become heated, and if it is permanently cooled, or kept below a certain temperature, true arc discharge is prevented; but at the same time this does not prevent a condenser discharge from taking place across the gap. The proof that the oscillations so produced are practically continuous is shown by the fact that Marconi found that the waves emitted from the sending antenna provided with such a discharger could not affect at a distance his magnetic detector, unless an interrupter was inserted in one of the circuits of the receiver.

Mr. Marconi has also devised another similar form of discharger with which he states the best results have been obtained over long distances, which provides a regular succession of feebly damped waves. This discharger consists of a disc, A, which can be rotated at a high speed (see Fig. 50), which carries upon its surfaces at regular intervals knobs or studs. These studs pass in the course of their revolution between two fixed or rotating discs, C_1, C_2, the rest of the electrical arrangements being, as already described in connection with Fig. 49. As this wheel rotates, each time a stud passes between the discs C_1, C_2 a discharge

of the condenser E takes place ; but any arc discharge which is formed is at once extinguished, and on the passage of the next stud the condenser E is again able to furnish a train of oscillations. In this way an extremely regular series of slightly damped wave trains is produced which, when picked up at the receiving end by a suitable oscillation detector employed with a telephone, produce a clear musical note in the telephone which the ear can quite easily differentiate from the noises created by atmospheric electrical disturbances or other vagrant waves. These trains succeed each other at the rate of several hundred per second.

These high-speed disc dischargers of Marconi have now stood the test of prolonged use, and proved of the greatest value in enabling long distance radiotelegraphic transmitting to be worked at a very high signalling speed and for long periods of time without intermission, both of which conditions are inseparable from success in commercial radiotelegraphy.

The theory of these Marconi discharges has been investigated by Reinhold Rudenberg in an article entitled "Die Erwarmung rotierender Elektroden, insbesondere beim Marconischen Generator fur kontinierliche Schwingungen"

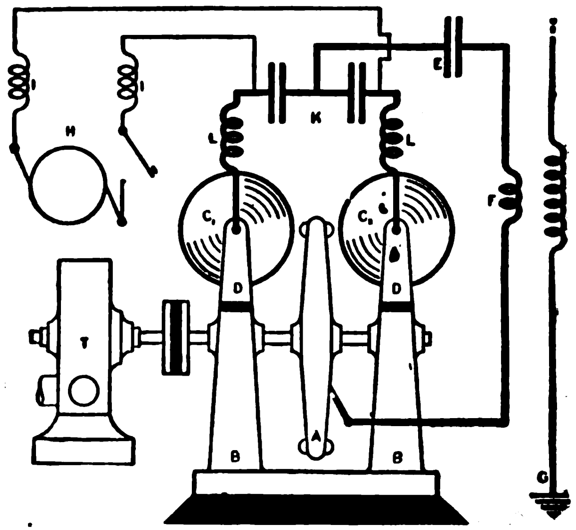
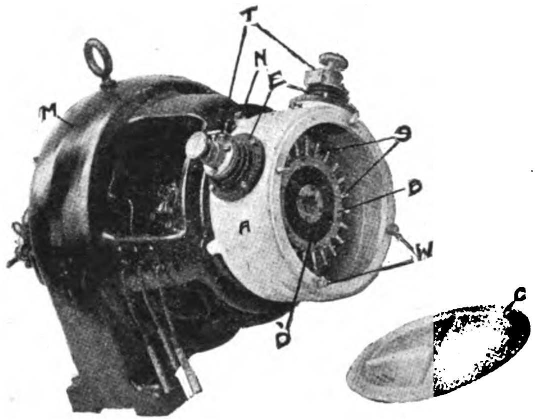


FIG. 50.—Marconi Studded Disc High-Speed Rotating Discharger.

(see *Jahrbuch der drahtlosen Telegrafie und Telephonie*, vol. ii. p. 18, 1908), to which the reader must be referred for details.

In the more recent forms of Marconi transmitter the rotating discharger is combined with the alternator by being fixed on the same shaft. Thus for ship transmitters, and for even some long distance transmitters, the alternator which provides the current for charging the condensers is driven by a direct coupled electric motor. Then on the opposite end of the shaft of this alternator is fixed a discharger consisting of a number of spokes like a wheel without a rim. These spokes just make grazing contact as they pass with two other fixed electrodes which may be metal discs, kept in slow rotation either by gearing or by a separate electric motor. This serves to close the condenser circuit and create the discharger. The discharger is enclosed in a box which serves to deaden the sound, and the air through this box is kept renewed by a fan. The violent movement of the air extinguishes at once any electric arc which forms, and therefore enables a very rapid and uniform rate of discharge to be kept up. Thus suppose the alternator revolves 3000 R.P.M., and there are 10 spokes on the discharger, this gives 500 discharges per second, and creates a quenched musical spark discharge. The general arrangement of such a plant is shown in Fig. 51.

For small laboratory dischargers it is now very usual to employ some form of high-speed rotating discharger. A convenient form consists of a brass disc



[*By permission of The Wireless Press, Ltd.*

FIG. 51.—Marconi High-Speed Rotating Musical Spark Discharger on the Shaft of the Alternator.

with short metal spokes screwed into its edge. These spokes nearly touch in their revolution two brass balls carried on insulating supports. The disc has a pulley on its shaft, and is rotated by an electric motor 1200 or 1500 revolutions per minute. The rotating spokes intermittently complete the discharge circuit by making grazing contact with the two balls.

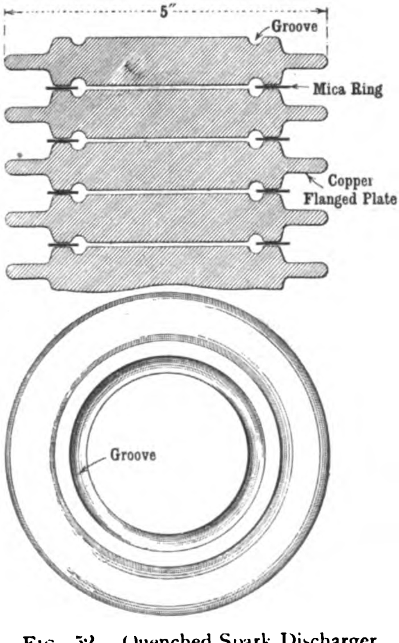
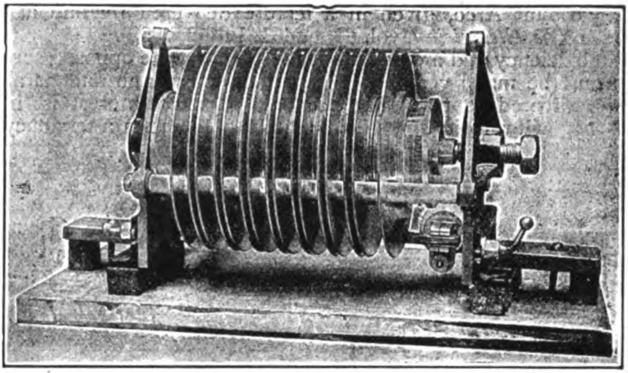


FIG. 52.—Quenched Spark Discharger, Plan of Copper Flanged Plate.

We have next to consider forms of discharger adapted for the method of excitation by shock or impact, the principles of which have already been explained (see Chap. III. § 15). It has been shown that in a system of two coupled oscillation circuits, in one of which damped oscillations are excited by means of a spark gap, the reaction of the two circuits on one another produces in both of them oscillations of two frequencies. It has also been mentioned that in the case of very short spark gaps the damping is extremely large, and that by availing ourselves of this fact it is possible to construct a transmitter producing damped waves in which the primary oscillation is damped out of existence after one or two oscillations, and the secondary is permitted to oscillate freely in a single period, and radiating, therefore, waves of a single wave-length. Starting from Wien's researches on this matter, the Gesellschaft für Drahtlose Telegraphie devised forms of discharger for conducting spark telegraphy on this quenched

spark system. In one form it consists of a series of copper discs or copper boxes with flat sides cooled with water, the outer surfaces of which are placed in contiguity, but separated by very thin rings of mica. The surface of the boxes or plates must be made extremely true and the interspace extremely small, about



[*Taken by permission from* "*The Electrician.*"

FIG. 53.—Perspective View of Quenched Spark Discharger.

0·15 or 0·1 of a millimetre, and ten or twelve of these discs or boxes may be placed in series. A groove is turned in the flat surface of the discs, and the mica ring extends half-way across this groove, as shown in Fig. 52. This series of discs or boxes takes the place of the ordinary spark balls, and the outer members of the series are connected to the secondary terminals of a charging

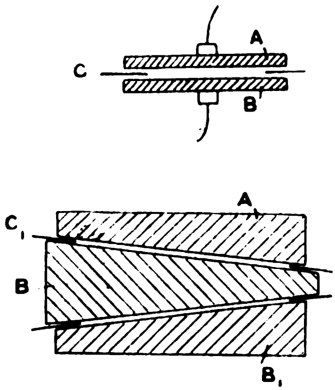


FIG. 54*a*.—Von Lepel Dischargers. A, B, metal plates or surfaces separated by a paper ring, C, or rings, C_1, C_2.

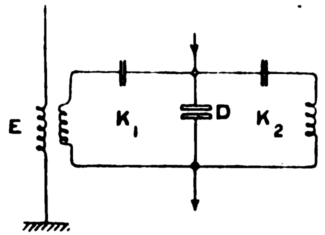


FIG. 54*b*.—Mode of Connection of the Von Lepel discharger D to the antenna E.

transformer fed by a high frequency alternator having a frequency of 300 to 500 or more. The extremities of the series of discs or boxes are also connected to the oscillatory circuit consisting of a condenser in series with one coil of an oscillation transformer, the other coil in series with the antenna. When the charging transformer is in action it produces a very large number—500 or more spark discharges in the discharger, which are quenched instantly, and these

are accompanied by an equal number of condenser discharges, each of which is quickly damped. The directly or inductively coupled antenna hence receives a very large number of shocks or impulses per second, which set up its free vibrations. In Fig. 53 is shown a perspective view of this discharger as made by the Gesellschaft für Drahtlose Telegraphie, taken from a description of their apparatus by Count Arco, given in a lecture delivered by him at Cologne in June 1909 (see *The Electrician*, vol. 63, p. 461, July 2, 1909).[41]

To obtain efficiency and a quickly quenched spark, the opposed surfaces of this discharger must be made very smooth and perfectly plane and parallel, and are best made of silvered copper. The practical point of interest is, however, the time which they will remain smooth in practice. There is a tendency to become

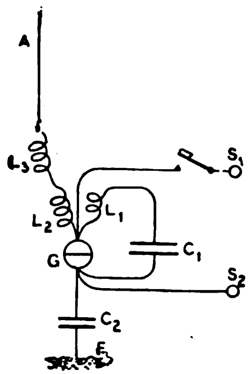


FIG. 55.—Connections of the Von Lepel Transmitter Circuits. G, Von Lepel discharger; C_1, C_2, condensers; L_1, L_2, L_3, inductances; A, antenna; S_1, S_2, supply Terminals.

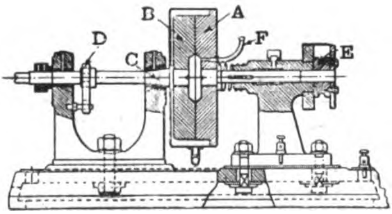


FIG. 56*a*.—Peukert Disc or Quenched Spark Discharger.

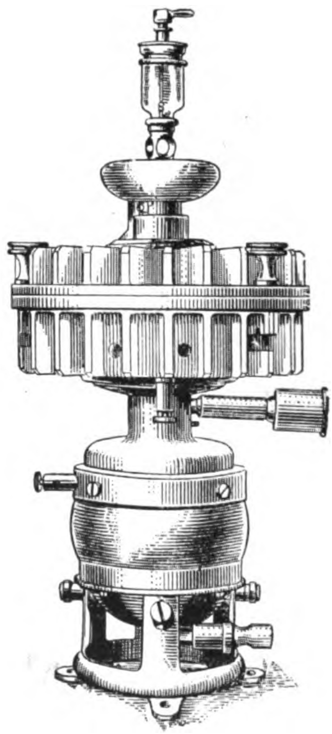


FIG. 56*b*.—Peukert Disc or Quenched Spark Discharger.

pitted, and the author has found that in this case the constancy and efficiency of the discharger fall off.

A somewhat similar discharger, composed of a pair of flat metal plates or concentric cones separated by a paper ring (see Fig. 54*a*), has been devised by Von Lepel.[42] These plates are connected to the terminals of a high tension direct current dynamo, and are shunted by a pair of oscillatory circuits containing an inductance and capacity which are syntonized. The antenna is inductively or directly connected to one circuit (see Figs. 54*b* and 55).

The continuous voltage produces a series of intermittent quenched discharges

[41] See British Patent Specification of W. P. Thompson, communicated by the Gesellschaft für Drahtlose Telegraphie, No. 6424 of 1909.

[42] See British Patent Specification of E. von Lepel, No. 17,249 of 1908.

between the plates, each of which is accompanied by a damped oscillation in the condenser circuits, and if an antenna is connected inductively or directly to one oscillatory circuit, free persistent slightly damped oscillations are set up in it.

Another form of quenched spark discharger was invented by W. Peukert, of Brunswick, which makes use of a film of oil instead of a thin air gap as in the Telefunken or Von Lepel dischargers (see *The Electrician*, vol. 64, p. 550, January 14, 1910).[43]

The Peukert discharger consists of a flat stationary metal disc, A, the face of which is kept flooded with oil through a small hole, F, in the disc (see Fig. 56*a*). Face to face with the fixed disc is another smooth metal disc, B, which can be made to revolve rapidly. The interspace between the discs is about 0·1 mm. The speed of the moving disc may be about 800 R.P.M. The discs are insulated from each other and form the two surfaces of the discharger. When a direct

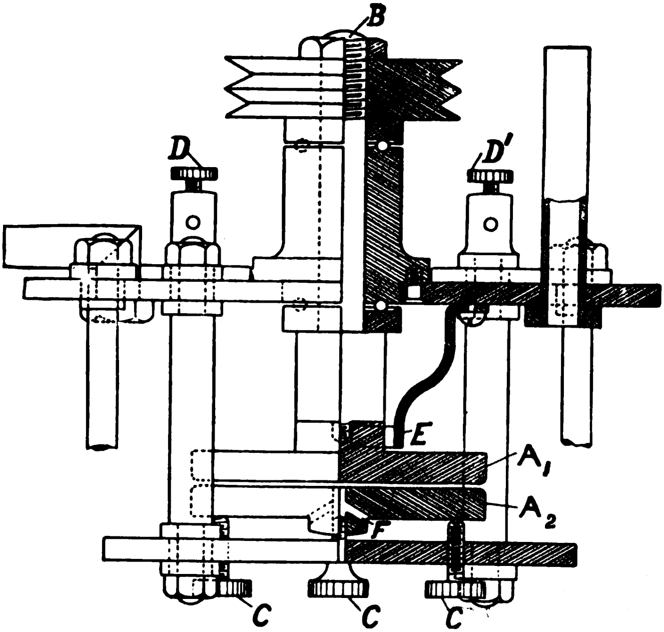


FIG. 57.--Section of Fleming Quenched Spark Discharger.

current voltage of about 200-400 volts is put on them the oil film is continually pierced, and if shunted by a condenser in series with an inductance, rapidly damped oscillations are set up in this circuit.

For wireless work the discs are best placed in a horizontal position, and one of them revolved by a small motor, and the discs may be flanged to keep them cool (see Fig. 56*b*). The discs are made of chemically pure copper, or copper plated with silver. Each gap or discharger must carry a current of not more than 4 amperes, but the voltage may be direct or alternating, and from 440 to 1500 volts.

The author has arranged a modified form of rotating plate discharger which has proved very convenient for use in the laboratory.[44] It produces a very dead-beat or quenched spark discharge, and, therefore, when placed in a primary circuit sets up in the secondary circuit free oscillations of a single periodicity, even if the coupling is close.

[43] See W. Peukert, British Patent Specification, No. 4762 of 1909.
[44] See British Patent Specification, No. 23,242 of 1910.

It consists of two round discs, A_1, A_2 (see Fig. 57), of polished steel turned extremely true, case-hardened and ground dead flat. One of these, A_2, has a hole, F, in the centre, the other, A_1, is fixed to a shaft, B, running true in ball bearings. This shaft is carried in a frame, to which the stationary disc is fixed by adjusting screws, C, C; the two discs are insulated from each other and are placed so that their surfaces are truly parallel and separated only by a quarter of a millimetre. The frame carrying the discs is placed in a glass vessel filled with paraffin oil, and the upper disc is revolved by an electric motor at a speed of 2000 revs. per min. The moving disc is connected to one insulated terminal, D, by a rubbing contact, E, on the shaft, and the fixed disc is connected to another insulated terminal, D′.

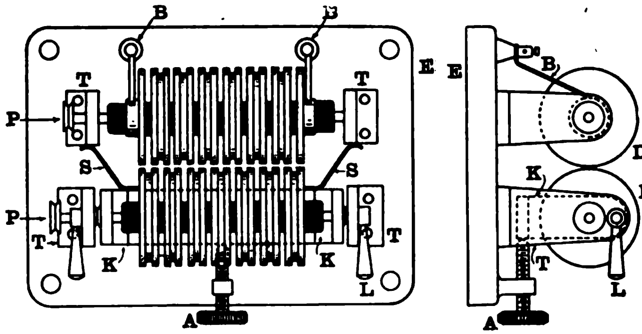
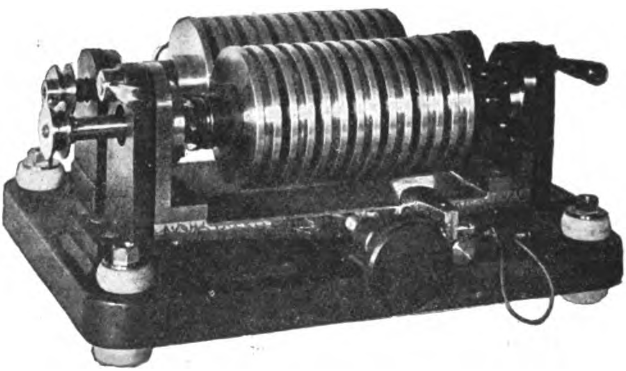


FIG. 58.—Lodge-Chambers Rotating Multiple Gap Discharger. Section.



[*Figs.* 58 *and* 59 *are taken by permission of the Proprietors from "The Electrician."*

FIG. 59.—Lodge-Chambers Rotating Multiple Gap Discharger. External View.

When the upper disc revolves at a high speed it flings the oil out between the discs and fresh oil is sucked in at a hole, F, in the lower disc. There is, therefore, a continual circulation of the oil between the plates, and by means of adjusting screws the lower disc is placed with its surface perfectly parallel with the under surface of the revolving disc. If, then, these discs are made the discharger in a condenser circuit, the discharge takes places perfectly uniformly over the whole surface of the two discs, that is to say, it does not take place continually at one spot. Moreover, it is a dead-beat discharge. The spark is damped out of existence instantly, and although the oil becomes carbonized it is continually being renewed between the discs, and the products of decomposition do not remain between the plates.

If two or more such dischargers are joined up in series, we have a very

efficient impact discharger, which will run for hours by the aid of a small electric motor without attention.

Several dischargers may be arranged in series, allowing, say, 800 volts for each gap.

The actual performance of any type of discharger can best be determined by the use of the author's photographic spark counter (see Chap. II. § 15), by which it is possible to determine at once how many sparks take place per alternation of the transformer, or per break of the induction coil primary, or per second. If the spark gap is very short a very large number of discharges of the condenser may take place at each interruption or alternate current period, as shown in the reproductions in Fig. 46(*a*), Chap. II., of spark photographs so taken.

It is, therefore, essential in any measurements of efficiency of the plant to apply the spark counter.

Several other forms of rotating discharger have been invented. One is that of Lodge and Chambers, constructed as follows :—

It consists of two sets of grooved copper discs like pulleys separated by ebonite washers and mounted on an ebonite shaft. These discs are so attached to a support that, whilst one set is fixed in position the other set can be moved by a screw parallel to the first set. The discs are set on their shafts pair and pair with spacing washers, but the discs on the two shafts are relatively so placed that the shortest distances compel an electric discharge to jump backwards and forwards from one set of discs to the other. Hence the whole spark gap is broken up into the sum of a large number of very short gaps ; whilst the surfaces between which the discharge happens are continually in movement. The arrangement will be easily understood from Figs. 58 and 59, which show the plan and elevation. The discs can be driven in opposite directions by a small electric motor. The action of this discharger when set in a condenser discharge circuit is to produce a highly damped or quenched discharge, but there is no arrangement for securing a true musical spark, that is a discharge in which the sparks happen at absolutely equal intervals of time.

19. Signalling Keys.—Another element of practical importance in the transmitting apparatus is the signalling key. In order to create the signals it is necessary to be able to close the primary circuit either of the induction coil or alternating current transformer used to charge the primary condenser for longer or shorter time, in accordance with the signals of the Morse alphabet. This is done by means of a primary key. When using an ordinary 10-inch induction coil, the primary current which has to be interrupted is a current of about 10 amperes. This can be easily done by means of a Morse key, having heavy platinum contacts and a long insulating handle. In order to quench the spark at the platinum contacts a large condenser may be placed across the break points, or else a magnet may be employed as a magnet blow-out to destroy the arc which tends to form on separating the points.

Mr. Marconi devised a key for induction coil working which renders it impossible to commence working the spark coil until the aerial is disconnected from the receiving apparatus. The key is shown in Fig. 60, where the black portions represent ebonite. When the key is not in use it rests on its back contacts, and the antenna is connected to the receiving instrument, ready, therefore, for reception. But as soon as the operator commences to send signals it automatically disconnects the antenna from the receiving instrument.

When alternating currents are employed to excite either an induction coil or an alternating current transformer, keys are now employed which are practically sparkless, because the contact cannot be broken until the instant when the current in the primary circuit passes through its zero value. This is achieved by making the primary current pass through an electromagnet, which holds down the contact piece when once it is pushed down by the key, and the primary circuit is not broken again, even if the key is raised, until the primary current passes through its zero value (see Fig. 61).

Another method of signalling with syntonic apparatus is to throw the secondary or antenna circuit into and out of tune with the primary condenser circuit by cutting out inductance, or to short-circuit the condenser in the spark circuit

by an impedance coil. This last method is to be preferred, as it intermits the spark. Automatic sending by punched tape has for power station purposes superseded hand sending entirely.

The type of signalling key mostly used in small and large power stations on the spark system is the relay key, in which a small continuous current is

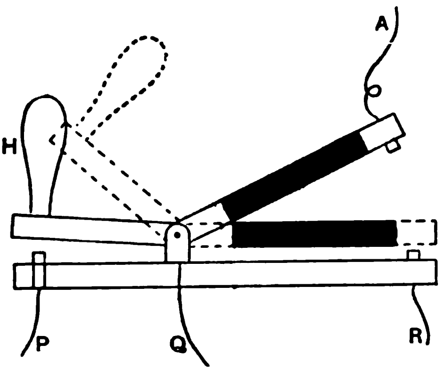


FIG. 60.—Marconi Signalling Key arranged to Automatically Disconnect the Antenna from the Receiver before Signalling. A, connection to antenna; R, connection to receiver circuit; P, Q, connections to primary circuit of induction coil. The black portions are ebonite.

manipulated by an ordinary telegraph or Morse key. This current serves to actuate an electromagnet which closes the circuit of the alternating currents in the charging transformer primary circuit.

The Marconi Company employ a relay key, the construction of which is shown

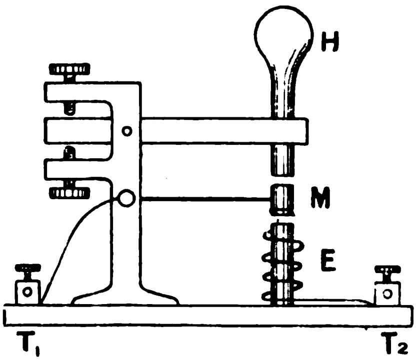


FIG. 61.—Non-sparking Key used with Alternating Currents. E, electromagnet; M, intermediate contact.

in Figs. 62 and 63.[45] In its simplest form it consists of an electromagnet, *b*, which is energized by an alternating current which is taken from an alternator, *a*. The coil *n* which is interposed may be the primary coil of a transformer or transformers which serve to charge the condenser of the transmitter. This circuit has two closing contacts, *e* and *j*, one closed by the attraction of an armature,

[45] See British Patent Specification, No. 25,381 of 1903.

d, and the other by pressing down a key, *g*. If the latter is depressed, the electromagnet *b* is energized and closes the contact *e*. If then the key *g* is raised, the contact *e* remains closed until the alternating current passes through its zero value when it flies up and opens the contact. In this way the contact *e* is opened without sparking.

The key *g* may in turn be pressed down by a second electromagnet, *l*, operated by the current from a battery and a key, *m* (see Fig. 63), at a distance. In this manner a key opening and closing a circuit conveying a very small direct current can be made to start and stop a large alternating current in a highly inductive circuit without much sparking. This spark can be reduced and the contacts at *e* kept cool by a blast of air thrown on to them from a blower. By the use of this key the Marconi Company operate high-power long-distance transmitters by means of a Wheatstone or Creed automatic transmitter working with punched tape.

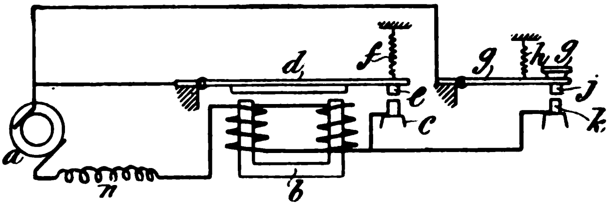


FIG. 62.

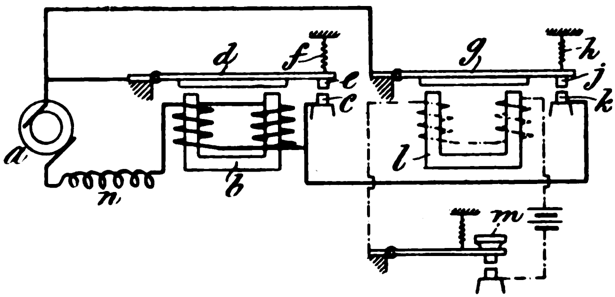


FIG. 63.

Relay Keys used by Marconi's Wireless Telegraph Company in Large Stations.

20. Receiving Apparatus in Radiotelegraphy: Tuners. — The electromagnetic waves sent out from the transmitter fall on receiving antennæ located at various places, and if these latter are syntonized or tuned to the frequency of these waves, oscillations are set up in the receiving antenna which resemble in type, but are much more feeble in intensity than, those sent out from the sending antenna. If the emitted waves or wave trains are cut up into groups to form Morse signals, then the oscillations set up in the receiving antenna are similarly divided up. For good reception the mean receiving antenna current near its base should have an R.M.S. value of 40 to 100 microamperes.

These antenna oscillations are caused to transfer their energy to another closed receiving circuit comprising an inductance and a variable capacity which is coupled to the antenna circuit either by a single coil or by a double coil oscillation transformer or jigger.

It is necessary, therefore, to provide means for tuning the antenna to the frequency of the incident waves, and also to tune to it the closed receiving circuit.

It is, therefore, requisite to have in the antenna circuit a variable inductance called a tuning inductance or variometer, and also in some cases a condenser of variable capacity, called an aerial tuning condenser, placed in series with the antenna.

In the Marconi system there is also a very short spark gap, called an earth arrester, inserted somewhere between the earth plate connection and the antenna

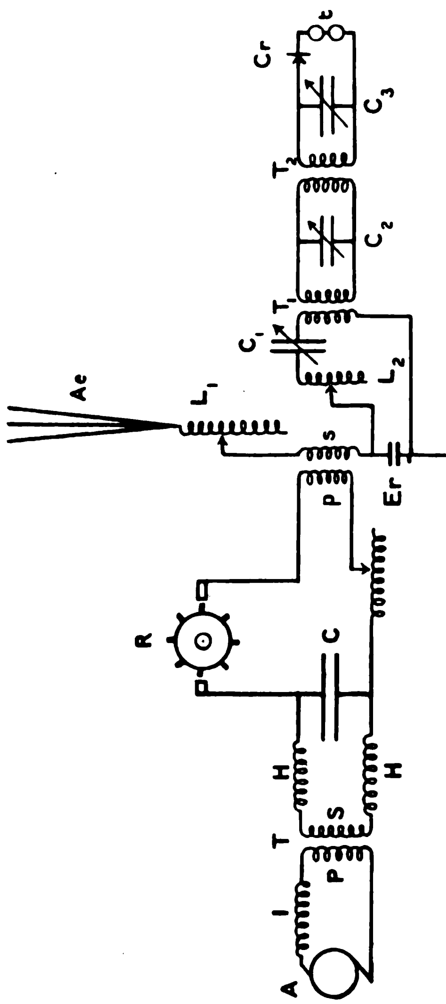


FIG. 64.—Diagrammatic Scheme of Circuits in Transmitting and Receiving Apparatus in the Marconi System. A, alternator; I, choking coil; T, charging transformer; H, H, chokers; C, main condenser; R, rotating discharger; P, S, sending jigger; Ae, aerial; Er, earth arrester; C_1, C_2, C_3, receiver condensers; T_1, T_2, receiver transformers; Cr, crystal detector; *t*, telephone.

proper. The receiving circuit is connected to the two sides of this spark gap. This gap is sufficient to compel the feeble oscillations induced in the aerial by the incident waves to pass through the aerial tuning condenser, and the primary of the receiving jigger or oscillation transformer.

When, however, powerful oscillations are set up in the antenna by the transmitting apparatus, this small spark gap is bridged by the spark and connects the antenna to earth whilst short-circuiting or cutting out the receiving circuits.

Hence there is no switching over of the antenna from transmitter to receiver. The whole arrangement is perfectly automatic. It will be understood from the diagram in Fig. 64, which shows the arrangement of sending and receiving appliances on the Marconi system.

A feature of the closed receiver circuit of the Marconi Company is the intermediate closed circuit in the receiver. Furthermore, it is necessary to be

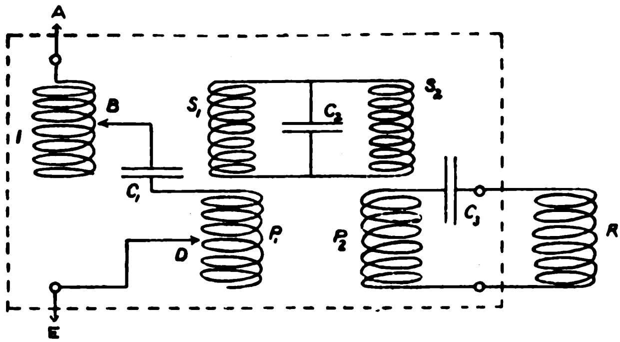


FIG. 65.—Circuits of the Marconi Tuner.

able to vary the coupling or mutual inductance of the coils which transfer the energy from the antenna to the closed receiving circuit. These various changes are effected by the use of appliances called "tuners" or receivers in which all the necessary changes can be made quickly by moving rotating switch contacts. As an example of such an arrangement, we shall describe here the tuner devised and used by Marconi's Wireless Telegraph Company.

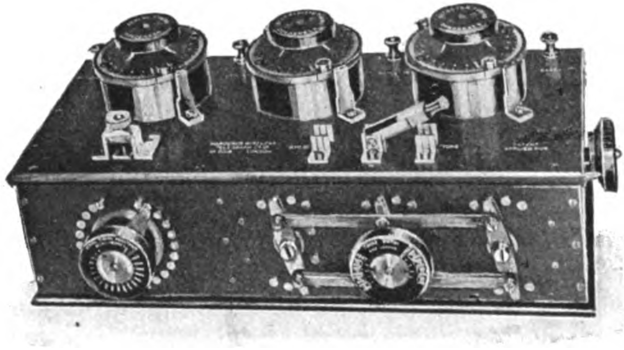


FIG. 66.—Perspective View of the Marconi Tuner.

This tuner consists of a box which is provided with three adjustable condensers and five adjustable inductance coils (see Fig. 65).[46] These condensers consist of semicircular metal plates separated by some dielectric such as ebonite, alternate metal plates being connected together to an axis, so that by rotating this axis one set of plates can be more or less moved in between the others, and the capacity of the condenser so formed varied, the actual capacity being

[46] See British Patent Specification, No. 12,960, June 4, 1907, granted to the Marconi Wireless Telegraph Company, Ltd., and C. S. Franklin.

indicated by a divided scale on the head. This instrument contains three such condensers, respectively called the aerial tuning condenser, the intermediate tuning condenser, and the detector tuning condenser, and these condensers are arranged in connection with inductance coils as shown in Figs. 65 and 66. In this diagram, A (Fig. 65) represents the antenna or aerial wire, and B represents an aerial tuning inductance in series with it, and C_1 represents the aerial tuning condenser, and in the same circuit is inserted any desired portion of another coil P_1, one point of which can be put to earth E. The coil P_1 is loosely coupled with another coil S_1 which forms part of the intermediate circuit comprising the intermediate condenser C_2 and the two coils S_1 and S_2; S_1 being loosely coupled with the coil P_1, and S_2 loosely coupled with the coil P_2, which last coil, together with the condenser C_3, called the detector tuning condenser, forms the circuit to which the receiver R is connected, which may be the coil of a magnetic detector or of any other suitable receiver. The oscillations set up in the antenna A set up oscillations in the coil P_1, which induce others in the coil S_1, and these again other oscillations in the coil S_2, and these in turn set up oscillations in the coil P_2 which finally affect the receiver. The coupling of the coils S_1 and P_1 can be altered, and also of S_2 and P_2, and the inductances and capacities are also variable, as described. The capacity of the condensers C_1, C_2, and C_3 can be continuously varied from zero to a maximum value of about 0·001 microfarad.

In adjusting the instrument for use, the antenna circuit, comprising the coil B and the condenser C_1 and a portion of the coil P_1, has to be varied until its natural period coincides with that of the waves to be received. The intermediate circuit, comprising the condenser C_2 and the coils S_1 and S_2, has then also to be tuned by varying the capacity until it has the same period, and in like manner the circuit containing the coil P_2 and the condenser C_3 has to be tuned until it has the same natural period. Looking at the perspective view of the instrument (see Fig. 66), the aerial condenser is the left-hand condenser, the intermediate condenser is the middle condenser, and the detector condenser is the right-hand condenser. The handles of the aerial tuning inductance and of the tuning switch are seen in the middle of the diagram, but those for varying the coupling of the oscillation transformers are not shown.

This instrument is mostly employed by the Marconi Company in connection with the Marconi magnetic detector. Hence there are four terminals on the instrument respectively marked *earth*, *aerial*, and *magnetic detector*. The aerial wire has in series with it the secondary circuit of the transmitting jigger and also a tuning coil. These are connected outside the terminal marked *aerial*. The *earth arrester* is shown at *d* in Fig. 67. In setting the aerial tuning inductance and aerial condenser, the best values to select, of course, depend upon the wave-length to be received. A little experience shows which are the best values to use at any particular station, but the following is the process for adjusting the instrument when signals from the station with which it is required to communicate are to be picked up.

1. Adjust the aerial tuning inductance, keeping the aerial condenser short-circuited, and then the aerial condenser must be adjusted until the strongest signals are obtained.
2. Set the intensifier handle to 90°.
3. Set the tuning switch to the wave-length roughly indicated by the amount of the aerial tuning inductance and the aerial condenser.
4. Throw over the changing switch to tune, and then vary the intermediate tuning condenser and the detector tuning condenser together until the best signals are obtained. It is necessary that these two condensers should be varied as nearly as possible together.
5. Adjust the aerial tuning condenser to give the strongest signals, and if any interference is found adjust the intensifier to a small value, and then adjust the condensers again. The further this intensifier handle is turned from 90° the sharper will the adjustments of the condensers become, owing to the looser coupling and the greater freedom from interference.

The variation in the coupling of the two circuits of the oscillation transformers

is effected by forming one coil of a short or squat cylindrical form, and the other in a concentric coil which can be rotated by a shaft, so as to set the axes of the two coils more or less in line, and thus vary their mutual inductance. In some cases the Marconi Company employ a pair of sliding tubes one within the other for the variable capacity condensers.

In most ordinary amateur receiving arrangements the circuits comprise merely a jigger or two-coil oscillation transformer, one circuit of which is inserted in the aerial circuit between the antenna and the antenna tuning coil, and the other circuit is connected to the terminals of a condenser of variable capacity. To the terminals of this condenser are attached as a shunt the ends of a circuit which comprises a high resistance double head telephone, having a crystal detector carborundum or else a "Perikon" rectifier in series with it. The telephone should have a resistance of at least 2000 and preferably 4000 ohms.

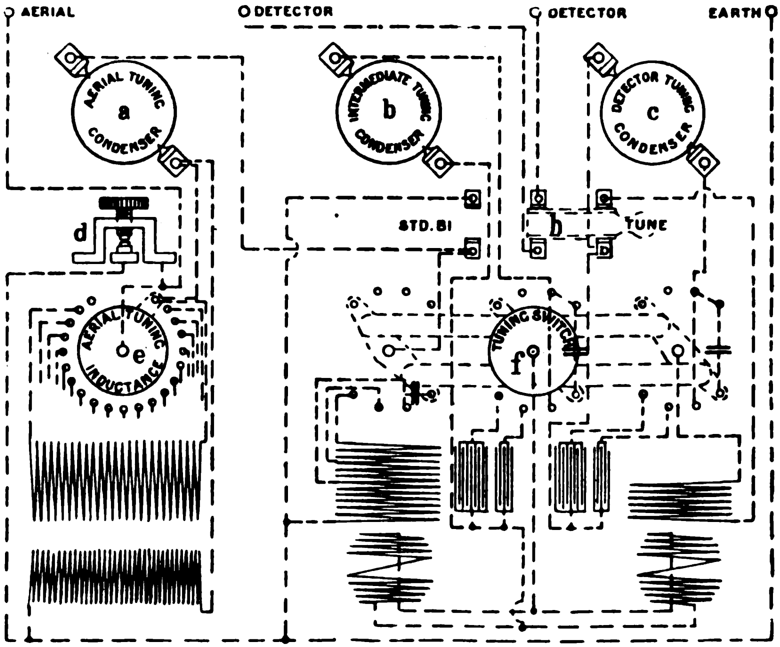
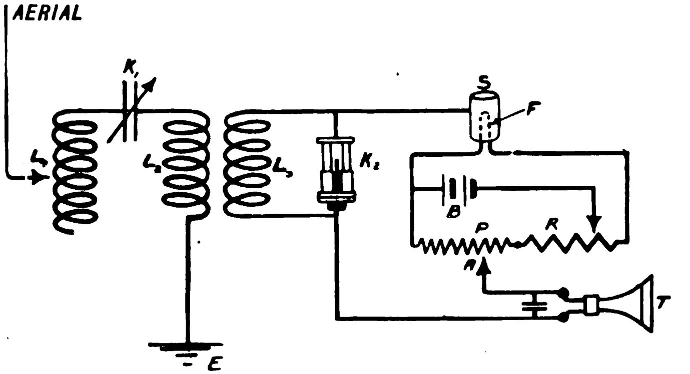


FIG. 67.—Details of the Connections of the Marconi Tuner.

The jigger should have its two coils separable, so as to be able to vary the coupling over a wide range. Whatever form of tuner is employed the oscillations in the antenna expend their energy in producing in the closed receiver circuit coupled to the antenna others which are a copy on a reduced scale of those in the sending antenna, and if these latter are cut up into Morse signals, so are the former. We have, then, to make these signals audible or visible. To do this requires the employment generally of some form of detector, and the mode of connection of it with the closed receiving circuit depends on the nature of the detector. If it is a magnetic detector the oscillation coil of the latter is inserted in series with the condenser in the closed receiver circuit. If, on the other hand, it is a high resistance detector, such as a rectifying contact or an ionized gas or vacuum valve detector, it is connected as a shunt across the terminals of the receiving circuit condenser (see Fig. 68). Detectors which are suitable for inserting in series with the condenser in the closed receiving circuit are sometimes called current

operated detectors, whilst those which are inserted as a shunt to the condenser are called potential operated devices. The real difference between them is, however, simply their impedance. High impedance or resistance detectors are inserted in shunt and low resistance in series. In Fig. 65 the coil R is the oscillation coil of a magnetic detector, and is, therefore, placed in series with the condenser C_3. In Fig. 64 a crystal detector Cr is used and it is, therefore, placed in shunt to the condenser C_3.

21. Receiving Arrangements for Undamped Waves.—The various forms of detectors we have described in Chapter VI., such as the magnetic, crystal, or rectifying valve, or the electrolytic detector, are only adapted when used in series with a telephone for receiving Morse signals by damped wave trains. If the waves are undamped then, when a telephone in series, say, with a rectifier, is inserted as a shunt to the receiving condenser, the telephone would be traversed by a feeble but practically continuous current. If the wave train is cut up at the sending end by a key into Morse signals, then all that would be heard in the telephone would be a *tick* or slight sound at the beginning and end of each signal. To secure the reception of properly defined Morse dots and dashes, the continuous or undamped waves must be divided in some way into equal groups, of which the frequency lies within the limits of audition, and is best somewhere near 500. This



[*By permission of the Wireless Press, Ltd.*

FIG. 68.—Arrangement of Receiving Circuit when using Fleming Valve Detector. L_1, K_1, aerial tuning coil and condenser; L_2, L_3, receiving jigger; K_2, receiving condenser; F, valve detector; T, telephone; B, valve battery.

will then create by itself a continuous sound in the telephone which is capable of being cut up into Morse signals by means of the sending key.

There are three well-known and ingenious appliances for the reception of Morse signals by a telephone when continuous waves are used, viz. the Poulsen tikker, the Fessenden heterodyne telephone, and the Goldschmidt tone wheel.

The Poulsen tikker in its original form consists of a small contact maker, in which a couple of thin gold wires are brought together and separated by some vibrating electromagnetic arrangement like an electric bell or buzzer.

This tikker is connected in series with a condenser of rather large capacity, and a telephone is connected in shunt across the terminals of the condenser. The whole arrangement is then placed as a shunt to the adjustable condenser C which is in the closed receiving circuit. The scheme of connections is as in Fig. 69. Hence no rectifier or detector in the usual sense of the word is employed. The operation of the tikker is as follows. Suppose the tikker contacts are open. There is then only the closed receiving circuit in connection with the antenna, and the antenna current creates and increases by resonance a certain current in the closed circuit and a certain potential difference at the terminals

of the adjustable condenser in the closed circuit. When the tikker contacts close, the capacity of the condenser shunting the telephone is added to that of the adjustable condenser, and an oscillation of new and lower period is set up in which the telephone shunt condenser takes part. When the tikker contacts open again the charge in the telephone shunt condenser flows through the telephone and causes a movement of its diaphragm which creates a short sound in the ear of the listener. If the tikker continues to operate, these sounds run together into a more or less continuous sound. If the continuous waves sent out from the sending station are interrupted so as to form Morse signals, then the sound made by the telephone will be correspondingly interrupted. The arrangement is, therefore, adapted for reception with continuous waves only. Moreover, the sound heard in the telephone has the pitch of the tikker frequency,

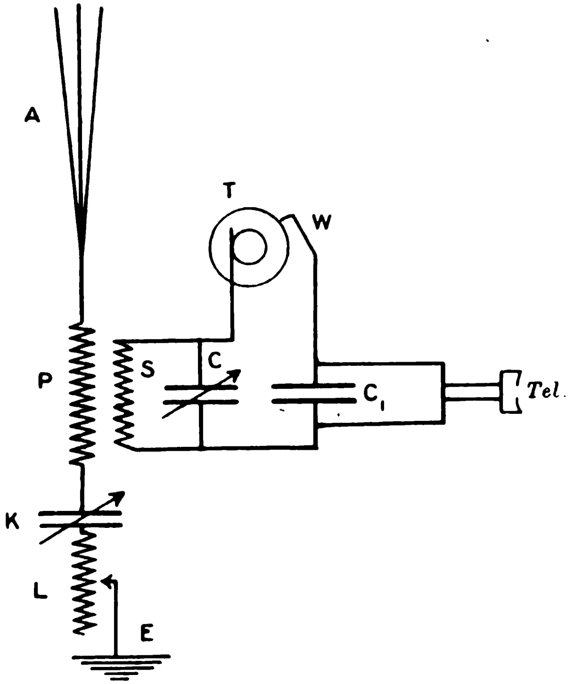


FIG. 69.—Mode of using a Tikker and Telephone for receiving Signals with Undamped Waves. T, rotating brass disc; W, steel wire contact; C, receiving condenser; C_1, telephone condenser; *Tel*, telephone.

and unless the vibration of the tikker is extremely uniform, is not a true high pitch musical note. Also since the amount of the charge in the condenser which is discharged through the telephone is not always the same, but depends on the instant at which the tikker contacts open, there is always a certain irregularity in the sound heard in the telephone. Moreover, its pitch is not in general high enough to be heard above and over atmospheric sounds. Nevertheless the receiver is a very sensitive one when used in connection with the continuous waves produced by the Poulsen arc generator. In recent forms of tikker the contacts are not vibrated, but a revolving brass disc, T, has the point of a steel wire, W, lightly pressing on its edge (see Fig. 69). This makes a chattering contact which answers better than the vibrating contact of gold wires. It is applied in the same manner.

The receiving instrument invented by Dr. R. Goldschmidt, which he calls

a tone wheel, is a highly ingenious, and very effective one. It consists of a wheel having teeth filled in with some insulating material against which one or more brushes press, so as to interrupt a circuit when the wheel revolves. The wheel is driven by an electric motor at a high speed, and there are arrangements for controlling the speed and preventing variations in it. The external view of the apparatus is as in Fig. 70. The wheel has a large number of teeth, say 800, on its circumference, and it is driven at a speed which makes the frequency of the interruptions approximately agree with that of the continuous waves used. Thus, for instance, let us suppose that the wave-length of the continuous waves used produced, say, by a high frequency Goldschmidt alternator is 6 kilometres, then the wave frequency is 50,000. Then the problem presented is to reduce this frequency, say to 500, so as to be audible in the telephone. If, then, the wheel has 800 teeth and revolves at a speed of 3750 revolutions per

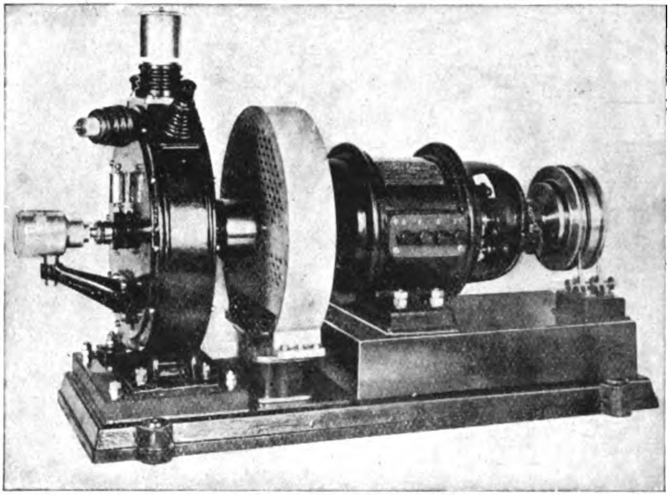


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FIG. 70.—The Goldschmidt Tone Wheel Detector.

minute, or 62·5 per second, 50,000 contacts will be made per second. Suppose the width of a tooth is equal to the space between the teeth, and that the receiving antenna current or one created inductively by it is passed from the wheel to the contact brush, and then through a telephone. The wheel as it revolves will cut off or pass all the half waves of current passing in one direction, and the telephone will be traversed by intermittent currents in the same direction, having a frequency of 50,000. The telephone will, therefore, emit no sound. If the speed of the tone wheel is slightly increased or diminished so as to be out of step with the frequency of antenna currents, then it is easy to see that there will be a kind of interference which will produce a variation in the current passing through the telephone, and that the frequency of this varying telephone current will be the difference between the tone wheel frequency and the antenna current frequency.

Thus, in the case just considered, let us suppose that the speed of the tone wheel is reduced from 3750 R.P.M. to 3694 R.P.M., then the frequency of the contact interruptions would be reduced from 50,000 to 49,250 nearly. Hence there would be 750 beats per second, and a shrill sound would be produced by the telephone having this frequency of 750. If, then, the continuous waves are cut up at the sending end into Morse signals, they would be heard at the receiving end by a listener at the telephone.

The tone wheel not only serves as a receiver but as a wavemeter as well. For if we know the speed of the wheel at which the sound in the telephone is just quenched, and if we know the number of teeth on the wheel, the product of these two numbers gives us the wave frequency and therefore the wave-length.

Also it serves as an interference preventer because it is generally possible to run the wheel at such a speed, that whilst the signals of the station in correspondence are heard well, those of other stations have such a high or such a low note that they are not heard. Again atmospherics or stray waves can also be cut out.

Another form of receiving device for continuous waves called the Heterodyne receiver has been invented by R. A. Fessenden. It involves the use of a telephone of particular construction, and is based upon a principle of interference analogous to that of the Goldschmidt tone wheel.

Suppose a telephone receiver, D, is wound with two independent coils (see Fig. 71), through one of them, C^1, is passed a high frequency alternating current, which is the current in a receiving antenna, A, or else one generated inductively by it. If this current had a frequency as high as, say, 50,000 it would not create

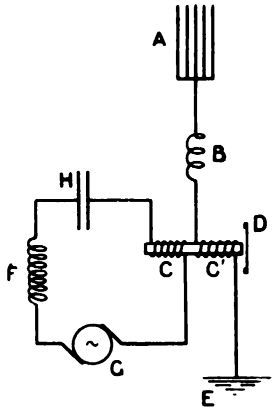


FIG. 71.—Fessenden Heterodyne Method of Reception.

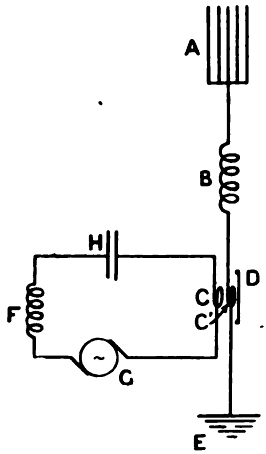


FIG. 72.

any sound in the telephone on account of the inability of such high frequency currents to pass through the telephone coil as well as of the ear to appreciate as sound vibrations of such high pitch. Suppose that through the other coil C is passed another high frequency locally generated current of approximately the same strength, but with a frequency differing from the first by, say, 500, or having a frequency of 49,500. Then these currents would interfere and the telephone diaphragm D would be acted upon as if a current having a frequency of 500 were passed through a single coil. It would, therefore, create a sound. In the Fessenden system of heterodyne reception one of these currents is created by a local high frequency alternator, G, of which the speed and, therefore, the frequency can be controlled by the receiving operator. The other current is the receiving antenna current, or else one generated inductively by it. The telephone used may have one coil on its magnets which is traversed by the local high frequency current, and another coil fixed to the inner side of the diaphragm which is traversed by the current in the receiving antenna, or by one induced by it. These two currents, then, cause the coils to attract and repel each other, and the resultant effect is that the diaphragm is moved as if by a current having a frequency equal to the difference of the frequencies of the antenna and the local current. The arrangement will be understood by the diagram in Fig. 72.

A modification of this electrodynamic method is found in the use of a static telephone in which the diaphragm is a very light disc moved to and fro by electrostatic attractions. This electrostatic receiver is shown in Fig. 73, in which A is the receiving antenna, D the diaphragm of the electrostatic telephone, and B a coil coupled inductively with the local high frequency circuit containing the H.F. alternator G. A still better arrangement is shown in Fig. 74. In this the antenna circuit is coupled inductively by one coil B″ with the local high frequency circuit containing the H.F. alternator G. It is also coupled by means of another coil B′ with a tuned circuit containing an ordinary crystal, valve, or contact rectifier L, and a pair of head magneto telephones, M, are included as usual in this circuit. If, then, high frequency undamped waves fall on the antenna, these create currents in it which interfere with the high frequency currents of different frequency created by the local H.F. alternator G. The ordinary rectifier and telephone circuit coupled to the antenna coil B′ is then affected by a current having a

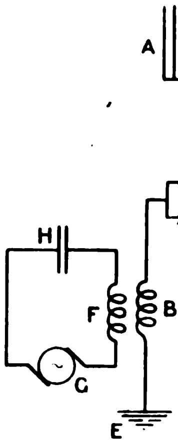


FIG. 73.

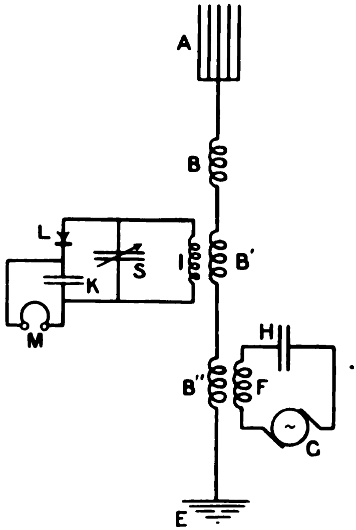


FIG. 74.—Scheme of Connections of the Receiving Circuit in the Fessenden Heterodyne Receiver.

frequency equal to the difference of the frequencies of the incident waves and of the local current. This last arrangement gives the best result. It is called the method of *beat reception*. The heterodyne receiver gives the operator full power to adjust the resultant frequency of the sound in the telephone to cut out and override the growl or noise due to atmospherics, and also can act as an interference preventer to cut out or render inaudible signals from stations with which communication is not desired if their wave-length is different from that of the station which it is desired to hear. With this heterodyne receiver signals have been easily read at a distance of 3000 miles.

The theory of this heterodyne receiver may be given as follows :—

Let $i_1 = I_1 \cos (pt - \theta)$ represent the current created in the antenna or coupled circuit by the incident waves.

Let $i_2 = I_2 \cos (qt - \phi)$ represent the current of different phase and frequency created by the local alternator. Then—

$$i = i_1 + i_2 = I_1 \cos (pt - \theta) + I_2 \cos (qt - \phi) \quad . \quad . \quad . \quad . \quad (62)$$

represents the resultant antenna current.

This may be written—

$$i = I_1 \cos(pt - \theta) + I_2 \cos(pt - \beta t - \phi) \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where $\beta = p - q$.

Expanding the right-hand side of this last equation and collecting terms in sin pt and cos pt we have—

$$i = [I_1 \sin\theta + I_2 \sin(\beta t + \phi)] \sin pt$$
$$+ [I_1 \cos\theta + I_2 \cos(\beta t + \phi)] \cos pt \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Hence by a well-known trigonometrical theorem the maximum value of $i = I$ is given by

$$I^2 = I_1^2 + I_2^2 + 2 I_1 I_2 \cos(\beta t + \phi - \theta) \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Accordingly the maximum current varies with a frequency $(p - q)/2\pi$. Also the square of the amplitude varies from

$$I_1^2 + I_2^2 + 2I_1I_2 = (I_1 + I_2)^2$$
$$\text{to} \quad I_1 + I_2^2 - 2I_1I_2 = (I_1 - I_2)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Hence the half difference of the above expressions, on which the loudness of the sound of the signal depends, is $2I_1I_2$. Accordingly it is possible to increase the loudness of the signal by increasing the local current.

This heterodyne receiver may also be applied to the detection of feebly damped waves. For if we suppose that a train of feebly damped waves is incident on the antenna, and if the group or wave train frequency is small, we can create *beats* in these damped trains by the superposition of continuous or undamped oscillations of suitable amplitude. By the use of the connections as in Fig. 74, we can create *beats* in the telephone which can have a higher frequency than the sound due to the frequency of the damped trains and therefore be distinguishable from it.

The heterodyne receiver is, however, a receiver more adapted for undamped than for damped waves.

22. Signals-making Appliances or Recorders.—In addition to the detector itself which is the device or appliance directly affected by the electric oscillations set up in the receiving circuit, we have also to associate with it some apparatus for recording this change in the detector, and making an audible or visual signal in the Morse or other code corresponding to the signals sent. The appliances used for such signal reception are: (i.) a Bell magneto-telephone either of low or very high resistance; (ii.) a Morse inker printing in dot and dash on telegraphic tape; (iii.) a Syphon recorder; (iv.) an Einthoven galvanometer sometimes including means for photographing on sensitive slip the movements of the fibre; (v.) more complicated apparatus for recording in alphabetic form or printing down the message signals.

By far the most commonly used arrangement is a double receiver Bell magneto-telephone of high resistance, say at least 1000 ohms, and better 2000 or 4000 ohms for each receiver. The telephones are attached to a steel band which fits on the head of the operator and brings the two magneto receivers opposite each ear. This leaves the operator's hands free. The magnetos are of the usual watch form in ebonite cases.

If the detector used is the Marconi magnetic detector in which the E.M.F. generated is small, then it is advisable to employ a telephone receiver, the coils of which have a resistance of only 100 or 200 ohms or so. If, however, the detector is a crystal of carborundum or a perikon or other rectifying contact or an ionized gas or Fleming oscillation valve, then the telephone coils should have as high resistance as possible, being wound with extremely fine copper wire in many turns. Receivers with resistance of 4000 ohms are commonly used. The crystal or rectifying contact or glow-lamp valve has generally in series with it a small direct E.M.F. provided by a voltaic cell shunted by a high resistance arranged as a potentiometer (see Fig. 68) and this is placed in series with the telephone and the two as a shunt across the receiver condenser. The operator has then to tune the closed receiving circuit to the antenna by shifting a contact on the inductance, or

better, by varying the capacity of the sliding or adjustable condenser. He may then have to vary the coupling of the two coils in the oscillation transformer which connects the antenna and closed receiver circuit. The operator does this with the telephones on his head. As each group of electric waves impinges on the antenna, it sets up damped oscillations in the receiving circuits, and these are rectified to that corresponding to each train of oscillations, assuming damped intermittent trains of oscillations are being used, a gush of electricity takes place, through the telephone making a short sound. If the trains succeed each other quite regularly these sounds run together to produce in the telephone a sound of pitch corresponding to the spark frequency. If that frequency is, say, 500 per second then the sound is a shrill musical note. The manipulation of the key in the transmitter circuits cuts up this sound into longer or shorter periods corresponding to the Morse dash or dot.

The operator then reads the letters by ear and writes them down as received. The operator soon learns to write down whole words at once. The process is the inverse of reading music at sight. In the latter the performer translates certain written or printed signs into audible sounds. In the wireless reception by telephone he translates the audible sounds into written letters or words. Such form of reception by ear is rendered difficult by reason of sounds called "strays," or "atmospherics," which are due to natural electric waves produced by atmospheric electricity or distant thunderstorms. These make themselves evident by squeaking or rustling sounds in the telephone. Also any signal waves which are due to other stations and are not tuned out cause interference.

When using the telephone as a signal-receiving instrument the operators are much assisted by the adoption of a suitable spark frequency and syntonized telephone. The sound made in the telephone on depressing the signalling key is really a series of very short sounds, each of which is produced by a single spark at the spark balls of the distant transmitter. Hence the note that is heard in the telephone is the result of the rapid reiteration of these short sounds. If the spark frequency is uniform and anything above a hundred per second, then, when the signalling key is continuously depressed at the transmitting station, the operator at the receiving end hears a musical note at the telephone corresponding to this spark frequency. This note only has a true musical character if the time interval between the sparks is perfectly uniform. As we have seen, this is very far from being the case when an induction coil with an ordinary or mercury break is employed, but when an alternating current transformer is used, then much greater uniformity in the spark frequency is possible. The ordinary telephone receiver is most sensitive, according to the researches of Lord Rayleigh and M. Wien, for some frequency lying between 500 and 1000. This is in part a physiological phenomenon depending on the qualities of the human ear and partly due to the telephone. Thus, Lord Rayleigh (see *Phil. Mag.*, vol. 38, 1894, p. 285, and "Theory of Sound," vol. i. p. 473) measured the alternating current in microamperes required to produce the least audible sound in a telephone receiver of 70 ohms resistance at various frequencies, and found values as follows :—

Frequency - - - - -	128	192	256	307	320	384	512	640	768
Least audible current in microamperes - - - - -	28	2·5	0·83	0·49	0·32	0·15	0·07	0·04	0·1

M. Wien found for a Siemens telephone somewhat different results, viz.—

Frequency - - - - -	64	128	256	512	720	1927	1500
Least audible current in microamperes - - - - -	12	1·5	0·13	0·027	0·008	0·013	0·024

Both, however, agree in finding a maximum sensitiveness for currents of a frequency between 600 and 700. This is partly due to the fact that the frequency of the actuating current then agrees approximately with the natural frequency of the ordinary size of telephone diaphragm. Hence, alternators for large-power radiotelegraphic stations are now designed to give currents with a frequency of about 300 or 600 alternations per second, so that when producing discharges of a condenser, the number of sparks per second may be at least 600, and fulfil the conditions for giving maximum sound in the telephone of the receiver per microampere.

Accordingly, by the adoption of this spark frequency, the sound made in the telephone is not only the loudest, but its musical character enables the operator to distinguish clearly between the sounds in the telephone, constituting the Morse signals, for which he is listening, and other sounds, irregular and often of a lower note, which are due either to atmospheric electric disturbances, or to vagrant waves of some other stations. The human ear possesses the peculiar power of paying attention to sounds of high pitch, and disregarding those of lower pitch which may be affecting it at the same moment. Attention was drawn to this by R. A. Fessenden (see U.S.A. Patent, No. 918,307 of 1908), and previously by Lord Rayleigh, in 1907 (see *Phil. Mag.*, November 1907). Fessenden states (*loc. cit.*) that, with a spark frequency of 900, messages can be read with great ease by telephone; when at a frequency of 250 they are unintelligible by reason of disturbing atmospheric discharges. One of the important characteristics of Marconi's high-speed rotating dischargers is that they can impart this high-pitched musical character to the sounds made by the distant spark in the telephone at the receiving end. It is also one of the advantages claimed for the quenched spark system when using a high frequency alternator. It is partly for this reason that the method of reception by telephone and ear has so largely superseded the method of receiving signals by the Morse printer, because the coherer detector used with the printer cannot distinguish between various classes of waves affecting the antenna in the same way that the human ear can, and it is therefore much more difficult to keep the true signals free from confusion by the interpolation of marks on the tape, due to atmospheric discharges.

In the early days of telephonic reception, when a spark frequency as low as 50 was not uncommon and when by the use of a hammer interrupter on the induction coil this frequency was very irregular, aural reception was a much more difficult thing than it is at present. The chief objection to the method now is that it leaves no automatic record of the message.

By far the larger number of commercial telegrams are sent in code, and hence if a single signal is dropped, the operator can obtain no assistance in replacing it from the sense of the context as is the case when ordinary language is used. Hence it has always been felt that an automatic record of the message was valuable. In his earliest work, Marconi employed a Morse inker to record the message which was operated by a balanced polarized relay actuated in turn by a coherer. The Morse inker consists of an electromagnet which draws down an armature, limited in play by two adjustable stops. This armature is on one end of a pivoted lever, the other end of which carries a metal inking wheel which just dips in a small vessel of printing ink. There is also a clockwork motion in a box which drives a strip of paper over the inking wheel in such fashion that, when the electromagnet draws down the armature, the inking wheel is raised, and prints on the paper tape a dot or a dash according to the duration of the current through the electromagnet. To work the printer well requires a current of about 40 to 50 multiamperes. Associated with this printer is a polarized relay.

The relay generally used is a slight modification of the Siemens polarized relay or else a British Post Office standard relay.

The construction of this polarized relay is as follows: A curved permanent steel magnet carries on one pole a pair of soft iron cores which are wound with magnetizing coils (see Fig. 75). On the other pole is pivoted one end of a light steel bar carrying a German silver prolongation. This bar is restrained from movement by two platinum tipped stop screws, and its end carries a hammer-

shaped platinum contact piece. The two stops are carried on a base movable by an adjusting screw. The permanent magnet magnetizes by induction both the soft iron cores and the pivoted steel armature bar. This latter is attracted, therefore, to one or other of the cores, whichever happens to be nearest, as determined by the position of the two stop screws, and will rest against one of them. If then a current is passed through the coils in such a direction as to weaken the polarity of the core nearest the armature bar and strengthen that of the other, the armature bar will fly over to the other stop screw. The steel armature bar is electrically insulated from the rest of the construction, but connected through its pivots with a terminal. The sliding base, which carries the stop screws, is connected also to a terminal, and one side of the end of the pivoted lever is insulated so that it makes electrical contact through platinized terminals with one of the stop screws.

Returning, then, to the action of the instrument, let us suppose that the stop screw carriage is so placed that the pivoted lever is pressing its insulated side

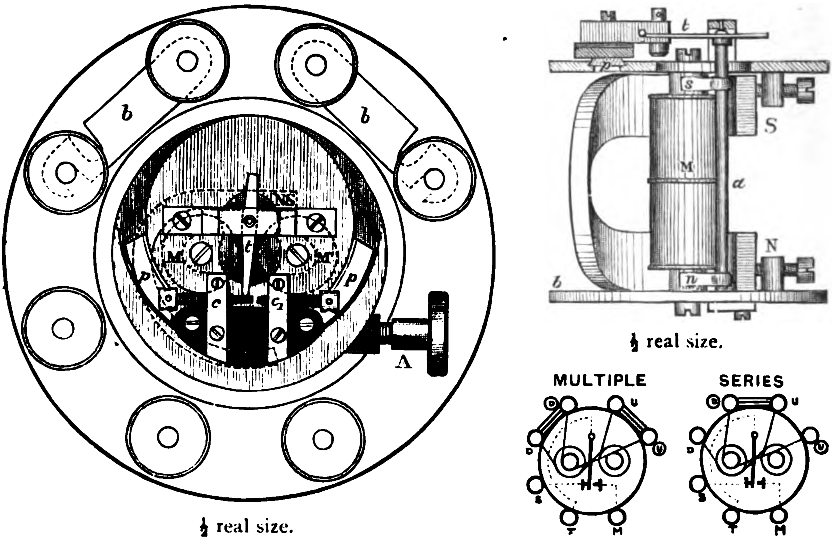


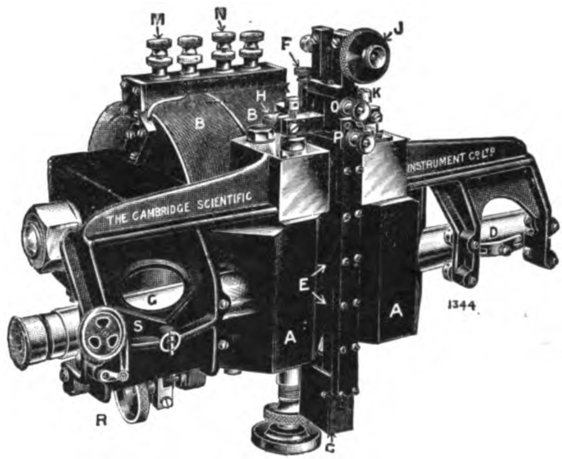
FIG. 75.—British Post Office Polarized Relay.

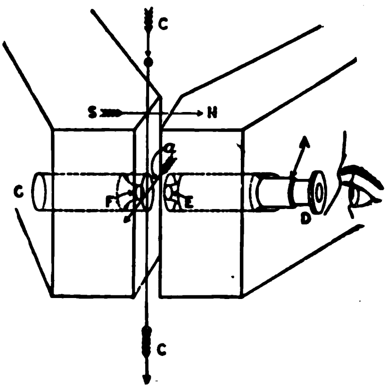
against one screw, and is being attracted most to the iron core on that same side. If a small current passes through the electromagnet coils in the right direction to weaken one pole and strengthen the other, the pivoted lever can be made to fly over against the other stop and make an electrical contact with it, and so close a circuit which sends a current through the magnet of the Morse inker. If this current, through the relay magnet coils, ceases, the pivoted lever will not go back against the insulated stop, but it can be made to do so by the tension of a very light spiral spring attached to the pivoted lever. By proper adjustments the relay contacts between the end of the pivoted lever and the screw stop contact can be made or broken by the passage of less than a quarter of a milliampere through the relay electromagnet coils. Hence a small current of this order may be made to close the circuit of 6 or 8 dry cells which can supply the current of about 40 or 50 milliamperes required to operate the Morse inker. Mr. Marconi adapted this relay for shipboard purposes by balancing the pivoted lever, so that the rolling or pitching of the vessel did not disturb its proper working. Also he enclosed it in an air-tight case so that the moisture of sea air could not deposit on the relay contact points and cause them to stick.

A rather more sensitive form of relay, known as the Post Office Standard

relay, is also used. It works on much the same principle as the above described relay. With care it can be adjusted to break and make contact with 0·1 of a milliampere through the relay magnet coils.

In the first few years of radiotelegraphy, Mr. Marconi used such a relay and inker in conjunction with his coherer and tapper (see Chap. VI. § 3), and the receiving circuits, to print down messages in Morse characters. The numerous adjustments of the tapper, relay, and Morse inker to get the best result required





[*By permission of the Cambridge Scientific Instrument Company.*

FIG. 76.—The Einthoven String Galvanometer. N, S, magnetic poles; C, C, silvered quartz fibre.

a good deal of skill, and in addition the intermixture of stray signs due to atmospheric discharges were not always easy to eliminate. Accordingly the employment of the telephone associated with a magnetic detector oscillation valve or rectifying contact, and the use of a high spark frequency, was hailed as a great improvement.

More recent recording arrangements, adapted for higher speed working, involve the use of an Einthoven galvanometer and photographic recording. As made by the Cambridge Scientific Instrument Company an Einthoven galvano-

meter consists of an electromagnet having a very strong field in a narrow interpolar air-gap (see Fig. 76). In this gap is held a silvered glass or quartz fibre. An optical arrangement enables an image of this fibre to be projected on a screen by means of a ray of light from an arc lamp which passes through suitable lenses, and through holes in the electromagnet pole pieces. If this fibre is placed in series with any oscillation rectifying device, such as a Fleming oscillation valve or a rectifying contact, oscillations can be converted into a practically continuous current flowing through the silvered fibre. The latter is then deflected or displaced across the steady magnetic field, and this minute movement is greatly magnified by the optical projection. In this manner a current of 30 or 40 microamperes can be made to produce a movement of several millimetres or even centimetres in the optical image of the fibre.

This movement can be made quite dead beat, and as the moving parts are extremely light, the fibre can follow with great rapidity the changes producing

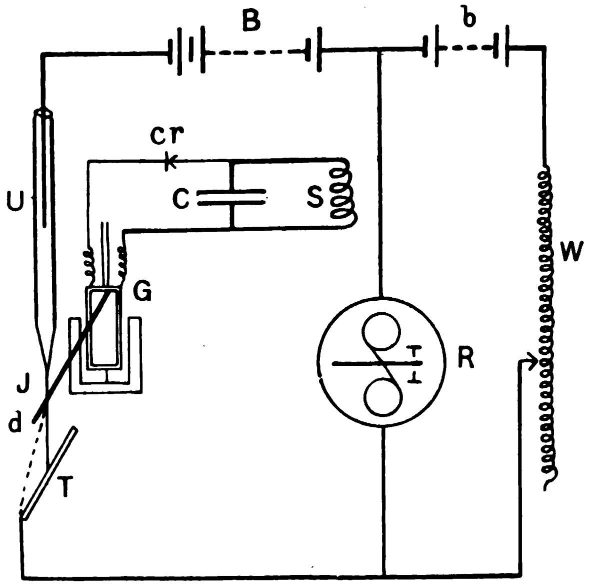


FIG. 77.—Axel Orling Jet Relay. U, jet tube; J, water jet; *d*, deflecting fibre; G, galvanometer coil; Cr, crystal rectifier; T, inclined plate; B, battery; R, electromagnetic relay; W, variable resistance.

the signals. When a *dot* signal is sent the fibre makes a sudden deflection to one side, and then springs back. When a *dash* is sent, the deflection is maintained for a short time. To record the signals the image of the fibre is allowed to fall on a strip of sensitive photographic film, a cylindrical lens being interposed to compress the image longitudinally. The film is moved forward by clockwork or a motor behind a slit in the light-tight chamber in which the film moves. It then passes through developing, washing, and fixing baths. If the fibre in the galvanometer does not move, then the film presents a white line running down the centre when developed. If the fibre is displaced by a signal, then long or short notches are cut out of this line and indicate the signals. The speed obtainable in this manner is from 60 to 100 words a minute.

Messages can also be printed down on the tape of a Morse inker by the employment of a sufficiently sensitive relay. Thus F. Ducretet and E. Roger of Paris have devised relays on the model of an ordinary moving coil galvano-

meter or syphon recorder which close a local circuit by the passage of even a current of 5 microamperes. Associated with this sensitive relay is an electrolytic detector and local cell which is set in action by the antenna current. By this combination the radiotelegraphic signals from long distance stations can be made to record themselves in Morse signals. An extremely sensitive relay has been invented by S. G. Brown. In this instrument a moving coil, suspended in a strong magnetic field, is deflected by the oscillations when rectified by any form of contact or ionized gas rectifier. The moving coil carries a long stylus ending in an iridium point, the end of which rests upon a metal cylinder. This cylinder

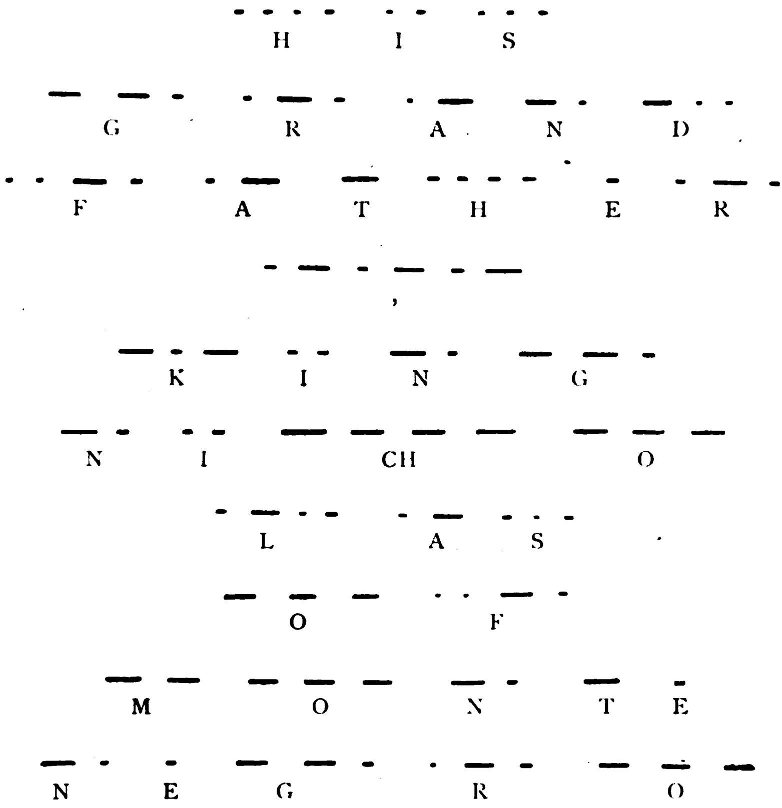


FIG. 78.—Part of a Wireless Message sent out from the Marconi Station at Coltano, near Pisa, and received at Seaford, Sussex, England, recorded by an Axel Orling Jet Relay.

is divided into two parts by a mica or other insulating sheet. This drum is continually revolved slowly by a motor, and hence any sticking of the contact point is prevented. The local circuit through the Morse inker is closed when the galvanometer coil deflects and so causes the point resting on the drum to deflect, and make contact with the metal of one side of the revolving cylinder.

Another very sensitive relay is one invented by Axel Orling, called a jet relay. This is constructed as follows: From a vessel containing slightly acidulated water, a stream of the liquid flows down a glass pipe, U, and issues as a very fine jet at the lower end. A very slender fibre of quartz is fixed into this nozzle so as to be

contained in the jet of liquid. Across the jet of liquid another quartz fibre extends, which is fixed like a bowstring across a rigid bent filament of glass. This last is in turn fixed to the moving coil of a syphon recorder or moving coil galvanometer, G (see Fig. 77).

Hence, if the coil moves ever so little, the bowstring quartz filament presses on the filament inside the jet of liquid and displaces the jet through a considerable

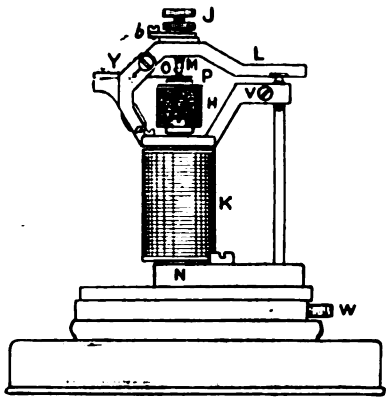


FIG. 79.—Telephone Magnifying Relay of S. G. Brown (Type A).

angle. This jet falls on to an inclined conducting plate, T, and it is easily seen that any displacement of the jet will alter the length of the liquid column which intervenes between the nozzle and the inclined conducting plate. This alters, of course, the electrical resistance of the column of liquid. This resistance change can be made to operate an ordinary relay, R, and Morse inker so as to print down Morse signals in response to currents of not more than 10^{-8} ampere passing

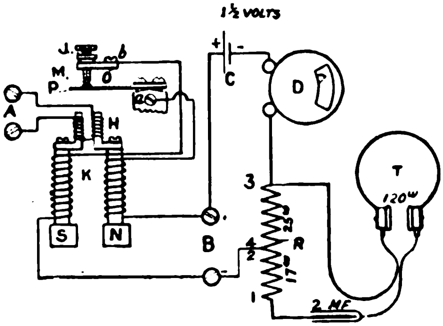


FIG. 80.—Connections of Brown Telephone Relay.

through the movable coil galvanometer, the coil of which carries the arm which deflects the liquid jet.

Hence by the use of a high resistance coil in series with a crystal rectifier, the two being shunted across the condenser in the receiving circuit of the radiotelegraphic apparatus, it is possible to print down in ordinary Morse characters the signals received even when very faint.

The illustration in Fig. 78 shows a reproduction of a part of a tape bearing such Morse recorded signals taken down with an Orling jet relay at a station in

Seaford, England, the signals being sent out from the Marconi station at Coltano, near Pisa, Italy.[47]

In cases in which it is not necessary to record signals, but when they are too weak to give good readable sounds in the telephone, a telephone relay, invented by Mr. S. G. Brown, is of great use. This relay is made as follows: The poles S, N of a permanent horse-shoe magnet carry soft-iron extension poles on which are wound two coils of wire, H and K (see Figs. 79 and 80). Through the coils H passes the feeble current to be magnified. Over the pole pieces is an elastic reed of nickel steel (invar) which has its free extremity over and close to the magnet poles. This reed carries a hard carbon block or button, O, on its upper surface, which is just touched by an iridium point, M. Through this contact

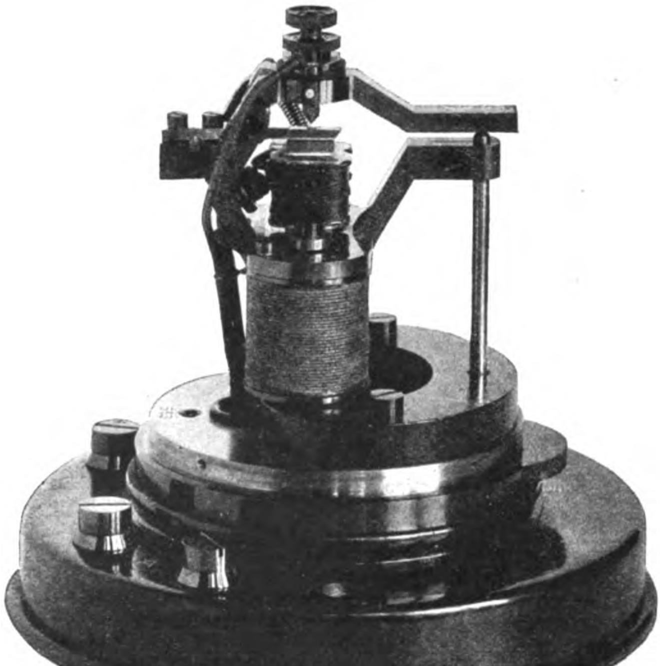


FIG. 81.—Telephone Magnifying Relay of S. G. Brown (Type A).

passes a current which circulates also round the magnet coil K, and through an ammeter and shunted telephone.

The scheme of circuits will be understood from the diagram in Fig. 80. It is clear that when the feeble currents circulate in the coils H the invar reed is set in vibration, and the variation of contact resistance at MO alters in like manner the currents through the telephone. Moreover, the current flowing across the contact helps to maintain it in adjustment just as the current through an arc lamp mechanism strikes and maintains the arc.

The external appearance of the form of relay, called type A, is shown in Fig. 81. The telephone used has a low resistance, and is connected in circuit with the relay through an auto-transformer, and a 2-mfd. condenser as shown in

[47] The writer is indebted for this sample of tape to the kindness of Mr. E. Raymond-Barker.

the diagram. The condenser excludes the continuous current and the transformer raises the voltage.

The connection with the receiving circuits and antenna is as shown in Fig. 82. The use of this relay magnifies feeble radiotelegraphic signals, so that they can be heard all over a room even when hardly audible at all without the relay. Mr. Brown has also invented another type of Telephone Relay for Wireless Telegraphy, not so portable as the A type but more sensitive.

In the latest pattern of Axel Orling jet relay (see Fig. 77) the jet falls upon a glass knife-edge and wets the two sides of it. These two liquid films then form

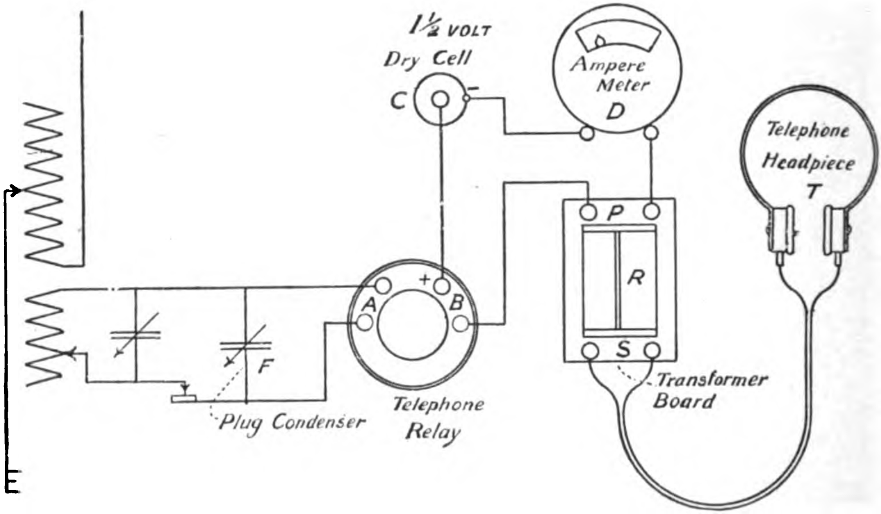


FIG. 82.—Scheme of Connections when using S. G. Brown Telephone Relay to Magnify Feeble Wireless Signals.

the two arms of a Wheatstone's Bridge, in the bridge circuit of which is placed the recording instrument. Invisibly small movements of the jet then alter the relative resistances of these two films and cause the recorder to make a signal.

These electromagnetic relays have now all been replaced as far as telephonic reception is concerned by varieties of the three-electrode thermionic valve or detector used in cascade as described in § 15 of Chap. VI. and § 4 of Chap. X. of this book. For additional information as to modern thermionic amplifiers the reader must be referred to a special treatise on the subject entitled "The Thermionic Valve in Radiotelegraphy," by the author of this book, published by *The Wireless Press*, Marconi House, Strand, London.

CHAPTER VIII

RADIOTELEGRAPHIC STATIONS

1. Radiotelegraphic Stations and Systems.—In this chapter we shall consider the combination of the various appliances which have been described in previous chapters into the complete plant for transmission and reception as required in radiotelegraphic stations. It has become the custom to speak of various types of radiotelegraphic plant as different "systems" of radiotelegraphy. The differentiation chiefly depends on whether the wave generation is dependent: (i.) on condenser discharges, (ii.) on the use of an electric arc, or (iii.) on the employment of a high frequency alternator. These methods are commonly called the spark, the arc, or the alternator systems of radiotelegraphy.

Strictly speaking, the fundamental distinction is the employment either of (i.) damped intermittent wave trains, or else (ii.) undamped or persistent waves.

In the case of the spark or damped wave train system we may employ (i.) strongly damped wave trains and a low spark frequency, a method now hardly ever used, or (ii.) feebly damped wave trains with high spark frequency, generally about 500, or else wave trains in close sequence to each other.

Besides these broad differences distinctions are sometimes made by affixing some inventor's name to a group of appliances, but this is not a scientific classification. A classification which is, however, of utility has reference to the range and object of the stations. Thus we speak of (1) coast or short distance stations intended to communicate up to 300 or 400 miles. (2) Long distance or high power stations for distances of 2000 or 3000 miles or more. (3) Ship stations on board ship. (4) Military or portable stations; and (5) Experimental stations for research work.

In order that the reader may understand the general arrangements used and methods of operating in each case, we shall describe more or less in detail the arrangements in certain typical stations of these various classes. It is no longer necessary to occupy space by descriptions of the early development of radiotelegraphic stations, as these have at present not much interest for practical workers.

2. Coast Stations.—In selecting the site for coast stations many considerations must have weight, but the locality is mostly determined by the nature of the work to be done, which is usually the establishment of communication with certain lines of shipping, or with another station across a channel. Accordingly, such radiotelegraphic stations are generally put as near as possible to the coast, frequently in isolated positions, and often situated on the summit of a high cliff. In any case a site must be selected which permits of substantial foundations for a mast and facility for erecting the same. The mast is generally a stout ship's mast constructed in three sections, and having a height of from 100 to 180 feet; steel lattice masts are also used in some places. In supporting the mast, if steel wire guys are employed, they must be cut up into lengths by insulators for the purpose of preventing oscillations of the same time period as the antenna being absorbed by them. One type of insulator employed consists of a series of balls of insulating material having grooves at right angles on them; the balls are connected by a well-tarred or paraffined rope, and the loop of the stay passes round the groove at right angles to that carrying the rope (see Fig. 1). In other cases a chain of ebonite or porcelain dead-eyes are employed, the dead-eyes being connected by a rope. The top of the mast sometimes carries a gaff, to the upper extremity of which is attached a pulley for raising the antenna. The aerial wire may consist of

hard-drawn copper or phosphor-bronze wire, preferably stranded. A single plain vertical antenna has not sufficient capacity for anything more than a short range, whilst the T-shape or L-shape antennæ have directive qualities and unsymmetrical radiative power. Hence a favourite form of antenna for coast stations is the umbrella antenna. This form of antenna has symmetrical radiation in all directions, but being a partly closed antenna its radiation decrement is not nearly so large as that of the single or multiple vertical antenna. Hence the oscillations in the umbrella form are more sustained, and the larger capacity bestows considerable energy on them.

In some cases the mast is replaced by a metal strut insulated at the bottom by being carried on a slab of marble or porcelain, and is made part of the antenna. In this case the stays supporting the strut must be insulated from it. In those cases in which a conductive earth is employed, copper or zinc plates must be sunk in the ground. The best form of earth plate is a number of radiating wires or strips of copper laid underground sufficiently deep to be in damp soil. It is always advisable to put down the earth plate in three sections not too close together for the sake of being able to make resistance tests. If, instead of an earth plate, an insulated balancing capacity is employed, it may take the form of a number of wires radiating from the mast, and it is best to stretch these at a height of about 8 feet from the ground, so as to be able to walk underneath them. These wires may conveniently radiate out from the mast to short poles about 8 feet high, arranged at a distance in a circle round it, and their outer ends are strained on to ebonite insulators.

FIG. 1.—Stay Insulator.

In some position near the mast a signalling house is erected, the size of which will depend upon the character of the work being done, and a residence for the operators will generally be erected in contiguity or not far from it. If the station is on the spark system, means must be provided for generating electric current for working either induction coils or transformers. If induction coils are used, then it will be convenient to have a small oil engine and dynamo by means of which secondary batteries can be charged, these batteries being employed to operate the induction coil. On the other hand, even in quite small coast stations it is becoming customary to employ alternating current transformers. In this case an oil engine must be put down, say, of 8 or 10 H.P., coupled direct to or driving by a pulley a small alternator, and also a direct current machine for charging cells and providing the exciting current for the alternator. This plant is conveniently kept in one room by itself under the charge of an engineer. The alternator should generate current at a pressure of one or two hundred volts with a frequency of not less than 100 and preferably 500 or 600. For the sake of security it is better to excite the alternator by means of current from storage cells which are regularly charged by a continuous current dynamo. In this way the oil engine may be employed to drive the direct current machine and charge the cells at times when it is not necessary to send messages. The cells then provide the current for exciting the alternator at any time when it is desired to signal. The current from the alternator is led through well-insulated leads into an adjacent room in which are placed the transformers, condensers, spark gap, and oscillation transformers, and to which room the aerial wire is also led. In small coast stations the condensers are preferably made with metal-coated glass plates placed in oil, as described in Chap. I. § 11. In many stations a type of tubular condenser is employed consisting of a large glass tube closed at one end and coated with silver inside and out to within a foot of the top, the glass being thicker between the silver edges than lower down, for the sake of giving greater security from puncture. If alternating current transformers are employed they must be oil insulated transformers, raising the voltage from 150 to 20,000 or 30,000 volts, and if higher voltages are required, it is more convenient to join the primaries in parallel and the secondaries in series to give the required voltage. The spark gap should be an enclosed spark gap in a silencing chamber. A third room contains a receiving

apparatus and the signalling key. It is convenient to have the switchboard in this room so that the operator when sending sees at once the voltage of his alternator and transformer and exciting current and speed of machine or frequency of alternator, and also can adjust these currents and voltages by means of appropriate rheostats and switches. He must also have a switch within reach by which he controls both the speed and voltage of the alternator and starts or stops its current. In stations on the arc system for producing undamped waves, in place of an alternator, a direct current dynamo is required giving a voltage of about 500 volts. In this case the arc apparatus, condenser, and oscillation transformer will generally be placed in the same room with the receiving apparatus, so as to be under the control of one operator. If the coast station is in communication with the General Post Office telegraph wires, then the ordinary telegraphic sending and receiving apparatus for telegraphy with wires will also be placed in the receiving room.

As a good illustration of a short distance station on the latest model, we may give the following details of the station erected by the British General Post Office at Bolt Head in South Devon, England, for communication with ships in the

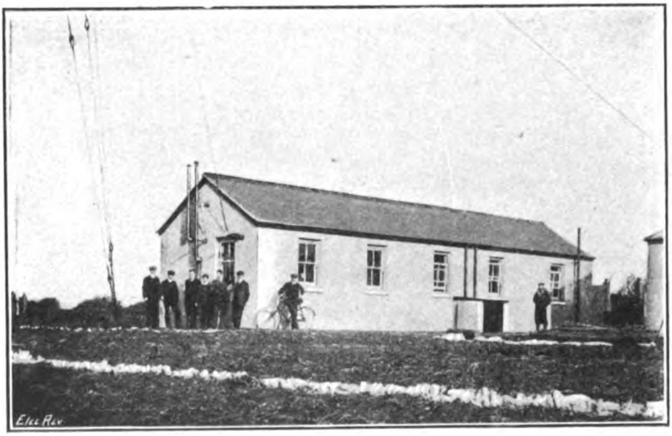


FIG. 2.—Bolt Head Post Office Radiotelegraphic Station.

Channel, opened for communication by the Postmaster-General at the end of 1908. The following account is taken, by kind permission of the proprietors, from an article in the *Electrical Review* for January 8, 1909, to whom we are also indebted for the use of the illustrations.

This station, although the seventh radiotelegraph station belonging to the British Post Office, was the first one to be opened to the public for communication with ships at sea. It is situated in South Devon, about 5 miles south of Kingsbridge, and is some 400 feet above sea-level. Its normal range of communication is 250 miles, but good communication is obtained with Scheveningen, in Holland, some 350 miles distant.

The construction and equipment of the station was carried out according to the specifications of the engineer-in-chief to the Post Office (Major O'Meara, C.M.G.), who placed contracts with Marconi's Wireless Telegraph Co. and the Westminster Engineering Co., the former for the supply of mast, aerial, etc., and the radiotelegraphic apparatus, while the latter supplied the power plant, which includes an oil engine, dynamo, secondary battery, switchboard, and the lighting of the building.

The work, which included the erection of a single-storied brick building (see Fig. 2) to accommodate the power plant and the telegraphic apparatus, was com-

menced on July 15, 1908. It was opened for work by the Postmaster-General on December 11, 1908.

Mast, Aerial, etc.—The mast is built of Oregon pine, and is in three sections: the lower mast being 73 feet in length, the topmast 56 feet, and the topgallant mast 54 feet, which, with the reduction for the overlap of the housing portions, provides a total height of 161 feet.

There are three sets of four flexible galvanized iron wire rope stays, each stay being divided into sections by porcelain insulators. The lower stays are 3 inches in circumference, are divided into two sections, and are insulated at three points; the intermediate stays are 2½ inches in circumference, are divided into three sections, and are insulated at four points; the upper set of stays are 2 inches in circumference, are divided into four sections and are insulated at five points. The separating insulators, which are shown in our view, are arranged to be in compression, so that if, for any reason, one of them should break, the stay would not part. Means are provided for regulating the tension in each stay.

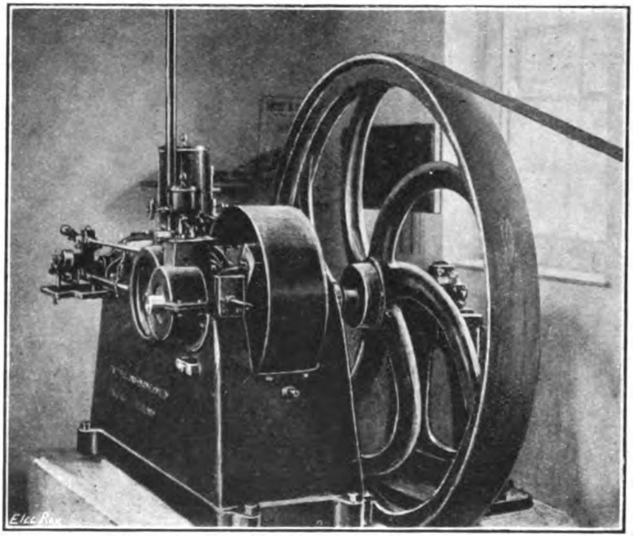


FIG. 3.—Engine in Bolt Head Station.

Four short poles about 30 feet in height are set some 150 feet distant from, and form the corners of, a rectangle around the mast, for the purpose of anchoring the aerial wires.

The aerial consists of two stranded phosphor-bronze conductors connected together at the operating room, but carried upward separately, one on each side of the mast. At the top, where they are attached to ebonite rod insulators, both conductors are bifurcated, and all four portions are then extended radially in a downward direction to within 50 feet of the four 30-feet poles already mentioned, to which they are attached by ebonite rod insulators and rope stays.

The "earth" connection is made by means of 52 copper leads, joined to 26 galvanized iron plates, 5 feet by 2½ feet in area, connected together and placed vertically in the ground, and forming a portion of a circle around the operating room.

For protection against damage by lightning, a sliding ebonite rod passing through the wall of the instrument room enables the operator to directly earth the aerial outside the building by bringing the wires against a brass ring connected to the earth plate.

The Power Plant.—A Campbell oil engine (Fig. 3), capable of developing a maximum of 10 B.H.P. when running at a speed of 265 R.P.M., is utilized to drive through a belt and friction clutch a 3-K.W. direct-current dynamo, coupled direct to an alternator. The clutch is required in order to avoid the removal of the belt when the engine is not in use, and the dynamo is being run as a motor from the secondary cells ; it also provides an easy means of starting the engine.

The D.C. dynamo (Fig. 4) provides current for charging the fifty-two secondary cells, for exciting the field of the alternator, and for lighting the building. It is also run as a motor from the secondary cells to drive the alternator when the engine is not in use.

The alternator furnishes 3 K.W. at 100 volts, at a frequency of 50.

The various connections are taken to a switchboard placed in the instrument room within easy reach of the operator, so that he can manipulate the different switches practically without leaving his seat at the operating table. A full diagram of the connections of the switchboard is shown in Fig. 5.

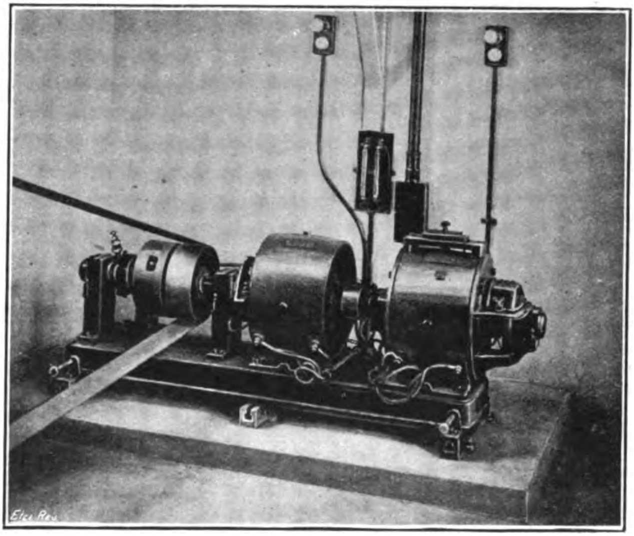


FIG. 4.—Alternator and D.C. Dynamo in Bolt Head Station.

The Radiotelegraphic Apparatus.—The switchboard connections of the radiotelegraphic apparatus are indicated in Fig. 5, and the scheme of circuit connections is shown in Fig. 6. The alternating current, controlled by means of a Morse key, is passed through a 3-K.W. transformer, placed in oil. In circuit with, and forming a shunt to the contacts of the Morse key, are four "magnetic keys" joined in parallel, which open when the alternating current is passing through its zero value, thereby preventing any injurious sparking at the key contacts, and ensuring a rapid break in the current.

A means of regulating the power used is provided by an iron-cored adjustable inductance A.

The secondary of the transformer is connected through iron-core choke coils B, and air-core choke coils C, to the battery of condensers D. These condensers are made up of thin iron plates separated by sheets of glass and immersed in oil. One side of each of the condensers is connected to the spark gap E, which is enclosed in a wooden box, while the other sides of the condensers are connected together and through the inductance F and primary G of the oscillation transformer to the other side of the spark gap.

The secondary H of the oscillation transformer is connected on one side to the aerial, and on the other to the earth plates through a small spark gap. This small gap is for the purpose of insulating the aerial in regard to received signals, whilst, as regards transmission, the spark makes the gap practically a direct earth connection.

Means are provided for varying the position of the secondary coil of the

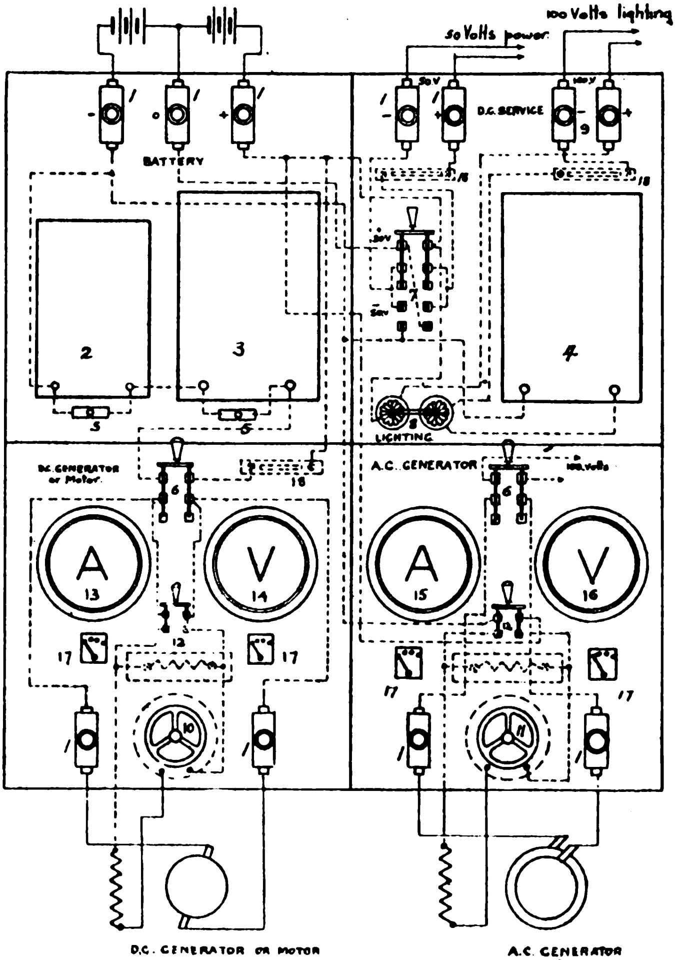


FIG. 5.—Switchboard Connections in Bolt Head Station.

oscillation transformer in relation to the primary coil, in order to vary the strength of coupling and thereby to obtain sharper resonance.

The wave-length of the transmitted signals is 600 metres, but as it may be necessary to signal with a wave-length of 300 metres, a second set of condensers J, spark gap K, tuning inductance L, and oscillation transformer M is provided. Both transformers are fitted with plug connections, so that the "aerial" and "earth" leads may be readily transferred from one to the other.

With the shorter wave-length, the oscillations from the 600-metre transformer charge the second set of condensers, which discharge through the second spark gap and the second oscillation transformer, to which the "aerial" and "earth" would in this case be connected.

The receiving apparatus is permanently connected to the transformer side of the small gap, which, as already stated, gives perfect insulation for reception purposes, and yet is practically a direct earth connection when sparks are passing. This arrangement would, however, give a loud click in the telephone whenever a spark passed, hence the Morse key is fitted with two small contacts normally separated, but which make connection and thereby short-circuit the telephone as the key is being depressed, and separate again after the sparking has ceased. Immediately these contacts open, any incoming signals pass through the receiving apparatus and actuate the telephone. Hence it is possible for the operator to be stopped in the middle of transmission if it should be necessary.

Received signals pass down the aerial through the secondary of whichever oscillation transformer is in use to "earth" *via* one or other of the two multiple

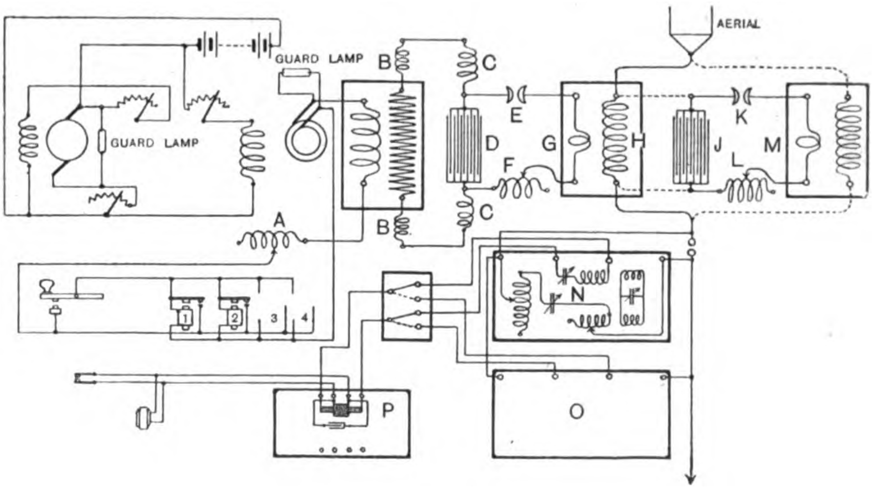


FIG. 6.—Circuit Connections in Bolt Head Station.

tuners, N or O, which are a series of adjustable inductances and capacities arranged for wave-lengths of 100 to 2000 metres in one case, and for 2000 to 6000 metres in the other.

Each tuner is provided with a switch, which connects the leads of the magnetic detector P to the "stand-by" or "tuning" positions. In the former position the coil of the detector is directly in circuit with the aerial tuning inductance, and in the latter position there is a double transformation between the aerial and the detector. The coupling of this double transformation can be varied by turning a handle at the side of the instrument. By this means it is possible to cut out to some extent the signals of another station which may be causing interference.

The magnetic detector is the well-known Marconi type of instrument, and is connected to a switch, which joins it to either tuner as desired. The headgear telephone receiver is connected to the detector in the usual way, and is also connected to the two additional contacts on the Morse key.

The coherer and Morse receiving apparatus are not used, as the station is always open and an operator is listening continuously when not transmitting.

The building (Fig. 2) is 55 feet long, 13 feet broad, and 14 feet high, and is divided into five rooms, as follows :—

Instrument room, 11 ft. × 7½ ft.
High-tension room, 13 ft. × 6½ ft.
Battery room, 13 ft. × 6½ ft.
Engine room, 24 ft. × 12 ft.
Ante-room, 11 ft. × 5 ft.

Messages are passed direct to Exeter by an overland telegraph wire.

The photographs from which these illustrations were prepared were taken by Mr. Coxon (of Major O'Meara's staff), who supervised the installation of the power plant at the station.

According to a statement made by the Postmaster-General (Mr. Sidney Buxton) in opening the Bolt Head Station, the cost of it amounted to £2000, and this may therefore be taken as an illustration of the capital outlay necessary to establish such a coast station capable of communicating 300 or 400 miles.

Since 1910, when the General Post Office took over the coast stations established in Great Britain by the Marconi Company, a good deal of work has been done by the G.P.O. in enlarging and bringing them up to date or replacing them by others. The simple induction coils used in these stations as originally established have been replaced by larger power plant so as to supply from 3 to 5 H.P., and new masts and modern receiving sets put in. As an illustration of a recent coast station plant, we may describe briefly the new G.P.O. stations at Fishguard and at St. Just, Cornwall, which have replaced the old Marconi stations at Rosslare and at the Lizard, and similar plants are, or will be, installed at North Foreland, Niton, Malin Head, and Valencia for public communication with ships.

At these stations a motor-driven alternator is installed of 3 to 5 K.W. size, having a voltage which can be regulated from 200 to 400 volts, and this supplies current to an iron core step-up transformer, which raises the voltage to 20,000 or so. The transformer is supplied through a variable choker or resistance, and has a key in its primary circuit for signalling. On the shaft of the alternator is fixed a Marconi studded disc discharger, having, say, 12 studs or arms (see Fig. 51, Chap. VII.). If, then, the alternator makes 2000 revolutions per minute, this discharger produces 400 discharges per second. The discharger is in series with a condenser consisting of metal plates suspended in oil, with or without glass plates between the metal ones. The condenser is also in series with the primary coil of the jigger, and the secondary coil is in series with the antenna. The capacity and inductance of these circuits is arranged so that by means of changing plugs or switches the wave-length radiated can be made either 300 metres or 600 metres at pleasure. The antenna at Fishguard is supported by a three-part wooden mast 150 feet high, and sustains a 4-wire fan aerial.

The receiving arrangements are as usual for telephonic reception, and circuits are so arranged that either a crystal detector or a magnetic detector can be used as desired.

The soil around the antenna is very rocky at Fishguard, and hence an extensive earth plate was put in. It consists of twenty-four plates of galvanized iron, each 5 feet by 2½ feet, bolted together and arranged to form two semicircles of 20 feet radius. Stout copper wires are connected to the plates, three to each plate, and these converge to a central earth terminal, and the earth wire is led into the station through an arrester gap. The station and mast are erected at the top of the cliffs near the harbour village, and the necessary power to drive the motor connected to the alternator is obtained from the Great Western Railway Company's generating station at the harbour. This station has a range of 250 miles by day and three or four times greater by night.

In cases where the station is not within reach of a supply of electric current from outside, a completely self-contained plant has to be put in. The arrangements at the new G.P.O. station at St. Just are then followed. In this case the mast is a steel lattice mast in three parts, 200 feet high, and at St. Just two such masts are erected (see Fig. 7). The sections of the mast telescope into each other, and the

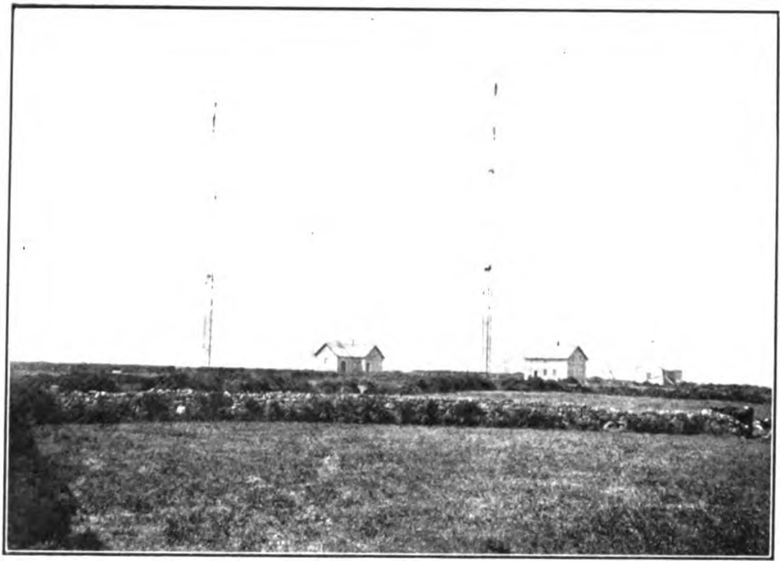


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FIG. 7.—G.P.O. Radiostation at St. Just, Cornwall.

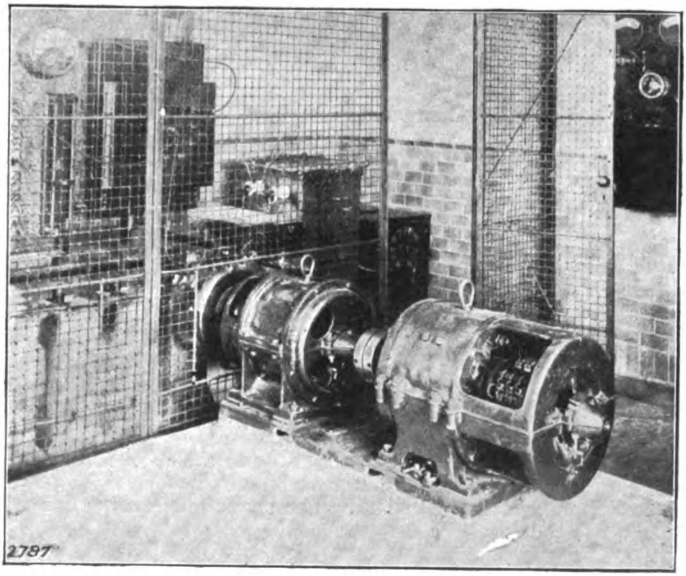

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FIG. 8.—Motor Alternator Plant in St. Just Wireless Station, fitted with Marconi Rotating Discharger on Alternator Shaft.

mast is stayed by 2½-inch steel ropes anchored in reinforced concrete blocks. One mast carries a 600-feet wave and the other a 300-feet wave antenna. These antennæ are made of copper stranded wire 7/19 in size, and are in the form of cages. The earth system consists of ten galvanized iron plates, 6 feet by 3 feet, arranged in two segments of a circle of 80 feet radius. The plates are buried vertically, and each connected to a copper wire which runs to the common terminal.

In the St. Just station the prime motors are two high-speed 10 H.P. oil engines with speed of 800 R.P.M. In each engine the carburettor is a single jet automatic with exhaust-heated vaporizer. It is arranged to start up with petrol, and then to change over to paraffin when warm. High tension magneto ignition is employed. Each engine drives a 5-K.W. D.C. dynamo, giving 100 to 140 volts at a speed of 700 revolutions per minute. This charges a battery of 52 cells, having 378 ampere-hours capacity at the 9-hour rate of discharge. The battery supplies power to the motor, driving an alternator of 5 K.W. size, carrying on its shaft a studded disc discharger. A view of this motor alternator plant is shown in Fig. 8.

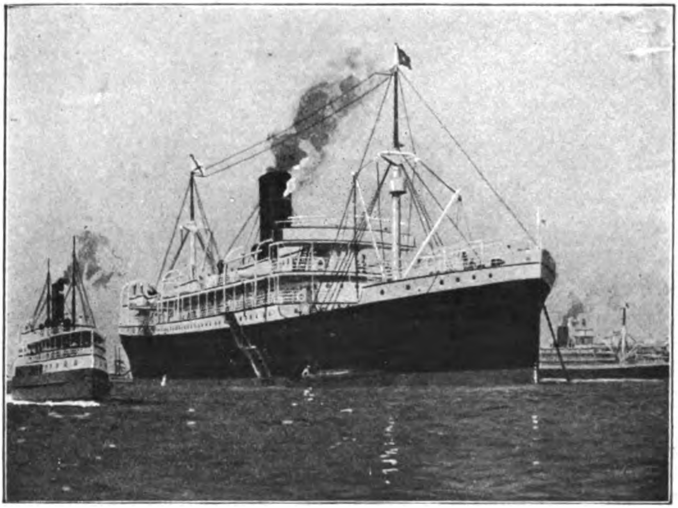

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FIG. 9.—The S.S. *Christopher* showing the form of Aerial used on large ships.

The receiving arrangements comprise a multiple Marconi tuner and crystal and magnetic detector, with telephonic reception. The receiving room is some little way from the motor alternator and dynamo room, so as to preserve silence.

The signalling is carried out by an electromagnetic key worked from the receiving room, which interrupts the current on the primary or low tension side of the alternating current step-up transformer. All the other arrangements of the transmitter are as described for the Fishguard station.

The reason for employing storage batteries to run the motor alternator is that without this arrangement the alternator could not be driven at a sufficiently uniform speed to give a musical spark. The spark frequency employed is from 300 to 600 in these stations.

3. **Ship Radiotelegraphic Stations.**—In ship installations, owing to the confined space, the wireless plant has to be very compact and also designed with the object of being thoroughly reliable in all cases. A cabin is therefore set apart for this work on deck and near the bridge. The aerials are generally of cage form and slung between fore and main mast with wires brought down to the cabin so as to form a T-aerial (see Fig. 9). On battleships or for long-distance transmission

the aerial is a six-wire cage and is often brought down some way fore and aft. The leading-down wires enter the wireless cabin through tubular ebonite insulators well protected from spray by metal hoods, and the aerial wires are sustained on the masts by capped ebonite insulators with ebonite flanges made as described in Chap. VII. The metal sheathing of the ship's hull forms the "earth." The aerial wires are generally of phosphor-bronze in the form of standard 7/20 or 7/22 cables, and when multiple wire cages are employed the wires are kept apart by strips of well varnished wood.

The aerial wires must be kept from contact with the rigging. The same aerial is used for sending and receiving, being normally always connected to the receiver.

As a general rule the transfer of the aerial from the sending to the receiving apparatus is perfectly automatic and managed as follows. The sending key has a back contact, and when it rests on the back contact the circuit of the receiving apparatus is closed. Also in circuit of the aerial wire there is a pair of spark points separated by a small air gap. The receiving circuit terminals are connected on either side of this gap. Hence when the sending key is not depressed the apparatus is in a condition to receive messages. If, however, the sending key is depressed the first thing is that the circuit of the receiver is opened. Then the E.M.F. in the sending transformer causes a spark to jump across the above-mentioned gap and puts the antenna to earth for sending, so that the mere manipulation of the sending key cuts out the receiver and sets up the oscillations in the antenna.

The signalling cabin is supplied with electric current from the ship's electric lighting circuit, which is almost always on the direct current system.

For small distances and small ships this current is used to charge sets of storage cells, which are then used to work an induction coil and interrupter and the usual form of spark discharger. In all ships a battery-worked induction coil is installed as an emergency set, so that in case of collision or stoppage of the machinery in the engine-room the operator has still the means of sending wireless messages.

In large ships, such as Atlantic liners, and in battleships and cruisers, an alternating current transformer plant is employed as in the modern coast stations.

The Marconi Company make such use of a plant called a 1½-kilowatt set, made as follows. It comprises a 4-pole rotary D.C. to A.C. converter; that is, a D.C. dynamo which takes continuous current from the ship's lighting circuits, and converts it to alternating current, drawn off by a pair of brushes pressing on slip rings on the shaft. The speed of the machine is 1500 R.P.M. Hence the frequency of the alternating current is 50. The machine may carry on its shaft a Marconi disc discharger which discharges the oscillatory circuit at exactly equidistant intervals 300 to 600 times per second, or it may have an independent spark gap. This machine is put into a silencing chamber in or below the signalling cabin (see Fig. 10). In this chamber is a step-up transformer in an iron case, and on the primary or low tension side of this transformer there is a choking coil, and also an electromagnetic or relay key for making signals, which is worked by a Morse key. This last key closes a circuit which permits the passage of a current through the coils of the electromagnet of the main key, and this causes an armature to be attracted and the circuit to be closed in the primary of the transformer. When the Morse key is raised there is no spark because the circuit is still closed, and the armature of the electromagnet does not fly up and open the circuit until the current passes through its zero value. The arrangement of this relay key will be understood from the diagram in Fig. 62 of Chap. VII.

The secondary circuit of the H.T. transformer is connected to a condenser through a variable inductance and through the primary circuit of the jigger.

The antenna is not permanently earthed, but there is at the base a small spark gap, as already explained in § 20 of Chap. VII., to either side of which the receiving circuits are attached. When the transmitter is operated a small spark bridges this air gap, and effectually earths the antenna.

The sending jigger is contained in a box, and the two circuits can be changed in position so as to vary the coupling.

The receiving arrangements are as described in § 20 of Chap. VII. for telephonic reception, and comprise a Marconi multiple circuit tuner and the Marconi magnetic detector, or a rectifying or valve detector is used with high resistance double-head telephones.

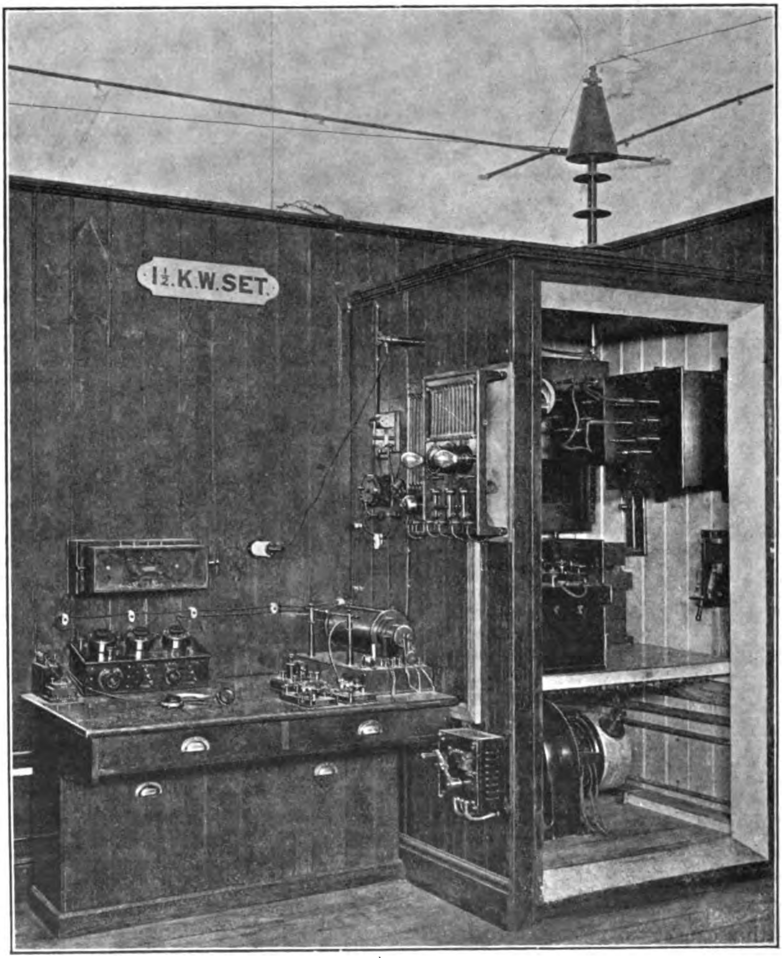

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FIG. 10.—View of 1½-K.W. Marconi Wireless plant as used on board large ships. The dynamo is seen in the silencing chamber on the right-hand bottom corner of the picture. The transmitting plant is above it and the receiving apparatus on the table to the left.

The appearance of the transmitting and receiving portions of the cabin are as shown on the right and left side of Fig. 10.

Since the reception is by telephone one operator must always be on duty to be

prepared to pick up any call which comes along. A useful addition is therefore some appliance for ringing up the operator if he happens not to be sitting with the telephones to his ears, to warn him that a message is coming.

The antenna current when rectified is too feeble to operate an ordinary electromagnetic relay, but a suitable call has been devised as follows:—

A high resistance movable coil galvanometer with balanced coil and index needle arranged so as not to deflect by the pitching or rolling of the ship has sufficient mass given to the movable part not to deflect sensibly unless the current through it is maintained for at least a second or two. This galvanometer is connected in series with a rectifying contact and joined across the condenser of the receiving circuit. The needle plays between two stops, and when it makes contact with one of them, it is made to close a local circuit and ring a bell. When the ordinary *dot* and *dash* signals are being received, the time during which the rectified current acts on the heavy galvanometer needle is too short to cause much deflection. If, however, 8 or 10 long *dashes* are sent in close succession the needle is deflected, and rings the call bell. The operator can then attach the telephone in place of the galvanometer and receive the message.

4. High Power or Long Distance Stations.—High power stations or long distance radiotelegraphic stations do not differ in essential principle from those on a smaller scale, such as ship and coast stations; but they differ in the magnitude and details of the appliances used. Apparatus which in a certain form may be called physical or laboratory apparatus has to be converted into engineering plant suitable for continuous work under all conditions of weather and time. The locality of a long distance station will generally be settled by the work to be done. In selecting the site for his first transatlantic stations, Mr. Marconi was naturally desirous of shortening the distance as far as possible between two stations in correspondence. Hence, after preliminary experiments, as already described, made at Poldhu, in Cornwall, England, and Cape Cod, Massachusetts, U.S.A., sites were selected on the west coast of Ireland at Clifden, in Connemara, and another at Glace Bay, in Nova Scotia. One consideration, which certainly ought to have weight in selecting the site for a large power station, is the liability to attack in case of war. The antenna of a power station must be carried by high towers 200 to 600 feet in height, and these, of course, are very conspicuous objects for a considerable distance off the land. As modern naval guns will send shells 10 to 20 miles, it is obvious that an enemy's ship might attack a power station when the ship is still considerably out of range of smaller guns on the coast. Hence, sites should be selected for long distance transoceanic stations which will preserve them from being the object of attack in case of war, when they might be of enormous use in communication with the national navy. A second important consideration is the nature of the ground in reference to the possibility of erecting the high masts or towers, and also the possibility of easily obtaining water and coal for steam engines and boilers. Hence, such a station should not be too far removed from railway systems.

The next step is the erection of the antenna supports. These are generally constructed of wood or steel lattice towers. Wooden towers may be constructed by bolting together a number of planks which break joint with one another, and are cross-braced by similar diagonal braces (see Fig. 11). Such towers have to be stayed with steel hawsers broken up into sections by insulators. In Paris use has been made of the Eiffel Tower for supporting the antenna of a large subterranean station near the base. If the antenna is to be of the umbrella form, a single tower suffices. If, however, it is to be a directive antenna, such as Marconi's, then at least two towers are required to support the vertical portion, and a number of masts placed in line to support the horizontal portion. As the erection of masts is a well-understood matter, it is unnecessary to give any details under this heading. The design and erection of a tower, whether in wood or metal, is a special piece of constructive work for which the aid of the civil engineer is generally necessary.

In some cases the towers are rigidly fixed at the base in foundations. In other cases the single lattice tower rests upon a ball and socket at the lower end, so that the tower is dependent on its rope stays for support, but has slight liberty to rock.

The metal lattice tower is sometimes insulated from the earth, but more generally in good conductive connection with it.

The Marconi Company have used of late years steel masts built up in flanged sections made of half cylinders bolted together. Each section is built up round a wooden mandrill, which is then raised up a stage, and serves to support at its head the tackle by which the next pair of half cylinders are hauled up and bolted round it. The mandrill is then elevated another stage, and so on until the steel mast is complete. Guys and stays of steel wire rope are used to support the mast, being attached at various heights as the mast rises. In some cases these masts are of sufficient diameter to admit of a ladder inside.

The reader will find some valuable information on the design and construction of guy-supported towers for radiotelegraphy in an article by Mr. Roy. A Weagant, in the *Proceedings of the Institute of Radio Engineers* for June 1915, vol. 3, p. 135. Also some information on wooden lattice masts by Mr. Cyril F. Elwell, in the same volume.

There are two types of antenna-supporting structures, viz. (1) the self-supported tower which, like the Eiffel Tower in Paris, relies on its own structure and breadth of base for its lateral stiffness, and (2) the guy-supported mast or tower which is held up by steel ropes. This latter may be of steel or wood; and if of steel it may be insulated from the earth, or it may be set in a concrete foundation. The insulated guy-supported strut has a ball and socket joint at the bottom, and rests on a slab of ferroconcrete or marble which is insulated from the earth.

There is an advantage in permitting a slight swaying of the tower with the wind. The relative advantages of wooden *versus* steel lattice towers has been much discussed. Wood is not suitable in some climates, and in any case the material must be very carefully selected.

The station buildings are, of course, erected in close contiguity to the base of the masts or towers, and must comprise an engine and boiler house, if steam is used, or engine and oil store, if oil is used; a transformer room; a condenser and oscillation transformer room, containing also the discharger if on the spark system, or the arc apparatus if on the arc system; and, of course, the receiving and operating room from which the messages are sent and received.

As regards the source of power, it is always desirable to employ steam engines if possible, as the turning moment of the alternator is then most uniform. The objection to a single cylinder oil engine as a source of power is that the load may be thrown on the engine by depressing the signalling key during the back stroke of the engine, or between explosions, and the result is then to impress a retardation on the engine. If oil is used, the engine ought certainly to be a high-speed multiple cylinder engine, securing a tolerable uniformity of turning moment. There is a certain advantage in the use of slow-speed steam engines with large fly-wheels, owing to the larger reserve of mechanical power stored up.

Where large supplies of water are available and a steam engine is used, it will, of course, be worked condensing. As the work is irregular, it is advisable to employ boilers of the water tube or locomotive type in which steam can be raised quickly. If the station is on the spark system the steam engine is employed to drive one or more alternators, the frequency of which ought not to be below 100 and preferably 500 or 600, unless some form of discharger is employed which multiplies spark frequency. The alternator should be of that type in which the armature is the fixed portion and the rotating portion the fields, and it should be excited by the current from secondary batteries which are charged at intervals by the current from a continuous current dynamo. The alternating current may be generated at 1000 or 2000 volts and then raised in pressure to a much higher voltage, 20,000 volts or more, by means of oil insulated transformers. It is better to have transformers of not too high a voltage, say 20,000 or 30,000 volts, and join the secondaries in series and the primaries in parallel rather than to have transformers of exceedingly high voltage in each instance. Oil insulated transformers are absolutely essential to prevent brush discharges in the interior destroying the insulation. Wires are laid from the alternator and transformer rooms to the operating room, so that indicating instruments, voltmeters, and ammeters may be placed there showing the operator at any instant the current coming out of his

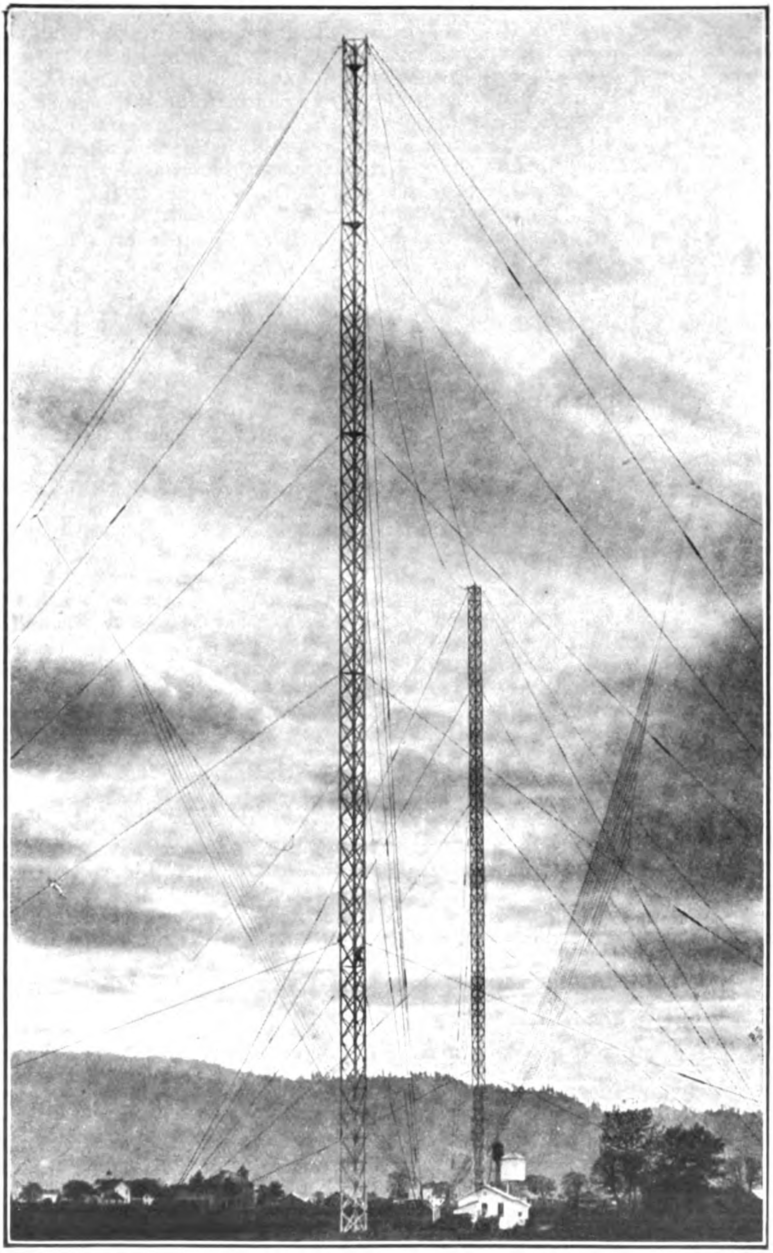

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FIG. 11.—View of Wooden Lattice Masts 300 feet high, used to support Antenna Wires or Aerial.

alternator, its voltage and frequency. It is generally best to separate the engine and boiler room a considerable distance from the transformer house, and also to separate the transformer house some distance from the condenser house. In this way any accident in the condenser house causing fire will not be so likely to spread to the transformer house, and in the same way any accident to the engine in the boiler room will not be so likely to damage the transformation and condenser plant. All parts of the plant must be, of course, duplicated or triplicated, so as to provide perfect security for continuity of working and opportunity for executing repairs. In fact, the arrangements of this part of the plant are simply those of a first-rate electric supply station, with the exception that the alternating current is supplied at a much higher voltage and higher frequency than is desirable in the case of lighting or power supply.

In the case of many large higher power stations the power is supplied electrically from some distant power station, being given in the form of low frequency polyphase currents. This can then be used to drive polyphase motors coupled either to alternators or to direct current dynamos.

The condensers in the case of the spark system will consist of a number of stoneware boxes containing metal plates separated by glass plates, the whole immersed in insulating oil, as described in Chap. I. § 11, or, where space is not of much importance, air condensers can with advantage be used, consisting of sheets of galvanized iron or zinc suspended on insulators six inches or a foot apart, these air condensers being joined up in series and parallel to give the required capacity and energy storage. On the arc system the condenser is, of course, much smaller and may be an oil condenser constructed of metal plates in oil, or a compressed air condenser, but in any case should be one in which the dielectric used has no energy absorbing quality. If the antenna is inductively coupled to a reservoir circuit, the oscillation transformer will have its primary circuit in series with the spark gap and the condensers, and its secondary circuit in series with the antenna and balancing capacity. This oscillation transformer generally consists of a rope of highly insulated finely stranded wire wound on a wooden frame, the two circuits being separated from one another by glass plates or ebonite cylinders, and the whole immersed in high insulating oil. It is of the utmost importance that the circuits of this oscillation transformer, and also of the connections of the condensers with one another and with the spark discharger, should be constructed of a suitably stranded tape consisting of a high conductivity wire not thicker than No. 40 S.W.G. single cotton-covered and twisted together into a plaited tape of sufficient width. The lowest possible ohmic resistance should be obtained in all parts of these circuits, and the high frequency resistance kept down by laminating, as above described, the conductors. The use of thick strips of metal or ordinary stranded cables for connection is a great source of energy waste in the oscillation circuits.

It will generally be necessary to insert certain choking coils in the primary circuit of the high tension transformers between them and the alternator, and also in the high tension circuit of these transformers between their secondary terminals and the spark gap. The circuit which includes the secondary circuit of the supply transformers, the condenser, and the primary circuit of the oscillation transformer, must be tuned to the frequency of the alternator; that is to say, to a comparatively low frequency, and this is done by inserting appropriate inductance coils close to the high tension terminals of the exciting transformers, and these choking coils serve the additional purpose, preventing oscillations during a discharge of the condenser travelling back into the exciting transformers. On the other hand, the circuit which contains the condensers, the primary and oscillation transformer, and the spark gap, must be tuned to the frequency of the antenna circuit comprising the capacity of the antenna, the secondary circuit of the oscillation transformer, and the earth wire. Proper means must be taken to prevent the establishment of any arc discharge across the discharger by employing either the Marconi rotating discharges, or some other equivalent device. The signals are made in the case of the power plant, either by short-circuiting one of the chokers H^1, H^2 in the low tension side of the supply transformers, as shown in Fig. 12, or else by short-circuiting a section of an inductance in series with the condensers, or by cutting out some of the condensers so as to throw the condenser

or reservoir circuit out of tune with the antenna. In any case, as little change in wave intensity should be made as possible; that is to say, a sufficient change should be made in the intensity of the emitted waves to cause the receiving station in correspondence at a distance to receive the necessary intelligible Morse signals. A large throw-over switch is generally worked by a lever, by means of which the operator changes over the antenna from sending to receiving; but as this limits the traffic capacity of the station, devices have been invented for duplexing the antenna and using it simultaneously for sending and receiving.

In a British patent specification (No. 13,020 of 1911) Senator Marconi described an extremely simple method of conducting duplex radiotelegraphy or simultaneous transmission and reception, which is only, however, applicable to long-distance working. It is, in fact, now used between his stations in Carnarvon (Wales) and New Jersey, U.S.A. The following description is taken from his specification. According to this invention the transmitting and receiving instruments at each station are placed a short distance apart, which, however, is only a small fraction of the distance over which messages have to be sent and received. Each receiver and transmitter is provided with its own antenna, but the antenna of each receiver

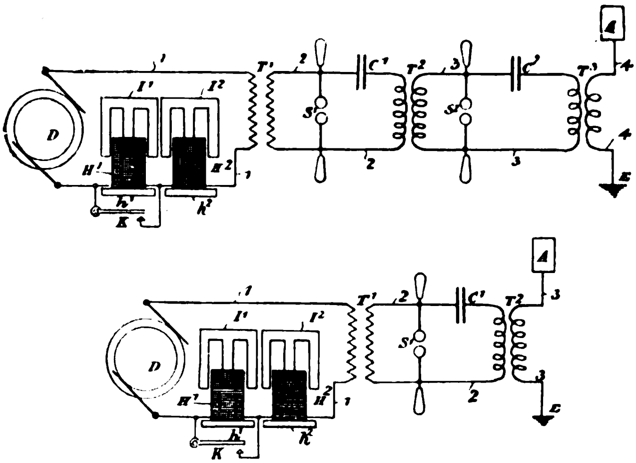

FIG. 12.—Method of Signalling by Short-circuiting Choking Coils placed in circuit of the Charging Transformer.

is twofold. Each receiver has an antenna suitable for receiving over the long-distance range, and it has also a second smaller antenna for receiving the signals from the adjacent transmitting station. These antennæ are generally bent antenna in accordance with Marconi's British patent, No. 14,788 of 1905.

The antennæ at the receiving station must have their horizontal portions set in a certain position relatively to a line joining the receiving station with the adjacent or nearest transmitter. The antenna receiving from the distant transmitter should have its horizontal part at right angles to the line joining its base to the nearer transmitter.

The other or second antenna should have its horizontal part in line with the adjacent transmitter and pointing away from it.

The two antennæ at each receiving station are called respectively the receiving antenna and the balancing antenna. These antennæ are so connected respectively through a double coil jigger with the receiving appliances that the powerful waves from the neighbouring transmitter produce two oppositely directed currents in the closed receiver circuit, and so do not affect the receiver. On the other hand, the feeble signals coming from the distant transmitter affect only one of the antennæ, viz. the larger one tuned to them and so create a signal. Simultaneous

transmission and reception can then go on at each station. In order that the oscillations produced in the two receiving antennæ by the proximate transmitter may exactly annul each other it is necessary that the phases should be in step. This is done by adjusting their distances from the nearer transmitter. It is also an advantage to tune the two transmitters to slightly different wave-lengths, so that each transmitter is not quite in tune with the adjacent receiver, but is in tune with the distant receiver.

It is usual to operate the transmitter in each station by electrical relay keys worked from the adjacent receiving station.

As regards short distance stations the solution of the problem is not quite so simple. In the ordinary method of working usual at present the operator at any one station uses the same antenna for both sending and receiving alternately. He switches this antenna on to the transmitting or receiving apparatus at pleasure to send or to receive. There is, however, always the difficulty that he may be trying to send at the same moment that his distant correspondent is sending to him, and hence confusion results. Accordingly, operators have to be careful to wait upon one another, and not to interrupt or change from receiving to sending or *vice versâ* without notice duly given.

The basis of several methods devised for simultaneous reception and transmission is to employ some mechanism for switching the antenna over rapidly and for short intervals from the transmitter to the receiver. The operator thus endeavouring to send a *dash* signal has not the antenna at his disposal for so doing continuously, but for a series of short fractions of a second in between which it is connected to his receiver, and he is then in a position to receive. In order that this process may be successful, some means have to be contrived for preventing clashing. That is to say, if there are two stations A and B in correspondence, the station A must be in a condition to receive when B is sending, and in a condition to send when B is connected for reception.

Many inventors, therefore, have endeavoured to devise synchronized mechanism which shall keep the stations in the above-mentioned alignment and correspondence. Any such mechanism is, however, extremely liable to get out of step, and to fail in its purpose.

As the author does not know of any successful practical application of such synchronizing mechanism, it is unnecessary to describe in detail any of the numerous plans proposed in patent specifications, but which have probably never been put into practice. Some methods have been devised which do not depend upon synchronization of revolving commutators at distant stations, but are of somewhat doubtful practicability.

One method was patented by J. S. Stone in 1901 (see U.S.A. Patent, No. 716,136, applied for January 23, 1901). In his specification the patentee proposes to employ two sending antennæ in which oscillations in opposite phases are excited. At a point midway between them a receiving antenna is set up, which will be influenced by arriving waves, but which will not, therefore, be affected by the opposed oscillations in the two local sending antennæ. These last, the patentee says, should preferably be placed half a wave-length apart. This, however, is quite impracticable in the case of the wave-lengths now generally employed. Also such a pair of antennæ would constitute a directive system, and radiate chiefly in the plane of the antennæ, and not at all in a direction at right angles. Hence, on these grounds the plan can hardly be considered a satisfactory solution of the problem.

Lee de Forest, in a U.S.A. patent, No. 772,879, applied for June 4, 1903, proposes another method which consists in inserting in the receiving circuit a revolving commutator which opens that circuit periodically. This commutator is driven by the alternator or interrupter which creates the spark discharges in the transmitter circuit, and is so arranged that the receiver circuit is to be open when the transmitter spark occurs, but is to be closed in the intervals between. Hence, when the sending operator closes the key to make a signal consisting of one or more spark discharges, his own receiving circuit is only closed at intervals between the sparks he is creating, and at those times can receive signals from a distant station.

By employing different spark frequencies at the two stations in correspondence, the chances of the sparks occurring at the two stations simultaneously are reduced to a very small amount. The assumption, however, which lies at the base of the proposal is that the spark intervals are perfectly regular. By the employment of the author's spark counter it can be shown that this may not be the case, and that an irregularity in spark intervals occurs even if the alternator frequency is constant.

A somewhat similar plan was proposed by R. A. Fessenden (see U.S.A. Patent, No. 793,652, applied for April 6, 1905).

The successful performance of simultaneous reception and transmission or duplex radiotelegraphy requires something more than means for switching the antenna over rapidly and alternately from the receiving to the sending apparatus, and it must dispense with any need for synchronization between commutators in distant stations if it is to be thoroughly practical.

Mr. Marconi clearly recognized this fact, and has devised a plan for use in conjunction with his own rotating discharger, which essentially depends on the fact that if the periods during which any one station is in a condition to receive are long compared with the time during which it is in a condition to send, no synchronization is necessary. This can be explained as follows: Let the long black lines in Fig. 13 represent the time intervals during which station A has its receiving apparatus connected to the antenna, and the short spaces between these lines the time intervals during which the same antenna is in connection with the transmitter at station A. Thus, if the last-named intervals are each 0·001 of a second, the longer intervals for reception may be 0·01 of a second.

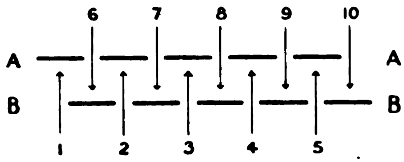

Fig. 13.

These periods for reception and transmission are made to succeed each other very rapidly and uniformly.

Suppose, then, at the station B a similar series of intervals of reception and transmission is taking place, during which the antenna at B is in connection with the receiving and sending apparatus respectively. These intervals at B may be following each other rather more quickly than those at A. We may represent this difference by supposing the interrupted line marked B in Fig. 13 to slide past the interrupted line marked A. It is clear, then, that at certain intervals of time, and for very short instants, the stations A and B will both be sending at the same instant; but if the periods of time during which they are each in a condition to receive are long compared with the time during which they can each send, the result will be that each station will, on the whole, for the greater part of the time, find the other in a receptive condition when it is sending.

Mr. Marconi's arrangement is specially adapted for use with his high-speed rotating disc discharger with studs on the disc in which short spark discharges occur at very frequent intervals, with much longer intervals between them. The method consists in using for reception all the idle intervals between the times at which sparks occur when the studs pass between the polar discs.[1] This is accomplished by rotating one or more commutators synchronously with the studded disc. These commutators consist of insulated wheels or discs C^1, C^2, having metal plates let into them and also brushes B^1, B^2 pressing against these plates (see Fig. 14) in such a manner that the brushes are short-circuited and an electric circuit closed at certain intervals during the time of rotation. In the diagram in Fig. 14 two separate antennæ (RA, TA) are shown for the sake of explanation, but it must be understood that in practice these may be one and the same. One of these, TA, is shown in connection with the condenser circuit which includes the rotating discharger, and the other, RA, is shown in connection with the receiver. If the key in the circuit of the exciting transformers is depressed for long or short periods, a series of discharges will take

[1] See Marconi's British Patent Specification, No. 16,546 of 1908.

place across the gaps between the side wheels of the discharger SD^1, SD^2, and the rotating disc D carrying the studs. It will be seen from the diagram that the receiving aerial is disconnected from the receiving apparatus, and that the receiving apparatus itself is short-circuited whenever a stud of the rotating disc is in the discharge position. The sending operator, therefore, so to speak, has the antenna in his possession for short periods of time at regular intervals, and if he depresses the key, making either a dash or a dot, a greater or less number of studs will pass the gap accompanied by discharges during that time. Thus, for instance, he makes a dot by depressing the key for a short time, then three or four studs may pass the gap in that time; but if he presses the key for a long time, making a dash, a dozen or more studs may pass, each passage of a stud being accompanied by a train of oscillations from the reservoir condenser. During the time that these oscillations are taking place the antenna is, as described, disconnected from the receiving apparatus, but in the intervals between the passage of the studs it is connected, and is therefore in a condition to receive

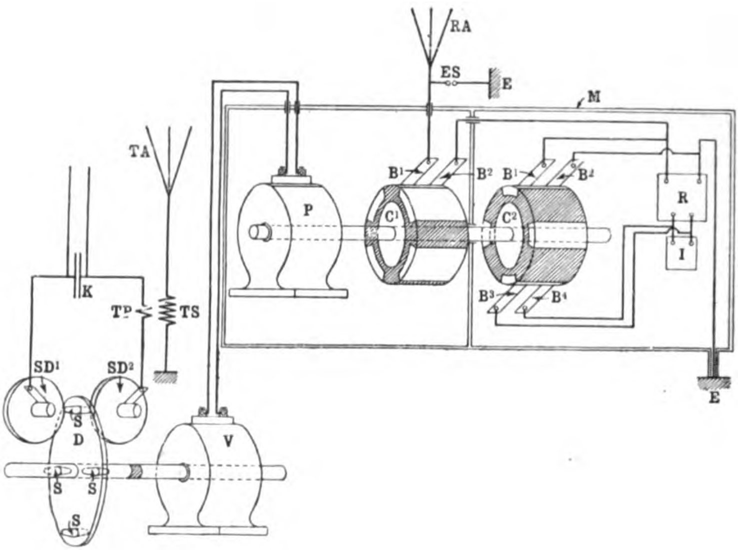

FIG. 14.—Marconi's Apparatus for simultaneously sending and receiving Radiotelegraphic Messages. V, P, electric motors coupled electrically or mechanically so as to run synchronously; D, Marconi studded disc discharger; C^1, C^2, commutators; B^1, B^2, B^3, B^4, brushes; R, receiving instrument; I, indicating instrument; TP, TS, transmitting jigger, which in actual practice is inserted in the antenna RA, just above the short-spark gap ES; the receiving instrument RI is short-circuited at the moment when a discharge is taking place at the studs S.

signals from the distant sending station. It is necessary, however, to make sure that there is a certain correspondence between the two stations in this respect, viz. that whilst one station is sending the other must be in a condition to receive, and *vice versâ*. As already explained, this can be achieved without any means of synchronizing the dischargers at both stations in correspondence, provided that the operative periods of the sending apparatus are much shorter than those of the receiving apparatus. In other words, any one period during which the antenna is connected to the receiver must be much longer than the period during which it is connected to the discharger. Hence, if a dot signal arrives to be received, a very short fraction of it may be lost owing to the operator at that station sending during certain fractions of the time represented by that dot signal, but enough will

be received to record an audible dot signal in the telephone or other receiver. There is, therefore, no necessity for any synchronization of the dischargers at distant stations. In fact, they must not rotate at the same speed, and the chances of their so doing and falling exactly into step are very small. Accordingly, by this ingenious plan Mr. Marconi evades all necessity for elaborate devices for synchronization, which, however well they look on paper, are not at all likely to give satisfaction in actual work.

We have already referred to the early work at the Poldhu station of the Marconi Company. This station has now (1919) been in active operation for more than eighteen years, and during that time its chief work has been sending Press and other messages for the Bulletins published on board the Atlantic liners. After the abolition of the early experimental plant in which an oil engine was used as motive power, steam plant was employed in the form of a tandem compound horizontal engine driving by belt a 75-K.W. single phase alternator having a frequency of 25. The condenser consisted of 400 glass plate condensers in a pattern designed by the author. More recently the steam plant has been increased by a turbine of 110 H.P. direct coupled to an alternator of 75-K.W. size having a frequency of 200. The alternator is geared to the turbine by a 10 : 1 helical step-down gearing.

Another 15-K.W. turbine supplies current for lighting and battery charging, and a third turbine of 5 H.P. is used for pumping condensing water to a reservoir. The new alternator charges a bank of 144 condensers constructed of zinc plates suspended in oil. In the old plant the slow speed rotating Fleming disc discharger was replaced in or about 1907 by a high speed Marconi discharger of the studded disc type which was separately driven. In the new plant the discharger consists of a series of metal spokes like a wheel without a rim which is attached through an insulating coupling to the shaft of the alternator. Each of the spokes of this discharger makes grazing contact as it passes with one of a pair of slowly rotating brass discs placed in diametrical positions. There are 10 spokes and the alternator speed is 2400 R.P.M. Hence it follows that there are 400 discharges of the condenser per second. The discharger is contained in a sound-proof well-ventilated box. The four wooden lattice towers originally erected to support the antenna have been demolished, and their place taken by two steel masts and four wooden ones which support a directional antenna. The two steel tubular masts are built up in 10-feet sections of pairs of half cylinders. They are 264 feet high. At a height of 250 feet they carry a triatic stay to which the aerial wires rise up from the station. The wires are then carried more nearly horizontally by triatic stays stretching between several pairs of wooden masts.

As an example of the general arrangements of modern power stations the following details may be given of those erected by Mr. Marconi for the Marconi Company, one at Clifden in Ireland, and the other at Cape Breton in Nova Scotia. A general view of the Clifden station is shown in Figs. 15 and 16, the engine and boiler house being on the right-hand side of the view in Fig. 16, the condenser house in the centre with the antenna masts above it and stretched away from it, and a residence for the operators in the foreground. The total engine power installed for spare and use is 1100 H.P.

The antenna at Cape Breton and Clifden consists of a number of wires rising 220 feet vertically, supported by masts, and then extending 1000 feet horizontally at a distance of 180 feet above the ground. The antenna is designed for a wave-length of about 20,000 feet. The condenser used has a capacity of 1·8 microfarad, and the spark length used is generally 18 to 20 mms., equal to a voltage of nearly 46,000 volts. The bent antennæ at Glace Bay, Nova Scotia, and Clifden, Ireland, are placed with their free ends pointing directly away from one another. The transmitting antenna contains 60,000 feet of wire, and the receiving antenna 18,000. The space covered by this directive antenna is about one-tenth of a square mile. The condenser employed is an air condenser formed of 1800 sheets of metal, 30 feet by 12½ feet, hung up on insulators, thereby avoiding the dissipation of energy inseparable from the use of glass condensers, and also the risks of stoppage of work involved in the puncture of a solid dielectric. The dischargers used as spark gaps in these stations are the high-speed revolving disc dischargers

invented by Mr. Marconi, already described. Owing to the regularity of the discharges, the Morse dash is heard in the telephone at the other side as a clear

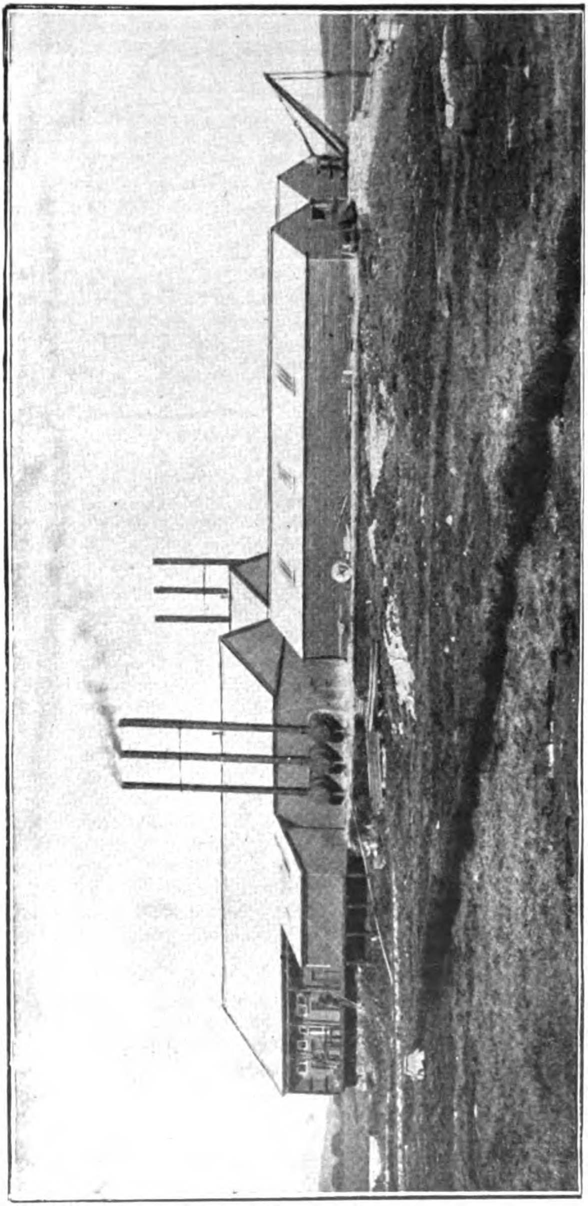

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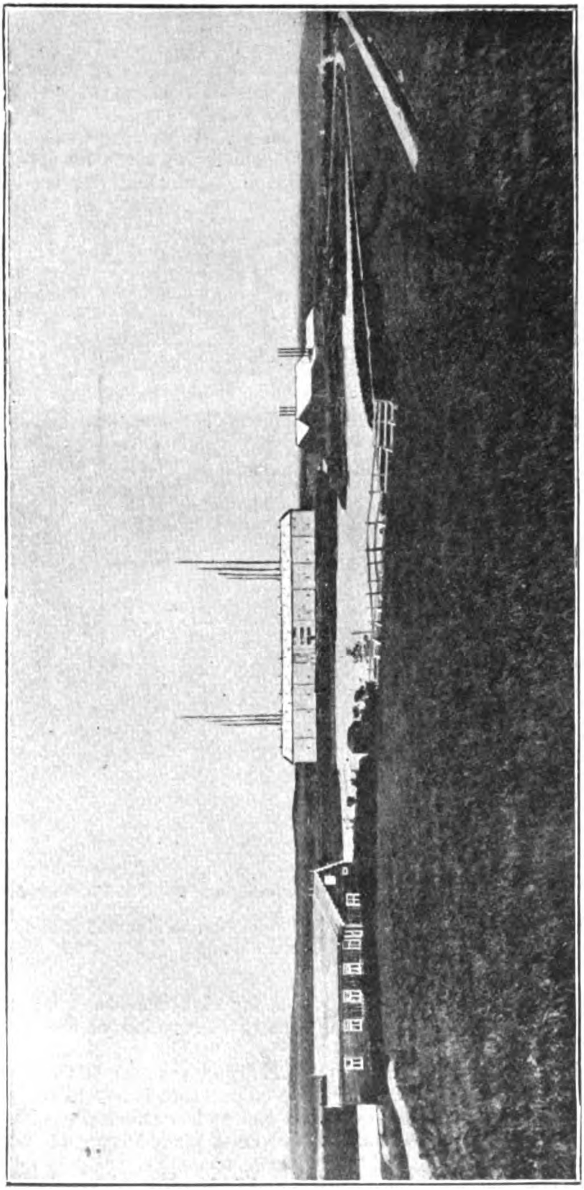
FIG. 15.—View of the Engine and Boiler-house of Marconi's Transatlantic Radiotelegraphic Station at Clifden, Ireland.

musical note, and the operator can distinguish easily between it and the irregular sounds due to atmospheric discharges.

In a lecture given on June 2, 1911, at the Royal Institution, Mr. Marconi gave the following details with regard to the Clifden station. He said that in his



[*By permission of Marconi's Wireless Telegraph Co., Ltd.*

FIG. 16.—View of the Condenser-house and Antenna of Marconi's Transatlantic Radiotelegraphic Station at Clifden, Ireland.

experience, when utilizing the best modern receivers, it was preferable to use intermittent discharges of feebly damped waves rather than continuous waves, and if the condenser discharges were perfectly uniform it was then possible not only

to tune the oscillation circuits to the wave frequency, but also the receiving telephone itself to the group frequency. Results almost as good were then obtainable as by continuous waves, provided the decrement (per half-period) of the damped waves was about 0·03 or 0·04. The condenser circuit at Clifden had a decrement of 0·015 to 0·03 for fairly long waves.

The interesting feature of the transmitting arrangements at Clifden is the employment of continuous current dynamos and a large storage battery to charge an air condenser. Several direct current generators joined in series giving a voltage of 2000 are employed to charge a battery of 6000 cells.

The capacity of each cell is 40 ampere hours. When the cells are charged the effective voltage is from 11,000 to 12,000 volts, and when being charged by the D.C. generators is about 15,000 volts. For a considerable portion of the day

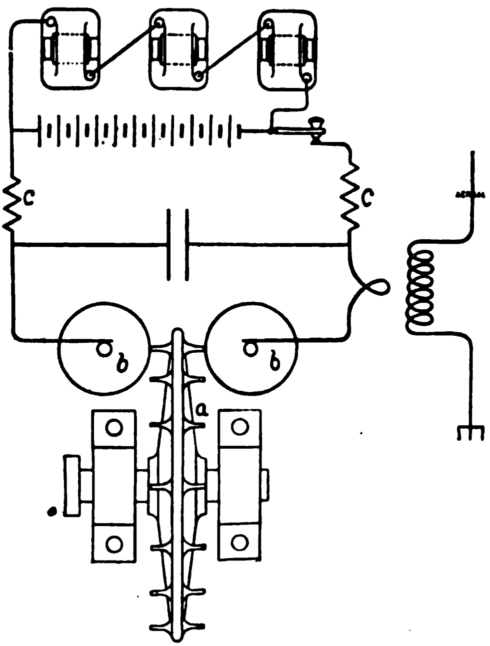


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FIG. 17.—Scheme of Connections in the Transmitter of Marconi Stations at Clifden and Glace Bay.

the storage battery alone is employed, so that for eighteen hours out of twenty-four no moving machinery is necessary, with the exception of the small motor running the disc discharger.

The potential to which the condenser is charged reaches 18,000 volts, whilst the battery alone only gives 12,000. This rise of pressure is due to the inductance of the circuit through which the condenser is charged. This high voltage battery is insulated by being divided into small sets of cells placed on stands suspended by insulators from the ceiling of the battery room.

The condenser is discharged through the jigger circuits by a studded disc rotating discharger made as described in § 18 of Chap. VII.

The scheme of connections is as shown in Fig. 17.

The studs on the rotating disc make grazing contact with the side wheels as they pass round, so that when the condenser is in the act of being discharged

the discharge circuit is a practically closed metallic circuit. The peripheral speed of the disc is 600 feet per second. Any arc which tends to form is therefore at once blown out, and the discharge circuit opened for the next condenser charge to take place. The primary or condenser discharge is therefore a highly damped or nearly quenched discharge, and the oscillations set up in the antenna circuit are its free oscillations. The energy thus transferred to the antenna nearly all goes out in effective radiation, and does not pass back into the spark gap. The wave-length of this radiation is from 5000 to 7000 metres, or about 22,000 feet, or say 4-5 miles.

Hence, since there may be perhaps 50 effective waves in each train, the wave train extends over a distance of 200 to 250 miles or more.

Since the condenser discharge is nearly quenched, the coupling of the transformer circuits can be fairly close without creating a double wave-length or making a double hump on the resonance curve. Hence the tuning at the receiving station is sharp.

The arrangements in the corresponding station at Glace Bay in Nova Scotia are similar. The directive antennæ of the kind shown in Fig. 26, Chap. VII., of the two stations on either side of the Atlantic are placed with their free ends pointing away from each other so that their radiative and absorptive powers towards each other are at a maximum.

In addition to the pair of stations at Poldhu in Cornwall, England, and Cape Cod in Mass., U.S.A., and the pair at Clifden in Ireland, and Glace Bay in Nova Scotia, the Marconi Company have recently erected two still larger stations—one at Carnarvon in Wales, and one in New Jersey, U.S.A., which are intended for the principal transatlantic correspondence between England and the United States.

The Carnarvon station is constructed for duplex working—that is, for simultaneous transmission and reception by the method employing two receiving antennæ which is described above.

As the distance across the Atlantic is nearly 3000 miles, the receiving station is placed at a distance of nearly 60 miles from the transmitting station, but the latter can be operated from the receiving station. The arrangements are, however, such that the receivers at the receiving station are not affected by the powerful waves sent out from the transmitting station, but only by the feeble waves arriving from the corresponding transmitting station on the other side of the Atlantic.

The transmitting station is situated a few miles east of Carnarvon, on the Cefn-du mountain, at a height of 680 feet above the sea.[2]

The receiving station is at Towyn, a seaside village in Wales, about 62 miles from Cefn-du.

Four wires connect these two stations, being carried on the same posts as the Post Office telegraph wires.

Power is supplied to the station by an overhead transmission line at 10,000 volts, three-phase currents being used. This power comes from the North Wales Power and Traction Company's station, situated at Cwm Dyli, about 11½ miles away. This power station obtains its power from water turbines supplied from a lake near Snowdon. The 10,000-volt pressure is reduced to 440 volts by a transformer station near the wireless station. The latter consists of a large building about 100 feet by 83 feet, divided into three parts (see Fig. 18).

The main transmitting sets which are in duplicate consist of a 300 K.V.A. single-phase alternator (see Fig. 19), supplying current at 1750 volts and 150 frequency, which is direct-coupled to a 500 B.H.P. three-phase motor taking current at 50 frequency and 440 volts from the above-mentioned power station. The motor also drives the D.C. exciter, giving 300 amperes at 40 volts. Also the Marconi disc discharger is coupled to the alternator shaft and driven in synchronism therewith.

The discharger is insulated from the alternator and from the earth, and con tained in a sound-proof chamber which is ventilated by a blower.

The alternator sends current to a bank of transformers, each of 75 K.V.A.,

[2] For the detailed description of this Marconi station the author is indebted to the courtesy of the proprietors of *The Wireless World*, published by the Wireless Press, Ltd., Marconi House, Strand, as well as for the permission to use the illustrations of it given here.

which raise the voltage, and each transformer is capable of being isolated on the primary and secondary side by switches.

The current for the motors and alternators is controlled from a main switchboard comprising ten panels, each 2 feet wide by 8 feet high, provided with all necessary cut-outs and rheostat controls for the motors.

The condensers are arranged on an upper floor, and consist of metal plates in oil.

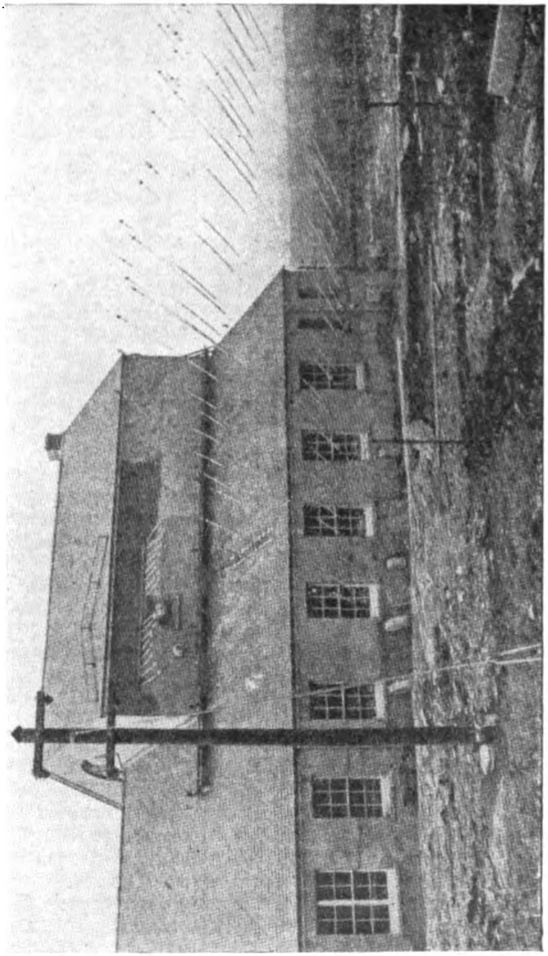

FIG. 18.—Part of Carnarvon Wireless Telegraph Station (Marconi), showing Entrance Point of Antenna Wires.

The transmitting plant is operated by relay keys from the receiving station, and these sending keys are in turn worked by automatic transmitters, in which the message is first punched out on telegraphic paper strip and then fed through the transmitter.

The main oscillation transformer or jigger is of the usual two-coil type wound with special stranded high frequency wire on insulated frames. Provision is made for altering the coupling between the circuits. The aerial tuning inductances are three in number, and provided with the necessary tappings for adjustment.

The antenna is a directional antenna which is approximately 3600 feet long and

500 feet in width. Ten steel tubular masts, each 400 feet high, form the supports. They are 3 feet 6 inches in diameter in the lower half and 2 feet 6 inches in the upper half, built in sections of 15 feet in length. Each mast stands on a block of concrete 12 feet by 12 feet by 6 feet, weighing nearly 48 tons. These gigantic masts are stayed by steel wire ropes, having insulators in them every 100 feet apart. These stays are anchored at the lower end to great blocks of concrete,

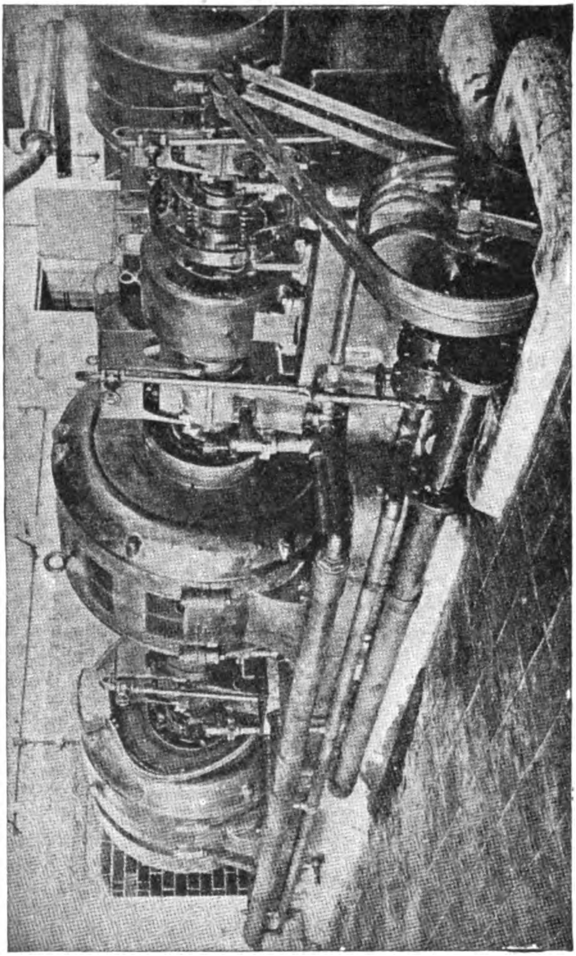

FIG. 19.—View of 300-K.W. Main Motor-Alternators in Carnarvon Marconi Transmitting Station.

each 12 × 12 × 12 feet, weighing 97 tons. Four sets of seven stays of 3-inch steel rope are used for each mast. The masts are placed in four rows of 3, 2, 2, and 3 masts 900 feet apart. The aerial wires diverge from a leading-in insulator to a line of rod insulators, supported by a triatic slung between the tops of the masts, and at the end an even tension is kept on each by a balance weight. The masts are built up the side of a mountain, and the long side of the aerial, therefore, follows the ground contour at a height of 400 feet above it (see Frontispiece).

The earth plate consists of one inner ring of metal plates surrounding the station, and a larger concentric outer ring, the two being connected by wires.

From the outer ring wires run up as far as the free end of the aerial, and from the inner ring wires are brought overhead to the main earth terminal.

The signalling switches in the transmitting station are relay switches, the arc at break being quenched by a powerful blast of air. These switches can either be worked in the transmitting station itself on an emergency, but are, as a rule, operated from the receiving station 60 miles away by means of Wheatstone automatic transmitters, controlled by punched paper, which work small current relays at the transmitting station, and these again set in action the main switches which break and make the main circuits controlling 300 K.V.A.

The whole process of sending a message by wireless across the Atlantic has thus been rendered nearly automatic by this wonderful combination of appliances, and can be conducted at the rate of 100 words a minute simultaneously in both directions. In *The Wireless World* for July 1914 the course of a message handed in at either of the Marconi Company's chief offices in London, viz. in Fenchurch Street or in the Strand, is thus described:—

Immediately it is handed in it is dispatched by special pneumatic tube to the instrument room, where it is transferred to punchers, who punch out in Morse code the message on paper tape, so that it resembles somewhat the perforated music-paper of a mechanical piano. This paper tape is then passed through an instrument called a Creed transmitter, which transmits the message electrically along a private line from London to the Marconi Station at Towyn, and the message is there reproduced automatically again in facsimile in punched paper tape. This paper tape is then passed through a Wheatstone transmitter, which operates by relays the switches at the transmitting station at Carnarvon 60 miles away, and the message is then conveyed across the Atlantic in the form of a stream of closely sequent electric wave trains interrupted in accordance with the Morse code, so as to spell out the message letter by letter. So rapidly and yet so surely does this chain of complicated pieces of apparatus work that the letters are flashed across the Atlantic at the rate of nine or ten letters a second, or 100 words a minute. These signals are physically represented by the trains of electromagnetic waves in the æther, each train composed of fifty or more waves 4 or 5 miles in length, built up into trains 250 miles long or so, the waves tailing away in amplitude. These flit over the ocean in groups of trains longer or shorter as the Morse code requires a dash or a dot. At the other end the process is reversed. The arriving waves give up their energy to the receiving antenna, and the signals as already described are received aurally by telephone, or photographically by the Einthoven galvanometer. In either case they are translated again on to punched tape by which they are transmitted to New York or other city, and ultimately reach the proper recipient.

Another interesting long distance radiotelegraphic station on the spark system is that at the Eiffel Tower in Paris. In order that the Champ de Mars, on which the Eiffel Tower stands, should not be disfigured by a building, the station itself is underground, and the Eiffel Tower acts as a support for the antenna. This station, for a long time before the European War of 1914, acted as a distributor for Greenwich time and for weather reports, as well as for news and French Government purposes.

There were then three equipments in the station, two being for low frequency spark methods and one for high frequency (musical) spark method. The main antenna consists of six galvanized steel cables which run fan-shape from the station to the top of the tower. The steel has a certain damping effect, but the cables have been found to last better than the bronze wires first used. These cables are carried by insulators at the top. The earth plate consists of a large number of zinc plates about 600 square metres in area, buried under the station foundations. There is also a shorter aerial of two wires for experimental purposes. The tower itself absorbs a certain amount of energy as the antenna wires are of about the same length, 1000 feet or so.

Power for the signalling is taken from the city supply lighting circuits in the form of alternating current at 220 volts and 42 frequency.

After passing through a switchboard and certain resistances it is fed into a transformer, or bank of transformers, which raise the pressure.

The antenna has in series with it an autotransformer coil consisting of 4½ turns of copper tube about 4 inches in diameter. The whole of this is in the aerial circuit, and a fraction, viz. about 1½ turns, is in the oscillation circuit. The condenser consists of 8896 Moscicki tubes, each able to stand 50,000 volts. These are arranged in racks (see Fig. 20). The spark gap consists of two fixed metal surfaces on which a blast of air impinges.

The signalling is effected by cutting out resistance in circuit with the feeding transformer. These resistances and transformers are shown in Fig. 21.

This main spark system employs as a rule 60 K.W., and can deal with 80 K.W. of supplied electric power. In addition to this large plant, there is a smaller one of 10 K.W. on the same low frequency spark system, and also one of 22 K.W. on the high frequency or musical spark system. This latter is operated by a Bethénod alternator, having a frequency of 600.

The high frequency spark system was recently increased by the addition of a 150-K.W. alternator, having a frequency of 1000, and the low frequency spark

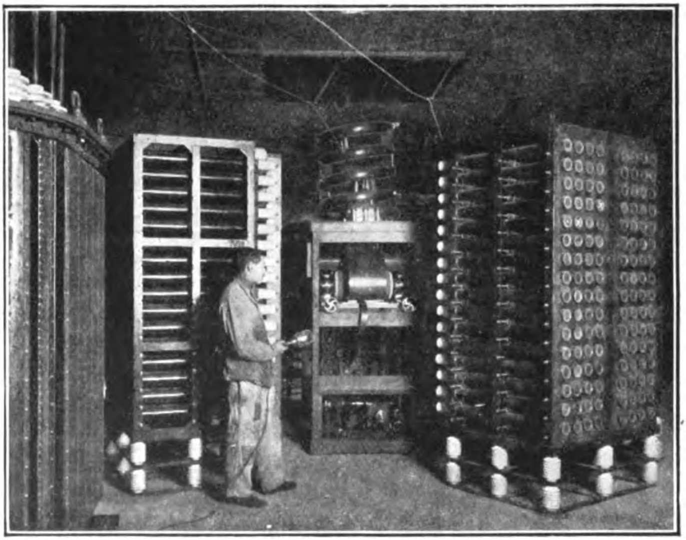

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FIG. 20.—Condensers and Spark Discharger in the Eiffel Tower Station, Paris.

system will be, no doubt, before long abandoned. The wave-length employed has generally been about 2000 metres.

To distribute time signals the following form of key, actuated by the clock of the Paris Observatory, was devised by Commandant Ferrié. A vessel containing mercury had in it a rotating electrode, R, and a mercury pump, *p* (see Fig. 22), the piston of which was depressed by a solenoid, E, through the agency of a relay by a current sent by the Standard Observatory Clock. At the instant the clock closed a circuit the pump caused a squirt of mercury to take place against the rotating electrode, and so short-circuited a resistance in series with the feeder transformer, and thus made the spark take place. Experiment showed that there was a constant delay of 0·2 second between the emission of the signal and the clock time. The vessel containing the pump is filled with coal-gas to prevent oxidation of the mercury.

This radiotelegraphic plant was employed to distribute Greenwich mean time over part of Europe and of the Atlantic, and also certain information collected by the Weather Bureau as to the barometric pressure, wind velocity, temperature, and weather at certain stations, viz. at Reykjavik (in Iceland), Valencia (in Ireland),

Ouessant (Ushant in France), Corunna (in Spain), Horta (in the Azores), and St. Pierre (in Newfoundland).

This information enabled ships at sea to obtain their longitude and certain

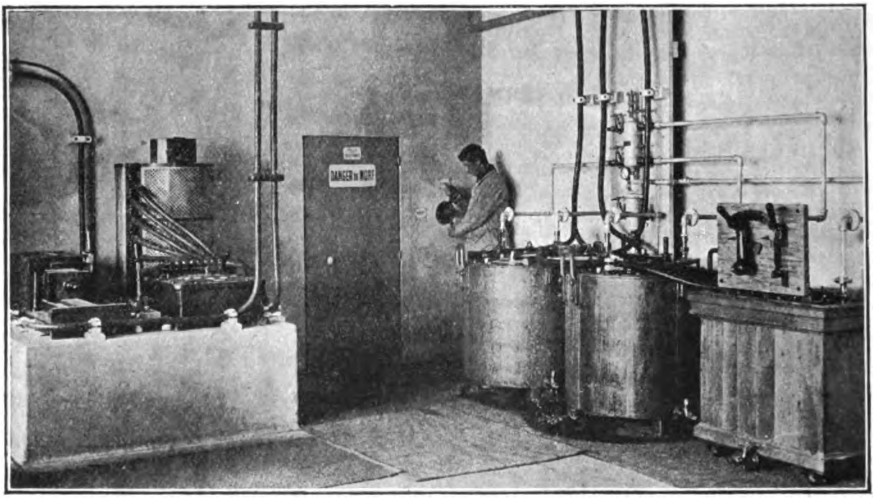

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FIG. 21.—Primary Inductances and Rheostats in Eiffel Tower Station, Paris.

useful information as to weather prospects. This system was, however, interrupted by the European War in August 1914.

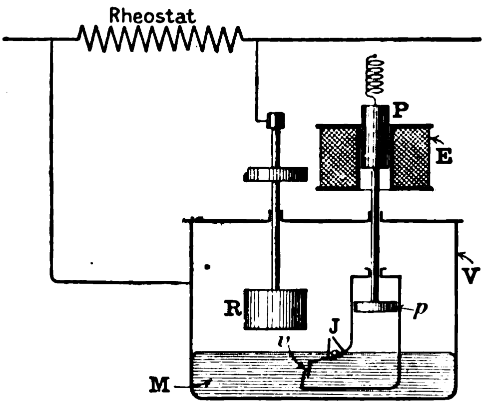

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FIG. 22.—Automatic Sending Key in the Eiffel Tower Station, Paris.

We turn, then, to consider another long-distance pair of stations established to operate with the Goldschmidt alternator, already described in Chap. I.

The typical arrangement of a high-power station on this system will be best understood from a description of the Tuckerton Station, New Jersey, corresponding with a similar station near Hanover, Germany.[3]

The principles of the Goldschmidt alternator have already been explained in Chap. I.

The diagram in Fig. 23 shows the scheme of circuits for a four circuit frequency transformation. On account of the ease of handling it is found best to incite with direct current. Hence the first alternator circuit consists of the stator carrying the D.C. current, the rotor being driven by an electric motor. This last draws its energy from two direct current generators in Ward-Leonard connection.

[3] See *Proceedings of the Institute of Radio-Engineers*, New York, U.S.A., March 1914, vol. 2, p. 69, in which a description of the Tuckerton Station is given by Mr. E. E. Mayer.

By regulating the voltage of these two dynamos very great ease of speed control and starting of the alternator is obtained. The motor at Tuckerton is of 250 H.P. and 220 volts, and has a speed of 4000 R.P.M. The Goldschmidt alternator coupled to it has an output of 200 K.W., and is intended for normal working at 100 K.W. The inductances of the tuning circuits of the alternator are cylindrical or flat spiral coils of copper tube. A general view of the alternator is shown in Fig. 24. They are in series with suitable condensers and arranged on the terminals of the stator and rotor according to the scheme of connection shown in Fig. 23. The normal speed of the rotor is such that the first A.C. current generated has a frequency of about 10,000 and this is multiplied up to 40,000 or 50,000 as explained in Chap. I.

To make the signals, an oil insulated relay switch operated by a key is inserted in the exciter circuit so as to interrupt the D.C. exciting current. The exciting power of the 200-K.W. alternator is between 5 and 10 kilowatts. To avoid changes of speed in making the signal the key not only closes the exciter circuit but also

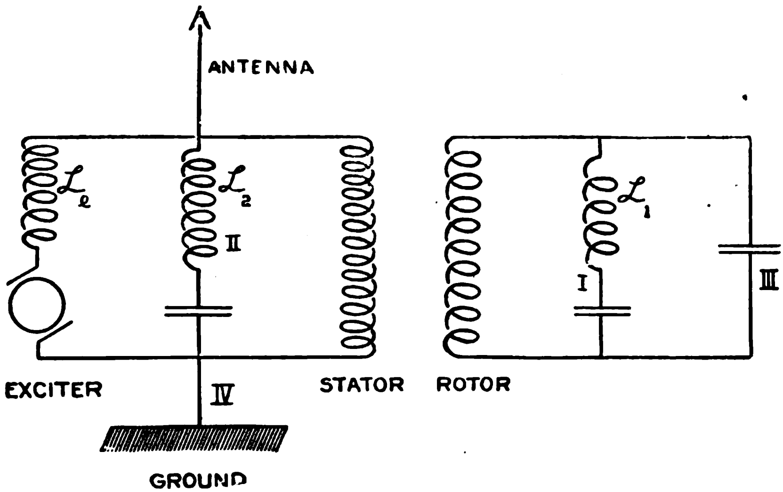

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FIG. 23.—Scheme of Circuits of Goldschmidt Alternator in Tuckerton Radio-Station.

at the same time weakens the field and speeds up the driving motor (see Fig. 25). This compensation prevents any drop in frequency on signalling. To secure efficiency the various inductive capacity circuits which take up the intermediate current harmonics are made with as large capacity and as small inductance and resistance as possible.

The antenna is supported by a steel tower 825 feet (=250 metres) high. This tower is insulated at the base on glass blocks, and there is also an insulating joint in the middle. The tower is stayed by steel guys. The antenna itself is a double cone. Around the steel tower at a distance of 1500 feet are arranged a circle of poles 40 feet high. A radial system of wires extends from the top of the tower about one-third of the way to the tops of these poles, the wires being continued by chains of insulators, and a second set of wires comes down to the base of the tower from the outer ends of the upper wires. The antenna is in series with a loading coil formed of a large copper spiral and with a variometer or variable inductance (see Fig. 26). The receiving apparatus comprises a tone wheel as already described in Chap. VII.

Using the devices above described, radiotelegraphy has been conducted over a distance of 4000 miles (=6500 K.W.) from Tuckerton to Eilvese, both by day as

well as night. A considerable difference has been found in the signal strength by day and night, and also from day to day.

Up to the present (1919) no regular commercial work has been carried on by these Goldschmidt stations.

5. Military and Portable Stations.—Wireless telegraphy has provided a new implement of great utility in military operations. It enables communication to

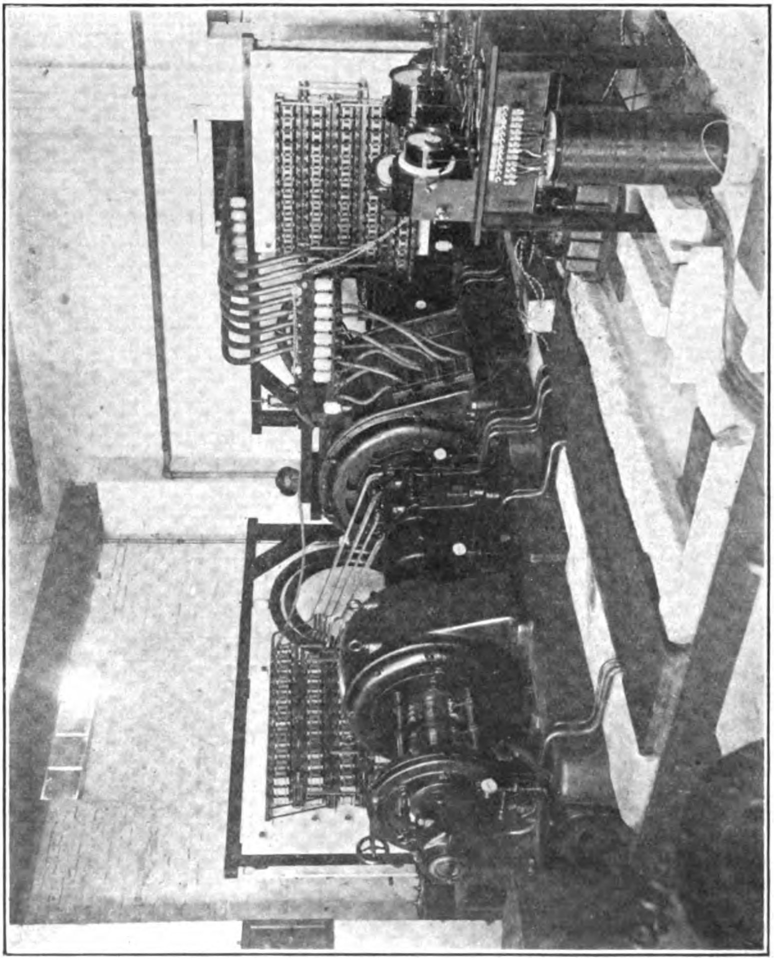

FIG. 24.—View of Goldschmidt Alternator in Tuckerton Station, U.S.A.

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be quickly established between far distant points in a country, and therefore is used to enable commanders in the field to apprise themselves of events and issue instructions without the need of laying telephone or telegraph wires as in former days. For this purpose radiotelegraphic apparatus has been designed of which all parts are made portable. Marconi's Wireless Telegraph Company have worked out in the light of special experience three types of military wireless plant, as follows :—

1st. Sets in which all parts can be carried on horseback, called cavalry or pack sets ;

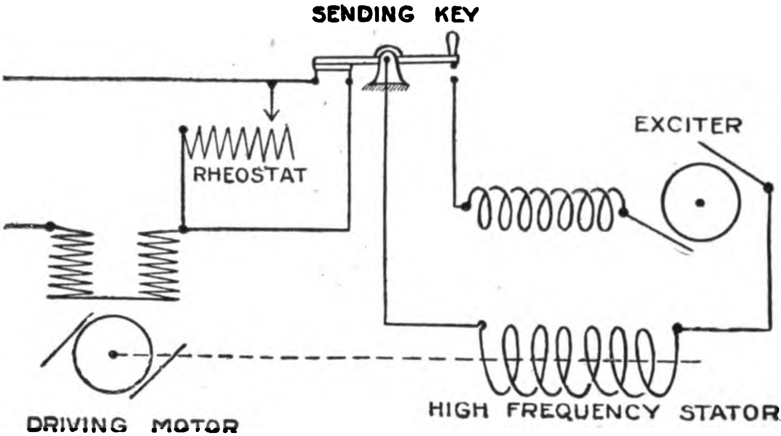

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FIG. 25.—Scheme of Connections in the Transmitting Circuit of Goldschmidt Alternator.

2nd. Sets in which all parts are contained in light carts or in motor cars, for longer distances ; and

3rd. Hand or knapsack sets in which the parts can be transported on soldiers' backs for scouting and short-distance work.

The cavalry or pack sets and small handcart sets are of such size as to enable them to communicate over 35 or 40 miles, and the larger motor-car or cart sets for 150 or 200 miles over ordinary flat country.

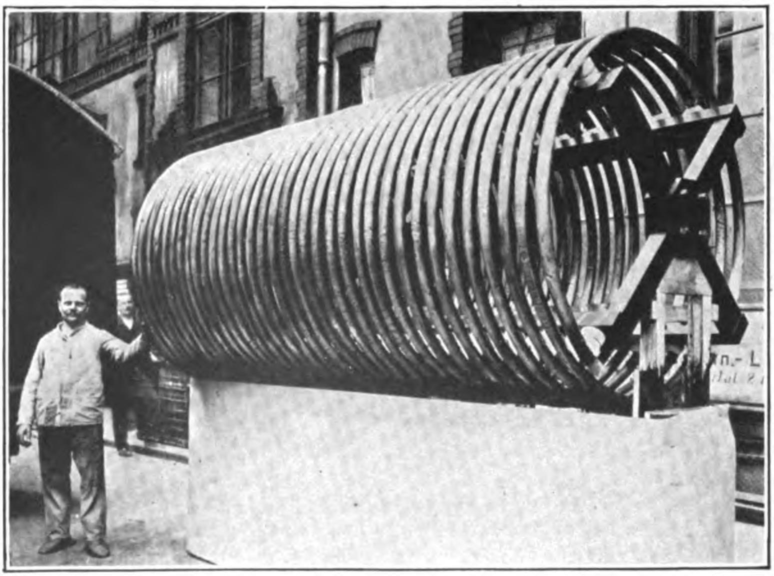

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FIG. 26.—Loading Coil at Tuckerton Station, U.S.A.

Masts used for antenna support in portable or military stations are either jointed tubes put together like fishing rods or else telescopic masts formed of a nest of concentric tubes which are extended by a screw mechanism. Preference

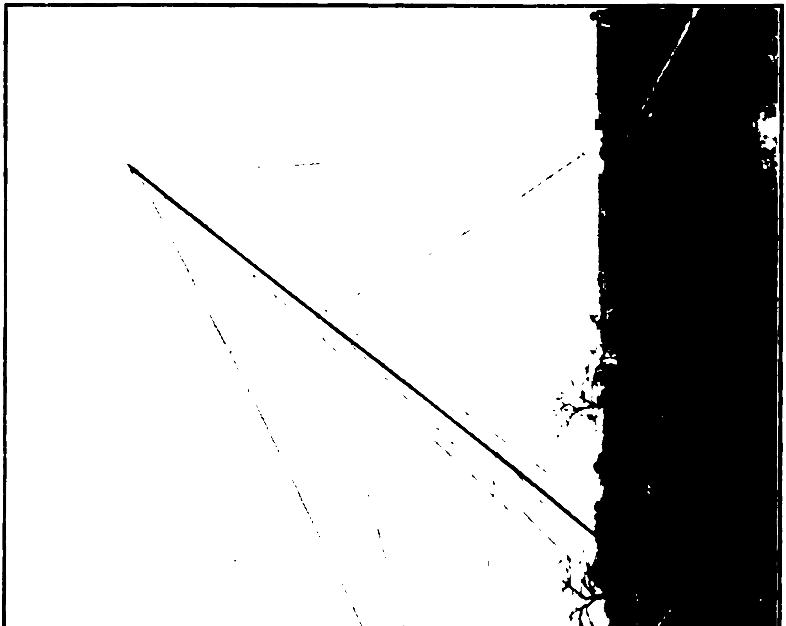

FIG. 27.—Erecting a Marconi Wireless Telegraph Military Mast in the Field.

is generally given to the former type, as less likely to give trouble by getting out of order. Also masts of moderate height sustaining directive antennæ are to be preferred to high masts carrying umbrella antennæ, since the former are less conspicuous from a distance. The Marconi Company employ steel tubes fitting together in six sections of 5-feet lengths, two such 30-feet masts being employed to sustain a multiple horizontal aerial of T or Γ shape. This provides an antenna

not requiring any large clear space of ground in which to erect it. Jointed masts of such moderate height can be put together when lying on the ground, and then raised by two men just as builders raise a ladder. A higher mast has to be erected

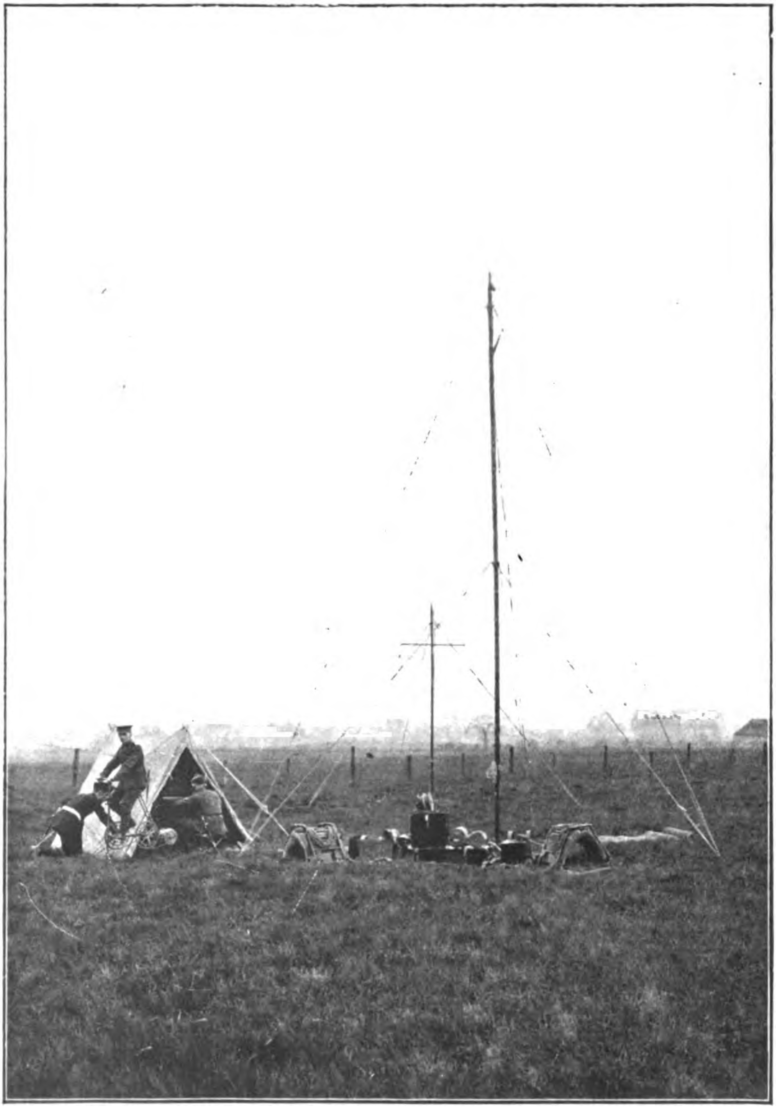

FIG. 28.—Marconi Directive Antenna and Radiotelegraphic Station for Military Wireless Telegraphy.

by jointing at the base a sprit at right angles to it and then connecting this sprit by guy ropes to the mast at various points. The mast is then raised by pivoting the right angle corner on the ground and getting a purchase on the outer end of the sprit and hauling it down with tackle. The mast is prevented from falling

over as it goes up by guy ropes at the side held by other men. The whole process will be understood from the illustrations in Figs. 27 and 28.

Turning then to the details of the apparatus employed we shall describe the pack cart and motor-car sets of the Marconi Wireless Telegraph Company as perhaps the best illustrations of plant carefully worked out in every detail in the light of experience in the field.

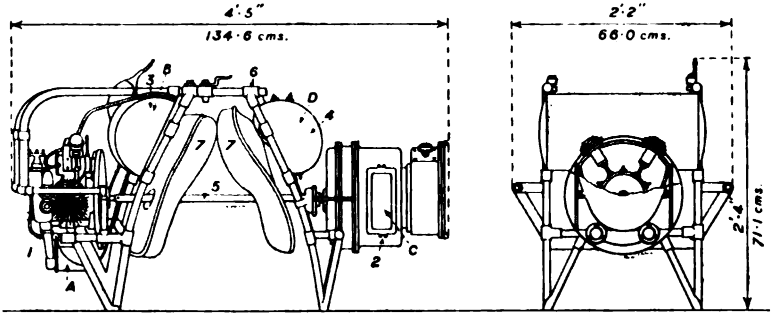

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FIG. 29.—Engine and Alternator Saddle of Pack Set of Marconi Wireless Apparatus.

The pack station is a 0·5-kilowatt size. The whole of the appliances are carried on four horses, each horse being loaded with about 200 lbs. weight (=115 kgrms.), 165 lbs. being plant and 40 lbs. saddle. Each load is divided equally into two parts to be carried on either side of the saddle. The *personnel* consists of six men or two as a minimum. The earth connection is made of several sheets

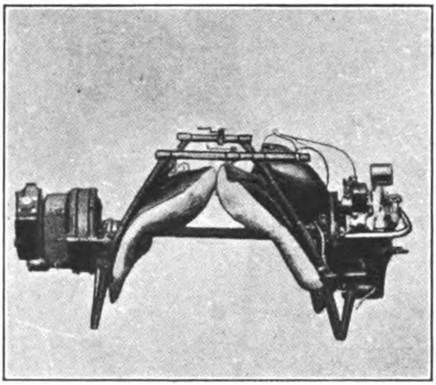

FIG. 30.

of copper gauze laid on the earth, and kept flat by a few stones laid on them, and if need be kept wet by a few buckets of water.

The generating set consists of a 3-H.P. two-cylinder petrol engine air cooled, which is permanently attached to one side of a saddle, the other side carrying a ½-kilowatt self-exciting alternator (see Figs. 29 and 30). If the saddle is lifted off the horse and placed on the ground, the engine and alternator can be connected across by a shaft, and the generating plant is then complete. The alternator carries on the outer end of its shaft a rimless wheel consisting of spokes set in a

hub. This is the revolving spark discharger. It is contained in a light aluminium case which is ventilated by a fan blower. The whole of the receiving and transmitting appliances comprising jigger or oscillation transformer, tubular Leyden jars or condenser aerial tuning inductances, step-up transformer and sending key, and also the complete receiving apparatus, are contained in two boxes carried on a second horse. These need only to be placed one above the other and coupled by flexible leads to the alternator aerial and to the earth plate to set up the

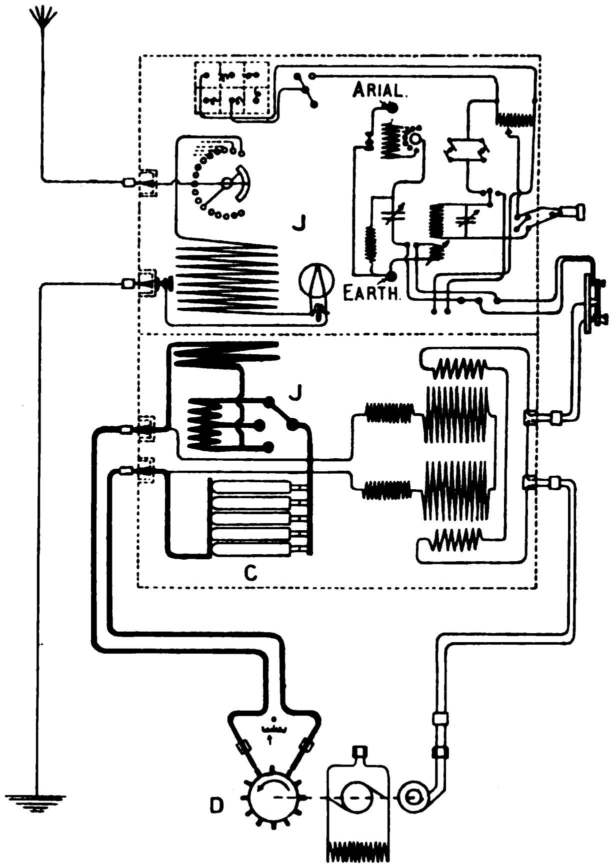

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FIG. 31.—Scheme of Connections of Marconi Pack Set of Wireless Apparatus.

D, High-speed rotating discharger; C, tubular condensers; J, jigger or oscillation transformer.

complete station. The other two horses carry the aerial wire supports and insulator and the two 30-feet steel masts divided into four loads, two for each horse.

Fig. 31 shows the scheme of connections of the whole of the receiving and transmitting plant. The detector used is a crystal (carborundum), and a pair are mounted with two-way switch in the receiver. A pair of head high resistance telephones are employed to receive.

The wave-lengths used are either 500, 600, or 700 metres, and by means of suitable subdivisions of the inductances and condensers the tune can be changed at pleasure instantly.

The same size of apparatus, 0·5 kilowatt, can be mounted up in a pair of light carts connected like a gun and its limber, arranged for traction by horse or man power.

In this case the petrol engine alternator and discharger are mounted up on one base which is carried with the masts on the rear cart whilst the apparatus boxes ride on the other.

For stationary work the whole sending and receiving apparatus can be arranged in a cabinet, as in Fig. 32.

The larger set, called a 1½-kilowatt set, is mounted up in carts or on a motor car.

The range is 150 miles over normally flat country.

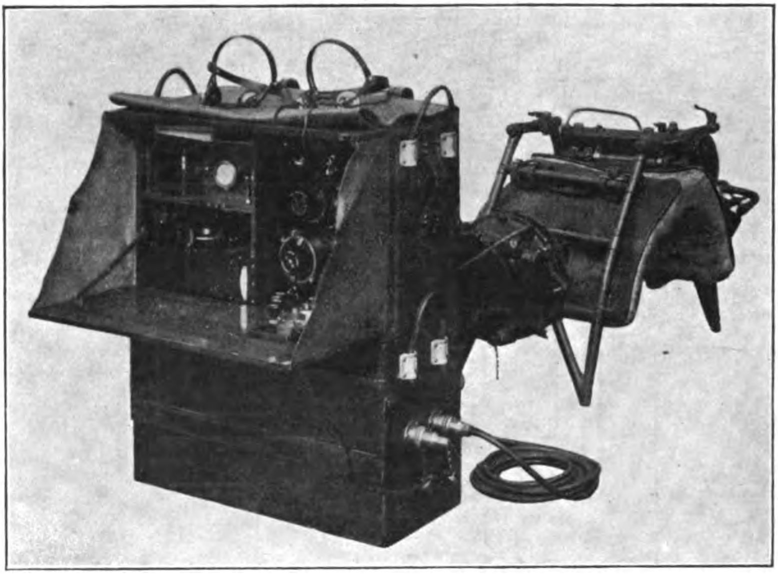

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FIG. 32.—Half-Kilowatt Marconi Wireless Pack Set with Musical Spark Transmitter, showing the Alternator and Engine on the Saddle and the rest of the Appliances in the Cabinet.

The station is carried on two carriages of the limber and waggon type drawn by four horses.

The *personnel* consists of eight men, though it can be worked by three. The station can be erected and in use within twenty minutes of halting. The masts are sectional, 70 feet in height, and two in number. They support a horizontal antenna of T-shape.

The generating set consists of a two-cylinder 8-H.P. petrol engine air cooled, which is coupled direct to a 1½-kilowatt self-exciting alternator. The shaft of the alternator also carries the revolving disc discharger as in the case of the small sets. The wave-lengths available are from 700 to 1300 metres. The total weight of the station is about 60 cwts. or 3 tons. This is divided between two double carts. The limber of the first carriage carries the generating set, complete with necessary oil tanks, switchboards, and connections. The waggon carries the

whole of the transmitting and receiving apparatus fixed and ready in two boxes. The limber of the second carriage carries spare oil tanks and tools, and the waggon carries the masts, derricks and aerials, and gear.

The alternator is a 12-pole self-exciting alternator, giving an output of 12·5 amperes at 175 volts, and the exciter part furnishes a direct current of 4 amperes at 20 volts, which is also used for charging the accumulator, used for lighting the

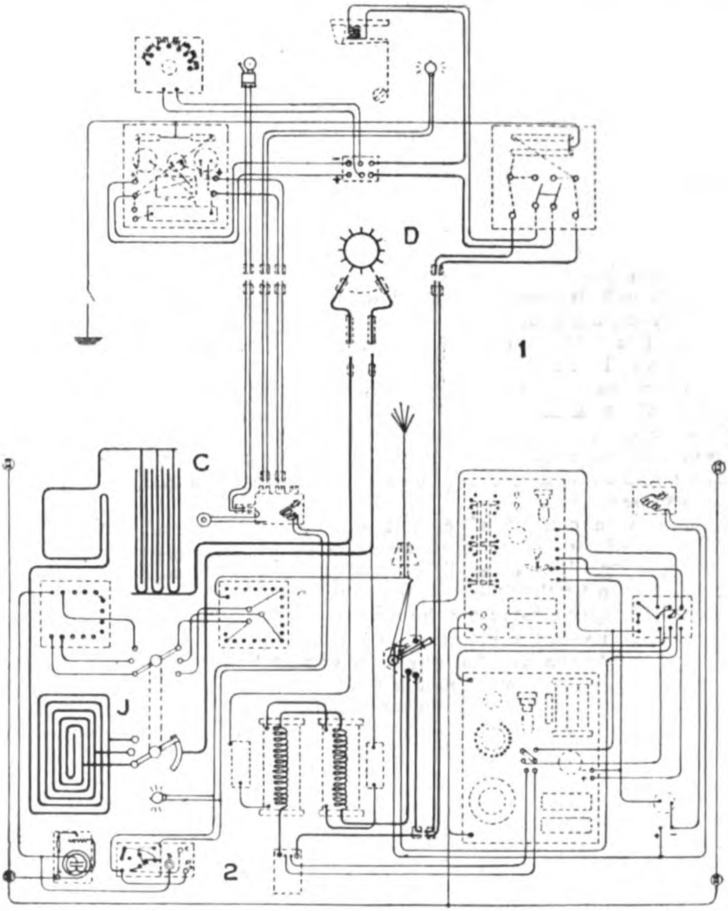

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FIG. 33.—Connections of the 1·5-Kilowatt Marconi Military Cart Set of Wireless Apparatus.

carriage lamps and igniting the filament of the Fleming valve glow lamp used as a detector in the receiver circuit.

The alternator voltage is raised by an iron core transformer to about 20,000 volts. The condenser consists of a number of Leyden jars having coatings of electrically deposited copper. The discharger consists of a disc of ebonite mounted on the end of the alternator shaft. The disc is embraced by a metal ring which carries spokes projecting from it. These spokes make grazing contact with two fixed electrodes which are so set that the discharge happens at the

instant when the E.M.F. of the alternator comes to a maximum value in its phase.

The detector used in the receiver may be either a Fleming glow-lamp valve, or a crystal detector at pleasure.

The circuit between the aerial tuning inductance and the earth is interrupted by a small spark gap. When the aerial is used for transmitting, this gap is at once bridged by a conducting spark.

The signalling key has a back contact which when raised to send a signal interrupts the receiving circuit. Hence on depressing the signalling key which is in the primary circuit of the step-up transformer, the receiver circuits are automatically cut out. If, however, the key is not depressed, then the receiver is in connection to receive. The whole of the receiving and transmitting apparatus is permanently fixed in the carts, so that all that has to be done in the field is to start the engine, connect up by flexible conductors the alternator, transmitter circuits, and aerial and earth, and also connect in the receiving circuits. The connections will be understood from the diagram in Fig. 33, and the appearance of the engine and instrument carts from Figs. 34 and 35.

A similar set is mounted up in a motor car, the only difference being that the engine driving the car is also used when the car is standing still to drive the 1½-K.W. alternator which is fixed on the chassis in a convenient position.

The masts in this case are carried on the roof of the car or on a separate trailer. They are made in sections, formed of steel tubes which fit together like a fishing-rod. The mast is erected by using spare sections as a derrick, in the manner shown in Fig. 27.

The aerial wires consist of a pair of stranded bronze wire cables.

6. Application of Radiotelegraphy to Aeroplanes and Dirigible Balloons.—In the case of large dirigible balloons there is no difficulty in carrying fairly high-power wireless plant. The propeller engines can be employed to drive a self-exciting alternator, and the type of plant above described for use in military work can also be carried.

The aerial wire can be a pendent wire let down from the car, and some insulated mass of metal or coil of wire can be utilized as an "earth" plate or balancing capacity. The chief difficulty arises from the additional weight to be carried, which even for the 0·5-kilowatt plant is nearly 800 lbs.

A source of danger arises from the possibility of sparks igniting the gas in the balloon. However well the spark gap may be enclosed, it is always possible to draw high frequency sparks from any piece of metal near the oscillator. Nevertheless, with care in the arrangement of the apparatus this danger can be reduced to a minimum. Plant sufficiently powerful for the communication over 50 to 100 miles can be carried, and has been used for communication with fixed land stations. In the case of aeroplanes there is much more difficulty.

Here every ounce of weight tells. Induction coils, jiggers, and other parts can, however, be constructed with aluminium wire, and a special form of high-tension dynamo can be carried to supply the current.

As regards the aerial wire, it is usual to employ an aerial wire floating or trailing in the air behind the machine; but it is necessary to be able to wind up the wire on a reel, or it may get entangled with the propeller or with trees and cause an accident. An alternative method is to support the aerial wire above the planes by very light stiff sprits. There is difficulty, however, in securing a counterpoise or balancing capacity; hence in some cases a double wire or Hertzian radiator has been tried with jigger inserted at the centre. (See Mr. Thorne Baker on "Wireless Telegraphy from Aeroplanes," *The Electrician*, vol. 66, p. 902, 1911.)

Many different forms of apparatus have been designed with a view to special lightness. Portable stations are made by the Marconi Company weighing 30 lbs., having a power of 40 watts and a radius of action of about 15-20 miles. More powerful sets weighing 82 lbs., having a power of 350-500 watts, have a radius of 50-60 miles. In the small 40-100 watt sets the power is derived from an 8 or 16 Vost storage battery coupled with an induction coil. In the 350-500 watt sets the source of power is a self-exciting alternator which is driven by a belt from the propeller engine. The alternator shaft has on it a rotating disc discharger. In

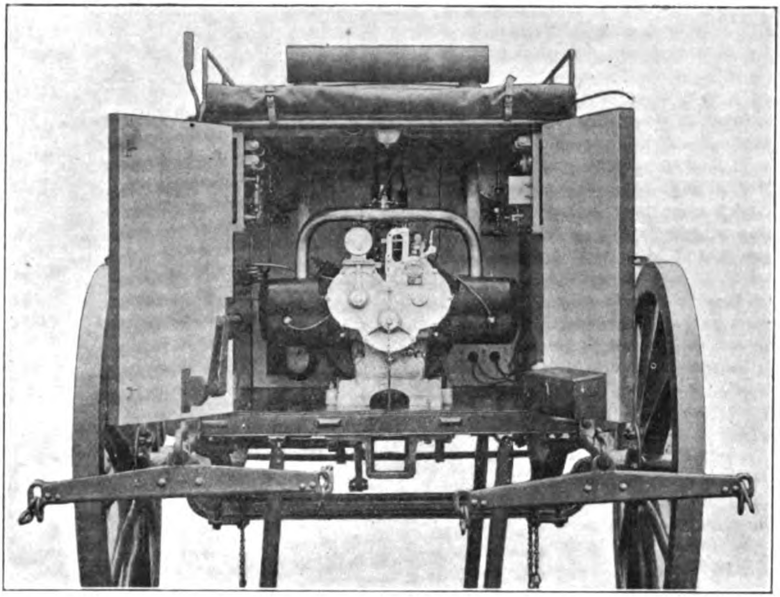

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FIG. 34.—Rear View of Interior of Engine Cart of Marconi 1·5-K.W. Military Set of Wireless Plant.

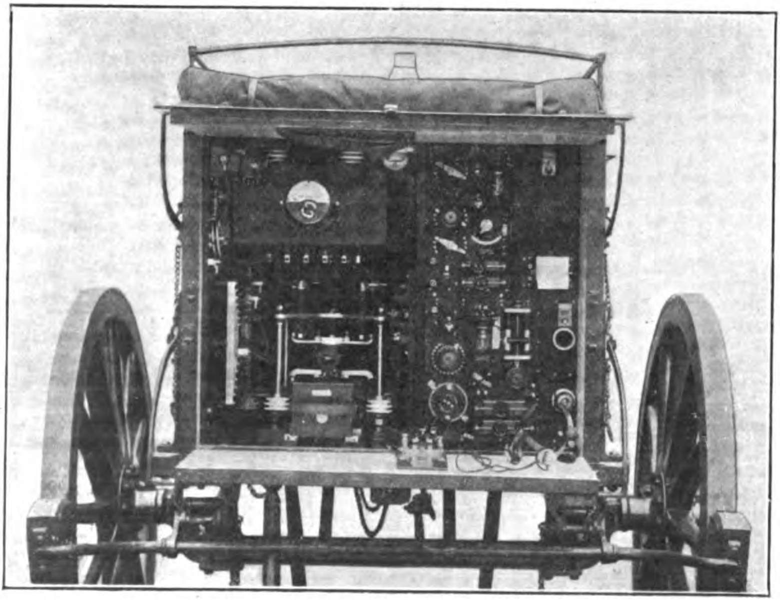

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FIG. 35.—Rear View of the Interior of the Instrument Cart of Marconi 1·5-K.W. Military Set of Wireless Plant.

both cases the key is an enclosed key to prevent any possibility of a spark igniting petrol vapour. This key is fixed near the pilot's or observer's seat. In Figs. 36 and 37 the external view of these Marconi aircraft sets of apparatus are depicted. Some difficulty is found in receiving signals owing to the noise of the engine drowning out the faint sounds heard in the telephone. This can partly be overcome by using a pair of telephones, one over each ear, and a thick pad as well to keep out sounds.

It would be a great advantage to have some simple form of receiver which gave a visible signal, as for instance illuminating a small lamp, in place of a telephonic sound; but invention has not yet provided a sufficiently simple and light relay for working a printer. There is room for much ingenuity in meeting these special requirements in aeroplane radiotelegraphy.

7. **Prevention of Interference.** — Radiotelegraphic intercourse between stations is subject to interference from three causes. First, the signals being

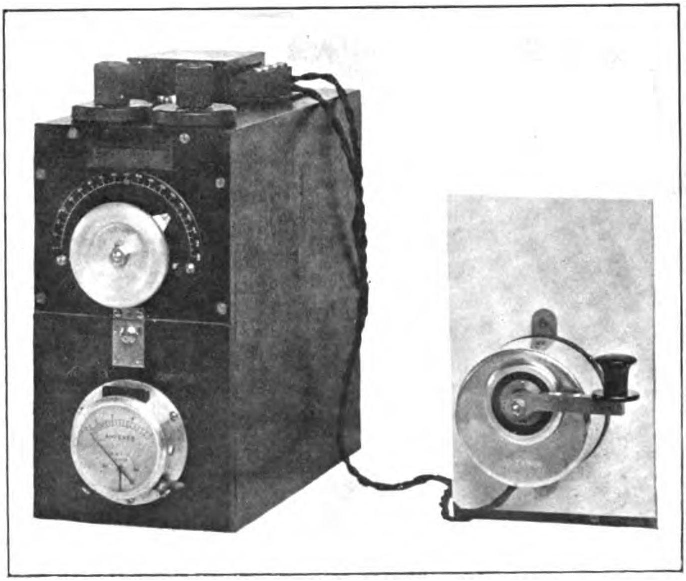

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FIG. 36.—40-Watt Aircraft Transmitter made by Marconi's Wireless Telegraph Company. The box contains the coil, spark gap, condenser, and jigger, and the sending key is in the enclosed round box on the right hand of the picture.

exchanged between other pairs of stations may be picked up unintentionally, that is to say, they may force themselves undesired upon receivers of the station in question. Secondly, stray waves, due to natural atmospheric discharges, may in the same way create false signals and confuse a message. Thirdly, there may be deliberate attempts to drown out or "jam" the signals of a certain receiving or transmitting station. All these disturbing effects can be more or less nullified by appropriate means, and a good deal of the art of radiotelegraphy consists in the operators getting round these difficulties, aided by suitable appliances.

In the early days of radiotelegraphy it was commonly thought that stations could be rendered immune from disturbance by merely tuning the receiver to the special wave-length sent out by the transmitter.

If the transmitter sends out feebly damped or undamped waves, and if their

amplitude is not too large, then it is possible to arrange receiving circuits of small decrement or damping, which will be responsive to these emitted waves if the receiver is exactly tuned to them, but will not respond if the natural frequency of the receiving circuit is altered by as little as one-half of one per cent.

On the other hand, if powerful or very strongly damped waves fall upon this receiver they will act like an impact excitation, that is, they will give a blow to the receiving circuit and set it in oscillation with its own natural frequency. Hence it follows that no simple tuning can exclude the effects of such strong, highly damped waves.

The great enemies of the wireless telegraphist are the natural atmospheric electric discharges called strays or X's. These are highly damped vagrant

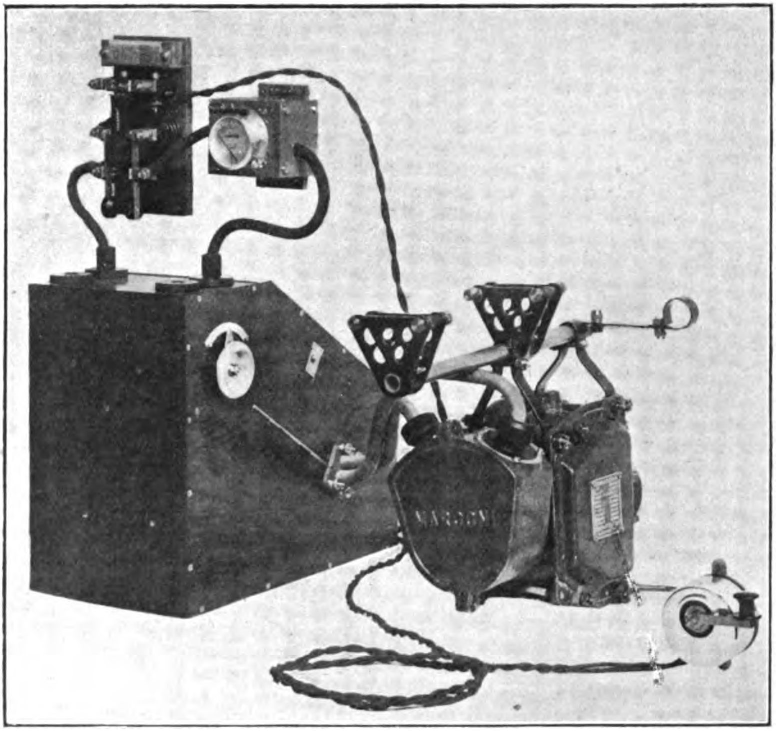

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FIG. 37.—350-Watt Aircraft Transmitter made by Marconi's Wireless Telegraph Company. The box on the left hand of the picture contains the condenser and jigger. The alternator and rotary disc discharger is in the centre, and the enclosed sending key on the right hand of the picture.

electric waves sent out from some region, originating perhaps in storms, and they cause disturbances or false signals in receivers.

One of the practical problems of wireless telegraphy is, therefore, to prevent these strays from making audible or visual signals in the telephone or other receiver used.

This problem has exercised the ingenuity of many inventors. One solution of it has been proposed by R. A. Fessenden, applicable when a telephone is used as

a signal receiver. This arrangement, which he calls an *Interference Preventer*, is as follows:—

Suppose that we couple to any antenna by suitable oscillation transformers three receiving sets, each comprising a closed condenser of variable capacity, an inductive circuit, and, say, crystal detector, shunted across the adjustable condenser. Then a magneto-telephone receiver is provided, having four coils on its magnet, which may be called C_0, C_0, C_1, C_2.

The coils C_0, C_0 are connected to one receiver, and the coils C_1 and C_2 to the other two, the connections being such that the currents in C_0 and C_0 respectively oppose those in the coils C_1 and C_2 on the telephone magnet. Now let the receiver to which the coils C_0 and C_0 are connected be tuned to the frequency of the wave it is desired to receive, and the receivers to which C_1 and C_2 are connected be tuned respectively to slightly longer and slightly shorter waves. Then suppose feebly damped waves of exactly the right frequency fall on the antenna. They create oscillations only in the receiver to which the telephone coils C_0 and C_0 are attached, and hence the observer hears the signals. If, however, a powerful strongly damped wave falls on the antenna it creates by impact excitation the free natural oscillations of all the three receivers to which the coils C_0, C_1, and C_2 are attached. But the current in C_1 opposes that in one coil C_0, and the current in C_2 opposes that in the other coil C_0, and hence the resultant effect on the telephone is small. In other words, not much sound is produced, or the stray wave does not much affect the telephone.[4]

It is easily seen that there are various ways in which the principle which underlies the above described method can be put into practice. If we couple two receiving sets inductively to the same antenna and tune one of them exactly to the free period of oscillation of the antenna and put the other slightly out of tune, then it follows that a feebly damped and feeble wave of exactly the antenna frequency will affect only the co-tuned closed and coupled receiving circuit. If, however, strongly damped waves fall on the antenna they will set up by impact oscillations in both receivers, these being of slightly different frequencies. If then we can arrange that the signal detecting instrument, say the telephone, shall be affected by the difference of these resultant currents, then we have a system sensitive to feeble but not to violent waves.

Another method which has been applied by Senatore Marconi and the engineers of the Marconi Company in the prevention of interference or the elimination of atmospherics depends on the form of the characteristic curve and on the properties of a certain class of oscillation detector, such as the Fleming oscillation valve or the crystal detector.

It has already been explained (see Chap. VI.) that a crystal of carborundum has an asymmetric conductivity. The characteristic curve (see Fig. 34, Chap. VI.) shows that for extremely small voltages applied in either direction the crystal has a very high resistance and passes very little current. Under larger electromotive forces of 1 or 2 volts the crystal conducts much better in one direction than the other.

Again, it has been explained that if we apply to the crystal a steady voltage in the direction in which it conducts best, of such magnitude as to correspond to a point of change of curvature or sharp bend in the characteristic curve, then the superposition of a small alternating E.M.F. will make a change in the mean value of the current, but not if we work at a part of the characteristic curve which is nearly flat.

Hence, by means of a rectifying crystal associated with an adjustable potentiometer and battery we can detect feeble electric oscillations which might not be detectable by the aid of the rectifying power of the crystal alone.

Suppose, then, that we arrange as shunts across the terminals of the condenser of a receiving circuit two crystals each provided with means for applying a boosting voltage by the aid of a battery and potentiometer, but place the crystals in opposite directions so that one conducts best in one direction and the other in the

[4] See *The Electrician*, vol. 61, p. 221, 1908. Also *Science Abstracts*, vol. 11, B, *abs.* 198. Also R. A. Fessenden, British Patent Specification, No. 4709 of 1907, Fig. 2.

other, and let them be both also in series with a coil to which a telephone is inductively coupled (see Fig. 38).

Next, suppose that in the case of one crystal we apply little or no boosting voltage, but in the case of the other crystal just the right voltage to give the loudest signals.

Then we have an arrangement called by the Marconi Company a balanced crystal receiver. The operation is as follows: If feebly damped waves of the right frequency fall on the receiver, then if they are not too strong, only one detector responds, viz. the crystal having applied to it a boosting voltage. For the E.M.F. produced in the receiving circuits by the message bearing signals is small, and it is therefore possible to arrange the two crystals so that for small

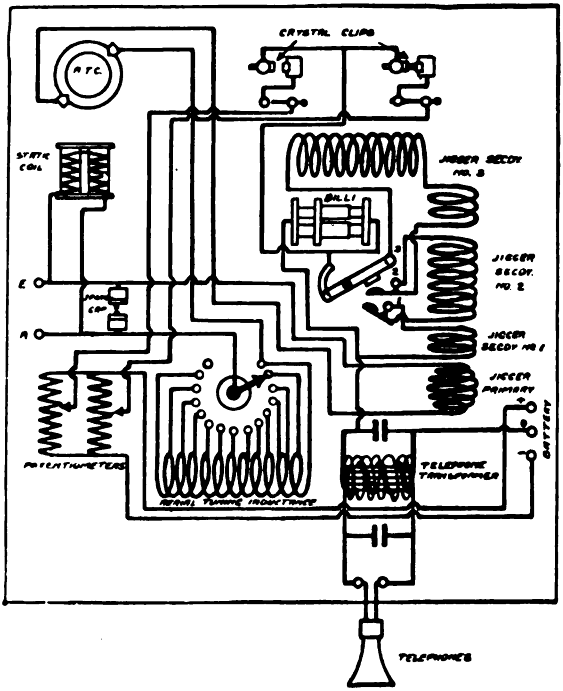

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FIG. 38.—Connections of the Marconi Balanced Crystal Receiver Set. The clips holding the two crystals in opposition are seen at the top of the diagram, and the telephone in series with them is at the bottom.

E.M.F. one rectifies and the other does not. If, however, a violent impulse is given to the receiver, then both crystals rectify because we are then working higher up on the characteristic. But the crystals rectify in opposite directions, which is equivalent to removing them both altogether. Accordingly such a receiving set may be made sensitive to the gentle feebly damped oscillations set up by the message signals, but violent atmospherics do not produce much sound in the associated telephone. A very similar arrangement has been devised by Mr. Marconi, described in a British Patent Specification, No. 4125 of 1909, and also in a Specification by Mr. H. J. Round, No. 20,441 of 1910, in which two Fleming oscillation valves are employed instead of two crystals.

The arrangement of circuits in this duplex Fleming valve receiver will be

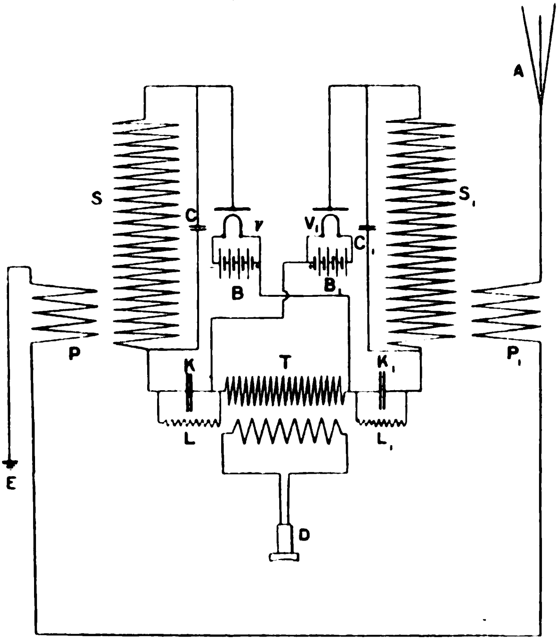


FIG. 39.—Marconi Receiving Circuit with Balanced Fleming Oscillation Valves arranged to be immune from Atmospheric Disturbances.

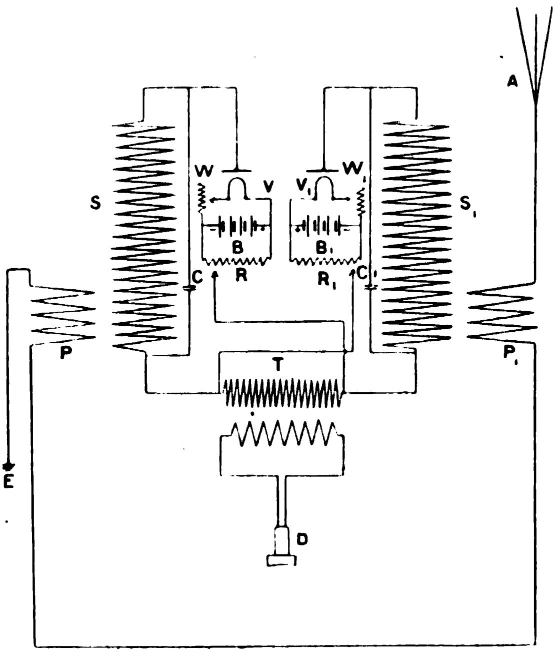
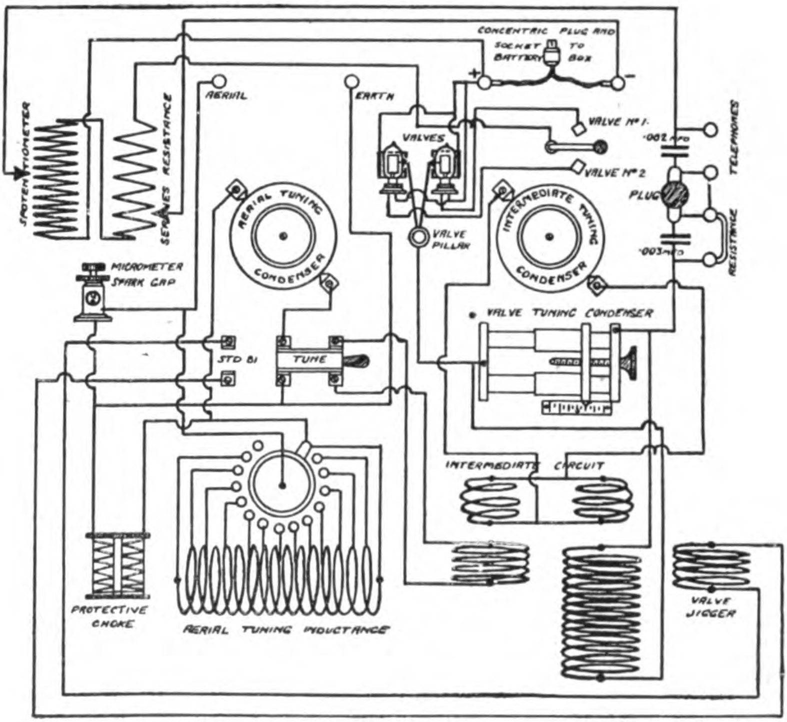


FIG. 40.—Marconi Receiving Circuit arranged to be immune from Atmospheric Disturbances.

understood from the diagrams in Figs. 39, 40, and 41. These give the theoretical scheme of connections for a duplex or balanced Fleming valve receiver and the actual connections for the same, which are rather more complicated, and which are employed by the Marconi Company.

8. The Efficiency of Radiotelegraphic Stations.—There are several points of view from which we may regard the performance of radiotelegraphic stations. We may in the first place consider merely the success of their operations regardless of questions of cost. The fact of being able to communicate between two places without any interconnecting wire is in itself an interesting and important feat, but



[*By permission of The Wireless Press, Ltd.*

FIG. 41.—Connections of the Marconi Balanced Valve Receiver Set.

when once it is done, we next ask whether the process can be continued with such regularity as to enable it to take its place amongst commercial means of communication. We may call this the attainment of efficiency of performance. Later on we may proceed to ask how far the process can be conducted in competition with other methods as regards cost, and this leads to the consideration of the energy expenditure, depreciation of plant, and cost of labour involved. The great obstacles which have interfered with efficiency of performance are the difficulties connected with the isolation of stations and atmospheric electrical disturbances; in other words, making the receiver immune from disturbance by electric waves due to natural electric discharges, or the operation of stations other than the one from which it is desired to receive signals. We have considered already the means so far adopted for the solution of this problem. Whilst it cannot be said that the difficulties are completely overcome, yet they are so far under control that

radiotelegraphy has taken its place as an indispensable means of communication, especially over sea and between ships. Aided also by the rules of the Convention and by national legislation, unnecessary disturbance of the æther is as far as possible prevented, and we may say that whilst the efficiency of performance of radiotelegraphic stations is not yet ideal in its perfection, it is well within those limits necessary for practically useful and for commercial work.

Accordingly, questions of cost or energy efficiency have next to be considered. At the outset one of the objections raised against radiotelegraphy, especially long distance, was the expenditure of energy needed for its operations as compared with that involved in telegraphy with wires. This, however, is merely part of a larger question. The cost of any undertaking in which scientific plant is utilized for commercial purposes may be properly divided into three parts: (1) The interest on the capital outlay necessary to establish the plant and appliances; (2) the cost of working it and conducting its operations; and (3) the allowance for depreciation or antiquation of plant, and for renewals and repairs. From a commercial or business point of view, it is the total cost under all three of the above headings which matters, and not merely one of them taken alone.

In the case of submarine cables the most important items are the capital outlay and the cost of repairs and renewals. The capital outlay on a deep-sea submarine cable averages about £200 per mile or more in initial cost, and repairs are frequently very expensive. A single repair has been known to cost even £70,000. On the other hand, the energy expenditure and actual cost of working is small Nevertheless, the whole cost of submarine cable-working and the charges which must be made for messages if the undertaking is to be commercial, depend upon all the costs taken together.

On the other hand, in the case of radiotelegraphy, the capital outlay on plant or apparatus is relatively to the cost of cables very small. Also the cost of repairs is not large, assuming the absence of exceptional events, such as fires or earthquakes, which may destroy a large station entirely. Hence the fact that there is a larger expenditure of energy in sending any given message radiotelegraphically compared with that by cable is only a limited aspect of the case, and does not imply that the total cost is any more. There is abundant evidence to show that on the whole it is much less. Nevertheless, the question of the sources of energy loss in case of radiotelegraphic apparatus is very important. Sufficient data have been gathered to show that the actual expenditure of energy in true æther wave-making is comparatively small and that in radiotelegraphic transmitters, as at present constructed, the internal energy dissipation is very considerable. There is, therefore, great room for improvement in this respect. It is important, therefore, to ascertain the sources of this loss, and to improve the energy efficiency of transmitters. Taking the case of the ordinary spark transmitter, we have first the internal losses in the alternator, or rotary transformer. Since the power factor of the transmitter plant is small, these internal losses in the alternator may be considerable unless the machine is designed specially for working on a low-power factor circuit.

In the next place energy losses are incurred in the step-up transformers, but these can be minimized by careful design. In the oscillatory circuit we have these energy losses in the condensers, and resistance losses in the inductance and in the spark. The condenser losses are by no means insignificant if glass condensers are used, and hence air condensers are preferable when space permits.

A serious source of loss is generally the bad design of the inductance and connecting circuits. These may be kept down, however, by a proper stranding of the conductors—making them of plaited silk-covered No. 40 H.C. copper wire, and avoiding the use of thick copper strip or coarsely stranded cables.

The energy losses in the spark are unavoidable, but are, no doubt, much diminished by the employment of quenched sparks instead of long single persistent sparks.

The power given to the step-up or charging transformer can be measured by a wattmeter in the usual way, and the internal losses in this transformer, viz. the iron core and copper or resistance losses, also determined for various values

of the current supplied. Hence we can find by difference the power supplied to the oscillatory circuit.

By the measurement of the decrement of this circuit we can ascertain, as already mentioned, the resistance losses, including those in the condenser and spark. Hence by difference again we can ascertain the power given to the antenna, and if the high frequency resistance of the antenna is ascertained, we can finally ascertain the power expended in radiation after deducting the resistance losses due to the earth connection, if any.

The reader may be referred for some efficiency measurements of the above kind or on small spark transmitter apparatus to a paper read by the author before the Institution of Electrical Engineers of London in November 1909, "On some Quantitative Measurements in Connection with Radiotelegraphy," and to one read at the same date by Dr. Eccles and Mr. Makower, "On the Efficiency of Short-spark Methods of Generating Electrical Oscillations."[5] In the first paper it is shown that for the spark transmitter used the power expended in making electric waves was rather less than 10 per cent. of that given to the charging transformer, and in the latter paper it is shown that in the case of a discharger of the Von Lepel type the fraction of the power given to the discharger which appeared as oscillations in the antenna did not much exceed 14 per cent.

The above measurements were, however, made with experimental plant not by any means as perfect as possible. Taking ordinary ship or coast or radiotelegraphic plant of 1½ to 5 K.W. capacity, we may say that in those cases in which a spark discharge between fixed surfaces is employed, and no special pains taken to quench it, the ratio of the total power given to the antenna to the total power given to the charging or step-up low frequency transformer will not exceed 10 to 15 per cent. On the other hand, an air blast on the spark will increase it to perhaps 20 per cent. If quenched spark or high speed rotating discharges are employed the efficiency may rise to 50 per cent.[6] In the case of the arc transmitter there is a much less economical transformation, not more than 25 per cent. of the energy supplied being transformed into energy of antenna oscillations. Any accurate statement of the efficiency of a radiotelegraphic plant should be presented in the form of an energy balance-sheet showing the power credited in watts to the whole appliance, and on the other side of the accounts the manner in which this is expended in the various stages of transformation. The author was the first to give such an energy balance-sheet for a radiotelegraphic transmitter.[7]

The power for this plant was supplied by a small 1½-K.W. rotary converter taking direct current from public supply mains and converting it to alternating current having frequency of 100.

This alternating current passed through certain choking coils to a small A.C. static transformer which raised the voltage to 20,000 volts. This was employed to charge glass plate condensers which were discharged through the primary circuit of a jigger and across a static spark gap with air blast. The secondary of the jigger was inserted in the circuit of an antenna which had the usual tuning coils in series and an earth connection. The currents both in the condenser circuit and on the antenna near its base were measured by Fleming hot-wire ammeters made as described in § 11, Chap. II. The decrement of the closed circuit was measured with the cymometer, and also that of the oscillations in the antenna by means of the Bjerknes method as described in § 13, Chap. III.

The energy supplied to the condenser by the charging transformer is dissipated in several ways—

(i.) As heat in the spark in the primary circuit;
(ii.) As heat in the metallic portion of the primary condenser circuit;
(iii.) In the condenser in dielectric hysteresis and brush discharge;
(iv.) As heat in the antenna and antenna circuit, and also in the earth plate and connections thereto;
(v.) As electric radiation from the antenna.

[5] See *Journal of the Institution of Electrical Engineers*, vol. 44, 1910, pp. 344 and 387.
[6] See Dr. W. H. Eccles, *The Electrician*, vol. 70, p. 522, 1912.
[7] See J. A. Fleming, "Some Quantitative Measurements in Connection with Radiotelegraphy," *Journal Institution of Electrical Engineers*, vol. 44, p. 383, 1911.

We can make an approximate estimate of the percentage of the stored energy which is radiated, as follows :—

Let us assume that the primary condenser has a capacity C_1 mfds., and is charged to a voltage V_1, as estimated from the spark length, and is discharged N times per second. Then the rate at which energy is given out by the condenser is $\frac{NC_1V_1^2}{2 \times 10^6}$ watts.

Let R'_1 be the high frequency resistance of the primary circuit, and r the resistance of the spark. Then, if L_1 is the inductance of the primary circuit, and δ_1 its resistance decrement, as modified by the presence of the secondary or antenna circuit, we have—

$$R'_1 + r = 4nL_1\delta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where n is the frequency.

In the same way, if δ_2 is the *resistance decrement* of the antenna circuit, and R'_2 its high frequency resistance, including in this any earth-plate resistance, if an earth connection is employed, we have[8]—

$$R'_2 = 4nL_2\delta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Let J_1 and J_2 be the R.M.S. values of the primary or condenser circuit currents and the antenna current respectively, this last being measured at the earthed end of the antenna. We can always determine these currents by the use of hot-wire ammeters inserted between the antenna and earth plate and in the condenser circuit. Accordingly, if we have in use a condenser in which we may fairly assume there is no dielectric loss, the difference between the power given out by the condenser and that dissipated as heat must be the power W radiated by the antenna. Hence the value of the expression —

$$W = \frac{NCV^2}{2 \times 10^6} - J_1^2 4nL_1\delta_1 - \tfrac{1}{2}J_2^2 4nL_2\delta_2 \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

gives us the radiation in watts, provided the earth-plate loss is negligible.

The factor $\frac{1}{2}$ is prefixed to the third term because the current varies up the aerial in accordance with a sine law, and is J_2 at the earthed end and zero at the summit of the aerial. Hence its mean square value is $\frac{1}{2}J_1^2$.

The frequency n is obtained from a measurement of the common oscillation constant of the two circuits.

The values of L_1 and L_2 are obtained when we know the capacities of the condenser in the primary circuit C_1 and that of the aerial C_2, for $C_1L_1 = C_2L_2 = O^2$ when the circuits are syntonized, O being the oscillation constant of either circuit.

If capacities are all measured in microfarads, inductances in centimetres, currents in amperes, and potentials in volts, the above expression for the rate of radiation of energy in watts is transformed into—

$$W = \frac{NC_1V_1^2}{2 \times 10^6} - \frac{O}{50}\left(\frac{J_1^2\delta_1}{C_1} + \frac{1}{2}\frac{J_2^2\delta_2}{C_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

The value of δ_1 will depend upon the spark gap length and capacity C_1 in the condenser circuit. The value of δ_2 will, in general, be small, since there is no spark gap in the antenna circuit, unless there is considerable dissipation of energy by earth-plate resistance, which ought not to be the case. The exact value of δ_1 can be obtained by means of a resonance curve, as already explained. For if we allow the condenser circuit to act inductively on another closed circuit of known resistance decrement, δ_3, we can, by varying the inductance of this last circuit, determine the sum of the decrements $\delta_2 + \delta_3$ by the resonance curve, and δ_3, being known, we thence find the value of δ_2. Thus, for instance, a primary circuit had in it a condenser of $\frac{1}{40}$ mfd., and a spark gap of 1·5 mm., charged

[8] It should be noted that we are not here concerned with the radiation decrement of the antenna.

50 times per second. The oscillation constant was 7·0, and the current J_1 was 10 amperes. If, therefore, $\delta_1 = 0{\cdot}03$, we have for the power W, reckoned in watts imparted to the coupled antenna, the value—

$$W = \tfrac{1}{2} \cdot \frac{36}{40} 50 - \frac{7 \times 3 \times 40}{50} = 5{\cdot}5 \text{ watts}$$

From this would have to be deducted the power loss due to the high frequency resistance of the antenna, and any earth-plate loss, and the balance is the power radiated.

We then notice that we have several different definitions of the transformation efficiency of such a coupled transmitter. We may consider in the first place how much of the power given to the antenna is radiated as electric waves, and how much is dissipated as heat in the circuits. The ratio of power radiated to total power given to the antenna may be called the *antenna radiation efficiency*.

Then we may consider how much of the power given to the closed condenser circuit is handed on to the antenna in the form of high frequency oscillations. Finally, we may ask how much of the power given to the plant in the form of continuous or alternating current is radiated as electric waves from the antenna. This last may be called the *overall efficiency* of the transmitter.

It is a somewhat difficult matter to make these measurements of efficiency accurately for any actual transmitter. It can approximately be done by determining the energy losses in each part of the system as follows:—

By means of a wattmeter, correct for low-power factor values, we can ascertain the mean power in watts given to the transformer or induction coil. Care must be taken that the wattmeter reads correctly on low-power factors. We have then to ascertain how this power is expended. We can measure in the usual manner the internal losses in the transformer due to copper resistance and iron core loss, and subtracting these, find the power given to the oscillation circuit. The next step is to determine the mean-square current in this primary oscillation circuit, which can be done by means of the author's thermoelectric high frequency ammeter, and also the primary circuit decrement, which can be done by taking a resonance curve with the cymometer, as explained in Chap. III. During this measurement the antenna must be detached so that no radiation takes place. This decrement measurement should be made for various values of the current in the primary oscillation circuit.

The inductance of this circuit must also then be measured by the cymometer or by any other suitable means, and then from the primary decrement δ_1 and inductance L_1 we can calculate as above shown (see equation (1)) the power loss due to resistance. This measurement includes any loss in the condenser, because this loss proportionately increases the decrement. The sum of the internal losses in the transformer and that in the oscillation circuit being deducted from the power supplied, must give the power taken up by the antenna. The amount taken up in antenna resistance and by any earth-plate resistance can be ascertained by making a measurement of the high frequency resistance by means of the differential electric thermometer described in Chap. II. of samples of the wire used to make the antenna and oscillation transformer circuit in series with it and any connecting wires.

If the current I_2 in the antenna circuit is measured by a hot-wire ammeter inserted in it at various points, we can calculate the value of $\tfrac{1}{2}R_1' I_2^2$ for the whole antenna. There remains, then, the earth-plate resistance loss, if any. In the case of ships at sea this loss will be very small, but not altogether negligible in the case of stations on shore. It can, however, be determined by substituting for the earth plate a metallic capacity insulated from the earth, but which is sufficiently large to create the same mean-square current at the base of the antenna, and when these two last power losses are deducted from the total power given to the antenna, the difference must be the power radiated. Such measurements as have been made by this method show that the power actually expended in radiation in a spark transmitter is always small compared with the total power given to it. The actual percentage will vary considerably with apparatus and conditions, but it is probably near the mark to say that when using the spark transmitter creating

damped waves the actual power radiated by an ordinary inductively coupled antenna is not more than 25 per cent. of the power given to the exciting transformer, even if as much. For further information on this point, the reader may be referred to a paper by the author on "Quantitative Measurements in Radiotelegraphy," read to the Institution of Electrical Engineers of London, December 16, 1909 (see *Journal Inst. Elect. Eng.*, vol. 44, p. 344, 1910).

Working in the above manner on a small experimental transmitter, the author gave in the above-mentioned paper an energy balance-sheet for it as follows :—

POWER BALANCE-SHEET FOR AN EXPERIMENTAL RADIOTELEGRAPHIC TRANSMITTER.

Cr.	Watts.	*Dr.*	Watts.
By power given to rotary transformer in form of direct current, 12 amperes (nearly) at 220 volts -	2620	To Rotary losses - - - - -	1139
		,, Line losses - - - - - -	71
		,, Choker losses - - - - -	610
		,, Transformer losses - - - -	355
		,, Oscillation circuit losses in condenser, spark gap, and inductance	351
		,, Power given to the antenna partly dissipated as heat in wire and unknown losses in earth plate and remainder radiated - - -	94
	2620		2620

The rotary converter and chokers were not especially made for this work, and hence the losses in them were abnormally large. The efficiencies of the various portions of the plant are shown in the chart on p. 607.

This experimental plant was in many respects very inefficient, and for a well-designed transmitter on the spark system, the overall efficiency might be as high as 25 per cent. when using an unquenched spark between stationary balls, and rather higher for a quenched spark. In some measurements made in 1910 Count Von Arco claimed 40 per cent. efficiency for a 2-K.W. Telefunken quenched spark transmitter. (See *Elektrotechnische Zeitschrift*, Heft 20, 1910; also *The Electrician*, vol. 65, p. 357, June 10, 1910.)

It would appear, however, that the antenna losses due to true resistance of wire are included in this efficiency.

In the case of the Poulsen arc method of generation the large heat-losses in the arc make the overall efficiency certainly lower than in well-designed spark systems.

The real difficulty of making these efficiency measurements is that of determining exactly the antenna radiation resistance apart from the high frequency resistance of the wire itself and that of the earth plate and connections. This earth plate and earth resistance varies with the frequency of the oscillations, that is with the wave-length, and hence any measurements made with low frequency or direct currents are likely to be extremely erroneous.

Attempts have been made to separate out the radiation resistance from the total resistance of an antenna by reducing the height of the antenna without otherwise altering its length or capacity, and then inserting in the antenna such a resistance as shall keep the current in it at its original value, and then assuming that this inserted resistance is the equivalent of the reduction in radiation resistance.

One difficulty is that we do not know whether the effect of the antenna in creating signals at a distant receiver is wholly due to a space electric wave or partly due to a surface electric wave travelling through the crust of the earth. On the whole it seems safer at present to confine our measurement of energy efficiency to determining the ratio between the whole power expended on or given to the antenna in setting up oscillations in it, and the power delivered to the low frequency charging transformer. The last quantity can be easily measured by a properly constructed wattmeter adapted for measurements on low-power factor. The power delivered to the antenna can be determined by substituting for the

EFFICIENCY CHART FOR SMALL EXPERIMENTAL TRANSMITTER.

Rotary intake
2620 watts.

Rotary output
1481 watts.

Rotary losses
1139 watts.

Rotary efficiency =57 per cent.

Transmission losses
71 watts.

Transformer intake
800 watts.

Choking coil losses
610 watts.

Choker efficiency =57 per cent.

Delivered to condenser
445 watts.

Transformer resistance
losses 302 watts.

Transformer iron
losses = 53 watts.

Transformer efficiency =56 per cent.

Primary oscillation circuit
losses 351 watts.

Delivered to antenna
94 watts.

Oscillation generator efficiency } =21 per cent.

Antenna resistance
losses 16 watts.

Antenna radiation
78 watts,
including earth-
plate losses.

Antenna efficiency =83 per cent.

Overall efficiency of plant } =10 per cent.

antenna a closed condenser circuit comprising an air condenser and a very low resistance inductance adjusted so as to have the same capacity and time of oscillation as the antenna, and also in series with it a non-inductive resistance so adjusted that when the primary coil of the primary condenser circuit is coupled to this substitutionary secondary circuit, it will induce in it oscillations of the same frequency and same R.M.S. value as those at the base of the actual antenna. This resistance is then equal to the total antenna resistance. Hence the product of this total antenna resistance and the mean square value of the antenna current near the base, gives the power expended on the antenna. The ratio of this last power to the power read by the wattmeter connected in front of the low frequency charging transformer gives us the efficiency of generation of the antenna oscillations. This may be taken as the measure of the effectiveness of design of the ransmitter.

Lastly, we may consider the fraction of the energy sent out by the transmitter which is captured by the receiver. Some measurements of this kind are recorded in M. Tissot's book, "Etude de la Résonance des Systèmes d'Antennes dans la Telegraphie sans fils." In the case of a certain plain antenna 50 metres high and 4 millimetres in diameter, on board a French battleship *Henri IV.*, corresponding with a similar antenna at a distance of 1·7 kilometres, M. Tissot found that when the sending antenna was charged fifty times a second to a potential equal to a 5-cm. spark the mean radiation was 36 watts, or $1{\cdot}8 \times 10^7$ ergs per spark. At a distance of 1 kilometre the energy picked up by a similar receiving antenna was 320 ergs per spark, or 6400 ergs per second. Hence the captured energy even at this short distance was only $\frac{1}{560}$ of 1 per cent. of that sent out from the transmitter, and a still smaller fraction of that given to the transmitter. M. Tissot notes the curious fact that the energy picked up by the receiving antenna is larger than that corresponding to its own surface. The figures given will, however, be sufficient to show the exceedingly small fraction of the power supplied to the transmitter which is represented by the oscillations set up in the receiving antenna by the radiotelegraphic appliances at present in use. The state of affairs is analogous to that of solar radiation. The earth only captures about 5×10^{-10} of the energy sent out from the sun, and the earth and the sun may be regarded as the receiver and transmitter in a system of short wave wireless telegraphy on a gigantic scale.

9. The Design of a Radiotelegraphic Station.—The design of a radiotelegraphic station, especially for long distances, calls for ordinary engineering and electrical engineering experience, such as is applied in the erection of an electric light or power station as well as for special radiotelegraphic knowledge. When we are given the distance of which it is required to communicate, experience or regulations will decide the wave-length which should be used. We have, then, to determine the power required, and to design the station accordingly.

Owing to the great variations of signal strength which take place by day and night when working over long distances, it is necessary to provide a large margin of power. The difficulty is, however, to get this power into the aerial. The achievement of signalling depends on the production of a current in the receiving antenna, which for good signalling must not be much less than, say, 30 microamperes when using telephonic reception. It is then possible to determine from certain experimental data which have been obtained the corresponding sending antenna current when working over various distances.

An empirical formula for this purpose has been given by Dr. L. W. Austin as the result of measurements made over sea by daylight up to distances of 1000 or 2000 miles (see Chap. IX. § 3).

Let I_1 be the current in the sending antenna and I_2 that in the receiving antenna, and let h_1 and h_2 be the antenna heights. Let D be the distance between the stations and λ the wave-length. Then for transmission by day it is found that

$$I_2 = 4{\cdot}25 \frac{I_1 h_1 h_2}{\lambda D} \epsilon^{-\alpha D/\sqrt{\lambda}} \qquad . \quad . \quad . \quad . \quad . \qquad (5)$$

where α is a constant which is not far from 0·0015 for over-sea working.

In the above formula I_1 and I_2 are measured in amperes, and λ_1, h_1, h_2, and d are measured in kilometres.

The formula can, of course, be used to determine I_1 when I_2, h_1, h_2, d, and λ are given. I_2 may be taken as 40 microamperes for good receiving through 25 ohms equivalent antenna resistance—that is, 4×10^{-8} watts. For further discussion of the problem of pre-determining the receiving antenna current under various conditions and for different formulæ for it, the reader must refer to Chap. IX. § 3.

TABLE I

Sending antenna current I_1.	Working distance.	
	By day.	By night.
amps.	miles.	miles.
1	75	90
2	135	180
3	180	270
5	235	450
7	280	630
10	345	900
15	420	1350
20	475	1800
25	525	2250
30	565	2700
40	630	3600
50	685	4500
60	725	5400

In the above equation (5) the constant 4·25 refers to flat top or T-shaped antenna such as is used on ships. This constant is determined by the form of the antenna.

Taking two ship T-antennæ of height $h_1 = h_2 = 130$ feet and a wave-length $\lambda = 1000$ metres, and assuming a receiving current of 40 microamperes, Dr. Austin has calculated the sending antenna currents corresponding to various distances as in the Table I. above.

The reason for the difference in range by day and by night is discussed in the next chapter.

Dr. Austin has also given the following calculated Table II. of sending antenna currents corresponding to certain distances and wave-lengths for two flat-top or T-antennæ 450 feet high:—

TABLE II

Distance in nautical miles.	$\lambda = 1000$ metres.	$\lambda = 2500$ metres.	$\lambda = 3750$ metres.	$\lambda = 6000$ metres.
	amps.	amps.	amps.	amps.
1000	1.5	13·5	15	17
1250	38·0	27·0	27	30
1500	91·0	49·0	44	46
1750	200·0	95·0	77	74
2000	490·0	155·0	122	105
2250	...	245·0	200	160
2500	...	470·0	314	235
2750	...	...	500	335
3000	...	...	775	500

The Table II. shows the advantage of lengthening the wave for long-distance working, as it reduces the sending antenna current.

Having obtained the value of the current in the sending antenna, we are able, by the aid of the information and formulæ given in the preceding section, to calculate the power thrown off from it reckoned in watts.

For if we assume a flat-top or T-antenna, we can then take from the Table in § 13 of Chap. V. the radiation resistance for the wave-length used.

To ascertain the power to be supplied to the antenna to create this radiation we have to take into account the ohmic resistance of the antenna, and also the earth-plate resistance. The total power to be supplied to the antenna may therefore be represented by the expression

$$P=(R_r+R_o+R_e)I_1^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where R_r is the radiation resistance above considered, R_o is the equivalent high frequency ohmic resistance of the antenna, and R_e is equivalent earth-plate resistance, and I_1 is the current at the base of the antenna.

The last two quantities depend on so many unknown and indefinable factors, *e.g.* form of antenna wires, form of earth plate, moisture of ground, and nature of neighbouring conductors, that it is impossible yet to give means for exact predetermination. All we can do is to find a factor of correction in the light of experience. It would be safe to say that we must at least double the radiation resistance to obtain the total resistance. This being the case, we have then the means of approximately determining the total power to be supplied to the transmitter, and hence the engine, alternator, and transformer plant to be laid down.

If the transmitter is a spark transmitter with quenched or musical spark, it must be remembered that the *power factor* of the condenser circuit which is charged by the step-up transformers is very low. It will seldom be greater than 0·5. Hence in the design of the alternator this must be taken into account. The alternator armature has to supply a large wattless current, and hence its armature must have sufficient current-carrying capacity.

In settling the engine and alternator power it is necessary to allow a large margin, and it is safe to provide power equal to at least twenty times the radiated power. That is, we do well to assume only a 5 per cent. efficiency between indicated engine power and power radiated as electric waves. Having decided in this manner the output, we can specify the engine, alternator, and transformer plant. The type of engine will be determined by the supply of water, access for coal, and other difficulties. Wherever possible steam engines should be employed. In some cases electric power can be purchased directly from a power supply system, and the voltage transformed by local transformers.

In any case these transformers must be oil-insulated transformers, and may raise the voltage to 20,000 or 30,000 volts.

The design and specification of the plant is so far entirely ordinary heavy electrical engineering. In the next place we have to consider the provision of the condenser plant.

There is a great advantage in using air condensers if possible. The objection to air at ordinary pressure is its small dielectric strength. The use of compressed air obviates this difficulty, but renders the condenser more expensive. The next best arrangement is metal plates in highly insulating oil, as the dielectric glass plate or glass tube condensers have advantage in small bulk capacity, but are fragile and liable to age in use.

When we have decided the spark frequency N, which may best be about 300 to 500 per second, and the charging voltage V, the capacity of the condenser C for a given power output W is determined by the equation $W=\frac{1}{2}NCV^2$. It is necessary here also to allow a large margin, as the efficiency of the oscillation transformer is in general not high, and energy is dissipated in the spark gap. Hence the possible condenser power delivery should be at least three times that of the power radiation of the antenna.

In the design of the oscillation transformer, antenna loading coil or variable inductance, very great care should be taken to reduce resistance loss of power to a minimum. This can be done to some extent by stranding the cable used, so

as to reduce high frequency resistance, and the added resistance which arises from unequal distribution of the current across the section of the conductor. The circuits of the oscillation transformer should be made of very fine H.C. copper wire about No. 36 S.W.G. enamelled or silk-covered, but not squeezed too near together as this causes the production of eddy currents in the wires equivalent to added resistance.

All condenser connections should be made with the same cable, or with very thin sheet copper in parallel insulated sheets or strips.

It must be borne in mind that the object of the whole transmitter is to get power into the aerial, and there radiate it as long electric waves, and that a radiotelegraphic transmitter is in a sense a sort of electric lamp, the efficiency of which is measured by the percentage ratio of the power radiated as long electric waves to that supplied to it as indicated power of the engine or other prime mover.

For some additional information on the design of radiotelegraphic stations, the reader may be referred to articles by Dr. Shunkichi Kimura in *The Electrician*, vol. 70, pp. 50, 95, 135, October, November 1912, in which a number of useful formulæ are presented in the form of curves.

In the selection of a wave-length it is well to bear in mind that Austin's formula for the received current above given shows that there is a certain wave-length which gives the maximum received current for a given distance between the stations, other things remaining the same. This optimum wave-length is easily found, as follows :—

Taking Austin's empirical formula in equation (5) above, viz.—

$$I_r = 4{\cdot}25\frac{I_1 h_1 h_2}{\lambda D}\epsilon^{-\alpha D/\sqrt{\lambda}}$$

we differentiate with respect to λ and obtain

$$\frac{dI_r}{d\lambda} = \frac{4{\cdot}25\, I_1 h_1 h_2}{D\lambda^2}\left(\frac{\alpha D}{2\sqrt{\lambda}} - 1\right)\epsilon^{-\alpha D/\sqrt{\lambda}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence the condition for a maximum is that

$$\lambda = \frac{\alpha^2 D^2}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and this wave-length gives the largest receiving current for a given distance D and given heights of antennæ.[9] In other words, for the same type of plant the wave-length used must increase as the square of the distance over which signals have to be sent.

On the subject of the Design of Radiotelegraphic Stations the reader may consult a paper by Mr. A. S. Blattermann in *The Electrician*, vol. 72, 1914, pp. 780 and 821. In this paper the writer assumes the use of a quenched spark transmitter giving rise to a single wave radiation from the sending antenna, and supplies a number of curves for facilitating calculations and examples of their use. The paper, however, must be read in connection with a criticism of it by L. B. Turner on p. 869 of the same volume of *The Electrician*.

10. Directional Wireless Stations.—We have already explained in § 8 and § 9 of Chap. VII. that a closed or nearly closed high frequency oscillating circuit has an asymmetry of radiation and radiates best in its own plane. In virtue of the general law of exchanges it therefore absorbs best in the same plane. If we construct a closed or nearly closed radiating-circuit consisting of two inclined separate aerial wires connected at their base by a horizontal wire and having their upper free ends much nearer together than their bases, as in Fig. 17 of Chap. VII., this type of antenna will radiate best and absorb best electric waves in its own plane. If, then, we can cause this nearly closed antenna to rotate round a vertical symmetrical axis, we can use it to determine the direction of the radiant point or sending station for electric waves travelling over the earth. All that is necessary is to couple inductively to the antenna a suitable detector circuit and then rotate the closed antenna round its vertical axis until the signals or telephone

[9] See *The Wireless World*, vol. i. p. 552, December 1914, where the above formula is given by Mr. L. Cohen.

sounds in the detector circuits are a maximum. We should then know that the plane of the aerial is in the direction of the radiant point or source of those waves. It would, however, be quite impracticable to rotate in this manner a large nearly closed antenna, but a determination of the direction of travel of the incident waves can be made by using two such closed or nearly closed antenna set with their planes fixed and at right angles to each other. If, then, the closed detector circuit which contains the variable or tuning condenser, and in series or shunt with this same form of detector, is coupled inductively with these two antennæ, in such manner that the secondary coil in the condenser circuit can be placed with its axis directed so as to have its maximum mutual induction with one or other of two primary coils inserted respectively in the two antenna middle points, or else in some intermediate position in which it has mutual induction with both these primary coils, we can move this secondary coil into a position in which the telephone associated with its detector gives a maximum sound. When this is the case it is quite easy to show that the plane of that secondary coil lies in the direction of propagation of the incident wave.

The practical form the arrangement takes is as follows :—

On a hollow cylinder made of insulating material two rectangular coils are wound in planes at right angles. The wires of these coils are well insulated, and

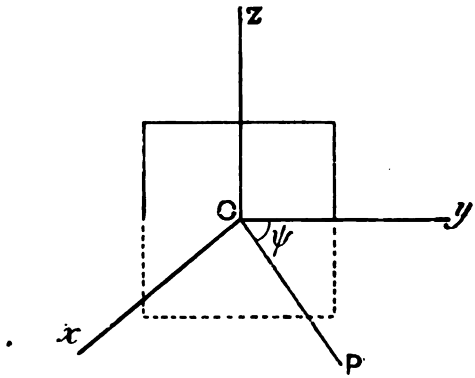


FIG. 42.

two sides of the rectangles lie close to the cylindrical surface parallel to the axis, and the other two pass diametrically across the circular ends of the cylinder. These coils are inserted respectively in the base or lower portion of the two nearly closed antennæ whose planes are at right angles.

Within the cylinder is placed another rectangular coil rotatable round the axis of the cylinder so that the plane of the latter coil can be placed parallel to that of either of the two fixed coils wound on the cylinder, or turned into some intermediate position. This movable coil forms part of the closed circuit of the receiver system, if it is a directional receiver, and part of the closed spark or oscillation-producing circuit, if it is a directional radiator.

The cylinder carries a divided circular scale and the rotatable coil an index needle so that its angular position with regard to the two outer fixed coils can be determined by inspection.

We can then explain the action of such a system of coils with variable mutual nduction as follows :—

Consider a small rectangular oscillatory circuit placed with its plane vertical to the earth (see Fig. 42). Let the plane of the rectangle be taken as the plane of yz and its centre be taken as origin. The axes of x will then be perpendicular to the plane of the coil. Let electric oscillations of high frequency be supposed to exist in this rectangular circuit. We have already, in Chap. V. § 13, equations

(72) and (73), given the expressions for the electric and magnetic forces due to those oscillations at any point at a distance r from the centre of the closed oscillator.

If, in these equations, we put $z=0$, we obtain the value of the forces in the equatorial plane xy. For distances from the oscillator, large compared with its dimensions, the values of the components of these forces are—

$$Z=\frac{AMnm}{r}\cos\chi\frac{y}{r}$$

$$\alpha=-\frac{Mm^2}{r}\cos\chi\frac{y^2}{r^2}$$

$$\beta=\frac{Mm^2}{r}\cos\chi\frac{xy}{r^2}$$

where Z is the electric force at distance r parallel to the axes of z, and α and β are the components of the magnetic force parallel to the axes x and y and χ, m and n have the meaning specified in Chap. V., M being the magnetic moment of the

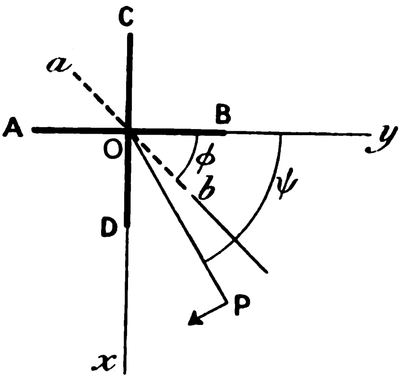


FIG. 43.

oscillator. If we write $\cos\psi$ for $\frac{y}{r}$ and $\sin\psi$ for $\frac{x}{r}$, and C_1 for $\frac{AMmn}{r}\cos\chi$ and C_2 for $\frac{Mm^2}{r}\cos\chi$, we can write the above equations in the form

$$Z=C_1\cos\psi,\ \alpha=-C_2\cos^2\psi,\ \beta=C_2\sin\psi\cos\psi.$$

Suppose we draw a radial line OP (see Fig. 42) in the plane of xy from the centre of the oscillator O which makes an angle ψ with the y-axis, and if the length $OP=r$, then it is clear that the electric force at P($=$Z) is parallel to the z-axis and has a value $C_1\cos\psi$, and the resultant magnetic force $(\sqrt{\alpha^2+\beta^2})$ is in the plane of xy and has a value $C_2\cos\psi$. Moreover, this resultant magnetic force at the point P is perpendicular to OP since $\beta\cos\psi=\alpha\sin\psi$.

Let us then suppose that two such rectangular coils are erected with planes perpendicular to the earth and at right angles to each other, one coil being in the plane yz and the other in the plane xz. Let the thick lines AB and CD, in Fig. 43, represent the plan or intersection of these coils with the xy plane. Let ab be the trace or section of another rectangular coil with its plane perpendicular to the xy plane, and let ϕ be the angle this latter rectangle makes with the coil AB, whilst ψ is the angle which any line OP drawn in the plane xy makes with the plane of the coil AB. Let oscillations be set up in the coil ab. These will induce other secondary oscillations in the two coils AB, CD. The induced oscillations will have amplitudes respectively proportional to C $\cos\phi$ and C $\sin\phi$ in the coils AB and CD where C is some constant. Hence the total electric and magnetic

forces produced by both coils AB and CD at the point P will be proportional respectively to,

$$CC_1 \cos\phi \cos\psi + CC_1 \sin\phi \sin\psi$$

and

$$CC_2 \cos\phi \cos\psi + CC_2 \sin\phi \sin\psi$$

or to

$$CC_1 \cos(\phi-\psi) \text{ and } CC_2 \cos(\phi-\psi).$$

Accordingly the forces at P will have a maximum value when $\cos(\phi-\psi)=1$ or when $\phi-\psi=0$ or $\phi=\psi$, that is when the plane of the coil *ab* coincides with the direction OP.

The same, therefore, is true of the absorption.

Hence, if the coils AB and CD represent the coils above mentioned wound on the cylinder and respectively inserted in the two nearly closed antennæ circuits which are at right angles to each other, the above proof shows that if the movable coil is connected to a detector circuit, and that coil is then rotated into a position in which the signal strengths received on it are a maximum, the plane of the coil will lie in the direction in which the incident electromagnetic waves are arriving.

We can thus construct what is called a *directional receiving aerial* with two fixed nearly closed receiving antennæ with planes and right angles, and the only movable part is the coil in the interior of the above described cylinder having wound on it the two rectangular coils with planes at right angles to each other. By this means it is possible to locate the direction of the sending station emitting the waves which are detected.

The greatest utility of such directional aerials arises when two stations at a known distance apart are both equipped with directional aerials made as above described.

These stations can then be used to determine the exact position of the transmitter emitting the waves they both pick up simultaneously, which transmitter may be situated on a ship or in a dirigible airship or aeroplane.

Thus, for instance, if there be two such directional stations at places X and Y, and if the distance XY is known and if the stations are in communication with each other by telephone or by radiotelegraph, then if a ship or other radio station at a distant point P sends out a wireless signal, each of the stations X and Y will pick it up simultaneously and locate the direction of the arriving waves. This will determine the angles PXY and PYX respectively. If, then, the distance XY is known, the length of the sides XP, YP can be immediately calculated or determined graphically by a simple device. The stations X and Y can then notify to P its position.

In this case the land stations are provided with diretional receivers and the sending station possesses an ordinary symmetrical transmitting aerial. This method was much employed during the latter part of the European War (1914-1918). It was used by Germans to locate the position of Zeppelin airships lost in fog or cloud. In this case land radio stations provided with directional receivers were called up by the lost airship and gave it its position in latitude and longitude, also enemy ships incautiously using wireless telegraphy ran risk of having their position determined by our land directional stations.

Such directional receiving aerials have also been placed on board ship with advantage to enable the ship, when in fog, to determine the direction of known sending coast radio stations, and by two such determinations at different distances the approximate position of the ship at sea.

In establishing such directional receiving aerials on board ship certain precautions were found necessary. The aerials were made completely closed at the top and formed of single wires, four such wires coming down from a single overhead insulator along lines forming the edges of a four-sided pyramid. The bottom ends of these wires were then brought in horizontally to a central receiving cabin. The necessary width at the bottom of the aerials can be obtained by connecting the bottom ends to the bulwarks of the ship. It was found necessary to place this directional aerial in a perfectly symmetrical position in the centre of the ship, between funnels and metallic masses. Also it was necessary to disconnect the ordinary T or fan-shaped aerial wire used in transmission. At the

centre of the lower horizontal portion of each of these closed aerials a condenser of variable capacity is inserted with inductance coils on either side by means of which it was coupled to the direction-finder already described above. The object of these condensers is to tune both the closed receiving aerials to the same frequency so that the oscillations created in the two fixed coils of the direction-finder are exactly in the same phase, and have the same amplitude at maximum. It was also found necessary to place the direction-finder close to the two aerials, and perfectly symmetrical with them.

Furthermore, since the observation of the position of maximum sound in the telephone on rotating the secondary coil of the direction-finder is not sharply marked, it was found better to observe the two positions of zero or minimum sound which occur at $\pm 90^\circ$ from the maximum, and then take the means of the two readings on the angular scale. By such means the direction of coast radio stations at distances of 100 miles or so can be determined on a ship provided with directional receiving aerials with an error of not more than 2° or $2^\circ\ 30'$.

On the east coast of Canada there have been established four direction-finding stations at Cape Sable, Cape Causo, Cape Race, Newfoundland, and at the mouth of Halifax Harbour, which are able to give to any ship asking for its position its latitude and longitude when the ship is lost in a fog, and hence enabling the ship to steer a safe course. For some results of determinations of positions of land stations by a ship provided with directional apparatus, the reader is referred to *The Electrician*, vol. 75, p. 778.

Another application of wireless telegraphy by ships to determine their relative position in fogs or their position as regards coast stations has been made by Professor J. Joly. If a ship or coast station is provided with the means of making audible signals, such as short blasts on a whistle or horn at equal distances of time, and at the same instant sending out a radio signal, a distant ship or coast station will receive the radio signal practically at the instant of its emission, but since the sound wave travels in air at the rate of 1100 feet per second, the sound wave arrives later than the radio signal by 5·5 seconds for every mile of distance between the sending and receiving stations. Suppose, then, that a coast station is provided with means for sending out sound signals and radio signals simultaneously at known intervals of time, a ship in the offing could, by observation of the lag of time of the sound signal, determine at once its distance from the coast. Professor Joly has also worked out ingenious methods by which two ships each so provided can determine their distance from each other.

For details the reader is referred to his papers in *The Proceedings of the Royal Society of London*, vol. 92, A, 1915-1916, pp. 170, 176, 252, and also to a paper by H. C. Plumer, p. 377, in the same volume.

CHAPTER IX

RADIOTELEGRAPHIC TRANSMISSION

1. The Function of the Earth in Radiotelegraphy.—In this chapter we shall consider some of the practical problems connected with the transmission of electromagnetic waves of long wave-length over the earth when employed for radiotelegraphic purposes. We have seen that when initiating practical radiotelegraphy Mr. Marconi's success was reached by employing nearly vertical aerial wires insulated at the upper ends, but at the moment of sending the signal these wires were both connected to the earth. At the sending station he used an aerial wire having a pair of spark balls inserted near the earthen end (see Fig. 1 (*a*)). When

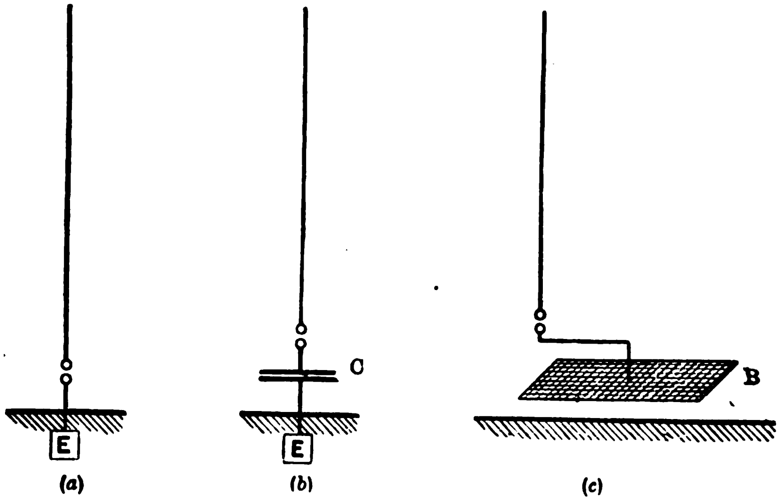


FIG. 1.—Various Modes of Connecting the Transmitting Antenna to the Earth. (*a*) By direct connection of the lower spark ball to an earth plate, E; (*b*) by connection through a condenser, C; (*c*) by a balancing capacity, B.

a spark passed across the gap between the balls the aerial wire was at that moment placed in conductive connection with the earth. At the receiving end the incident electric waves created oscillations in the receiving aerial, and this wire likewise had its lower end in connection with the earth. Hence the two aerial wires at the moment of sending the signal were joined together through the earth. In all his subsequent improvements he never departed from this earth connection at both ends, and maintained it was essential for successful long-distance working.

Subsequently other inventors inserted a large capacity or condenser, C, between the spark balls and the earth plate (see Fig. 1 (*b*)), and others adopted the use of a balancing capacity or counterpoise, B (see Fig. 1 (*c*)), consisting of a sheet of metal or network laid over the earth, but insulated from it. It cannot be considered that the insertion of the condenser between the earth plate and the spark balls makes any essential scientific difference, although it may be done for

the purpose of reducing the antenna capacity. If the conductively earthed antenna (*a*) is employed, then, as we have seen, the distribution of current in it must be such that there is a node of current at the upper end, and an antinode at the earthed end. In other words, a current must flow into and out of the earth plate and earth. If we sever the metallic connection to the earth plate and introduce a condenser of large capacity, we do not make any change in this respect. In place of a current of conduction we have a dielectric current through the condenser, and if its capacity is sufficiently large, this dielectric current may be as great as the current of conduction which flowed when the metallic conductor was complete. Hence (*b*) is not different in principle from (*a*), and the antenna may with equal truth be said to be "earthed" at the lower end in both cases.

On the other hand, great differences of opinion exist whether the technical advantages lie on the side of the "earthed" antenna, or with the completely insulated antenna with balancing capacity (*c*). It is clear that in some cases the balancing capacity cannot be employed; for instance, on board ship, where there is no room for it. Hence, the hull of the vessel, making good earth, is invariably used. In other instances, such as large land stations, the necessary balancing capacity, if used, would be very inconvenient owing to its size, in that it would impede access to the base of the towers or masts carrying the antenna, and necessitate a larger area of ground for working.

In the case of antennæ of very large size and capacity, such as those used for long-distance stations, the size of the insulated balancing capacity required would make its use often inconvenient. Hence it is, so to speak, put underground, in which case it becomes an earth plate.

It has sometimes been urged that the use of the balancing capacity in place of the earth plate prevents dissipation of energy by currents created in the earth, but this is not the case, because, even when a balancing capacity is used, there is still a movement of electricity in the earth around the antenna. Mr. Marconi has always preferred to employ the direct conductive earth connection for antenna, both sending and receiving. In his British Patent, No. 12,039 of 1896, the 15th and 16th claims are as follows:—

15. A receiver consisting of a sensitive tube or other imperfect contact inserted in a circuit, one end of the sensitive tube or other imperfect contact being put to earth whilst the other end is connected to an insulated conductor.

16. The combination of a transmitter having one end of its sparking appliance or poles connected to earth, and the other to an insulated conductor, with a receiver as is mentioned in claim 15.

The reference is to arrangements as shown in Fig. 2 of § 2 in Chap. VII. We have seen that the first demonstrations of electric wave telegraphy were obtained with this earthed system, and others who followed Marconi agreed with him in approving and employing it.

The following extract, taken from a paper by Admiral Sir Henry Jackson, endorses this opinion [1]:—

"A point of interest, which has also great effect on the signalling distance, is the efficiency of the earth connection of both the transmitting and receiving instruments. Fortunately for the system, on board a modern ship there is no difficulty in obtaining an almost perfect earth connection when the ship is at sea. In dry dock, however, there is, in fine weather, a great difficulty in doing so, and the effects of the bad earth with the ship in dock, on the signals, are extremely marked, both for transmitting and receiving, reducing the distance as low as to 25 per cent. of the distance with the ship afloat.

"A similar effect, due to drought, has been observed with some shore stations, where, according to my experiences, the maximum signalling distances have always been obtained during wet seasons of the year.

"A typical example is given:—

"On one particular occasion, towards the end of a very dry summer (last year), the maximum signal distance between a certain ship and station, 500 feet above the sea, was 38 miles, the usual distance having previously been 68 miles. Two days later, during which

[1] See Admiral Sir H. B. Jackson, R.N., F.R.S., "On Some Phenomena affecting the Transmission of Electric Waves over the Surface of the Sea and Land," *Proc. Roy. Soc. Lond.*, 1902, vol. 70, p. 254.

time no alterations whatever had been made to the adjustments of the instruments, but which included twenty-four hours of heavy rain, the maximum distance obtained was 70 miles, which has since been maintained.

"Repeated experiments with and without earths on the transmitter and receiver have shown that, in the open sea, signals may be obtained up to 50 or 60 per cent. of the full distance, without earths on the receiver, though such a large proportion is unusual, the average being 30 per cent. A condenser of suitable capacity acts nearly as well as a good earth; without an earth on the transmitter, the percentage of distance has never exceeded 15 per cent. Using good earth, but no aerial wire whatever on the receiver, or near it, signals have never been obtained over 3 miles. With no aerial wire on the transmitter, I have never known a signal to be received on board another ship over 2 miles distant.

"My experience demonstrates most clearly, and with no marked exception, that, for signalling any distance beyond a few miles, the combination of aerial wires and good earths is essential, for both transmitting and receiving instruments."

On the other hand, Sir Oliver Lodge has expressed strong opinions on the disadvantage of the direct connection of the antenna to earth, stating that the result of so doing is to damp out the oscillations set up in it sooner, and bestow on the trains of radiated waves a large decrement.[2]

He maintains that the earth connection is inimical to "good tuning," which means to the production of prolonged trains of oscillations in the antenna. He says (*loc. cit.*), "If the earth were a perfect conductor it would presumably act like a mirror preventing the waves from spreading in that direction, and thereby doubling the intensity of any radiator above it; except that in certain places there would be liable to be interference bands where the difference between the source and the image was half a wave-length. Such interference, however, chiefly occurs in the case of those long trains of waves appropriate for tuning. For single pulses —that is to say, the snaps needed for untuned signalling—the effect of a perfectly conducting earth would probably be good, and in so far as the sea is a moderately good conductor, connection with the sea may be advantageous for such signalling; but for tuned relation between the stations it is becoming clear that instead of prolonging the oscillations, its resistance wipes them out and kills them. It is far better to ignore the earth and work independently of it both at the sending and receiving end, taking care to keep everything insulated. We hereby gain the advantage of being independent of fluctuations in the quality of the soil in respect both of permanent geological quality and of variable heat and moisture, and we also get far better tuning."

"On the train of waves passing between distant stations the earth probably has no particular influence except by reason of its irregularities and obstruction; but over great distances it is possible they may be reflected advantageously in the good conducting regions of the upper atmosphere. But with extremely great distances Mr. Marconi has chiefly dealt. My object has been to perfect the tuning for moderate distances."

But these opinions, even although coming from a great authority on the scientific side of the subject, are not supported by the evidence of experience in practice. As already pointed out, a balancing capacity is impossible in the large majority of cases, ships and power stations, and very inconvenient even in large shore stations.

The opinion that the earth in between the sending and receiving station exercises no particular effect is directly negatived by the researches of J. Zenneck and Brylinski, and by the everyday experience of radiotelegraphists, who are well aware of the great differences between over-sea and over-land radiotelegraphy, especially when using short wave-lengths.

The bases for the above opinions are to be found in the paper published by Sir Oliver Lodge and Dr. Muirhead, in which experiments are described made between stations at Down and at Elmers End in Kent, 7 miles apart. The station at Elmers End sent out radiation of about 440 metres wave-length, with an applied power of 400 watts. The station at Down received this on an aerial consisting of two capacity areas, each composed of four loops of wire on a horizontal plane, the

[2] See Sir Oliver Lodge and Dr. A. Muirhead, "On Syntonic Telegraphy," *Proc. Roy. Soc. Lond.*, vol. 82, A, p. 227, 1909.

centres being connected by a vertical wire. The upper area was elevated 60 to 67 feet above the earth, and the lower one at various heights above the earth. At the centre of the vertical wire was a receiving instrument which, in the case of these measurements, was a Duddell thermal ammeter.

A variable inductance or capacity was also inserted by means of which the receiving aerial could be put more or less out of tune with the incident wave, and a resonance curve could then be plotted from the observed deflections of the thermal ammeter It was found that when the lower capacity area was completely insulated and some way above the earth (even only 6 feet up) the resonance curve plotted out with a very sharp peak, thus indicating small damping both in the sending and receiving circuits. If, however, the lower capacity area was near to or on the earth, the resonance curve was flat and presented no well-marked peak. The experiments undoubtedly show that the earth at the receiving antenna does produce a sensible effect in damping the free oscillations set up in the receiving aerial, but they do not give proof that the earthing at the sending end is equally injurious. Nor do they prove that if the receiving or wave detecting instrument is inserted in a suitable closed oscillatory circuit inductively connected to the receiving antenna, the damping out of the oscillations in the aerial wire will be also accompanied by an equally quick damping out of the oscillations in the closed coupled circuit.

The conclusion of Sir Oliver Lodge and Dr. Muirhead that the earth connection is always a disadvantage is not supported by the opinion of other workers, who have carried out similar experiments with earthed and non-earthed antennæ. Thus J. S. Sachs,[3] in 1905, conducted experiments on the function of the earth in wireless telegraphy, in which he used Braun's form of apparatus, the energy transmitted and received being measured thermoelectrically. In some cases the stations were in resonance, and in others not. He came to the conclusion that the radiation from a system with an aerial and direct earth connection is three or four times greater than when the earth is replaced by a balancing capacity. He also tried the effects of raising the transmitter and receiver high above the ground, and found it an improvement. From these experiments he concluded that the earth between the stations exercises an absorptive action and does not much reflect the waves. He found that the energy received varies very approximately inversely as the square of the distance.

W. Burstyn (see *Science Abstracts*, vol. 10, B, 1907, *abs.* 222) also concludes, from a theoretical discussion of the influence of the size and position of the balancing capacity with respect to the earth, that unless the station is a small or temporary one, or built on hard dry rock, it is more economical to employ a direct earth connection. For military stations or those constantly moved about, he thinks it is better to employ a balancing capacity, as it ensures the equivalent of a good earth, and a proper earth connection cannot be obtained in dry sandy soil or very rocky ground. On the other hand, practical experience on a large scale does not give any warrant for the conclusion that the direct earth connection is always bad; on the contrary, long-distance work is impossible without it.

The reader may also be referred to a paper by Mr. Charles A. Culver in the *Physical Review* for September 1907, p. 200, entitled "A Study of the Propagation and Interception of Energy in Wireless Telegraphy," in which the writer examines the question of earth connection, and comes to the conclusion that "the earth plays a highly important part in the transmission of energy in wireless telegraph circuits, particularly the 'ground' at the transmitting station." He considers that the earth connection greatly increases the effect on the receiver. He even suggests that the effects at great distances are due to electrical disturbances, propagated through the earth's crust, and not directly to the effects of a free Hertzian wave through the space above it. There are, however, special difficulties connected with this view, though on the other hand there is some experimental evidence that the action at a distance due to an earthed transmitting antenna is not wholly the result of a space wave, as signals can be received without the use of any high

[3] See J. S. Sachs, "On the Function of the Earth in Wireless Telegraphy," *Elektrotechn. Zeitschr.*, vol. 26, p. 951, October 1905; or *Science Abstracts*, vol. 8, B, 1905, *abs.* 1589.

aerial wire at the receiving station, provided there is a good earth connection to the receiver.

The reader may also be referred to some quantitative experiments by Prof. C. Tissot (see "Résonance des Systèmes d'Antennes"), in which measurements were made with the bolometer receiver of the current in a receiving antenna, collecting radiation from a distant sending antenna. This last was so arranged that it could be connected at pleasure to various earth plates having a "bad" earth, a "dry" earth, and a "damp" and "very good" earth. The deflections of the bolometer galvanometer were respectively 10, 26, 28, and 34 scale divisions, thus showing the improvement in the receiving antenna current with improvement in the "earth" at the sending end. Again, M. Tissot measured the receiving-end current when the earth plate was 1 metre square and 30 metres square, and found bolometer deflections respectively of 30 and 55 divisions. He also found that at the sending antenna under the same conditions the mean square value of the current at the base of the antenna increased with the area of the earth plate at the sending end up to a certain area, whilst on board ship, where the contact with "earth," or rather sea, wàs perfect, the mean square sending antenna current had a still larger value than for a similar transmitter on shore with an earth plate. His conclusion is that the earth connection absorbs a certain fraction of the energy imparted to the sending antenna.

Apart altogether from the insulation or non-insulation of the antenna from the earth, whether conductively connected to it or connected through a condenser, or, on the other hand, united to an insulated balancing capacity, the nature of the earth's surface, whether sea or land, damp or dry soil, between the sending and receiving stations, exercises a great effect upon the range of radiotelegraphy possible with any given apparatus. We must, therefore, consider the function of the earth generally in this matter.

In the earliest days of electric wave telegraphy it was found that the waves employed of about 100 to 300 metres wave-length travelled much better over sea than over land. Also very dry land seemed to offer greater obstruction than damp soil, and an improvement in overland radiotelegraphy was often noticed in those days after the occurrence of wet weather.

These facts showed that the nature of the terrestrial surface between the stations was not without effect upon the transmission of radiotelegraphic waves. Hence we must consider from a theoretical point of view the propagation of an electric wave over a conducting surface.

It has already been shown in Chap. II. that high frequency alternating currents or oscillations are chiefly confined to the surface of the conductors conveying them. The penetration of the current into the conductor is less, the greater the conductivity and the greater the magnetic permeability of the material of which it is made.

This can be illustrated by an experiment shown by the author in a discourse at the Royal Institution on June 4, 1909, as follows :—

An oscillatory circuit is constructed, consisting of a condenser comprising one or more Leyden jars, a rectangular wire circuit having a gap in it which can be bridged, and a spark gap. The gap can be closed by inserting in it a short wire spiral, consisting of a copper, iron, or brass wire about No. 14 S.W.G. gauge, and each of the same length, viz. 30 to 40 cms. wound up in short open spirals of 8 or 10 turns. Oscillations are set up in this circuit by an induction coil as usual. Alongside of this circuit is placed a cymometer with neon tube as indicator, the brightness of the glow serving as an index of the amplitude and damping of the oscillations in the primary circuit. If then the spirals are successively placed in the primary circuit, and the cymometer circuit adjusted to resonance, we may place the cymometer so near the primary that when the copper or brass spiral is in circuit the neon tube glows brightly, but when the iron spiral is in circuit the tube hardly glows at all. If then a galvanized iron wire of the same thickness and length is substituted, it will be found that the tube glows as brightly as when the copper spiral is employed. This shows that the thin layer of zinc put on the iron is sufficient to prevent penetration of the oscillations into the iron ; in other words, that they are confined to the surface.

If, however, we paint the iron spiral or even cover it with a thick layer of badly conducting plaster of Paris, it still damps the oscillations as much as a bare iron wire, showing that the oscillations penetrate through the badly conducting layer of plaster or paint.

We have already, in Chap. II. § 1, given the fundamental equations for the density of a current established in a conductor at any point of which the coordinates are x, y, z, viz.—

$$\frac{4\pi\mu}{\rho}\frac{du}{dt}=\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and two similar equations in v and w, where u, v, and w are the rectangular components of the current density, and μ and ρ are the permeability and resistivity of the conductor.

The meaning of this equation will best be understood by applying it to a particular case. Let us suppose a plane surface to separate a conductor of conductivity $\frac{1}{\rho}$ and permeability μ from a dielectric of unit permeability and zero conductivity. Let x be the direction of an axis measured from any point in the plane perpendicular to the surface, whilst the y and z axes lie in that plane. If then a current is established in the conductor, it will begin at the surface and diffuse inwards by a process resembling the conduction of heat. If the current is alternating, it may be represented as proportional to the real part of the function ϵ^{jpt}. Hence, since there is no variation in the direction of y and z, the equation (1) reduces to

$$\frac{4\pi\mu p}{\rho}ju=\frac{d^2u}{dx^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and if a^2 denotes $\frac{4\pi\mu pj}{\rho}$, we have—

$$\frac{d^2u}{dx^2}=a^2u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

as the equation expressing the current diffusion into the conductor.

The solution of this is—

$$u=A\epsilon^{-ax}+B\epsilon^{ax} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and since $u=0$, when $x=\infty$, the constant $B=0$, or $u=A\epsilon^{-ax}$

Now $(1+j)^2=2j$, where $j=\sqrt{-1}$.

Hence
$$a=\sqrt{\frac{2\pi\mu p}{\rho}}(1+j) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

And therefore
$$u=A\epsilon^{-\sqrt{\frac{2\pi\mu p}{\rho}}x}\epsilon^{j\left(pt-\sqrt{\frac{2\pi\mu p}{\rho}}x\right)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Accordingly, the current u decreases in amplitude and changes in phase as we penetrate into the conductor. The current is reduced to $\frac{1}{\epsilon}$ of its value at the surface at a depth equal to $\frac{\sqrt{\rho}}{\sqrt{2\pi\mu p}}$.

Thus, for instance, if the conductor is copper we have $\mu=1$, $\rho=1600$, and if the frequency of the alternations is 10^6, then—

$$\sqrt{\frac{2\pi\mu p}{\rho}}=2\pi\sqrt{\frac{n}{\rho}}=157.$$

The current, therefore, is reduced to $\frac{1}{\epsilon}=0\cdot367$ of its amplitude or strength at the surface at a depth of $\frac{1}{157}$ of a centimetre, or about $\frac{1}{400}$ of an inch.

At about four times this depth the current is reduced to ϵ^{-4} of that at the surface,

viz. to about 2 per cent. of its surface value. Hence the current is practically confined to a skin of thickness $\frac{2}{\pi}\sqrt{\frac{\rho}{\mu n}}$. If the conductor is of iron, then for a frequency 10^6 we may take $\mu = 1000$ and $\rho = 10{,}000$. Hence the skin thickness would then be about 0·002 cm. It can be shown that the resistance of the conductor in a direction parallel to the surface to alternating currents of a frequency n is the same as that of strips of thickness $\frac{1}{2\pi}\sqrt{\frac{\rho}{\mu n}}$ to steady currents. Hence, in the case of copper strip a thickness of about $\frac{1}{16}$ of a mm. or $\frac{1}{400}$ of an inch thick would present the same resistance to steady currents as does the slab of infinite thickness to currents of a frequency of 10^6 flowing parallel to the bounding surface. If then we consider a strip of metal of finite thickness and ask the question, For what limiting frequency has the strip practically the same resistance for alternating as for steady currents? it is easy to see that the upper limit of the frequency n is given by solving the equation—

$$\frac{1}{\pi}\sqrt{\frac{\rho}{\mu n}} = t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where t is the given thickness. Thus, for oscillations of frequency 10^6, if a strip of copper has a thickness not exceeding $\frac{1}{25}\,\frac{1}{\pi}$ cm. or about $\frac{1}{8}$ mm., it will have the same resistance as for steady currents. In other words, its high frequency resistance will be the same as its ohmic resistance.

The same principles are applicable in the passage of an electric wave over a conducting dielectric. The wave is an alternation of electric force perpendicular to, and magnetic force parallel to, the surface. The magnetic force diffuses into the surface and there dissipates as heat energy which is drawn from that of the wave. Hence the operations taking place when an ordinary low frequency current is established in a conductor and when an electric wave moves over its surface are identical in this respect that the conductor absorbs and dissipates energy.

This penetration depends, as we have seen, upon the conductivity of the surface over which the wave glides.

If the surface is a very good conductor, the wave penetrates into it very little, but glides over the surface. If it is a poor conductor, the wave penetrates into it to a greater extent, and the worse the conductivity the deeper the penetration.

The materials of which the earth's crust is composed, with some exceptions, owe their electric conductivity chiefly to the presence of water in them. They are called electrolytic conductors. Substances like marble and slate when free from iron oxide are fairly good insulators. Dry sand or hard dry rocks are poor conductors, but wet sand and moist earth are fairly good conductors. Sea water, owing to the salt in it, is a much better conductor than fresh water. The table on p. 623 gives some figures which, however, are only approximate, for the specific resistance of various terrestrial materials in ohms per metre cube. It will be seen that dry sand or soils are of very high specific resistance, and damp or wet sand or clay fairly low.

The values for the resistivity of earth crust materials in Table I are only very rough approximations. There is good reason for believing that some of these materials are very much better conductors for high frequency alternating currents than for continuous or low frequency currents.

If our earth's surface had a conductivity equal, say, to that of copper, then the electric radiation from an antenna would glide over the surface without penetration. In the case of the actual earth there is, however, considerable penetration of the wave into the surface, and therefore absorption of energy by it.

We are in the habit of speaking of electric or Hertzian wave telegraphy as "wireless" telegraphy, regardless of the fact that some of the functions of the wire in ordinary conductive telegraphy are taken by the earth in case of radiotelegraphy.

TABLE I

APPROXIMATE CONDUCTIVITY AND DIELECTRIC CONSTANT OF VARIOUS TERRESTRIAL MATERIALS.

Material.	Specific resistance in ohms per metre cube.	Dielectric constant. Air = 1.
Sea water	0·25 to 1	80
Fresh water	100 to 1000	80
Moist earth	10 to 1000	5 to 15
Dry earth	10,000 and upwards	2 to 6
Wet sand	1 to 1000	9
Dry river sand	very large	2 to 3
Wet clay	10 to 100	...
Dry clay	10,000 and upwards	2 to 5
Slate	10,000 to 100,000	...
Marble	5,000,000	6
Mercury	0·000001	infinity

In the older method the wire serves as a guide to the energy, but at the same time, so to speak, charges a commission for this service in the shape of the energy dissipated in it as heat. In the latter case the earth's surface acts to some extent as the guide, and it also takes toll for that office by dissipating some of the wave energy. Electromagnetic waves of long wave-length, generated at the transmitting station, thus lose energy as they travel over the earth's surface by penetration into and absorption by the terrestrial surface. It has long been known that radiotelegraphy is conducted with greater ease over sea than over land, but the reasons for this difference were not at once apparent. An important contribution was, however, furnished by a theoretical investigation made by Dr. J. Zenneck, in a paper entitled "The Propagation of Plane Electromagnetic Waves over a Plane Conducting Surface with Reference to Wireless Telegraphy."[4]

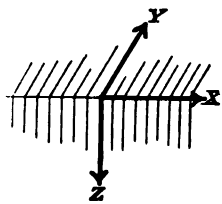


FIG. 2.

Dr. Zenneck considers the case of a plane electric wave travelling without divergence over a surface bounding two media of different conductivity and dielectric constant. It is obviously desirable to simplify the problem by leaving out of account at first the diminution of wave amplitude by mere distance, and also that due to curvature of the bounding surface. Let the direction of propagation be taken as that of the x axis, whilst the direction of the z axis is downwards into the denser medium or soil, and the direction of the y axis is away from the reader (see Fig. 2). This convention as to axes may be called the German system, as opposed to the English, in which the direction of the z axis would be upwards. The German system has the advantage that it correctly represents the relation between the electric and magnetic forces in the wave and the direction of propagation. For in this case, if x is the direction of wave propagation, then y is the direction of the magnetic force, and z that of the electric force of the wave, these vectors being in the plane of the wave.

The following symbols will then be used:—Let K denote the dielectric constant of the medium in the C.G.S. system reckoned in electrostatic units, and let μ denote the magnetic permeability in electromagnetic units. Hence for air $K=1$, $\mu=1$. Then let s be the specific conductivity in electrostatic units,

[4] See J. Zenneck, "Über die Fortpflanzung ebener elektromagnetischer Wellen längs einer ebenen Leiterfläche und ihre Beziehung zur drahtlosen Telegraphie," *Annalen der Physik*, vol. 23, p. 846, 1907.

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so that if ρ is the resistivity in ohms per centimetre cube, then $s=\frac{9\times10^{11}}{\rho}$. Also let the frequency be denoted by n and $2\pi n$ by p. Let $u=3\times10^{10}$ be the wave velocity, λ the wave-length, and $\frac{2\pi}{\lambda}=q$. We shall employ the letter j to denote $\sqrt{-1}$. Hence the expression for any vector is in the form $a+jb$.

Let the axial components of the electric force E be denoted by X, Y, and Z, and those of the magnetic force H by α, β, and γ.

Consider, then, a small rectangular element of volume taken in the medium close to the bounding surface, and with one corner at the origin (see Fig. 3). Let the side parallel to the x axis have a length δx, that parallel to the z axis have a length δz, and that parallel to the y axis a length unity. Then through the surfaces of this volume the electric and magnetic forces create displacement D and magnetic flux F per unit of area. If K is the dielectric constant of the material, then $D=\frac{KE}{4\pi}$, or, writing k for $\frac{K}{4\pi}$, we have $D=kE$; also $F=\mu H$, where μ is magnetic permeability. These fluxes and forces are connected in accordance with the two circuital laws of electromagnetism as follows:—

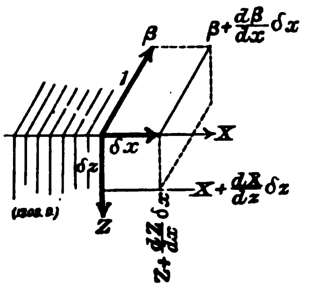


FIG. 3.

(i) The line integral of magnetic force round the boundary of a curve taken in the dielectric is numerically equal to 4π times the total electric currents through that area.

(ii) The line integral of electric force round any area is numerically equal to the time rate of decrease of magnetic flux through that area.

Let us apply the above theorems to the sides of the small element of volume. If s is the conductivity in electrostatic units of the material and $k=\frac{K}{4\pi}$, then for the side $1\times\delta x$ normal to the z axis, the electric force Z produces through it a conduction current $sZ\delta x$ and also an electric displacement $kZ\delta x$, and therefore a displacement current $pkZ\delta x$, which is in quadrature as regards phase with the conduction current. Hence the total current is $(s+jpk)Z\delta x$.

To reduce this current to electromagnetic units we must divide by $u=3\times10^{10}$, and, therefore—

$$\frac{4\pi}{u}(s+jpk)Z\delta x \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

is equal by the first law to the line integral of the magnetic force round the area $1\times\delta x$. This latter is equal to

$$\beta-\left(\beta+\frac{d\beta}{dx}\delta x\right) \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

since the magnetic force is wholly in the plane of the wave, and therefore β is the only component concerned. Accordingly we have—

$$\frac{4\pi}{u}(s+jpk)Z=-\frac{d\beta}{dx} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

Now the electric and magnetic forces in an electric wave are pulsating vectors which we shall assume are simple harmonic functions of the space and time. Therefore, mathematically we can take the components of the electric and magnetic forces as proportional to the real part of $\epsilon^{j(pt+qx)}$, because this function is equal to $\cos(pt+qx)+j\sin(pt+qx)$ and $\cos(pt+qx)$ represents a wave motion,

since it is a function which is periodic, both with regard to x and t, or space and time. Accordingly, if $\beta = A\epsilon^{j(pt+qx)}$, then $\frac{d\beta}{dx} = jq\beta$, and we have—

$$\frac{4\pi}{u}(s+jpk)Z = -jq\beta \qquad (11)$$

In the same way, if we take the total current through the area $1 \times dz$, and parallel to the x axis, we obtain the equation—

$$\frac{4\pi}{u}(s+jpk)X = \frac{d\beta}{dz} \qquad (12)$$

In the next place apply the second law to the area $\delta x \delta z$.

The line integral of electric force round this area is—

$$Z\delta z - \left(Z + \frac{dZ}{dx}dx\right)\delta z - X\delta x + \left(X + \frac{dX}{dz}dz\right)\delta x = \left(\frac{dX}{dz} - jqZ\right)\delta x\delta z \qquad (13)$$

The time rate of change of the magnetic flux through this area is $\mu\frac{d\beta}{dt}\delta x\delta z$

Hence we have—

$$u\left(\frac{dX}{dz} - jqZ\right) = jp\mu\beta\,;$$

or dividing both sides by 4π and putting μ' for $\frac{\mu}{4\pi}$, we have—

$$\frac{dX}{dz} - jqZ = \frac{4\pi}{u}jp\mu'\beta \qquad (14)$$

If we write v for $\frac{u}{4\pi}$ our equations take the form—

$$(s+jpk)Z = -vjq\beta \qquad (15)$$

$$(s+jpk)X = v\frac{d\beta}{dz} \qquad (16)$$

$$\frac{dX}{dz} - jqZ = \frac{1}{v}jp\mu'\beta \qquad (17)$$

Eliminating X and Z from the above equations, we have—

$$\frac{d^2\beta}{dz^2} = \left\{q^2 + j\frac{p\mu'(s+jpk)}{v^2}\right\}\beta \qquad (18)$$

The solution of (18) is $\beta = \epsilon^{-jBz}$, where

$$B^2 = -\left\{q^2 + \frac{jp\mu'(s+jpk)}{v^2}\right\}$$

or

$$B^2 + q^2 = -j\frac{p\mu'(s+jpk)}{v^2} \qquad (19)$$

Hence, since β varies as $\epsilon^{j(pt+qx)}$, we have for the complete expression—

$$\beta = A\epsilon^{-jBz}\epsilon^{j(pt+qx)} \qquad (20)$$

where A is some constant.

From (15), (16), and (17) we then easily find that—

$$X = -jBA\frac{v}{s+jpk}\epsilon^{-jBz}\epsilon^{j(pt+qx)} \qquad (21)$$

$$Z = -jqA\frac{v}{s+jpk}\epsilon^{-jBz}\epsilon^{j(pt+qx)} \qquad (22)$$

Suppose, then, that we apply these equations and solutions to two small solid rectangles of side-lengths δx, 1, and δz taken in the direction of the axes, but one

taken in the dielectric below the bounding surface and one in the air above (see Fig. 4).

In the air the equations take the form—

$$\left.\begin{aligned} (s+jpk)Z &= -vjq\beta \\ (s+jpk)X &= -v\frac{d\beta}{dz} \\ \frac{dX}{dz} - jqZ &= \frac{1}{v}jp\mu'\beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In the dielectric as above—

$$\left.\begin{aligned} (s'+jpk')Z' &= -vjq\beta' \\ (s'+jpk')X' &= v\frac{d\beta'}{dz} \\ \frac{dX'}{dz} - jqZ' &= \frac{1}{v}jp\mu'\beta' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where the accents denote quantities in the dielectric. The solutions are—

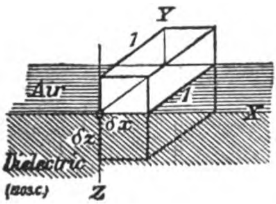


FIG. 4.

$$\left.\begin{aligned} \beta &= A\epsilon^{-jBz}\epsilon^{j(pt+qx)} \\ X &= -jBA\frac{v}{s+jpk}\epsilon^{-jBz}\epsilon^{j(pt+qx)} \\ Z &= -jqA\frac{v}{s+jpk}\epsilon^{-jBz}\epsilon^{j(pt+qx)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$\left.\begin{aligned} \beta' &= A'\epsilon^{-jB'z}\epsilon^{j(pt+qx)} \\ X' &= jB'A'\frac{v}{s'+jpk'}\epsilon^{-jB'z}\epsilon^{j(pt+qx)} \\ Z' &= -jqA'\frac{v}{s'+jpk'}\epsilon^{-jB'z}\epsilon^{j(pt+qx)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Then, as shown above, the values of B and B′ are given by the equations—

$$\left.\begin{aligned} B^2 + q^2 &= -jp\mu'\frac{s+jpk}{v^2} \\ B'^2 + q^2 &= -jp\mu'\frac{s'+jpk'}{v^2} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

At the bounding surface the horizontal components of the magnetic force—viz. β and β'—are identical. We have, then, when we put x, z, and $t=0$ in the above equations (26) and (27), $A=A'$. Again, since $X=X'$ when x, z, and $t=0$, we have—

$$\frac{B'}{s'+jpk'} = -\frac{B}{s+jpk}$$

For brevity let us write T for $s+jpk$, and T′ for $s'+jpk'$, and also P for $\frac{p\mu'}{v^2}$.

Then the above relations may be written—

$$\left.\begin{aligned} B^2 + q^2 &= -jPT \\ B'^2 + q^2 &= -jPT' \\ \frac{B}{T} + \frac{B'}{T'} &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Hence
$$B^2 + jPT = B'^2 + jPT'$$

and
$$\left.\begin{aligned} B^2 &= -jP\frac{T^2}{T+T'} \\ B'^2 &= -jP\frac{T'^2}{T+T'} \\ q^2 &= -jP\frac{TT'}{T+T'} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Also we have from equation (25)—

$$\frac{X}{Z}=\frac{B}{q}=\sqrt{\frac{T}{T'}}=\sqrt{\frac{s+jpk}{s'+jpk'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

In the case of air we may consider the conductivity zero. Accordingly—

$$\frac{X}{Z}=\sqrt{\frac{jpk}{s'+jpk'}}=\sqrt{\frac{j\frac{pk}{s'}}{1+j\frac{pk'}{s'}}}.$$

Let m stand for $\frac{pk}{s'}$ and m' for $\frac{pk'}{s'}$. Then—

$$\frac{X}{Z}=\sqrt{\frac{jm}{1+jm'}}$$

Suppose 2ϕ is an angle whose tangent is $\frac{1}{m'}$, then we have by a well-known theorem—

$$\epsilon^{j2\phi}=\cos 2\phi+j\sin 2\phi$$

or

$$\epsilon^{j2\phi}=\frac{m'}{\sqrt{1+m'^2}}+j\frac{1}{\sqrt{1+m'^2}}=\frac{j+m'}{\sqrt{1+m'^2}}$$

or

$$\epsilon^{j2\phi}=\sqrt{1+m'^2}\frac{j(1-jm')}{(1-jm')(1+jm')}$$

$$=\frac{j}{1+jm'}\sqrt{1+m'^2}$$

Therefore

$$\frac{X}{Z}=\sqrt{\frac{jm}{1+jm'}}=\sqrt{\frac{m}{\sqrt{1+m'^2}}}.\epsilon^{j\phi} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Accordingly, X and Z are two vectors which differ in phase by an angle ϕ such that $\tan 2\phi=\frac{s'}{pk'}$.

We are now prepared to apply these formulæ to numerical calculations. It must be remembered that k stands for $\frac{K}{4\pi}$, where K is the dielectric constant as usually measured. Also s stands for the conductivity in electrostatic units, and is therefore equal to $\frac{9\times10^{20}}{r\times10^{9}}$, where r is the specific resistance in ohms per centimetre cube. Accordingly—

$$\tan 2\phi=\frac{1}{m'}=\frac{4\pi\times9\times10^{20}}{2\pi n K r\times10^{9}}=\frac{18\times10^{11}}{nKr}$$

Suppose, then, that we select a wave-length of 1000 feet or 300 metres as our radiotelegraphic wave, and consider the wave to be travelling in air and over the surface of sea water for which $K=80$ and $r=100$. Then $n=10^6$, and we have $\frac{1}{m'}=225$, or $2\phi=90°$ and $\phi=45°$. Also $m=\frac{1}{18{,}000}$.

Hence

$$\sqrt{\frac{m}{\sqrt{1+m'^2}}}=\frac{1}{134} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

This shows, therefore, that when waves 1000 feet in length travel over sea water the horizontal component X of the electric force in the air is negligible, since

$$\frac{X}{Z}=\frac{1}{135}\epsilon^{j\frac{\pi}{4}}$$

Therefore the electric force at the sea surface is nearly perpendicular to that surface, and is a nearly pure alternating force. The same applies to the magnetic vector.

Suppose in the next place that electric waves of the same length are being propagated over very dry land. In this case we should have $K=2$ and $r=10^5$ nearly. Hence, if $n=10^6$ we have—

$$\tan 2\phi = \frac{1}{m'} = \frac{18\times 10^{11}}{2\times 10^{12}} = 0{\cdot}9$$

or

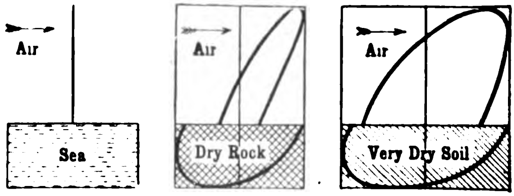
$$2\phi = 42^\circ \text{ or } \phi = 21^\circ \text{ (nearly)}$$

Also $\frac{1}{m} = 1{\cdot}8$. Hence—

$$\sqrt{\frac{m}{\sqrt{1+m'^2}}} = 0{\cdot}625 \text{ (nearly)}$$

In this case, therefore, the horizontal component X of the electric force in the air is $62\frac{1}{2}$ per cent. of the vertical component in magnitude, and they differ in phase by 21°.

When two simple periodic quantities differ in phase and amplitude they compound into a pulsating vector represented by the rotating radius vector of an ellipse.



$r=1$ ohm / metre cube. $K=80$. (*a*)

$r=100,000$ ohms / metre cube. $K=2$-3. (*b*)

$r=10,000$ ohms / metre cube. $K=1$-3. (*c*)

r = Resistivity. K = Dielectric constant.

FIG. 5.

The electric vector in the wave travelling over dry land is therefore not by any means perpendicular to the surface, but is inclined to it, and there is a considerable rotating component. The resultant electric force in the air above the ground may be represented by the rotating radius vector of an ellipse, the major axis of which is inclined forward in the direction in which the wave is travelling (see Fig. 5).

We have to consider, in the next place, the loss in amplitude or intensity of the wave as it travels along due to the absorption of energy by the surface over which the waves are travelling.

The amplitude of either wave vector is expressed by a function of the form $M\epsilon^{j(pt+qx)}$.

If we call the amplitude at the origin M_0, then $M_0 = M\epsilon^{jpt}$. At a certain distance x' along the x axis the amplitude will have fallen to $\frac{1}{\epsilon}$ of that at the origin; we then have

$$M\epsilon^{-1}\epsilon^{jpt} = M\epsilon^{j(pt+qx')} \quad . \quad . \qquad (33)$$

Now q is a complex quantity; let it be written in the form—

$$q = a + jb$$

Then

$$jqx' = jax' - bx'$$

Therefore we have—

$$\epsilon^{-1}\epsilon^{jpt} = \epsilon^{-bx'}\epsilon^{j(pt+ax')}$$

Hence—

$$bx' = 1$$

or

$$x' = \frac{1}{b}$$

Accordingly, if we can express the value of q in the form $q = a + jb$, then we see that $\frac{1}{b}$ is the distance in which the amplitude decays to $\frac{1}{\epsilon}$ of that at the origin by reason of the absorption of energy by the surface over which the waves travel. We have, therefore, to express the value of q in the form $a + jb$. Dr. Zenneck has shown how this may be done as follows:—

Referring to equations (29) we see that

$$q^2 = -jP\frac{T+T'}{TT'}$$

where $$P = \frac{p\mu'}{v^2}$$

and $$T = s + jpk, \text{ and } T' = s' + jpk'$$

If the waves travel in air, then we may put $s = 0$, and we have—

$$q^2 = \frac{p^2\mu' k}{v^2}\frac{s' + jpk'}{s' + jp(k+k')} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The following theorem will then be found useful—

If $a + jb$ is any vector, and if $\tan\phi = \frac{b}{a}$, then it is clear that—

$$\sqrt{a^2 + b^2}\,\epsilon^{j\phi} = a + jb$$

This follows at once from the known exponential values of sin ϕ and cos ϕ, and the geometrical signification of $a + jb$.

Accordingly, we may write the expression $s + jpk$ in the form—

$$\sqrt{s^2 + p^2k^2}\,\epsilon^{j\phi}$$

where $$\tan\phi = \frac{pk}{s}$$

Therefore, from (34) we have—

$$q = \sqrt{\frac{p^2\mu' k}{v^2}}\sqrt{\frac{\sqrt{s'^2 + p^2k'^2}}{\sqrt{s'^2 + p^2(k+k')^2}}}\,\epsilon^{j\left(\frac{\phi_1 - \phi_2}{2}\right)} \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where $$\tan\phi_1 = \frac{pk'}{s'}$$

and $$\tan\phi_2 = \frac{p(k+k')}{s'}$$

Let $\sqrt{s'^2 + p^2k'^2}$ be represented by the letter R, and $\sqrt{s'^2 + (k+k')^2}$ by R′; then, remembering that $v = \frac{(3 \times 10^{10})}{4\pi}$, and $k = \frac{K}{4\pi}$, and $\mu' = \frac{\mu}{4\pi}$, where $K = 1$ and $\mu = 1$ for air, we have—

$$a + jb = q = \frac{p}{3 \times 10^{10}}\sqrt{\frac{R}{R'}}\left\{\cos\frac{\phi_1 - \phi_2}{2} + j\sin\frac{\phi_1 - \phi_2}{2}\right\}$$

Hence—

$$b = \frac{p}{3 \times 10^{10}}\sqrt{\frac{R}{R'}}\sin\frac{\phi_1 - \phi_2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where $$\tan\phi_1 = \frac{pk'}{s'} \text{ and } \tan\phi_2 = \frac{p(k+k')}{s'}$$

Accordingly, the value of $\frac{1}{b}$, or the horizontal distance at which the wave amplitude falls to $\frac{1}{\epsilon}$ of that at the origin, can be numerically calculated when the values of p_1, k', and s' are given. In the case of air $k=\frac{1}{4\pi}$. Thus for very dry soil we might have values as follows: $s=9\times10^{20}\times10^{-16}$, $k'=\frac{2}{4\pi}$, and we may take p to be $2\pi\times10^6$. It can then be shown from equation (36) that $\frac{1}{b}=4$ kilometres.

In the above manner Dr. Zenneck has calculated the distances for diminution of amplitude to $\frac{1}{\epsilon}$ for terrestrial surface materials of various conductivities and dielectric constants, and for an assumed wave-length of 300 metres, corresponding to a frequency of 10^6, and set out the results in curves reproduced in Fig. 6.

It is, then, at once seen that there is a certain soil conductivity which produces the maximum loss of amplitude for a given distance. It is clear that this should

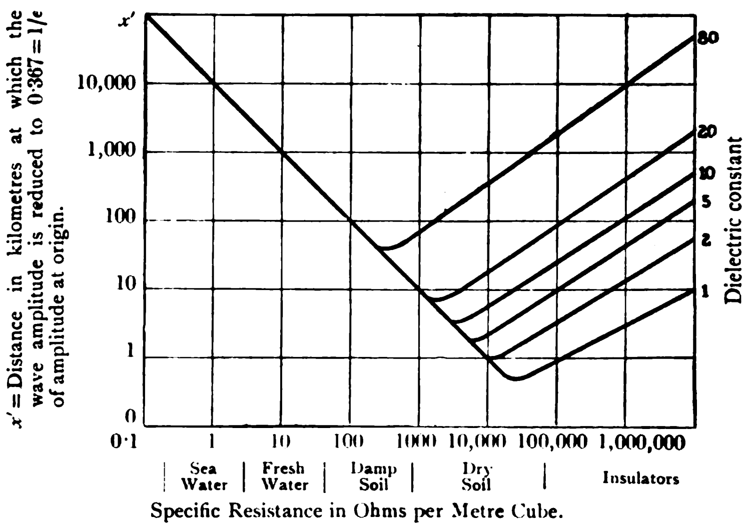


FIG. 6.—Curves showing Distance in which Electric Waves 1000 Feet (300 Metres) in Length have their Amplitude reduced to $1/\epsilon$ by travelling over various Surfaces. (Dr. Zenneck.)

be the case, for if the terrestrial surface were a perfect conductor the waves would not penetrate into it at all, whilst if it were a perfect non-conductor there would be penetration, but no dissipation of energy as heat.

From our equations other important deductions can easily be made as to the depth at which the amplitude of the waves penetrating into the earth is reduced to an assigned fraction, say $\frac{1}{\epsilon}$ of that at the surface. For if we refer to equations (26), it is seen that the magnetic force β' is a function of z the depth below the surface. Hence, if we put $z=0$ we have—

$$\beta_0'=A'\epsilon^{j(pt+qx)}$$

where β_0' is the force at the surface.

Now, the exponent B is a complex quantity of the form—

$$-(c+jd)$$

and if z' is a certain depth at which the amplitude is $\frac{1}{\epsilon}$ of that at the surface we have—

$$\beta' = \beta_0' \epsilon^{j(c+jd)z'} = \beta_0' \epsilon^{jcz'} \epsilon^{-dz'}$$

If, then, $z' = \frac{1}{d}$ we have—

$$\beta' = \beta_0' \epsilon^{\frac{jc}{d}} \epsilon^{-1}$$

or $\frac{1}{d}$ is the depth at which the wave amplitude is reduced to $\frac{1}{\epsilon}$ of that at the surface.

Referring to equations (29), it is seen that—

$$B^2 = -jp\frac{T'^2}{T+T'}$$

and also that—

$$q^2 = -jp\frac{TT'}{T+T'}$$

Hence—

$$B = \sqrt{\frac{T'}{T}} \cdot q$$

But we have shown in equation (35) that q can be expressed in the form—

$$q = \frac{p}{u}\sqrt{\frac{\sqrt{s'^2+p^2k'^2}}{\sqrt{s'^2+p^2(k+k')^2}}}\epsilon^{j\left(\frac{\phi_1-\phi_2}{2}\right)}$$

It follows, then, that since $T' = s' + jpk'$ and $T = s + jpk$, that we have—

$$B = \frac{p}{u} \cdot \frac{\sqrt{s'^2+p^2k'^2}}{\sqrt{\sqrt{s^2+p^2k^2}\sqrt{s'^2+p^2(k+k')^2}}}\epsilon^{j\left(\phi_1-\frac{\phi_2}{2}-\frac{\phi}{2}\right)} \quad . \quad . \quad . \quad (37)$$

where $\tan\phi_1 = \frac{pk'}{s'}$, $\tan\phi_2 = \frac{p(k+k')}{s'}$

and $\tan\phi = \frac{pk}{s}$

Now we know that—

$$\epsilon^{j\left(\phi_1-\frac{\phi_2}{2}-\frac{\phi}{2}\right)} = \cos\left(\phi_1-\frac{\phi_2}{2}-\frac{\phi}{2}\right) + j\sin\left(\phi_1-\frac{\phi_2}{2}-\frac{\phi}{2}\right)$$

and if $B = c + jd$, then equating real and unreal parts, and remembering that $s = 0$ for air, we find for the value of d the expression

$$d = \frac{\sqrt{p}}{3\times10^{10}} \frac{\sqrt{s'^2+p^2k'^2}}{\sqrt{k}\sqrt{\sqrt{s'^2+p^2(k+k')}}} \sin\left(\phi_1-\frac{\phi_2}{2}-\frac{\phi}{2}\right) \quad . \quad . \quad . \quad (38)$$

and $\frac{1}{d}$ gives us the depth at which the wave amplitude is reduced to $\frac{1}{\epsilon}$ of that at the surface.

The expressions (36) and (38) are of great practical utility.

For example, let us suppose electric waves, having a wave-length of 300 metres or 1000 feet, are travelling over sea water. The question is, How far does this wave penetrate into the water before its amplitude is reduced to $\frac{1}{\epsilon}$ of that at the surface?

The air has a dielectric constant $K = 1$ and a conductivity zero. Sea water has a resistivity, say, of 100 ohms of centimetre cube. Hence for air—

$$k = \frac{1}{4\pi} \text{ and } s = 0$$

for sea water $\quad k' = \dfrac{80}{4\pi}$ and $s' = \dfrac{9 \times 10^{20}}{10^{11}}$

Also $\quad p = 2\pi \times 10^6 \quad u = 3 \times 10^{10}$

Therefore $\quad \tan \phi = \dfrac{pk}{s} = \infty \quad \therefore \phi = 90^\circ$

and $\quad \tan \phi_1 = \dfrac{pk'}{s'} = \dfrac{4}{900} \quad \therefore \phi_1 = 0^\circ$

and $\quad \tan \phi_2 = \dfrac{p(k+k')}{s'} = \dfrac{4}{900} \quad \therefore \phi_2 = 0^\circ$

Accordingly

$$d = \frac{\sqrt{2\pi \times 10^6}}{3 \times 10^{10}} \sqrt{\frac{\sqrt{81 \times 10^{18} + 1600 \times 10^{12}}\,12\cdot5}{\sqrt{81 \times 10^{18} + 1640 \times 10^{12}}}} \frac{1}{\sqrt{2}}$$

which is very nearly to $\frac{1}{50}$. Hence, $\frac{1}{d} = 50$ cms., or the amplitude of the wave would be reduced to 0·367 of the amplitude at the surface at a depth of ½ metre. Hence, below a depth of 4 or 5 metres there could be no amplitude at all. In

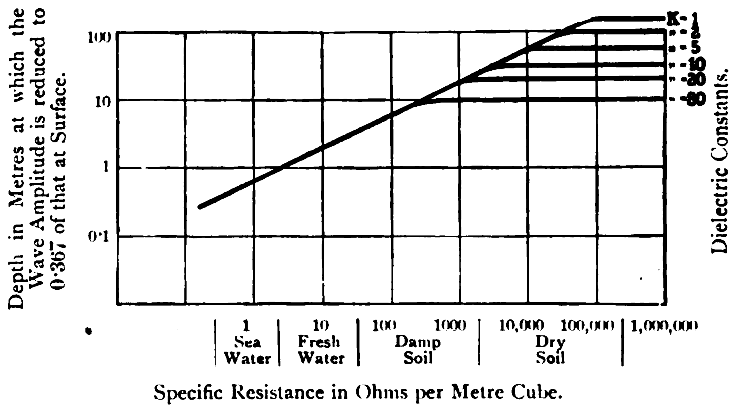


FIG. 7.—Depth of Penetration of Waves 1000 Feet in Length. (Dr. Zenneck.)

other words, when such waves travel over sea water their effect is wholly confined to a surface layer a few feet in thickness.

If, however, we consider the propagation to take place over very dry soil, for which the conductivity in electrostatic units might be as small as $9 \times \frac{10^{20}}{10^{16}}$, we should find that the value of the distance $\frac{1}{d}$ might then amount even to 100 metres or more, showing that the penetration of the wave into soil of small conductivity and small dielectric constant is very considerable. Dr. Zenneck set out the results of calculations for various cases in a series of curves as given in Fig. 7.

The conclusions to which the above investigation leads are that in the case of radiotelegraphy the nature of the earth's surface material between the sending and receiving stations must exercise a very important influence on the wave energy captured by a given receiving antenna at a given distance; and that the transmission is effected with the least loss over sea. This is entirely in accordance with experience. Again, the results show that the effect of dry soil in reducing the wave amplitude is as much due to its small dielectric constant as to its small conductivity.

Dr. Hack has shown in another paper that underground water or moisture, not on the surface layer, is of assistance in reducing the loss of wave amplitude.[5]

A matter of great practical importance is the consideration of the effect of wave-length on the dissipation of wave energy by soil absorption. If, for instance, we propagate over sea and land electric waves 1000 feet, and also of 10,000 feet in wave-length, what difference in the loss of wave amplitude by soil absorption will be produced? This question can be answered by the help of equation (36). We will consider the case of transmission (1) over sea; (2) over moist land; (2) over very dry land.

1. *Transmission over Sea.*—The resistivity of sea water may be roughly taken as 100 ohms per centimetre cube. Hence, $s' = 9 \times 10^9$ electrostatic units. The dielectric constant of sea water $= K = 80$; therefore $k' = \frac{80}{4\pi}$. The dielectric constant of air $= 1$; therefore $k = \frac{1}{4\pi}$.

We shall take the case of waves 300 metres and 3000 metres long.

(*a*) $\lambda = 300$ metres, $n = 10^6$, $p = 2\pi \times 10^6$, $pk' = 40 \times 10^6$, $p(k+k') = 40{\cdot}5 \times 10^6$

$$\tan \phi_1 = \frac{pk'}{s'} = 0{\cdot}00444, \quad \tan \phi_2 = \frac{p(k+k')}{s'} = 0{\cdot}0045.$$

Therefore
$$\sin \frac{\phi_1 - \phi_2}{2} = \tfrac{1}{2} 10^{-5}$$

Now, $s'^2 + p^2k'^2$ is nearly equal to $s'^2 + p^2(k+k')^2$. Hence from (36)

$$b = \frac{2\pi \times 10^6}{3 \times 10^{10}} \cdot \frac{1}{2} \frac{1}{10^5} = \frac{1}{10^9} \text{ (nearly)}$$

or $\frac{1}{b} = 10{,}000$ kilometres.

(*b*) Suppose $\lambda = 3000$ metres, $n = 10^5$, $p = 2\pi \times 10^5$, $pk' = 4 \times 10^6$, $p(k+k') = 40{\cdot}5 \times 10^6$, $\tan \phi_1 = \frac{pk'}{s'} = 0{\cdot}000444$, $\tan \phi_2 = \frac{p'(k+k')}{s'} = 0{\cdot}000445$.

Therefore
$$\sin \frac{\phi_2 - \phi_1}{2} = \tfrac{1}{2} 10^{-6}$$

$$b = \frac{2\pi \times 10^5}{3 \times 10^{10}} \frac{1}{2} \frac{1}{10^6} = \frac{1}{10^{11}} \text{ (nearly)}$$

or $\frac{1}{b} = 1{,}000{,}000$ kilometres.

We see, therefore, that although a wave 300 metres long travels over sea water with small absorption, the lengthening of the wave has yet a very beneficial influence in reducing the loss of amplitude.

2. *Transmission over Ordinary Land Surface.*—Since the degree of moisture, and therefore conductivity, of land surface soils differs very much, it is difficult to give a single number which can be taken as the value either of the conductivity or of the dielectric constant of dry land. We may, however, for the sake of an example, consider a case in which the specific resistance of the soil is 10,000 ohms per centimetre cube and the dielectric constant 5.

Then in this case we should have—

$$s' = 9 \times 10^7 \; k' = \frac{5}{4\pi}, \text{ and } k + k' = \frac{6}{4\pi}$$

If we take $\lambda = 300$ metres, then we have—

$$p = 2\pi \times 10^6 \; pk' = \tfrac{5}{2} 10^6 \; p(k+k') = 3 \times 10^6$$

Hence

$$\frac{pk'}{s'} = \frac{5}{180} = 0{\cdot}028 = \tan 1° 36'$$

[5] See F. Hack, "Die Ausbreitung ebener elektromagnetischer Wellen langs eines geschichteten Leiters, besonders in der Fällen der drahtlosen Telegraphie," *Annalen der Physik*, vol. 27 p. 43, 1908.

and $$\frac{p(k+k')}{s'}=\frac{6}{180}=0{\cdot}033=\tan 1°\ 54'$$

Therefore $$\sin\frac{\phi_1-\phi_2}{2}=0{\cdot}0026$$

$$b=\frac{2\pi\times 10^6}{3\times 10^{10}}\times\frac{26}{10,000}=\frac{52}{10^8}$$

since $s'^2+p^2k'^2$ is so nearly equal to $s'^2+p^2(k+k')^2$, and, therefore, $\frac{1}{b}=20$ kilometres.

If, then, we take $\lambda=3000$ metres, we find in the same manner $\frac{1}{b}=2000$ kilometres. This shows clearly that lengthening the waves from 300 to 3000 kilometres greatly reduces the wave absorption over land.

3. However we take the case of an exceedingly dry surface soil having a resistivity of 10 megohms per centimetre cube, we should find that lengthening the waves from 300 to 3000 metres produced hardly any appreciable improvement in the loss of wave amplitude.

Accordingly we can draw the following conclusions as to the effect of wave-length upon radiotelegraphic transmission.

1. In the case of transmission over sea, the absorption for waves of 300 metres long is not very large; but, nevertheless, increasing the wave-length to 3000 metres is an advantage.

2. In transmission over land the absorption of waves 300 metres long is very sensible, and increasing the wave-length to 3000 metres produces a very beneficial effect.

3. In the case of extremely dry soil the terrestrial absorption is very large, and increasing the wave-length from 300 to 3000 metres produces no very marked improvement.

The final conclusion is that the superior transmission over sea is due to the relatively high conductivity and high dielectric constant of sea water, and that in the case of transmission over land it is necessary to employ very long electric waves to obtain efficient transmission.

The subject has also been fully discussed by Brylinski (see *Science Abstracts*, vol. 10, A, p. 103, 1907, or *The Electrician*, vol. 57, p. 970, October 1906).

Brylinski examines the general theory of the penetration of the wave into the soil when a plain wave passes over it with magnetic force parallel and electric force perpendicular to the surface. He takes the average resistivity of soil to be 66 ohms per metre cube, which is rather low, and that of sea water to be 3·73, which is much too high. For a frequency of $\frac{10^7}{2\pi}$, he shows that the current or wave would penetrate into such soil about 50 metres, and that 95 per cent. of it would be within 6 metres of the surface. In the case of sea water it would be about 0 25 metre.

Brylinski has considered the effect of the damping of the oscillations on the terrestrial absorption. He found that damping causes a somewhat complicated distribution of the current at various depths. In the upper layer the current density diminishes somewhat more rapidly than the undamped currents, and then at a certain depth more slowly.

Owing to the rapidity of decrease of current density with depth, the current is practically confined to a certain strip or layer, the resistance of which, in spite of the infinite extension of the soil downwards, is a perfectly definite quantity, which increases with the resistivity, frequency, and permeability of the soil and with the damping of the oscillations. Thus, for a soil of resistivity 66 ohms per metre cube, and a frequency of $\frac{10^7}{2\pi}$, the current density at a depth of 5 metres is only 21 per cent. of that at the surface, and at a depth of 10 metres only 4 per cent., and at 15 metres less than 1 per cent., assuming that undamped waves are employed. This implies that for these waves the penetration is confined to a depth of about 15 metres. In practical experience it is found that the soil in certain places is particularly absorptive, and hinders very much the propagation over it of waves of certain wave-lengths.

Thus, Dr. L. W. Austin,[6] conducting experiments at Brant Rock, U.S.A., for the United States Navy, with the aid of the two cruisers *Birmingham* and *Salem*, noticed the following facts: It was found that whilst the *Birmingham* lay off Newport, signals received at Brant Rock 48 miles away, sent from the ship with wave-length of 1000 metres, were very weak and lost 95 per cent. of their energy in reaching Brant Rock receiver. If, however, signals were sent with waves of 3750 metres length they arrived without sensible loss. This, as Dr. Austin remarks, shows that the soil round the north of Newport, which is not far from New York, absorbs very powerfully waves of 1000 metres wave-length.

The following table gives the results of the experiments :—

TABLE II

Wave-length in metres.	Sending antenna currents in amperes.	Receiving antenna current in microamperes.		Absorption of signals in per cent.
		Observed.	Calculated.	
1000	28·7	1050	5400	95
3750	26·3	1600	1550	0

The calculated or predicted value of the received signals was made with the aid of the Austin-Cohen formula given in the last section of Chap. VIII., based upon measurements made at a known short distance from the transmitter when using the same aerial current.

In this connection it is important to notice that our knowledge of the true electric conductivity of various earth-crust materials and of sea water for high frequency currents is still very imperfect. It has been proved by experiments made by the author and the late Lieut. Dyke that the true conductivity of most dielectrics for alternating currents is very much greater than for steady currents; and for frequencies up to 5000 the conductivity of many dielectrics is a linear function of the frequency. Experiments undertaken at the author's suggestion by Dr. Bairsto in the Pender Electrical Laboratory of University College, London, have shown that for frequencies of the order of 10^6 the conductivity of certain earth-crust dielectrics, such as slate or marble, is still greater than for telephonic frequencies and reaches a maximum for a certain frequency. Thus in the case of dry slate the author's experiments showed that at a frequency of 920 a certain sample of dry slate had a specific resistance of 20 megohms per centimetre cube; at 3000 it had a resistivity of 7·5 megohms, and at 5000 of 5 megohms, whereas at a frequency of $2{\cdot}5 \times 10^6$ Dr. Bairsto found resistivity of only 0·4 megohm. At still higher frequencies the resistance tended to increase again.

Hence we can say that, generally speaking, the earth's crust is a much better conductor for currents of radiotelegraphic frequency than for low frequency currents. This will tend to reduce the dissipation of energy of electric waves moving over it, and also of the penetration of the wave into the earth's crust.

2. The Effect of Obstacles and of Atmospheric Conditions between the Sending and Receiving Antennæ.—Although earlier observations seemed to show that hills, trees, and buildings did not form an insuperable barrier to telegraphic communication by electric waves, yet later quantitative measurements have proved that the effects produced by the interposition of such obstacles is quite sensible, and in some cases very pronounced.

The diminution of signalling distance due to the interposition of hills and cliffs of various materials and heights was carefully investigated by Admiral Sir Henry Jackson, for the British Navy, and his results were communicated to the Royal Society of London in 1902.[7]

[6] See Dr. L. W. Austin on Radiotelegraphy, *The Journal of the Washington Academy of Sciences*, vol. i., November 1911.

[7] See Captain (now Admiral Sir) H. B. Jackson, R.N., F.R.S., "On some Phenomena affecting the Transmission of Electric Waves over the Surface of Sea and Earth," *Proc. Roy. Soc. Lond.*, **1902**, vol. **70**, p. **254**.

TABLE III

OBSERVATIONS BY ADMIRAL SIR HENRY JACKSON ON THE EFFECT OF INTERPOSED OBSTACLES ON ELECTRIC WAVE TELEGRAPHIC COMMUNICATION

Reference number to fig. in Plate VII.	Height of aerial wire above sea.	Distance from the land.	Particulars of the land.				Height of aerial wire above sea.	Maximum signal distance.		Percentage of maximum distance over land to over sea.
			Maximum height.	Total thickness.	Formula, strata, etc.	Surface.		At sea.	Over the land.	
	Feet.	Miles.	Feet.	Miles.			Feet.	Miles.	Miles.	
1*a*	158	2	150	4	Shale	Pasture, wet . .	178	62	50	81
1*b*	158	1¼	250	7	Sandstone and slate . .	Ditto	178	62	45	73
		Yards.								
2	125	130	200	2¼	Porous sandstone . .	Building, dry . .	160	65	50	77
3*a*	125	250	250	6	Porous coral sandstone .	Cultivated, wet . .	160	25	20	80
3*b*	...	220	500	6	Ditto, over limestone . .	Ditto	...	...	17	68
3*c*	...	500	700	8	Ditto	Ditto	...	...	15	60
3*d*	..	1000	500	7	Ditto	Ditto	...	...	16	64
		Miles.								
3*e*	160	3	1083	6	Gritstone and marl . .	Bare and dry, with scrub	125	...	17	68
3*f*	...	4	1400	9	Ditto	Ditto	...	...	14	56
3*g*	...	3	120	3	Semi-crystalline limestone .	Ditto	...	...	15	60
3*h*	...	1 to 2½	5250	7	Ditto	Ditto	...	...	No signals.	Very small
4*a*	160 + 500	On the land	500	Over 9	Porous sandstone . .	Cultivated, wet . .	110	45	35	78 (T.)

The reference numbers in column 1 refer to the diagrams in Plate VII. at p. 640.

TABLE IV

Reference number to fig. in Plate VII.	Height of aerial wire above sea.	Distance from the land.	Particulars of the land.				Height of aerial wire above sea.	Maximum signal distance.		Percentage of maximum distance over land to over sea.
			Maximum height.	Total thickness.	Formation, strata, etc.	Surface.		At sea.	Over the land.	
	Feet.	Miles.	Feet.	Miles.			Feet.	Miles.	Miles.	
4b	160 + 500	...	600	Over 8	Porous sandstone, over limestone	Cultivated, wet . .	110	45	30, but none from 10 to 14	67 (T.)
4c	,,	...	400	22	Sandstone	Ditto	130	135	85 and over	Over 63
5	154 + 330	3	500	17	Limestone and iron ores .	Cultivated, wet and dry	125	105	40	39
6	125	Yards. 100	800	$\frac{1}{4}$	Ditto	Cultivated, wet . .	110	45	None at 18	Less than 40 (T.)
7a	85	Miles. $3\frac{1}{2}$	834	$2\frac{3}{4}$	Limestone	Scrub and wood, wet .	55	20	10	50 (T.)
7b	85	3	432	$1\frac{3}{4}$	Limestone and much iron ore	Ditto, dry . . .	55	20	None at 7	Less than 35 (T.)
7c	85	3	260	$1\frac{1}{2}$	Sandstone	Ditto	55	20	14	70 (T.)
8a	125	1	1800	22 16 plain	Limestone. Valleys between hills	Ditto, wet and dry .	130	50	28	56
8b	125	2	1200	4	Limestone and iron ores .	Bare, wet and dry .	160	65	15	23
8c	125	2	2060	6	Ditto	Ditto	160	65	None at 15	Less than 23

The reference numbers in column 1 refer to the diagrams in Plate VII. at p. 640.

The experiments were conducted between ships of the British Navy provided with apparatus on the Marconi system, the cymoscope used being a metallic filings coherer, and the test employed being the maximum distance at which good Morse signals could be sent between two ships. The transmitting and receiving apparatus were tuned, and the wave-length employed was the fundamental one used in the British Navy. The wave-length used is not precisely stated in the paper, but was probably 500 or 1000 feet. In describing the results, we shall quote freely from Admiral Jackson's paper. The observations proved that the interposition of land, especially rocks of certain kind, greatly reduces the maximum signalling distance between ships equipped with wireless telegraph apparatus as compared with the distance over open sea for the same equipment. The results are collected in the tables on pp. 636 and 637. These Tables, III. and IV., and accompanying diagrams 1 to 8 given in Plate VII. (see p. 640), are, by kind permission, taken from Admiral Jackson's paper.

In reference to the above observations, Admiral Jackson says :—

"An examination of these results shows the marked difference between the effects due to the various natures of the intervening land.

"Summarizing them for soft rocks, hard limestone, and limestone containing a large roportion of iron ores respectively, the percentage of maximum signalling distance through hem compared to the open-sea distance is as follows :—

	Soft sandstone, shale, etc.	Hard limestone.	Iron ores.
Maximum distance	81	68	Less than 40
Minimum ,, ...	56	25	,, 23
Mean ,, ...	72	58	,, 32

"Consider, firstly, the soft rocks: The two maxima percentages of distance (81 and 80) are over rather low land of no great thickness; the minimum, 56 per cent., is over high land, half as thick again as in these cases.

"Secondly, the limestone: The maximum percentage (68) is over the thinnest layer recorded of limestone (see 3*b* in Table III. p. 636, and corresponding diagram in Plate VII. p. 640), the minimum (less than 25) is over a precipitous high mountain through which no signals could be passed at any distance, though they were obtained without difficulty over a low promontory of the same island and of the same formation, when both ships had moved to such positions as to bring the low instead of the high land between them (3*h* and 3*g*).

"Thirdly, the rocks containing iron ores: In all these cases a greater loss of proportional distance is recorded than in the others—and it was exceptional to receive any signals at all—and the best result recorded in several trials was but 39 per cent. of the open-sea distance.

"The results shown in Fig. 6 are the most conclusive that I have obtained in proving the screening effect of hard rocks containing iron ores on the passage of electric waves through land. The pinnacle of rock shown therein represents an extremely precipitous, narrow, but high promontory jutting out from the mainland and rising abruptly out of the sea, to which it is *steep to*, so that the ship could pass close to it in perfect safety at a distance of about 100 yards.

"To ascertain the effect of this wedge-like obstruction, the ship was steered close to the land, and her position was carefully noted when signals ceased or commenced. These signals were being sent continuously from another vessel (distant 18 miles) during the whole period of the trials, the letter F (- - —— -, in Morse Code) being made by her at the rate of twenty-five per minute by my syntonic transmitter.

"The results showed that the signals ceased and commenced abruptly at the moment that the aerial wire passed the tangent from the transmitting ship to the edge of the cliff; the action was so abrupt, that, on one transit, the latter part of the long sign in the 'F' was the first indication of signals that was received; and on another transit, in the opposite direction, the long of the 'F' was the last sign received, the short being dropped; these were unusual results, as the signals generally die away gradually, the long signs breaking up, thus: (. . . . . .), and the shorts appearing as dots (.), before any signs are actually lost.

"Another point that may now be considered, is the case shown in (4*b*), when signals could not be exchanged when the ship was close under the land, but could be when clear of the land and in the same direction as before; the trial was repeated on several occasions for verification.

"Possibly the case previously considered is of the same class, as it is noteworthy that when the ship was further off the promontory and also from the transmitting ship, though

the two ships were still masked by high land of much greater thickness than before, a few stray signals were received occasionally, which evidently passed over, not round, and not through the land, as the ship was then in a land-locked bay.

"Referring now to 3*e* and 3*f*, where the intervening land was both higher and thicker, and yet did not stop signals at longer proportional distances, it may be concluded that the waves of electrical induction, which must pass from ship to ship in order to record signals, may in certain cases pass through the land. Thus: one of the ships was lying alongside a perpendicular cliff of considerable height, and yet only experienced a loss of distance of about 12 per cent.

"8*a* gives a typical case of waves passing through valleys, and the results were so marked and so frequently repeated with different ships and on separate occasions that eventually the track of a vessel, proceeding at a known speed, could be roughly estimated, though distant 25 miles, by noting the intervals between the times when signals were lost and when received, and comparing these intervals with the time taken by the ship to cover the distances between the valleys, which were well delineated on the chart, and through which the waves could evidently wind their way with less obstruction than by any other route.

"We have thus obtained evidence that the waves of electric induction may pass (1) through land, (2) over land, (3) round land, but that a large proportion of their energy is lost in doing so. (4) That the screening effect of the land varies with its nature, and is greater for iron ores than for limestone alone, and that for this latter it is greater than for soft rocks. No effects which could be attributed to interference of waves, due to reflection from a hilly background, have been recorded by me."

Admiral Jackson then describes his observations on the effect of varying conditions of the atmosphere on the effective distance working of electric wave telegraphy. He says—

"Some of these conditions constitute a most serious obstacle to the effective transmission of electric waves over medium distances, and are, in consequence, a source of error likely to be encountered, and which cannot be foretold nor allowed for in wireless telegraphy.

"These effects are much less frequently noticed in temperate than in subtropical regions. In the Mediterranean Basin they seem to be particularly prevalent, and most persistent in summer and autumn.

"Owing to their sudden advent and their equally sudden cessation, it is most difficult to carry out systematic or prearranged experiments."

He therefore confines his remarks to observations made in various parts of the Mediterranean Sea. Speaking of these atmospheric effects, he says (*loc. cit.*):—

"The first case is that due to the effects of lightning discharges, which may or may not be visible at the station where its effects are noticed. As a rule, with the instruments in normal adjustment, the effect of every discharge is to record a signal, the exceptions being very few.

"The method adopted to observe this was to fit an electrical bell, worked by the receiving instruments, close to the observer, and at night observe the flashes and note if the bell rang.

"For detailed observations, it was found more convenient to record the effects on the tape, and this was the method subsequently adopted. On the approach of the area of disturbance towards the ship, the first visible indication generally is—the recording of dots at intervals varying from a few minutes to a few seconds: secondly, the recording of three dots with a space between the first two, thus: (- - -), or *e i* in the Morse Code, and this is the sign most frequently recorded by distant lightning; thirdly, the recording of dashes; the intervals between these then gradually decrease and merge into irregular signs, which have sometimes spelt words in the Morse Code; the effects generally die out more suddenly than they appear.

"They are much more frequent in summer and autumn than in winter and spring—in the neighbourhood of high mountains than in the open sea—in southerly than in northerly winds (in the Mediterranean Sea)—in the front of a cyclonic disturbance of the atmosphere than in the rear, and with a falling barometer than with a rising one. In settled fine weather, if present, they reach their maxima between 8 and 10 P.M., and frequently last during the whole night, with a minimum of disturbance between 9 A.M. and 1 P.M.

"The next cause which is ultimately connected with the above is the shorter distance at which signals can usually be received, when any electrical disturbances are present in the atmosphere, compared to the distance at which they can be received when none are present. The distance varies from about 30 to 80 per cent. compared with that obtained in fine clear weather. It does not in any way decrease with the increase of the number of lightning

discharges which register their effect on the instruments, at any given time, but rather the reverse, the loss in distance generally preceding the first indications, on the instruments, of the approaching electrical disturbance.

"A very marked case is given as an example: Two ships whose instruments were in perfect order, and whose sea-signalling distance was about 65 miles, opened their distance from each other on a fine, calm, bright day; when they were 22 miles apart the signals died away, though there was no intervening land or other apparent cause for this, but it was noticed that the barometer was falling; the ships closed and got into communication again. Atmospheric disturbances were then registered on both sets of instruments, and on the ships opening out again, no signals were obtained over 20 miles. The trials were concluded shortly after, owing to intervening land. A few hours later a heavy winter gale came on, and its approach had evidently been foretold by the falling barometer, the loss of distance in signalling, and the electrical disturbances in the atmosphere, as shown by the signals received on the instruments. No lightning flashes were observed.

"On another occasion, during a period of strong but intermittent atmospheric effects, no signals were obtainable between two ships up to the usual maximum signal distance. When separated 50 per cent. beyond this distance, and immediately after a particularly strong and persistent series of electrical discharges, the latter half of a signal, which was being transmitted very slowly, was correctly deciphered at a distance then considered phenomenal, with the instruments employed at the time. A few minutes later the atmospheric effects vanished, and with them all signs of further signals, till the ships had closed to their usual signalling distance. This demonstrates that the actual electrical discharges do not of themselves reduce the signalling distance or transmission of the waves at all times, but that they may, under some circumstances, assist that transmission, possibly by a cumulative effect of the waves emitted by the discharges on the waves emitted by the transmitter, these combining and increasing the effect in the receiver.

"Another observed effect which reduces the usual signalling distance is probably due to the presence of material particles held in suspension by the water spherules in a moist atmosphere.

"The Mediterranean Sea is, for days together, frequently exposed to the force of the scirocco wind; this south-easterly wind is laden with damp, and often charged with salt from spray, and dust particles from the African coast. During the continuance of these winds, the maximum signal distance is generally less than in winds (wet or dry) from any other quarter, the proportional distance being from about 60 to 80 per cent. The effect of a scirocco wind can be and is allowed for in practical wireless telegraphy."

In reference to this question of the effects of obstacles, we may make mention of some further interesting observations made by Admiral Jackson, R.N., in course of work done in wireless telegraphy for the British Navy. They are concerned with the production of areas of weak reception when ships which are signalling to each other are placed in certain relative positions. Admiral Jackson, in the paper above mentioned (*loc. cit.*), says:—

"This phenomenon manifests itself by the gradual weakening and occasionally by the total cessation of signals, as the distance between the two ships increases, up to a certain point, and their reappearance as the distance is still further increased; in the majority of cases the weakening of signals occurs at, or about, half the signalling distance in the open sea, under the same circumstances, which circumstances include the direct connection of the aerial wire to one ball of the induction coil used for transmission.

"The three following examples are typical cases. Units of distance are given in lieu of nautical miles.

"(*a*) A ship, A, steamed away from a station, B, to ascertain the maximum distance at which she could receive signals in the open sea.

"At 48 units of distance the signals weakened, at 57 they ceased, at 65 they appeared again, and were kept up to 100 units of distance.

"(*b*) Four ships, C, D, E, F, steered as shown in the diagram, the maximum signalling distance between each pair being about 100 units distance.

"(The results of the signals transmitted by D are those specially to be considered.)

"In position (1) D's signals were received by E, F, not by C.

" ,, ,, (2) ,, ,, ,, ,, ,, F, ,, E, C.

" ,, ,, (3) ,, ,, ,, ,, ,, C, F ,, E.

"C did not commence signalling before reaching (2), and her signals were received by D and E, and maintained by them to position (3), when the trial was finished.

"E's signals, which were few in number, were received by C and D in (3), but not by D in (2).

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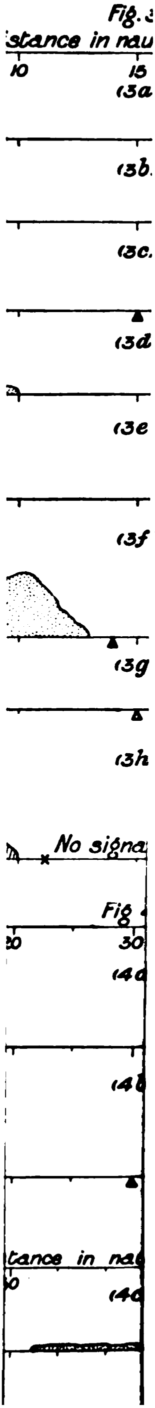


"(c) In the third example, ships D and F carried out a similar trial independently. Between 45 and 55 similar units of distance no signals could be exchanged either way, though at 60 units and above, and below 40, the signalling was perfect.

"To further verify that it was the system of transmission that was the cause of this cessation of signals, a syntonic method, of the same approximate frequency of transmission, though of rather less power, was used alternately with the other system. Signals were exchanged perfectly with the syntonic method, but on reverting to the other method the signals again ceased.

"This was tried repeatedly with identical results. Many other similar cases have been recorded, but the effects are not always so equally well marked, even under identical circumstances."

In reference to the cause of this effect, Admiral Jackson says :—

"I consider this effect is due to want of synchronism in the oscillatory discharge between the spark balls of the transmitter. This want of synchronism has also been observed by others in the photographs of oscillatory spark discharges. C. Tissot [8] especially remarks that, in his apparatus (presumably used for a wireless telegraph transmitter), the images of the successive sparks are not equidistant, and that the first interval is always greater than the other intervals, which also decrease very slightly. This implies that the first wave emitted is longer than the second, and so on. Owing to the rapid damping of our form of transmitter, probably only the first two or three waves emitted are of any practical value in exciting the coherer in wireless telegraphy at a distance of 30 miles; and to excite it at such distances with the power used in these transmitters, it is probably essential that the effects of the successive waves should be cumulative in their action, and for them to be so they must syntonize with the natural period of oscillation of the receiving circuit, which period, in the cases under notice, was the mean frequency of the waves emitted by the transmitter as nearly as this could be practically adjusted.

"Consider the first two waves emitted, or the interval between the first and fifth sparks of the oscillatory discharge, when the third one is not spaced midway between them; the resulting waves, differing but little in length, and moving with equal velocities and in the same direction, leave a point O (the spark gap), the second starting a mean wave-length behind the first one, and in the same phase; at some fixed point, P, in their path, owing to the difference in their length, the two waves will pass that point in the opposite phase, and at a point Q, approximately double the distance from O that P is from O, they will pass Q, again in the same phase, and so on, as at all points the second wave is a mean wave-length behind the first one. *To excite the coherer* under the conditions presumed to be necessary for long distances, the impulses due to these waves must syntonize with the natural period of oscillation of the receiving circuit, and therefore these successive waves must pass by that circuit (wherever it may be), with the second following in the same phase as the first, or nearly so, otherwise the tendency of the second one will be to weaken or annul the effect of he first one.

"At the point P, therefore, when the waves are in opposite phase, it may be expected that signals will be weak, and at Q, when they are in phase, they *may be* strong, but, owing to Q's distance from O being double that of P, the effect of each individual impulse at Q is only half its effect at P, and Q *may be* the maximum distance from O at which the cumulative effect of the successive waves will excite the coherer, even when they are in phase and in perfect syntony with the receiver circuit.

"I have not yet been able to investigate the exact cause of the non-synchronous emission of the waves, but I attribute these 'zones of weak signals' (as I term them) to this non-synchronous emission of the waves, and to the rapid damping of this form of transmitter, and would observe that when using my syntonic transmitter, in which the damping is less rapid, I have never noticed these effects."

He finally sums up his conclusions in this important paper in reference to wireless telegraphic communication over sea between ships as follows :—

"(1) That intervening land of any kind reduces the practical signalling distance between two ships or stations, compared with the distance obtainable in the open sea, and that this loss in distance varies with the height, thickness, contour, and nature of the land; and that, based on the results of these observations, it may be concluded that some of the waves of electric induction, transmitted by wireless telegraphy, may pass through, over, and possibly round the land, and are comparable to the passage of ocean waves through or over a reef, or round high land, which waves proceed along their course with diminished energy, after passing such obstructions.

[8] *Comptes Rendus*, March 25, 1901, vol. 132, p. 763; and December 2, 1901, vol. 133, p. 929.

"(2) That material particles, such as dust and salt held in suspension in a moist atmosphere, also reduce the signalling distance, probably dissipating and absorbing the waves.

"(3) That electrical disturbance in the atmosphere also acts most adversely to the regular transmission of these waves, in addition to affecting the receiving instruments by lightning discharges.

"(4) That a system of transmission in which the oscillations are rapidly damped is irregular in its action on distant receivers, owing to the irregularity of the train of waves giving rise to different types of disturbance at different parts of their path, which may not have at certain points the necessary cumulative effect on the receiving circuit.

"(5) That the earth's function in the transmission of waves is most important; but that its importance is secondary to that of the aerial wire, or capacity insulated in the air above the surface of the surrounding sea or earth."

Observations have been made by Colonel George O. Squier, of the United States Army, on the absorption of the electromagnetic waves by trees and other vegetable organisms.[9] These originated in the discovery that a very good earth can be obtained for a military telephone line by merely driving an iron nail into the roots of a large tree. It was found that the conductivity for telephone currents of a growing tree in a healthy state is such that the nail need not be driven in quite at the root of a tree, but may be put into the trunk 30 feet or more above the ground with equally good results. This showed that the electrical connection between a tree and the earth is very good, and that the mass of growing widely spread roots of a large tree constitute a "good earth." This led to an attempt to make use of tall trees as wireless telegraph antennæ, by connecting some point near the base of the tree, by means of wire, with some point higher up on the tree trunk, the point near the base of the tree being either close to the trunk or a little way removed from it. It was shown by careful experiments that the tree trunk really did play the part of an antenna, and that the effect was not merely due to the elevated wire. Experiments were then made to ascertain if there was any screening effect from neighbouring trees in the line with the receiving station. The wave-length used in most of the tests was about 300 feet in length. No numerical results were given, but it is asserted that marked absorptive effects due to trees in the mass were noticed.

The author has also frequently noticed the obstruction to long electric waves by the buildings of a city. This obstruction is much greater to waves of about 300 feet in wave-length than to much longer waves.

3. The Effect of Earth Curvature on Long Distance Radiotelegraphy.—As long as radiotelegraphy was confined to distances of not more than 100 or 200 miles it could be reasonably assumed that the effect of earth curvature was negligible. When, however, distances of 2000 to 3000 miles came in question this assumption was not legitimate. The transmission of signals across the Atlantic by Marconi at once raised the question how far diffraction is concerned in this long distance working. It is a familiar fact that waves of sound bend round an obstacle to a greater or less extent. The same thing happens to a much less degree in the case of light waves. The general explanation of light wave diffraction is as follows:—

Let C (Fig. 8) be a radiant point emitting circular waves. At any instant let the wave have arrived at the position P. Let O be any point further on. Then by Huyghen's principle the effect or disturbance at O is due to the sum of all the secondary waves propagated outwards from all points in the circular wave front which has reached P. With O as centre describe a number of arcs of circles, the radius of each being greater than the next one by $\frac{1}{2}\lambda$, where λ is the wave-length, and let these arcs cut the wave front into sections or elements PM, MM_1, M_1M_2, etc., which we shall call half wave-length elements. It is clear that the length of the arc PM is greater than the length of the arc MM_1 and so on, and that as we move outwards along the wave front these arcs tend to become equal. Corresponding to every point in the arc MM_1 there is, therefore, a point in the arc PM such that the difference of their distances from O is

[9] Excerpt from Major-General Arthur MacArthur's Report to the War Department, U.S.A., on the Military Manœuvres of the Pacific Division, 1914. See also British Patent Specification, No. 25,610 of 1904, of F. W. Howarth, a communication from George Owen Squier.

equal to $\frac{1}{2}\lambda$. Hence the waves sent out by these two points will be in opposition as regards phase. We may assume that the number of wave-making points or centres of disturbance in each little arc is proportional to the length of the arc. It is therefore evident that the further the arcs are taken from the pole P of the wave, the more completely they neutralize each other's effects at O when taken pair and pair consecutively. Accordingly, the disturbance at O will be chiefly due to the middle portion of the wave, viz. that which lies near P, and will be destroyed by any opaque object placed near the central portion of the wave.

In the case of light waves the wave-length is very small compared with the distances OP, CP, and hence the effective portion of the wave front is very closely confined to a small area round the pole P. If, therefore, an opaque screen is placed so as to cut off half the wave, only a very small part of the space within the geometrical shadow will receive light, viz. up to such a limiting distance OD, bounded by a point D, from which, if radii increasing by $\frac{1}{2}\lambda$ are drawn, the lengths of the arc PM_1, MM_1 cut off on the wave front differ by a sensible amount. The depth OD (see Fig. 9) will increase with the wave-length, because the greater the wave-length the further round the wave front

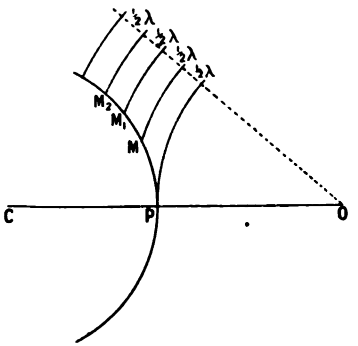


FIG. 8.

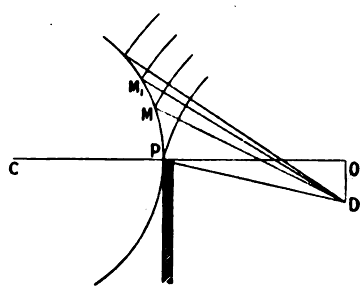


FIG. 9.

does the inequality in length of the half wave-length elements extend. Accordingly, in the case of light waves there is an illumination which extends slightly within the boundary of the geometrical shadow, but fading off, and in any case very small. On the other hand, if we are dealing with long electric waves, or with sound waves of which the wave-length is not small compared with the distances PO or PC, then this bending round or diffraction is very sensible, and there is no sharply defined edge to the shadow.

In the case of radiotelegraphic electric waves having a length of 1000 feet or more, natural objects, such as cliffs or hills, do cast what may be called electromagnetic shadows; but the longer the wave-length the less well defined is this shadow. The wave-lengths at present employed in Transatlantic and other long distance radiotelegraphy are of the order of 20,000 feet. The transmission of these over distances of 3000 to 4000 miles round the earth is an everyday feat, and readable radiotelegraphic signals have been sent and received over distances of 6000 miles, or about one-quarter of the circumference of the earth. Such waves are about 4 miles in length or one-thousandth of the earth's radius. This is nearly the ratio between the wave-length of rays of yellow light, and the radius of a very small sphere about half a millimetre in diameter. The question then at once arose, whether pure diffraction could account for the whole of the effect at a distance in the case of radiotelegraphy? Mathematicians, therefore, began to consider the problem of electric wave propagation round a conducting sphere.

The problem was first attacked by Prof. H. M. Macdonald in 1903,[10] but his analytical methods were criticized by Lord Rayleigh and the late M. Henri Poincaré.[11]

In a subsequent paper Macdonald corrected his work, and gave a table showing the decay in amplitude of electric waves 1000 feet long in being propagated round the earth.[12] The question was then discussed by Dr. J. W. Nicholson in a series of papers in the *Philosophical Magazine*,[13] and also reconsidered by M. H. Poincaré[14] in some lectures.

The conclusion to which Macdonald, Nicholson, and Poincaré all arrived was that the ratio of reduction of wave amplitude involved an exponential factor, the exponent varying inversely as the cube root of the wave-length.

In a more recent paper (see *Proc. Roy. Soc. Lond.*, vol. 90, series A, p. 50, 1914), Prof. Macdonald has reconsidered the whole subject, and given a table for the diffraction of waves varying in wave-length from 320 to 5000 metres over distances up to 1260 miles round the earth.

Meanwhile Prof. A. Sommerfeld[15] had treated the problem of the propagation of electric waves from an oscillator placed vertically at the plane boundary of two dielectric media of different conductivities and had proved that there must be not only a space wave but a surface wave propagated along the boundary of the two media.

This is somewhat analogous to the surface wave which Lord Rayleigh showed in 1885 must exist at the surface of an elastic solid, when a disturbance is made in its interior.

Thus in the case of the earth any internal shock will create two types of waves, one due to the elastic resistance of the material to compression, and the other due to its resistance to distortion These waves are space waves, but travel through the medium with different velocities, viz. about 10 km. and 5 km. per second respectively in the case of the earth. But owing to the fact that the surface molecules are in an exceptional position there is also a production of a surface wave which travels, in the case of the earth, with a velocity of about 3 km. per second. In the same manner at the bounding surface of two dielectrics of different qualities there is a surface electric wave which spreads out from any point at which an oscillator exists. Sommerfeld shows that the amplitude of this surface decreases inversely as the square root of the distance.

Sommerfeld suggested that long distance radiotelegraphy is conducted chiefly by these surface electric waves. H. W. March[16] and W. v. Rybczynski[17] also discussed the propagation of electric waves over the surface of a spherical earth.

Rybczynski agrees both with Nicholson and Poincaré in the conclusion that in the expression for the amplitude of the electric wave as effected by diffraction there is a factor $\epsilon^{-A\lambda^{-\frac{1}{3}}}$. In other words, the amplitude diminishes exponentially and the exponent varies inversely as the cube root of the wave-length. We have then to mention a careful and able examination of the problem of diffraction round a sphere by Prof. A. E. H. Love.[18] He examines critically the latest results by Macdonald, and agrees with them. He also brings them into comparison with the analysis of Nicholson and of Rybczynski and finds a certain difference, which he exhibits in the following way[19]:—

Let H be the magnetic force at the surface of a perfectly conducting sphere

[10] See H. M. Macdonald, *Proc. Roy. Soc. Lond.*, vol. 71, A, p. 251, 1903; also vol. 72, A, p. 59, 1904.

[11] See Lord Rayleigh and M. Poincaré, *Proc. Roy. Soc. Lond.*, vol. 72, A, 1903.

[12] See H. M. Macdonald, *Phil. Trans. Roy. Soc.*, vol. 210, A, p. 113, 1910.

[13] See J. W. Nicholson, *Phil. Mag.*, February, March, April, May 1910, vol. 19, pp. 276, 435, 516, 757.

[14] See H. Poincaré, *La Lumière Électrique*, vol. 4, 2nd ser., November 28, December 5, 12, 19, 1908. See especially December 12, p. 323, 1908.

[15] See A. Sommerfeld, *Annalen der Physik*, vol. 28, p. 665, 1909.

[16] See H. W. March, "Ueber die Ausbreitung der Wellen der drahtlosen Telegraphie," *Annalen der Physik*, vol. 37, p. 29, 1912.

[17] W. v. Rybczynski, *Ann. der Phys.*, vol. 41, p. 191, 1913.

[18] A. E. H. Love, *Trans. Roy. Soc. Lond.*, vol. 215, A, p. 105, 1915.

[19] See A. E. H. Love, *Phil. Trans. Roy. Soc. Lond.*, vol. 215, A, p. 125, 1915.

due to an oscillator placed at an angular distance θ measured along a great circle. Let H_1 be the force at another distance θ_1 from the oscillator. Then the ratio H/H_1 is expressed by the following formulæ as obtained from the results of Macdonald, Nicholson, and Rybczynski respectively :—

(Macdonald) $$\frac{H}{H_1}=\frac{\cos \frac{1}{2}\theta}{\cos \frac{1}{2}\theta_1}\sqrt{\frac{\sin \frac{1}{2}\theta_1}{\sin \frac{1}{2}\theta}}\,\epsilon^{47 \cdot 80\,\lambda^{-\frac{1}{3}}(\sin \frac{1}{2}\theta_1 - \sin \frac{1}{2}\theta)} \quad . \quad . \quad . \quad (39)$$

(Nicholson) $$\frac{H}{H_1}=\sqrt{\frac{\sin \theta}{\sin \theta_1}}\,\epsilon^{23 \cdot 8\,\lambda^{-\frac{1}{3}}(\theta_1 - \theta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

(Rybczynski) $$\frac{H}{H_1}=\sqrt{\frac{\theta_1 \sin \theta_1}{\theta \sin \theta}}\,\epsilon^{11 \cdot 3\,\lambda^{-\frac{1}{3}}(\theta_1 - \theta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The latest investigation of the problem is that by Dr. G. N. Watson (see *Proc. Roy. Soc. Lond.*, vol. 95, A, p. 83, 1918) who agrees substantially with Nicholson in his results.

The only point on which the above analysts are in accord is that in the exponential term the exponent is inversely as the cube root of the wave-length.

Before discussing these formulæ it is desirable to refer to the experimental work conducted on the measurement of the strength of radiotelegraphic signals at various and very large distances from a sending antenna.

4. Experimental Determination of the Law of Decrease of Electric Wave Amplitude and Energy at Various Distances from an Oscillator.— Hertz showed, as we have seen in Chap. V., that at a distance from a small Hertzian oscillator the electric and magnetic forces in the emitted wave vary inversely as the distance from the oscillator. The energy in the wave must decrease inversely as the square of the distance, and as the wave energy varies as the square of the amplitude, the amplitude must vary inversely as the distance.

The same law holds good for an earthed vertical radiotelegraphic antenna provided we are dealing with distances of not more than a few hundred (say up to 400) miles or so. The law according to which the field of such an antenna decreases was first experimentally investigated by Messrs. Duddell and Taylor.[20] They operated with direct-coupled antennæ, and measured with a Duddell thermal ammeter the current in the receiving antenna. They used a wave stated to be 400 feet in wave-length, but which was probably somewhat longer, and they measured the current with the Duddell thermal ammeter (see Fig. 30, Chap. VI.) both in an antenna untuned, and in one tuned to the incident wave. The results of some of their observations, giving the distances between the stations in feet and the R.M.S. value of the current near the base of the receiving antenna in micro-amperes in the case of the tuned and untuned antenna, are set forth in the table on p. 646, and graphically in the curves in Figs. 10 and 11.

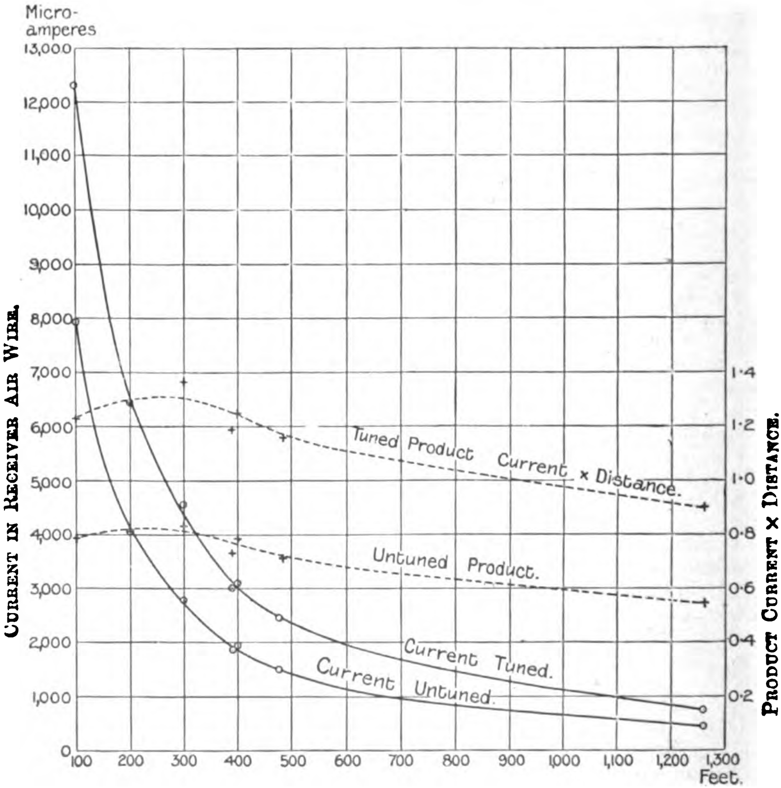
The interposition of trees was found to affect the result law of variation sensibly, but the general result is to show that the currents in the receiving antenna varied rather more rapidly than they should have done in accordance with the law of inverse distance, but less rapidly than in accordance with the law of the inverse square of the distance. The curves clearly indicate that at close quarters the current in the receiver varies more rapidly than at greater distances. Within the distance of a wave-length there is a very rapid decrease, which tends at much greater distances to come more nearly in accordance with the law of the inverse distance. This is quite in agreement with the deductions from Hertz's theory. He showed, as explained in § 7 of Chap. V., that for a lineal oscillator the electric force at a great distance varies inversely as the distance from the oscillator, but that this law of variation does not hold good at relatively small distances. M. C. Tissot (see *The Electrician*, 1906, vol. 56, p. 848) has made similar experiments with a bolometer cymoscope inserted in the receiving antenna circuit, and confirmed the fact that the effective or R.M.S. value of the antenna current at the receiver station approximately varies inversely as the distance from the transmitting station.

[20] See "Wireless Telegraph Measurements," by W. Duddell and J. E. Taylor, *Journ. Inst. Elec. Eng.*, London, 1905, vol. 35, p. 321.

CURRENT IN THE RECEIVING ANTENNA WHEN DISTANCE BETWEEN THE SYNTONIC RECEIVER AND TRANSMITTER IS VARIED.

Height of Receiving Antenna, 56 *feet. Height of Transmitting Antenna,* 42 *feet.*

Distance in feet between antennæ.	Currents in antennæ. Transmitter. Amperes.	Currents in antennæ. Receiver. Microamperes.	Product of distance and current in receiving antenna.
100	0·501	12320	1·232
200	0·507	6435	1·287
300	0·558	4548	1·364
400	0·541	3108	1·243
1260	0·541	715	0·901
2420	0·506	283·5	0·686
3700	0·517	105	0·388
4600	0·558	96·5	0·444
6220	0·563	69·5	0·432

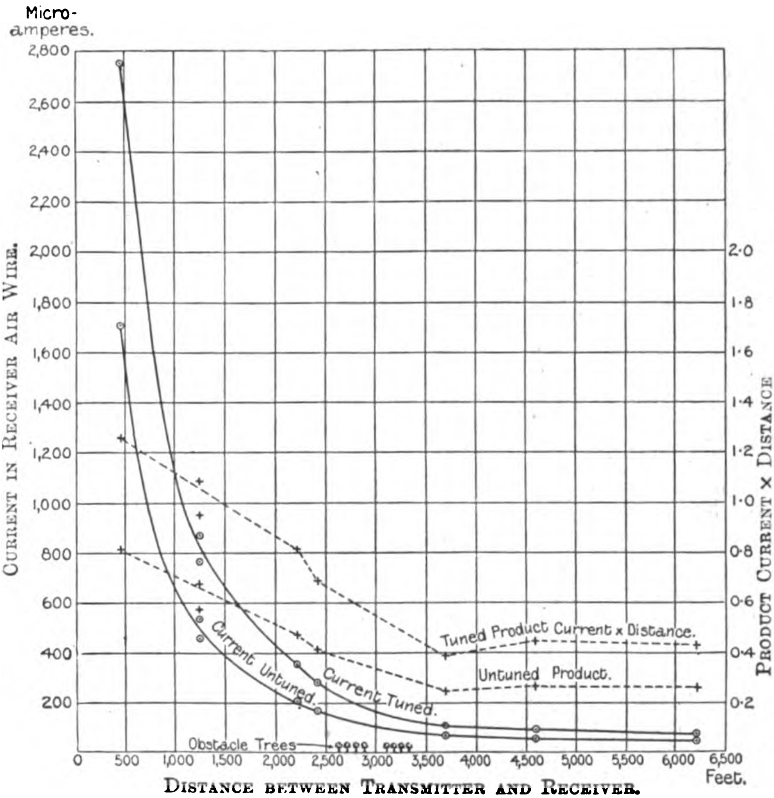


DISTANCE BETWEEN TRANSMITTER AND RECEIVER.

[*From the "Journal of the Institute of Electrical Engineers."*

FIG. 10.—Curve showing the Variation of R.M.S. Value of the Current in the Receiving Antenna as the Distance between the Transmitter and Receiver is varied. From experiments by Messrs. Duddell and Taylor.

Over larger distances a valuable series of experiments on this subject was made by Dr. L. W. Austin, and published in 1911 in a paper on "Some Quantitative Experiments in Long Distance Radiotelegraphy."[21] This work was carried out in 1909 and 1910 by the United States by the Navy Department with the aid of two cruisers, *Birmingham* and *Salem*, and the co-operation of the high-power radio station at Brant Rock, about 20 miles south of Boston. This station is equipped with a steel tower insulated at the base and 420 feet high. From the top four arms extend, and from each of these a pair of 4-wire cage aerial was



[*From the "Journal of the Institute of Electrical Engineers.*"

FIG. 11.—Curves showing the Variation of R.M.S. Value of the Current in the Receiving Antenna as the Distance between the Sending and Receiving Stations is varied. From experiments by Messrs. Duddell and Taylor.

suspended so as to make an umbrella antenna having a capacity of 0·0073 mfd. The antenna was loosely coupled to an oscillation circuit of the condenser spark type so that a single wave of 3750 or else one of 1000 metres length was radiated. The spark frequency was 500 per second, and the power available 50 to 60 kilowatts.

The cruisers were equipped with flat top or T antennæ, the length of the top being 116 feet and the height about 130 feet, and these were increased by fore and aft cages so as to bring up the capacity to about 0·0025 mfd. The ships were also furnished with motor alternators giving 10 kilowatts at a frequency of 500. The

[21] *Bulletin of the Bureau of Standards.* Washington, 1911. Reprint 159. Vol. 7, No. 3.

spark gap used at both shore and ship stations was a revolving one, and the condensers were metal plates in compressed air.

The receiving arrangements comprised hot wire and thermoelectric ammeters, and by means of these other detecting arrangements were calibrated, such as a zincite-chalcopyrite contact rectifier in series with a galvanometer. This last arrangement was calibrated by means of a thermoelectric detector and buzzer circuit which could be tuned to the frequency and to the wave-length used. On board ship a shunted telephone appears to have been used as the means of measuring the signal strength.

The experiments then consisted in measuring at Brant Rock the receiving aerial currents through a total resistance of 25 ohms corresponding to certain sending aerial currents on board the cruisers. At the same time the distance of the ships was known.

The received current was first measured when the ships were quite close, viz. at 22 miles distance.

For the 1000-metre wave-length it was found that the received current was 10,500 microamperes corresponding to 33 amperes antenna current on the *Birmingham* and 11,000 microamperes for 27 amperes on the *Salem*. For a wave-length 3750 metres the values were 3200 microamperes for 27 amperes sending currents, and 4100 microamperes for 24 amperes sending from the *Birmingham* and *Salem* respectively. The ships then went off to various distances, and similar measurements were taken by day and by night up to a distance of 1200 miles.

The results were then set out in curves in which the abscissæ denote distance between the sending and receiving station in nautical miles, and the vertical ordinates the receiving antenna current in microamperes. Observations made by night were denoted by the letter N put against them (see Fig. 12). The consideration of these results showed—

(1) That up to 100 or 200 miles the received antenna currents were nearly the same by night as by day, and varied very nearly inversely as the distance from the sending station ;

(2) That beyond this distance the night signals were much stronger, but more irregular than the day signals ;

(3) That the values of the receiving antenna currents for day signals lay fairly well on a curve, but decreased much more rapidly in strength than the inverse distance law predetermines.

An attempt was then made to find an empirical formula for the received current which should fit the observations.

Austin seems to have regarded the falling off in the signal strength with distance more rapidly than the inverse-distance law as due to "absorption" by the atmosphere which he thought would follow an exponential law. Hence his formula was obtained on the assumption that the actual received antenna current could be represented as the product of two factors ; one a factor following the Hertzian law of the inverse distance, and the other an exponential factor the exponent in which varied as the distance. This assumption, however, is not justified by a careful examination of the diffraction problem.

Let us then take the following symbols for the quantities concerned :—

I_s = sending current at base of antenna measured in amperes.
I_R = receiving current at base of antenna in amperes.
I = receiving antenna current at distance of 1 kilometre from sending station.
D = distance in kilometres between sending and receiving stations
h_1, h_2 = heights of sending and receiving antennæ.
λ = wave-length in kilometres.

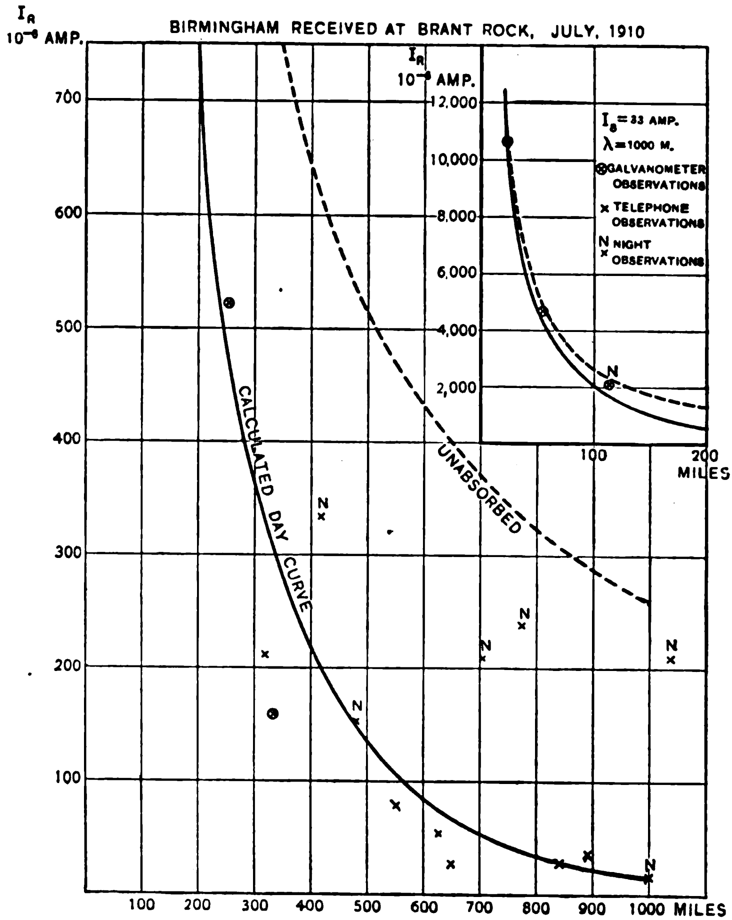
Accordingly Austin assumed that the receiving antenna current would be expressed by the formula—

$$I_R = \frac{I}{D}\epsilon^{-AD} \quad \quad (42)$$

From a discussion of the observed receiving antenna currents at various large distances, L. Cohen deduced the empirical formula—

$$A = \frac{a}{\sqrt{\lambda}}$$

Austin also concluded from his observations that the quantity I was proportional to $I_s h_1 h_2$ and inversely as λ.



[From Bulletin of the Bureau of Standards, Washington, U.S.A., Vol. 7, No. 3.

FIG. 12.—Observations taken by Dr. L. W. Austin, showing Variation of Radiotelegraphic Signal Strength over long distances. Abscissæ in miles, ordinates in microamperes. The ordinates of the dotted line marked "unabsorbed" indicate what the received current should have been if it had been strictly inversely as the distance. The firm black line is a curve calculated by equation (43).

Finally he embodied all the results in a single empirical formula as follows :—

$$I_R = 4 \cdot 25 \frac{I_s h_1 h_2}{\lambda D} \epsilon^{-0 \cdot 0015 D \lambda^{-1/2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

This formula gives the current in the receiving antenna through a resistance of 25 ohms in terms of the current in the sending antenna, both being measured in amperes, and all heights h_1, h_2, wave-length λ and distance D being measured in kilometres. The formula applies only to radio sending by daylight and over sea water, and for antenna heights from 37 to 130 feet, and wave-lengths from 300 to 3750 metres, and distances up to 1000 miles or so.

This is the formula employed in Chap. VIII. in § 9, concerning the design of radiotelegraphic stations.

With the exception of the exponential term, the factor $\frac{I_s h_1 h_2}{\lambda D}$ in Austin's formula is consistent with the expressions given by Hertz (see Chap. V. § 7 (31)) for the electric and magnetic force at a considerable distance from a small oscillator. For the forces are shown by Hertz to be proportional to $\frac{\phi m^2}{r}$ or $\frac{\phi mn}{r}$, where ϕ is the electric moment of the oscillator, and $m=2\pi/\lambda$ and $n=2\pi/T$, where λ is the wave-length, and T is the periodic time of the oscillator, and r is the distance from the oscillator to the point in question. The electric moment ϕ is the product of the maximum charge Q of the oscillator and its length, and the current at the centre of the oscillator is proportional to Qn. Hence $\frac{\phi m^2}{r}$ is proportional to $\frac{I_s h_1}{D\lambda}$ in the case of the antenna. Again, the received current is proportional to the product of the length of the receiving antenna and the magnetic or electric force of the oscillator at that point. Hence the Austin-Cohen formula is equivalent to the statement that the magnetic force at any point at a large distance on the earth from a sending antenna is proportional to the product of the electric or magnetic force of the oscillator in free space, and to an exponential attenuation factor $\epsilon^{-AD/\sqrt{\lambda}}$.

In other words, the ratio of the forces at two distances D_1 and D, measured along a great circle of the earth, or else at two angular distances θ_1 and θ, are assumed to be given by the empirical formula—

$$\frac{H}{H_1}=\frac{D_1}{D}\epsilon^{0\cdot0015\lambda^{-1/2}(D_1-D)}=\frac{\theta_1}{\theta}\epsilon^{(9\cdot6)\lambda^{-1/2}(\theta_1-\theta)} \quad . \quad . \quad . \quad (44)$$

There is, therefore, a considerable difference between this empirical formula and the expression derived by the mathematical analyses of Macdonald and Love.

Rybczynski[22] has examined Austin's experimental results, and expresses the opinion that if the kilometre is taken as the unit of length, then the exponential term in his (viz. Rybczynski's formula, see equation (40)) becomes $\epsilon^{-0\cdot0018D\lambda^{-1/3}}$, and that this agrees better with Austin's experimental results than the empirical formula given by Austin himself.

Another similar set of long distance experiments of great value were carried out by J. L. Hogan, Jr., and are described in *The Electrician* for August 8, 1913, vol. 71, p. 720. These experiments were conducted between the United States naval radio station at Arlington, Va., and the U.S.A. cruiser *Salem* at various distances up to 4000 kilometres (see Fig. 13).

Hogan expressed his results in terms of distance between the sending and receiving stations reckoned in kilometres, and an "audibility factor" defined as follows:—The signals were received on board ship with some kind of electrolytic detector or liquid barretter and a telephone.

The telephone has a certain impedance R for the received currents, and a shunt having an impedance S is put across its terminals and reduced until the signals are only just audible, so as to distinguish dots and dashes. Then the audibility factor A_f is defined by the expression—

$$A_f=\frac{R+S}{S}$$

where S is the value of the shunt which reduces the signals to bare audibility.

[22] See Rybczynski, *Annalen der Physik*, vol. 41, 191.

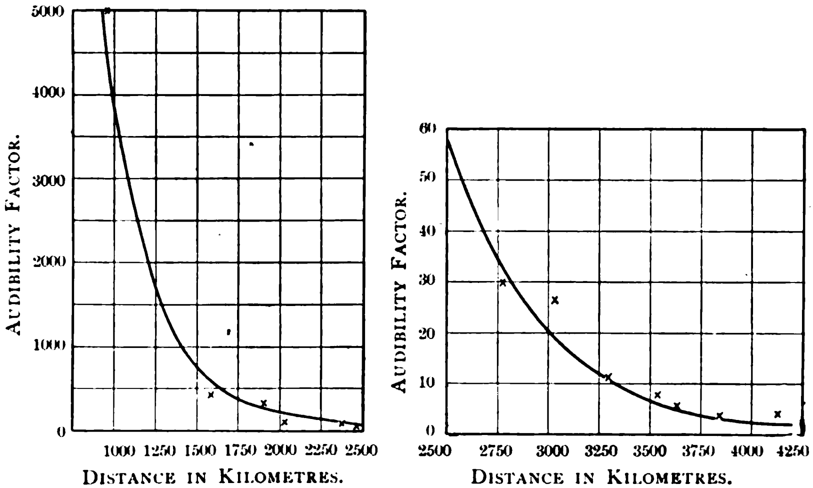
Hogan asserts that this audibility factor is proportional to the square of the receiving antenna current. He also alters Austin's empirical formula so as to express antenna heights in feet, distance in kilometres, wave-length in metres, sending antenna current in amperes, receiving antenna current in microamperes, and obtains an equivalent equation as follows :—

$$I_r \text{ (microamperes)} = \frac{395 I_s h_1 h_2}{D\lambda} \epsilon^{-0.0474 D\lambda^{-1/2}} \quad . \quad . \quad . \quad (45)$$

For arithmetic calculations we can write the above formula in the form[23]—

$$I_R = \frac{395 \, . \, I_s h_1 h_2}{D\lambda K}$$

where $$\log_{10} K = \frac{0{\cdot}0474}{2{\cdot}3026} \frac{D}{\sqrt{\lambda}} = \frac{0{\cdot}0215 D}{\sqrt{\lambda}}$$



[*By permission of the Proprietors of "The Electrician."*

FIG. 13.—Curves showing the Variation of Signal Strength with Distance during Daylight (Hogan).

In Hogan's experiments the effective height of the Arlington antenna was 450 feet, and that of the *Salem* 130 feet. The average sending current I_s was 23 amperes on the *Salem*, with wave-length 2000 metres, and 110 amperes with wave-length 3800 metres when sending from Arlington.

In Austin's experiments in 1910 he found that a receiving antenna current of 10 microamperes, or received energy of 1/400 of a microwatt, gave a just audible signal, but that a received current of 40 microamperes, or received energy of 1/25 of a microwatt, was necessary for good signals.

In Hogan's experiments, owing to improved apparatus, a received current of about half the above values gave the same results.

Austin states the value of the received currents in microamperes, whereas Hogan gives only the "audibility factor." It appears as if the value in microamperes of the received currents in Hogan's experiments can be obtained by multiplying the square root of his "audibility factor" by 4, but this is not quite certain.

[23] In the formula as given in *The Electrician* of August 8, 1913, p. 721, there is a mistake in the constant, which is given as 3·92, whereas it should be 395.

To gain additional light on some of these disputed questions another set of radiotelegraphic measurements was made between the station at Arlington, U.S.A., and the cruiser *Salem*, in February and March 1913. In these tests shunted telephone observations were taken in the daytime up to a distance of 1920 nautical miles or 3550 km., and tests with the Fessenden heterodyne receiver up to 2100 nautical miles or 3800 km. The wave-length used was 3800 metres, and the average sending current was 100 amperes. The height to the flat top of the *Salem's* antenna was 130 feet (39·6 m.), and the height to the centre of capacity 114 feet (34·5 m.).

The similar measurements for the Arlington antenna were 470 feet (143 m.) and 400 feet (122 m.). The results of observations were set out in charts and compared with the values predetermined by certain formulæ.

Dr. Austin modified his semi-empirical formula (43) for the receiving antenna current I_R as follows :—

$$I_R = 377 \frac{h_1 h_2 I_s}{\lambda D R \sqrt{1 + \frac{\delta_1}{\delta_2}}} \epsilon^{-0{\cdot}0015 D \lambda^{-1/2}} \quad . \quad . \quad . \quad . \quad . \qquad (45a)$$

where I_R is the current in the receiving antenna, and I_s that in the sending antenna, both measured in amperes. h_1 and h_2 are the heights of the sending and receiving antennæ in kilometres reckoned up to the centres of capacity. D is the distance of the stations in kilometres, λ the wave-length in kilometres, R the high frequency resistance in ohms of the receiving system, and the constant 377 is nearly 120π, whilst δ_1 and δ_2 are the decrements of the sending and receiving systems.

In the above formula the factor $\sqrt{1 + \frac{\delta_1}{\delta_2}}$ is introduced because damped oscillations were employed. If E is the electric force at the receiving antenna due to the sending antenna current, then the received current would be proportional to Eh_2/R if undamped waves were used, but if damped waves are employed then the current is obtained by dividing this last by the quantity $\sqrt{1 + \frac{\delta_1}{\delta_2}}$. This is quite easily proved as a deduction from equations (142) and (11) of Chap. III. Austin endeavoured to decide the question whether the exponential factor in the expression for the received current should be $\epsilon^{-0{\cdot}0015 D \lambda^{-1/2}}$ or $\epsilon^{-0{\cdot}0019 D \lambda^{-1/3}}$. The former is the empirical factor derived from his measurements in 1910, and the latter is, as we have above seen, the factor indicated by the diffraction theory.

In the measurements made between Arlington Station and the cruiser *Salem* in 1913, the receiving system resistance R was 50 ohms made up of inductance 25 ohms, antenna and ground 3 ohms, and resistance due to coupled secondary circuit 22 ohms. The decrements were $\delta_1 = 0{\cdot}05$, $\delta_2 = 0{\cdot}14$. The wave-length $\lambda = 3800$ metres, and sending current $I_s = 100$ amperes. The telephones used gave an audible response for 7 microamperes of antenna current through 25 ohms. Hence the energy required for an audible signal was $12{\cdot}25 \times 10^{-4}$ microwatt. Austin set out the results of these tests in a chart in which horizontal distances are miles or kilometres, and vertical distances the received current in microamperes.

It is not quite clear how Dr. Austin obtains the value of this received current in microamperes from the observations made with the shunted telephones on board the ships. Austin, however, concludes that the observations agree better with the empirical formula in which the exponent is inversely as the square root of the wave-length and not inversely as the cube root as predicted by theory. His conclusion is that the received current can be predetermined by the formula

$$I_R = I_s \frac{377 h_1 h_2}{\lambda D R} \frac{1}{\sqrt{1 + \frac{\delta_1}{\delta_2}}} \epsilon^{\frac{-0{\cdot}0015 D}{\sqrt{\lambda}}}$$

where the letters have the signification above given, and that this formula holds for long distances 1000 to 3000 miles over sea for daytime transmission with feebly damped waves.

Professor A. E. H. Love has, however, pointed out (see *Phil. Trans. Roy. Soc.*, London, vol. 215, A, p. 105, 1915) that however closely such a formula may represent the results of observations it can have no value except as an empirical expression. It is impossible to admit that the law of decrease of wave amplitude round a sphere is a combination of the law of decrease through empty space combined with an attenuation factor of an exponential form expressing the "absorption" or "scattering."

Professor Love has also critically examined Austin's results, and arrived at the conclusion that the "audibility factor" as defined by Hogan is nearly proportional to the square of the receiving antenna current for large values of the latter, but simply as the current for small values. Hence he concludes that the true antenna currents were simply proportional to the audibility factor in Hogan's experiments, and not to its square root. Assuming this, he finds that Macdonald's formula (39) enables a very fair prediction to be made of the received current at various distances over sea by day. The conclusion Love draws is that the diminution in strength of radiotelegraphic signals with distance is due simply to the combination of true diffraction with the expansion of the energy over larger areas, and that by day and over sea diffraction will account for the observed values of the received currents. This conclusion is not, however, confirmed when Macdonald's or Nicholson's formulæ ((39) and (40) § 3) for the ratio of the magnetic forces at two distances are tested by arithmetic and experience. These formulæ are not valid very near the oscillator. If we take H_1 to be the force at an angular distance θ_1 corresponding to a meridian arc of 500 miles and arbitrarily assume for H_1 a value of 10^6 we shall find that the force H at an angular distance $\pi/2$ is less than 10 by both formulæ. But very good radio signals have been received from large stations at angular distances of 90° and even 180°, and this requires far less attenuation than the above diffraction formulæ predict.

One thing, moreover, is certain, that diffraction will not account for the greater and more irregular strength of radiotelegraphic long distance signals received by night, and that we have to bring into consideration some other agency besides that of diffraction to explain it. There are also to be considered certain facts which seem to point to the co-operation of a surface electric wave with a true diffracted space wave.

5. The Effect of Daylight upon Radiotelegraphic Communication.—We are now in a position to consider the question of the causes of the difference between the signal strength of received signals by day and by night, to which brief reference has already been made.

This important fact was discovered by Senatore Marconi in 1902. In one of his voyages across the Atlantic, when receiving signals from Poldhu on board the ss. *Philadelphia*, he noticed that the signals were received by night when they could not be detected by day.[24] In these experiments, Senatore Marconi instructed his assistants at Poldhu to send signals at a certain rate from 12 to 1 A.M., from 6 to 7 A.M., from 12 to 1 P.M. and 6 to 7 P.M., Greenwich mean time, every day for a week. He states that on board the *Philadelphia* he did not notice any apparent difference between the signals received in the day and those received at night until the vessel had reached a distance of 500 statute miles from Poldhu. At distances of over 700 miles the signals transmitted during the day failed entirely, while those sent at night remained quite strong up to 1551 miles, and were clearly decipherable up to a distance of 2099 miles from Poldhu. Mr. Marconi also noted that at distances of over 700 miles the signals at 6 A.M. in the week between February 23 and March 1 were quite clear and distinct, whereas by 7 A.M. they had become weak almost to total disappearance. This fact led him at first to conclude that the cause of the weakening was due to the action of the daylight upon the transmitting aerial, and that, as the sun rose

[24] See *Proc. Roy. Soc.*, June 12, 1902, "A Note on the Effect of Daylight upon the Propagation of Electromagnetic Impulses over Long Distances," by G. Marconi.

over Poldhu, so the radiated energy diminished, and he suggested as an explanation the known fact of the dissipating action of light upon a negative charge.

Further consideration showed, however, that the reduction must be due to some influence acting in the space between the transmitter and receiver, as it was essentially a long distance effect and hardly noticeable at short distances or much under 200 miles. The next suggestion made was that it was due to some absorptive action of air ionized by sunlight upon the electric waves passing through the space so ionized by daylight.

It has been shown mathematically by Professor Sir J. J. Thomson, that small gaseous ions or electrons must be set in motion in the direction of propagation by a long electric wave travelling through them, and they must, therefore, absorb energy from a long electric wave when passing through a space through which such electrons are scattered.[25]

It is well known that ultra-violet light of wave-length shorter than about 1350 Angström units (1 A . U $=10^{-7}$ mm.) has the power of ionizing gaseous molecules or separating from them electrons. Hence, that portion of the earth's higher atmosphere which is facing the sun will have present in it more electrons or gaseous ions than that portion which is turned towards the dark space, and if these can absorb the wave energy it may, therefore, be less transparent to long Hertzian waves.[26] In other words, clear, sunlit air, though extremely transparent to light waves, may act as if it were a slightly turbid medium for long Hertzian waves. The dividing line between that portion of the earth's atmosphere which is impregnated with gaseous ions or electrons is not sharply delimited from the part not so illuminated, and there may be, therefore, a considerable penetration of these ions into the regions which may be called the twilight areas. Accordingly, as the earth rotates, a district in which Hertzian waves are being propagated is brought, towards the time of sunrise, into a position in which the atmosphere begins to be ionized, although far from as freely as in the case during the hours of bright sunshine.

Professor Sir J. J. Thomson has shown that if electric waves of length λ, and having a maximum magnetic force H, are travelling in a medium containing free electrons, each electron having a negative charge e and mass m, then the electron is moved forward in the direction of propagation. The maximum velocity w imparted to each electron in the direction in which the wave is moving is given by the expression—

$$V^2-(V-w)^2=\frac{\lambda^2H^2e^2}{4\pi^2m^2}$$

where $V=3\times10^{10}$. If e is reckoned in electromagnetic units, the ratio of charge to mass for an electron $=\frac{e}{m}=10^7$, and $\pi^2=10$ nearly. Hence, if w is small compared with V, we have—

$$w=\frac{1}{2}\cdot\frac{\lambda^2H^210^{14}}{4\pi^2 . 3\times10^{10}}=\frac{\lambda^2H^2}{24}1000$$

The magnetic force H of any wave is always a numerically small quantity in the electromagnetic system of units, and hence the velocity w is small unless λ is very large.

Accordingly, the presence of numerous free electrons in a space through which long electric waves are passing will rob these waves of energy. The energy imparted to the electron to give it a maximum velocity, w, is $\frac{1}{2}mw^2$, and from the above expression this is seen to be equal to $\frac{\lambda^4H^4e^4}{128\pi^4m^3V^2}$.

If there are N electrons per cubic centimetre, then, since the wave energy per

[25] See *Phil. Mag.*, August 1902, ser. 6, vol. 4, p. 253, J. J. Thomson, "On Some Consequences of the Emission of Negatively Electrified Corpuscles by Hot Bodies."

[26] The opinion that ionization of the air by sunlight is a cause of obstruction to Hertzian waves propagated over long distances was suggested also by Mr. J. E. Taylor. See *Proc. Roy. Soc.*, 1903, vol. 71, p. 225, "Characteristics of Earth-Current Disturbances and their Origin."

unit volume for a medium of unit permeability is $\frac{1}{2}H^2$, we see that the energy taken from the wave per cubic centimetre of space is—

$$\frac{N\lambda^4 e^4 H^2}{64\pi^4 m^3 V} \cdot \frac{H^2}{2} = \frac{1}{a} \cdot \frac{H^2}{2}$$

By means of an apparatus devised by Ebert and by Gerdien it is possible to determine the conductivity of the air, and hence the number of ions N per cubic centimetre. With this apparatus Boltzmann found during a voyage across the Atlantic 1150 positive and 800 negative ions per cubic centimetre. Also over the same ocean, A. S. Eve found from 600 to 1400 positive and 500 to 1000 negative, the ratio of positive to negative varying from 1·04 to 1·83. These numbers do not differ greatly from those found over large land areas, such as Germany or Canada. The above expression shows that the wave weakening is reduced by using a wave of small amplitude or small magnetic force H, and for that reason it should be largest in the neighbourhood of the sending antenna.

Although an absorption of wave energy by the above process is undoubtedly possible, it seems tolerably certain that this action is not capable of accounting for the observed difference between radiotelegraphy by day and by night. It is true that ionization of the air imparts electric conductivity to it, but the actual observed electric conductivity of the atmosphere by day at such heights as can be reached by us does not rise to the required value to produce the degree of absorption necessary to account for the difference between day and night transmission.

It is easy to obtain from the fundamental Maxwellian equations an expression which enables us to decide this matter. Let E be the electric force and H the magnetic force at any point in a space traversed by electric wires, and let μ be the magnetic permeability, σ the electric conductivity, and K the dielectric constant of the medium. Then by Maxwell's equations for a pure dielectric (see Chap. V. § 2) we have—

$$K\dot{E} + 4\pi\sigma E = c \text{ Curl } H \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$-\mu\dot{H} = c \text{ Curl } E \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where $c = 3 \times 10^{10}$. In the above equations the quantities E, σ, and K are to be measured in electrostatic units, and μ and H in electromagnetic units. If E and H vary harmonically or as the real part of ϵ^{-jpt} where $p = 2\pi n$ and n is the frequency, and if we write

$$k^2 = \frac{\mu K p^2 + j4\pi\mu\sigma p}{c^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

we can transform the above equations (46) and (47) into

$$k^2 H = \text{Curl}^2 H \text{ and } k^2 E = \text{Curl}^2 E.$$

Now k is a complex quantity and can be represented by $\alpha + j\beta$. Hence from (48) if we put $\mu = 1$ we have

$$\beta^2 = -\frac{p^2 K}{2c^2} \pm \sqrt{\frac{16\pi^2\sigma^2 p^2 + K^2 p^4}{4c^4}} \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Then by an easy transformation we can prove that

$$\beta = \frac{2\pi n}{c}\sqrt{\frac{K}{2}}\sqrt{\sqrt{\left(1 + \frac{4\sigma^2}{n^2 K^2}\right)} - 1}$$

If σ^2/n^2K^2 is small compared with unity, as it is when n is large and σ small, the above expression reduces to

$$\beta = \frac{2\pi\sigma}{c\sqrt{K}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

If then we consider a plane wave the amplitude of which is represented by $\epsilon^{j(kx-pt)}$ it attenuates to $1/\epsilon$ of its initial amplitude in travelling a distance $1/\beta = (c\sqrt{K})/2\pi\sigma$.

If ρ is the resistivity of the medium in ohms per centimetre cube, then $\rho=(9\times10^{11})/\sigma$ and we have finally as the distance x in which the wave attenuates to $1/\epsilon$ of its initial value—

$$x=\frac{\rho\sqrt{K}}{60\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

Now for air $K=1$, and if ρ has any such value as a million megohms per centimetre cube, then we should have—

$$x=\frac{10^{12}}{60\pi}=40,000 \text{ kilometres nearly.}$$

Hence for such a resistivity the wave would have to travel 25,000 miles to be reduced in amplitude by absorption to about one-third of its initial value. Some measurements of the resistivity of air were made by Messrs. A. J. Makower, W. Makower, W. M. Gregory, and H. Robinson in 1910 at Ditcham (see *Quarterly Journal of Royal Meteorological Society*, vol. 37, October 1911) by elevating a kite or captive balloon and measuring the potential of the balloon when insulated and the current flowing to earth when it is connected to earth by a wire. If C is the electrostatic capacity of the balloon, and V its potential when insulated, and if I is the current flowing to earth from it, and if S is the total conductivity of the surrounding air, then it can be shown that $4\pi C/\rho=S=I/V$. Hence $\rho=4\pi CV/I$. The above observers found that a kite having a capacity of about 100 E.S. units when flown at a height of 1400 feet on a certain day indicated a total resistance of 1000 megohms. This gives an air resistivity of 1·25 million megohms, which is not very high.

Nevertheless a plane electric wave would have to travel 50,000 kilometres in such air to be attenuated to $1/\epsilon$ of its initial value by conductivity alone. These figures are sufficient to show that air conductivity due to ionization cannot be a sufficient cause of the attenuation of long electric waves by daylight, at any rate near the earth's surface. No such electric conductivity of the atmosphere has been observed near the earth's surface as to account for the relative diminution of signalling distance by day by true absorption resulting from air conductivity.

The conductivity of the air would have to be 100,000 times greater than it is to explain such attenuation. We are, therefore, led to consider that the daylight transmission may be the normal transmission, and that the night signals are increased in strength by some means, rather than regard the night signal strength as the normal value and the day signals as attenuated by absorption due to conductivity. In any case, however, there are extraordinary variations in strength which it is necessary to mention. On these matters Mr. Marconi made some interesting remarks in his Nobel Prize Lecture, delivered December 11, 1909. He says:—

"It has been observed that an ordinary ship station, using about half a kilowatt, the normal range of which is not greater than 200 miles, will occasionally transmit messages across a distance of over 1200 miles. It often occurs that a ship fails to communicate with a near-by station, but can correspond with perfect ease with a distant one." Also he remarks, "Although high-power stations are now used for communicating across the Atlantic, and messages can be sent by day as well as by night, there still exist short periods of daily occurrence during which transmission from England to America, or *vice versâ*, is difficult. Thus, in the morning and evening, when, in consequence of the difference of longitude, daylight or darkness extends only a part of the way across the ocean, the received signals are weak and sometimes cease altogether. It would almost appear as if electric waves in passing from dark space to illuminated space and *vice versâ* were reflected in such a manner as to be diverted from their normal path."

"Another curious result on which hundreds of observations continued for years leave no further doubt, is that regularly for periods at sunrise and sunset, and occasionally at other times, a shorter wave can be detected across the Atlantic in preference to the longer wave normally employed. Thus at Clifden and Glace

Bay, when sending on an ordinary coupled circuit arranged so as to simultaneously radiate two waves, one 12,500 feet and the other 14,700 feet, although the longer wave is the one usually received at the other side of the ocean, regularly about three hours after sunset at Clifden and three hours before sunrise at Glace Bay, the shorter wave is alone received with remarkable strength so regularly that the operators tune their receivers to the shorter wave at the times mentioned as a matter of ordinary routine."

Again, it is a constant experience that ships provided with radiotelegraphic plant of no great power, say 1·5 kw., will occasionally be able to transmit signals 1000 miles or more, and in the same manner their receivers will pick up signals coming from very large distances. These freak transmissions and receptions as they are called are very common experiences, and indicate that there is on some occasions an exceptional facility in the transmission of long electric waves over the earth in certain directions. These effects taken in conjunction with observed regular diurnal and annual variations in the intensity of message signals and also of the stray waves due to atmospheric discharges prove that we have to look for the source of these vagaries of transmission in the structure of the atmosphere itself. Modern researches have shown that the earth's atmosphere may be broadly divided into two portions: first, a lower portion in which the chemical percentage composition remains tolerably uniform, since the mechanical mixture of its gases is maintained by winds and convection currents; but the pressure and temperature, however, fall as we rise upwards. Secondly, a higher region beginning at a height of about 10 miles or so when temperature ceases to fall and becomes nearly constant for an unknown further height. This region of constant temperature is called the *stratosphere*, and the lower region of gradually falling temperature the *troposphere*. The lower region is the locus of clouds and water vapour. Above a height of about 10 or 20 miles the atmosphere is in a state of perpetual calm, and the gases which compose it begin to sort themselves out in order of density. The highest upper regions are composed entirely of the lighter gases such as hydrogen and helium. At a height of about 60 kilometres oxygen begins to be greatly diminished, and above 100 kilometres oxygen and nitrogen are absent and hydrogen and helium are the only atmospheric constituents. There is not only a rapid fall in pressure as we rise to great heights but a change in composition and in refractive index as well.

Furthermore, owing to the transparency of these upper regions of the atmosphere to ultra-violet light in the sun's rays there is a considerable ionization of these gases, that is a liberation of positive and negative ions from the atoms This takes place to a very limited extent in the lower regions because the water vapour and especially the oxygen absorb the short wave-length actinic rays. This ionization converts the gases into conductors of electricity more or less. Hence we must consider the upper regions of the atmosphere as possessing probably a large but unknown amount of electric conductivity.

This ionized layer will begin at a lower level during the day time, thus in the night time and moreover at the boundary line between light and dark or in the twilight area there will be a more or less unhomogeneous condition of the upper layers of the atmosphere. We have then to consider that long distance radiotelegraphy involves the propagation of a wave through a dielectric of varying dielectric constant and conductivity at various heights, and one that is more or less strongly ionized in certain regions.

Dr. W. H. Eccles has shown that when an electric wave is propagated through a region containing heavy ions so that they only make small excursions under the action of the electric forces in the wave as it passes over them, then the wave velocity will be increased by the ions.[27] As the proof of this is important we reproduce it here in the form given by him.

Let e be the charge and m the mass of each ion and n the number per c.c. at a point whose co-ordinates are x, y, z. Let the waves advance in the direction of the x axis with electric force Z and magnetic force β in the plane of the wave

[27] See W. H. Eccles, "On the Diurnal Variation of the Electric Waves Occurring in Nature and on the Propagation of Electric Waves Round the Bend of the Earth," *Proc. Roy. Soc. Lond.*, vol. 87, A, p. 79, 1912.

at right angles to the x axis. Then if μ is the permeability and K is the dielectric constant of the medium we have from the Maxwell equations (see Chap. V. § 2, the equations

$$\frac{dZ}{dx}=\mu\frac{d\beta}{dt} \text{ and } \frac{d\beta}{dx}=K\frac{dZ}{dt}+4\pi ne\frac{du}{dt} \qquad (52)$$

where u is the displacement of the ion from its position produced by the waves. The above equations are merely the statement of the fact that the curl of the electric force is the negative rate of change of the magnetic flux and the curl of the magnetic force is 4π times the total current through the unit of area.

The equation of motion of the ion is

$$m\frac{d^2u}{dt^2}+f\frac{du}{dt}=eZ \qquad (53)$$

where f is a frictional coefficient such that the resistance to motion is proportional to the velocity of the ion. If Z and u vary in a simple harmonic manner or proportionally to ϵ^{jpt}, then the last equation becomes

$$\frac{du}{dt}=\frac{eZ}{mjp+f} \qquad (54)$$

Eliminating β and u from the three equations we reach

$$\frac{d^2Z}{dx^2}=\mu K\frac{d^2Z}{dt^2}+\frac{4\pi\mu ne^2}{mjp+f}\frac{dZ}{dt} \qquad (55)$$

This last has a solution of the form

$$Z=\epsilon^{-lx+jp(t-x/v)} \qquad (56)$$

for waves of frequency $p/2\pi$ of the wave velocity v is given by

$$v=\frac{mp}{gf}\sqrt{\left(\frac{2(1-g)}{\mu K}\right)}\left\{\sqrt{\left[1+\left(\frac{gf}{mp(1-g)}\right)^2\right]}-1\right\}^{\frac{1}{2}} \qquad (57)$$

and the absorption factor l by

$$l=\frac{\mu Kgf}{2m}v \qquad (58)$$

where g is put for

$$\frac{4\pi ne^2m}{K(m^2p^2+f^2)} \qquad (59)$$

Now g is smaller than unity and also $gf/mp(1-g)$. Hence, approximately we have

$$v=\frac{1}{\sqrt{\mu K}}(1+\tfrac{1}{2}g) \qquad (60)$$

That is to say, the velocity of the wave is increased by the presence of the heavy ions.

The verbal explanation of this is as follows:—The electric force causes a displacement or electric-flux in a dielectric. The ratio of displacement to force is called the dielectric coefficient of the medium. If the medium contains positive and negative ions the electric force tends to separate these and hence to produce what may be called a counter-electric force which diminishes the resultant displacement or flux. Accordingly the presence of the ions tends to virtually diminish the dielectric coefficient. It therefore causes an increase in the velocity of the wave, since this is inversely proportional to the square root of the dielectric coefficient.

If, therefore, the upper layers of the earth's atmosphere are ionized this will cause the portion of an electric wave passing through it to travel faster than that part passing through the lower unionized layers Accordingly there must be a

change in the form of the wave front, and therefore in the direction of propagation of the rays which is normal to the wave front at every point. It is, therefore, obvious that if at a certain high level in the earth's atmosphere we enter a strongly ionized layer of gases this overhead horizontal curtain or screen will cause electric rays sent up to it to be refracted down again by an action which resembles the optical phenomenon called the mirage. In this last case when a layer of still air near the hot earth is itself much heated its refractive index is diminished, and if the boundary surface of this hot air is defined fairly sharply rays of light falling on it obliquely will be turned through a considerable angle and bent upwards again.

Hence they will form an inverted image of distant objects, and therefore suggest a water surface to the traveller.

In the case of electric radiation the effect is as it were an inverted mirage.

It will be seen, therefore, that this *ionic refraction* in the atmosphere is in the right direction to carry a ray of electric radiation more or less round the earth. In fact it may even overdo it, and bring a ray down to the earth's surface too soon and hence tend to shorten the effective range.

In addition to this ionic refraction we have to a small degree the normal refraction due to the decreasing density gradient of the atmosphere which acts in the same way. This has been pointed out both by Dr. Eccles [28] and by the Author.[29]

The refractive index of a gas is proportional to the square root of its dielectric coefficient. Also by Gladstone and Dale's law the refractive index μ is connected with the density d by the relation $(\mu-1)/d$ is constant. Hence as we ascend in the atmosphere the refractive index and therefore dielectric coefficient become less. Therefore if a plane wave with vertical wave front starts from any point the upper part of it travels faster than the lower and the wave front tends to follow round the earth's curvature more or less.

The radius of curvature at starting can be calculated as follows :—

If we assume a homogeneous atmosphere and neglect the variation of gravity with height, and assume temperature to be constant or to decrease upwards according to some straight line law, it is a comparatively easy matter to calculate the density at any height in a column of gas in equilibrium under the forces of gravity and its own elasticity. Thus, if p_0 and q_0 are the pressure and density of the gas at the earth's surface, and p and q the same at any height h above it, and if T is the absolute temperature, g the acceleration of gravity, and G the gas constant, then we have

$$p=\mathrm{GT}q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

If we consider a horizontal slice of thickness δh, of a vertical column of the gas of unit cross-section, then the equation of equilibrium of the element is

$$-\frac{dp}{dh}\delta h=gq\delta h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

or

$$\frac{dq}{q}=-\frac{g}{\mathrm{GT}}dh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

If R is the mean radius of the earth, and if the mean value of gravity at the surface is 980, then $g=980\mathrm{R}^2/(\mathrm{R}+h)^2$ nearly. Since the variation of gravity is only about 5 per cent. in a height of 100 miles we make no great error in considering it as a constant. There is more difficulty in finding a function of h to express the temperature.

If we consider the temperature and gravity constant at various heights the integral of equation (63) is

$$q=q_0\epsilon^{-\frac{g}{\mathrm{GT}}h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

showing that the density decreases in accordance with an exponential law.

[28] See Dr. W. H. Eccles, "Atmospheric Refraction in Wireless Telegraphy." A Paper read to Section G of the *British Association*, September 16, 1913; also *The Electrician*, September 19, 1913.

[29] See Dr. J. A. Fleming on "Atmospheric Refraction and its Bearing on the Transmission of Electric Waves round the Earth's Surface." *Proc. Physical Soc. Lond.*, vol. 26, Part V., August 1914, p. 318.

We are now in a position to consider the refractive effects of the atmosphere at various heights. In gases the refractivity $(\mu - 1)$ is connected with the density q with fair accuracy by Gladstone and Dale's law $(\mu - 1)q = A =$ a constant. The constant A may be called Gladstone and Dale's constant. Its value for various gases is given in Table V.

TABLE V

Gas.	Refractive index μ_0 at 0° C. and 10^6 dynes/cm.^{2} pressure.	$A = \frac{\mu_0 - 1}{q_0}$.
Hydrogen	1·000138	1·56
Helium	1·000035	0·197
Neon	1·0000687	0·0777
Nitrogen	1·000300	0·242
Oxygen	1·000272	0·193
Argon	1·000285	0·161
Krypton	1·000424	0·117
Xenon	1·000693	0·123
Air	1·000293	0·227

We can then calculate the curvature of a ray passing through the atmosphere as follows:—

The atmosphere may approximately be considered as formed of concentric layers of gas having refractive indices which decrease as the altitude increases.

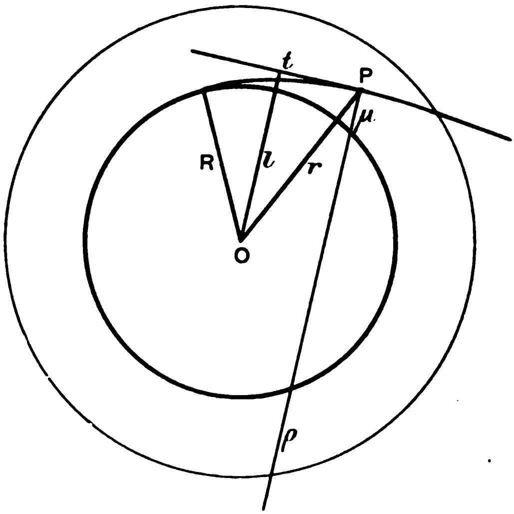


FIG. 14.—Refraction of a Ray of Light by the Earth's Atmosphere.

Let us take the centre of the earth as origin and consider the curved path of any ray travelling to or from the earth (see Fig. 14). Then if a tangent is drawn at any point P to this curve, and if l is the length of the perpendicular let fall from the origin on the tangent, and μ the refractive index at the point of contact of the tangent with the curve, it can easily be shown that all along the path of the ray the product $l\mu$ is constant (see Parkinson's "Optics," p. 109).

Again, if r is the radius vector to that point, and if ρ is the radius of curvature at the same point, it is easily shown that $\rho = r dr/dl$.

Also by Gladstone and Dale's law, the quotient $(\mu - 1)/q = A$, where q is the density at that point, will be a constant different for each gas. Taking the expression (4) to represent the variation of density with height, we have the following three equations :—

$$l\mu = C \quad \ldots \quad (65)$$

$$\frac{\mu - 1}{A} = q = q_0 \epsilon^{-\alpha(r-R)} \quad \ldots \quad (66)$$

$$\rho = r\frac{dr}{dl} \quad \ldots \quad (67)$$

where C and A are constants and r is the radius vector from the centre of the earth to any point on the ray at which the refractive index of the air is μ, whilst l is the length of the perpendicular from the centre of the earth on the tangent to the ray at that point. Also ρ is the radius of curvature of the ray at that point, and R is the mean radius of the earth, and $(r - R)$ is the height above the earth's surface. The symbol α stands for g/GT, where g is the acceleration of gravity, G is the gas constant, and T the absolute temperature.

From (67) we have

$$\rho = r\frac{dr}{dl} = r\frac{dr}{d\mu}\frac{d\mu}{dl} \quad \ldots \quad (68)$$

and from (65)

$$\frac{d\mu}{dl} = -\frac{C}{l^2} = -\frac{\mu^2}{C} \quad \ldots \quad (69)$$

Hence

$$\rho = -\frac{r}{d\mu/dr} \cdot \frac{\mu^2}{C} \quad \ldots \quad (70)$$

But by (66) we have

$$\mu = 1 + Aq_0\epsilon^{-\alpha(r-R)} \quad \ldots \quad (71)$$

Therefore

$$\frac{d\mu}{dr} = -\alpha A q_0 \epsilon^{-\alpha(r-R)} \quad \ldots \quad (72)$$

Hence substituting (72) in (70) we have

$$\rho = \frac{r\epsilon^{\alpha(r-R)}}{C\alpha A q_0}(1 + Aq_0\epsilon^{-\alpha(r-R)})^2 \quad \ldots \quad (73)$$

or by (66)

$$\rho = \frac{r}{C\alpha}\frac{\mu^2}{\mu - 1} = \frac{r}{l\alpha}\frac{\mu}{\mu - 1} \quad \ldots \quad (74)$$

Now

$$\alpha = \frac{g}{GT} = \frac{gq_0}{p_0} = \frac{980q_0}{10^6}$$

Also q_0 the density of the air at the earth's surface under a pressure of 10^6 dynes per square centimetre is nearly equal to $1/800$. Hence, $\alpha = 98/(8 \times 10^7)$.

The dimensions of α are inversely as a length. Hence, if we take the centimetre as unit of length $1/\alpha = 8 \times 10^5$ nearly. If, however, we reckon lengths or heights in kilometres then the numerical value of $\alpha = 1/8$ nearly.

Accordingly the following simple expression gives the radius of curvature of the ray :—

$$\rho = \frac{8r}{l}\frac{\mu}{\mu - 1} \quad \ldots \quad (75)$$

Consider, then, a ray starting horizontally from a point on the earth's surface. We have at that point $r = l = R =$ the earth's radius, and $\mu = 1{\cdot}000294$. Hence, $\mu/(\mu - 1) = 10{,}000/3$ nearly, and the radius of curvature of the ray at starting

$=80{,}000/3=26{,}666$ km., or 16,000 miles, or about four times the radius of curvature of the earth.

As the point considered is taken farther from the source the radius of curvature of the ray at that point increases. For we have

$$\rho=\frac{r}{la}\frac{\mu}{\mu-1}=\frac{r\mu}{la\mathrm{A}q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

But μ is very nearly unity, and A and a are particular constants for each gas, whilst l is never very much greater than r, and the density q continually diminishes with increasing altitude.

Hence, ρ rapidly increases with increase of distance of the point on the ray considered, as we take it farther from the source on the earth's surface.

Since a is equal to gq_0/p_0, it is clear, if lengths are measured in kilometres, that $a=98q_0$. Also $\mu_0-1=\mathrm{A}q_0$. Accordingly, taking the source at the earth's surface, where l and r are equal, and considering the ray emitted tangentially by the earth's surface, we have

$$\rho=\frac{\mu_0}{98\mathrm{A}}\frac{1}{q_0^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

But μ_0 for all gases is very nearly unity, and A has values for various gases as in Table V. above.

The constant A may be called the Gladstone and Dale constant, and q_0 is the density of the gas at 0° C. and 10^6 dynes per square centimetre pressure.

In the formula (77), if we substitute the values for air, viz. $\mu_0=1{\cdot}000293$, $\mathrm{A}=0{\cdot}227$ and $q_0=0{\cdot}001293$, we find $\rho=27{,}000$ km. as before, or 4·1 times the earth's radius.

Again, if we suppose the atmosphere consisted wholly of hydrogen, then inserting in formula (77) the values $\mu_0=1{\cdot}000138$, $\mathrm{A}=1{\cdot}56$ and $q_0=0{\cdot}00008837$, we find $\rho=880{,}000$ km. or 136 times the earth's radius.

If, however, the atmosphere consisted wholly of krypton, for which $\mu_0=1{\cdot}000424$, $\mathrm{A}=0{\cdot}117$ and $q_0=0{\cdot}0036125$, we find $\rho=6682$ km., or about the same as the earth's mean radius.

Finally, if the atmosphere consisted wholly of xenon, for which $\mu_0=1{\cdot}000693$, $\mathrm{A}=0{\cdot}123$, $q_0=0{\cdot}005643$, then we find $\rho=2768$ km., which is much less than the earth's radius.

On the assumption, therefore, above made as to uniform temperature we find the following remarkable result.

If the earth's atmosphere consisted wholly of krypton a ray of light sent out tangentially to the earth's surface would be refracted round the earth parallel to the surface, and never escape at all from the atmosphere. If, therefore, the atmosphere consisted entirely of krypton, wireless telegraphy right round the earth might be easily possible in consequence of this circular refraction of a tangentially emitted ray.

For the atmosphere as at present constituted this refractive bending is, however, very small. Nevertheless, it acts in the same direction as the ionic refraction in causing an electric ray emitted horizontally at any point on the earth's surface to follow round more or less the earth's curvature.

6. General Conclusions as to the Mode of Propagation of Long Electric Waves round the Earth.—Summing up the conclusions so far reached by radiotelegraphists, we may say that the effect produced by a radiotelegraphic transmitter at a great distance, say, 2000 to 6000 miles over the surface of the earth, is a complex one in which several different actions play a part.

There is, first, a propagation through the æther of a true space electromagnetic wave which is diffracted round the earth. The extent to which this contributes to the whole effect is, perhaps, greater than was formerly supposed, but is yet an undetermined quantity. Some mathematicians are now inclined to attribute to it the major portion of the transmission by day.

Then in the next place there is undoubtedly a contribution made to the effect

by waves which have suffered a refraction equivalent to a reflection by ionized air at high altitudes, and a very small effect due to the decrease in refractive index of air as we ascend upwards.

These causes tend to make the ray follow round the curvature of the earth and so assist as it were diffraction. It is to this variable ionic refraction that we must attribute the diurnal and annual variations in signal strength, and also the greater signalling distance by night as well as the irregularities attending the transition times of sunrise and sunset.

Then in addition we may inquire how far any contribution is made by a surface wave of the type investigated by Sommerfeld, which is equivalent to an electric wave propagated through or along the earth.

It has been definitely proved that we can receive signals from stations hundreds of miles away without any high receiving aerial, but merely by connecting one terminal of the receiving circuit to earth, and the other terminal to any large well-insulated mass of metal, whether inside or outside of a house does not matter. As far back as 1900 or 1901 Mr. Marconi received signals at Poole, near Bournemouth, from a station in the Isle of Wight, using as a collector merely an insulated zinc cylinder, attached to one terminal of his receiver, the other terminal being connected to earth. Then Mr. Campbell Swinton received in London signals sent out from the Eiffel Tower Station in Paris by using as the metal mass a bedstead in an upper room.[30] Mr. P. J. Ryle employed a bicycle out of doors,[31] and the author a zinc dustbin set on insulators on a table in a room in University College, London.[32]

Again, Mr. Marconi in 1906 received signals in Ireland from Poldhu in Cornwall, employing as a receiving antenna a long insulated wire lying on the ground with its free end pointing away from the receiving station.

More recent experiments have been made by Dr. F. Kiebitz[33] on the same lines. He used as receiving antenna long wires carried on insulators placed in ditches 1 metre deep. The ends of these wires were earthed through condensers. The receiving appliance was placed in the centre of the wire.

By such antennæ he found he could receive signals from all the principal radio stations in England, France, and America.

All these experiments show that the non-use of high or vertical antenna which can be cut across by a space wave does not entirely prevent the reception of radiotelegraphic signals, provided there is a good earth connection.

It seems probable, therefore, that some fraction of the effect of a transmitter at a distance when using earth-connected aerials may be due to a true surface wave, but it cannot contribute more than a small fraction of the total effect.

It is now, therefore, always the custom to reckon the range of a station solely by the distance of good working during the day and during the summer part of the year. This normal working is, however, much affected by the natural atmospheric or vagrant waves which we next proceed to consider.

7. Atmospheric Stray Waves and their Effect on Radiotelegraphy.—The high antennæ or aerial wires used in radiotelegraphy make the receiving instruments very susceptible to the effect of atmospheric electrical discharges, because these latter produce vagrant electric waves called "strays."

It was long ago suspected that lightning discharges were oscillatory in character. Sir Oliver Lodge pointed out this probable quality of them even before the publication of Hertz's researches on electric waves, guided thereto by a study of the phenomena attending the discharge of a Leyden jar.

In 1895 Popoff, in Russia, began to experiment with a lightning-conductor connected to a coherer as a means of detection of distant storms, and his work showed that these discharges of lightning created natural electric waves which were picked up by antennæ.

These natural electric waves are called "strays," "atmospherics," "X's," or

[30] See Campbell Swinton, *The Electrician*, vol. 71, p. 501, 1913.
[31] P. J. Ryle, *The Electrician*, vol. 71, p. 1025, 1913.
[32] J. A. Fleming, *The Electrician*, vol. 71, pp. 460 and 1065, 1913.
[33] See F. Kiebitz, "Recent Experiments on Directive Wireless Telegraphy with Earthed Antennæ," *The Electrician*, March 8, 1912, vol. 68, p. 868.

"Statics." From the earliest days of radiotelegraphy they gave trouble by printing down on the Morse inker tape occasional dots or groups of dots, which rendered the interpretation of the message signals difficult.

When the telephonic method of reception was adopted, using with it a low frequency spark, the stray waves produced irregular or occasional sounds like clicks or prolonged rattling or fizzling sounds in the telephone, and often made the message signals unintelligible. The introduction of the high frequency musical spark was a great improvement, as it imparted a shrill musical tone to the message signals and enabled the operator to concentrate attention on them to the exclusion of the lower pitched sounds due to strays. The interference of the strays is, however, always possible in connection with methods for visibly recording or printing the signals. A study of these stray waves has shown that they obey laws of their own.

Following on the first experiments of Popoff,[34] in 1895, using a lightning rod and coherer and Morse inker, Boggio Lera, in 1898, improved on Popoff's apparatus, and arranged that strong and weak signals should be recorded separately. Then Feriyi,[35] in 1901, showed that thunderstorms within 100 miles recorded themselves on his apparatus. Lastly, Turpain,[36] in 1903, made extensive observations and proved that such records were of use in forecasting weather hours or days in advance.

As radiotelegraphy came more into use, every station was forced to take note of strays, and it was found that they were more troublesome and persistent at some times than at others. W. H. Eccles introduced a simple method of recording them by hand, viz. to listen at a telephone connected with a receiver and antenna and make a vertical mark on a slip of paper, distances along which represented the flow of time, and the height of the mark, roughly, the loudness of the telephone sound.

Eccles and Airey [37] made a number of observations simultaneously in London and Newcastle in 1911, proving that 60 to 80 per cent. of the strays were audible at London and Newcastle simultaneously, and that the intensity of these roughly corresponded in such fashion as to show that they were due to the same disturbance created probably at places many hundreds of miles from both observing stations.

In England the departure from regularity in the strays is greatest in the summer, due no doubt to the origin of many in local thunderstorms. During the winter months the number and intensity of the strays agree better over larger areas, thus showing a non-local origin. Omitting these local strays, it is found that the rest are more numerous during the night than during the day.

Dr. Eccles [38] has thus described the phenomena occurring in connection with them about twilight. He says, "Starting to listen about half an hour before sunrise, the strays heard in the telephone are loud and numerous and much as they have been all night; then about 15 minutes before sunrise a change sets in, the strays get weaker and fewer rather quickly, till at about 10 minutes before sunrise a distinct lull occurs of perhaps a minute's duration. At this period there is sometimes complete silence. Then the strays begin to appear again, and within 10 minutes of the lull they have settled down to the steady stream proper to the daytime. These day strays are weaker and fewer than the night strays, except on rare occasions. The lull is sometimes very pronounced, and at other times there is no lull at all. It is usually more marked at sunset than at sunrise." Similar phenomena occur at sunset.

Since the cessation of the strays is a local effect determined by sunrise or sunset, and therefore taking place progressively as the twilight zone sweeps over the earth, it is obvious that it must depend upon some condition of the atmosphere at that locality at that time. For strays which are not heard at a place where the

34 Popoff, *Journ. of Russian Physico-Chemical Society*, 1895, vols. 28-29, p. 899.
35 Feriyi, *Comptes Rendus*, January 27, 1902.
36 Turpain, "La Télégraphie sans fil," p. 314.
37 W. H. Eccles and H. M. Airey, "Note on Electrical Waves occurring in Nature," *Proc. Roy. Soc. Lond.*, vol. 85, A, p. 145, 1911.
38 W. H. Eccles, "On the Diurnal Variations of Electric Waves occurring in Nature," *Proc. Roy. Soc. Lond.*, vol. 87, A, p. 79, 1912.

sun is setting can be heard by going a short distance either into the still daylight area or the night area. It seems probable that the atmosphere may be regarded as roughly divisible into three layers as regards ionization. There is an upper layer permanently in a state of strong ionization, as first suggested by Heaviside. This may be brought about not chiefly by solar ultra-violet light, but by the projection of radiant matter such as electrons from the sun, in virtue of the light wave pressure, as explained by Maxwell. Then below this there may be a middle region which is ionized by sunlight, in which the ionization fluctuates, being considerable by day, but disappearing more or less by night. In the lower regions there is also a fluctuating number of ions. On this hypothesis there is a gradually increasing number of ions in the air as we ascend, and therefore a certain ionic gradient. The upper or permanently ionized layer has been called the Heaviside layer, and we may call the region beneath it in which the density of the ions varies by day and night the variable layer. According to Eccles' theory the ionic gradient in the variable layer bends round the electric rays emitted from any long distance station, and compels them to follow more or less the curvature of the earth, and hence enables long distance radiotelegraphy to be conducted better by night than by day. The same effect causes the increase in the number of strays by night, because they are able to travel from greater distances, and affect the receiver at the observing station to a larger extent. The upper or permanently ionized layer may act as a reflecting layer by an inverted mirage action, and hence signals and strays reach a very distant point by an action which resembles that by which a very distant conflagration or search-light makes itself visible when really below the horizon by "lighting up the sky," to use Dr. Eccles' expression.

This hypothesis of a high level permanently ionized region in the air is not merely a supposition called forth by phenomena in wireless telegraphy, but is suggested by astronomical and meteorological facts. Simon Newcomb, the eminent American astronomer, in 1901 made measurements of the total light received from the sky on moonless nights, which showed it to be in excess of the sum of that received from stars. This result has been confirmed by other observers. W. W. Campbell showed in 1895 that a certain green line characteristic of the aurora spectrum, viz. $\lambda=5770$, can be detected on moonless nights in all parts of the sky. The conclusion is that there is always a certain aurora glow at high levels which indicates strong ionization. Prof. A. Schuster (see *Phil. Trans. Roy. Soc.*, vol. 208, A, p. 182, 1907) finds that his theory of the diurnal variations of magnetism demands an electric conductivity of the upper air of the order of 10^{-13} electromagnetic units, or a resistivity of 10,000 ohms per centimetre cube at a height of 100 kilometres.

In an interesting paper, "On Certain Phenomena Accompanying the Propagation of Electric Waves over the Surface of the Globe," W. H. Eccles[39] starts from this deduction of Schuster, and assumes that the conductivity is inversely as the pressure, and has a value of 10^{-24} electromagnetic units at the earth's surface. Taking the number of ions per c.c. as 10^4 at the earth's surface, it follows that there are 10^9 per c.c. at a level of 100 km.

Dr. Eccles hence deduces that the quantity g in his formula (60) for the velocity of a wave through ionized air (see p 658) will be unity at 60 km. high for waves of frequency 150,000, and at 40 km. for waves of 50,000 frequency.

The rate at which the wave velocity changes becomes very great when g approaches unity, which means that in the daytime the downward refraction of the electric rays will be so sharp as to amount to reflection.

This theory, therefore, shows us that the ionic refraction may either enable electric waves to travel further round the earth's surface—in other words, improve long distance transmission—or it may do the reverse, and bring them down to the earth too soon. Which of these effects will take place depends on the wavelength, and upon the distance and upon the height of the effective ionic layer.

The theory has, therefore, the merit that it is capable of adapting itself to the

[39] See *The Electrician*, September 27, 1912. This paper of Dr. Eccles was a contribution to the discussion on the scientific problems of wireless telegraphy at the British Association at Dundee opened by the author.

observations which are made in many ways in connection with long distance radiotelegraphic work.

We may in the next place collect together here some of the well-ascertained facts with regard to radiotelegraphic signalling, leaving out of account for the moment the "atmospheric" or "stray" waves.

Roughly speaking, one may say that ordinary ship transmitting and receiving apparatus has two or three times greater range by night than by day when using the 600 metre wave-length. This, however, is subject to great irregularities and exceptions.

For much longer waves the difference may even be reversed. Mr. Marconi stated in a Royal Institution Lecture (June 2, 1911) that for waves 4000 to 5000 metres long the transatlantic signals are sometimes stronger by day than by night.

He has also noticed in the signal strength very marked minima occurring in relation to the times of sunrise and sunset at Clifden (Ireland) and Cape Breton (Nova Scotia). He embraces the facts in the following statement:—

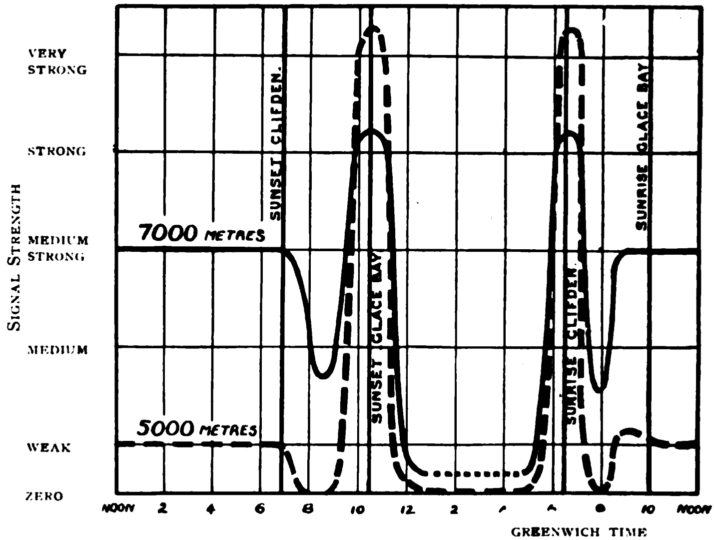


FIG. 15.—Curves representing the Diurnal Variation of Signal Strength at Clifden Marconi Station (Ireland), due to Long Electric Waves (5000 and 7000 metres) sent out from Cape Breton Marconi Station (Nova Scotia) as given from observations by Mr. Marconi.

"Waves of about 4000 metres length crossing the Atlantic from west to east yield strong and steady signals all day at Clifden, which gradually weaken after sunset at Clifden, reaching a minimum about $1\frac{1}{2}$ hours afterwards. The signals at Clifden then gradually increase in intensity till after sunset at Cape Breton, when they attain a maximum which is occasionally very high. During the night they are very variable in strength. Slightly before sunrise at Clifden the signals grow stronger, and sometimes pass quickly to a high maximum. They then dwindle to a marked minimum about 2 hours after sunrise at Clifden, and then return to the normal day strength."[40] These diurnal variations are delineated in the curve in Fig. 15, which is taken from the lecture of Mr. Marconi.

It appears, therefore, that there is a minimum value in the signal strength when the shifting boundary line between light and dark or day and night has reached a point about half-way across the Atlantic.

[40] See Senatore G. Marconi, on Radiotelegraphy, *Proc. Roy. Institution*, vol. xx. p. 202, 1911.

Eccles has confirmed this on measurements made of the Clifden signals in London. He found they sink to a minimum value at about twenty minutes after sunset in London—that is, when the sun is setting at a place about half-way between London and Clifden.

It appears as if the boundary plane between darkness and light in the earth's atmosphere formed by the cone of shadow projected by the sun acts as a reflector in some sense, and also is imperfectly transparent to the long electric waves. Eccles explains this by the suggestion that when the sunlight is withdrawn partial recomposition of ions takes place, but owing perhaps to the manner in which the positive and negative ions have drifted apart, this recomposition is not uniform, but there are patches or clouds of ions left in places. The boundary has, therefore, a structure something like transparent ice filled with air-bubbles. This imparts to it a certain opacity, because light is lost by innumerable reflections and refractions. In the same way the boundary zone, where sunrise and sunset is taking place, becomes by this want of ionic homogeneity more or less opaque to the long electric waves. This would account for the fall in intensity of the strays, and the drop in signal strength when such boundary intervenes between sending and receiving station. In other cases it seems to act as a reflector, and strengthens the signals when it comes behind a receiving station, and on the opposite side of it to the sending station.

Similar observations were made in 1912 by Messrs. Round and Tremellen[41] at Chelmsford (Essex, England) on the signals arriving from the Marconi stations at Clifden (Ireland) and Glace Bay (Nova Scotia) throughout twenty-four hours, beginning at midday (see Fig. 16).

The strength of signals received at Chelmsford remained nearly constant until about an hour before sunset. They then increased in strength very quickly to about four times normal strength. This happened a little after sunset at Clifden. This rise was followed by a sudden fall in strength, and the signals reached a minimum value about an hour after sunset at Clifden. An hour later a very sudden increase set in, which carried up the signal strength to nine or ten times its minimum day value. This continued with some irregular variations during the night. About an hour before sunrise at Chelmsford there was another sudden decrease in signal strength, followed again by a rise, and then by a fall to normal day strength soon after sunrise at Clifden. The strength of atmospheric strays followed nearly the same variation as shown in the lower curve of Fig. 16.

Other observers, such as G. W. Pickard, have noted similar variations of signal strength at times at or near sunrise or sunset at either the sending or receiving station. It seems, therefore, that when the boundary surface of the earth's shadow is near the sending or receiving station it may act as a reflector, and strengthen the signals. Also when coming between the sending and receiving stations it may act as a screen and weaken the signals.

In the above-mentioned Royal Institution Lecture on June 2, 1911, Mr. Marconi also called attention to a curious difference between the facility with which signals could be transmitted at about that time in a north-south direction and an east-west direction. The former is greater than the latter.[42] Eccles explains this by the assumption that the ionic clouds which lie in the boundary region are of the nature of thin sheets lying parallel to the shadow boundary surface, so that they give greater transparency in that region in the north-south direction than in the east-west direction.

In evidence given before a Parliamentary Committee in 1913,[43] Mr. Marconi again stated the facts as regards transatlantic transmission as follows :—

"Although as a rule messages can be sent at all times of the day and night between Clifden (Ireland) and Glace Bay (Nova Scotia), there still exist periods of fairly regular occurrence during which the received signals are at a minimum. Thus, in the morning and evening, when in consequence of the difference of

41 See *The Marconigraph*, vol. ii. p. 310, 1912.

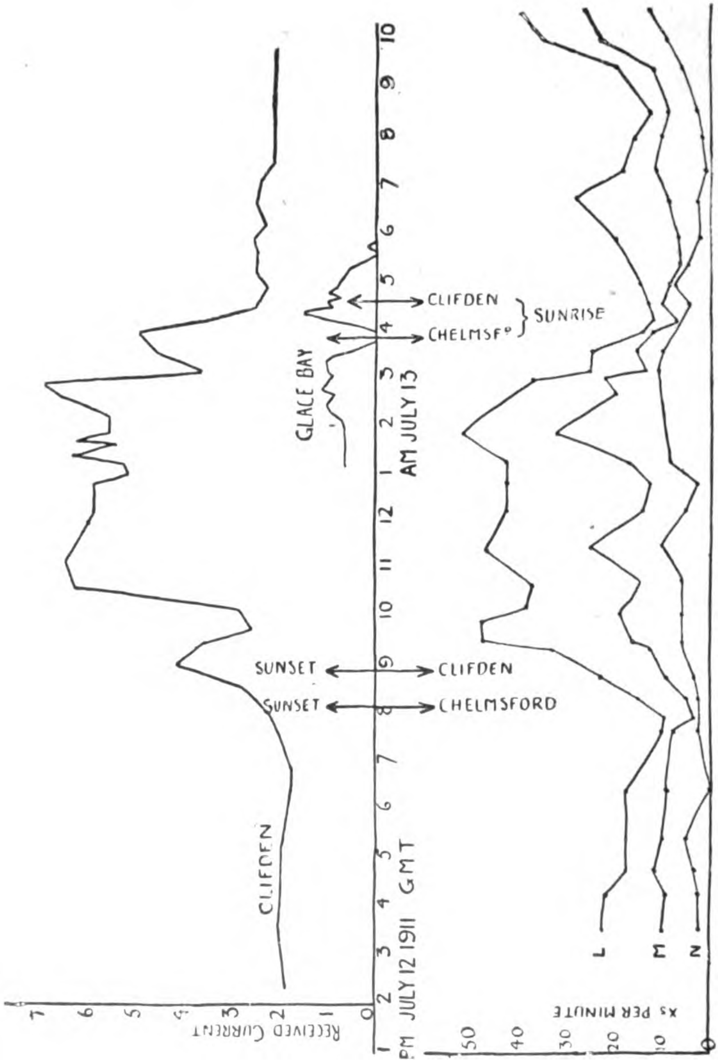
42 See Commendatore G. Marconi on Radiotelegraphy, *Proc. Royal Institution*, vol. xx, p. 202, 1911.

43 See *The Wireless World*, June 1913, vol. 1, p. 164, 1913.

longitude daylight or darkness extends only a part of the way across the ocean the signals are at their weakest.

"These variations seem to be less in a north-southerly direction than in an east-westerly one.

"The strength of the received waves remains as a rule steady during the



[By permission of the Wireless Press, Ltd.

FIG. 16.—Curves delineating Observations made at Chelmsford by Messrs. Round and Tremellen on the Strength of Signals received from Clifden (Ireland) and Glace Bay (Nova Scotia) Marconi Stations; and on the Strength of Atmospheric Strays at Chelmsford at the same time. The upper curve denotes the variation of signal strength and the lower curve that of the atmospheric strays during the same day. It will be seen that approximately the two curves vary in the same manner.

daytime. Shortly after sunset at Clifden they become gradually weaker. About two hours later they are at their weakest. They then begin to strengthen again and reach a high maximum about the time of sunset at Glace Bay They then return gradually to normal strength, but are variable through the night. Shortly before sunrise at Clifden the signals begin to strengthen again steadily and reach

another high maximum shortly after sunrise at Clifden. The received energy decreases again until it reaches a very marked minimum a short time before sunrise at Glace Bay. After that the signals come back to normal day strength."

In addition to this diurnal variation Mr. Marconi has given curves showing the monthly variation throughout the year (see Fig. 17) in the mean value of the transatlantic signals. Furthermore superimposed on the diurnal and monthly changes there are irregular variations in signal strength at corresponding times during the day.

If observations are taken of the signal strength received from any station with constant sending current and recorded at the same time of the day, great differences will be found in signal strength from day to day.

Thus daily observations began to be made by the author in London during

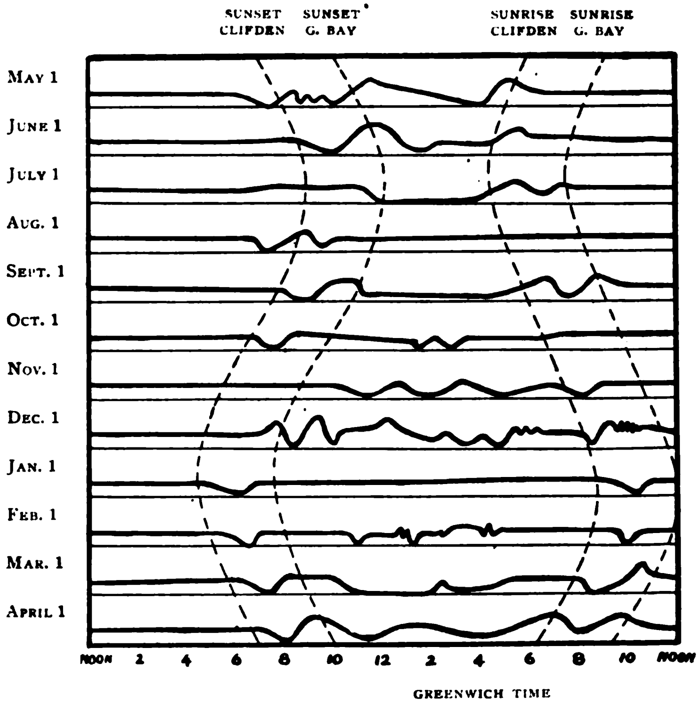


FIG. 17.—Mean Monthly Variation in Signal Strength of Signals exchanged between the Transatlantic Marconi Stations at Clifden (Ireland) and Cape Breton (Nova Scotia) (Marconi).

July 1914, of the strength of signals received from the Eiffel Tower Station, Paris, at 11 A.M. each day. These observations were unfortunately interrupted by the European War in August 1914.

As far as they went they showed that there was a curious and marked fall in signal strength on certain days, and rise in others as represented by the curve in Fig. 18. Prof. W. H. Marchant had made similar observations at Liverpool before the author, but for details of his work we must refer the reader to his interesting paper on "Conditions affecting the Variations in Strength of Wireless Signals," *Journal of the Institution of Electrical Engineers*, 1915, vol. 53, p. 329.

In addition any such exceptional changes in solar illumination as are caused by solar eclipses have been found to affect the strength of radiotelegraphic signals.

We might anticipate that when the lunar shadow sweeps through the earth's

illuminated atmosphere some of the phenomena that attend the daily passage of the earth's shadow are repeated; particularly those that occur on the boundary surface. For some remarks on this matter the reader may be referred to the contribution made by Dr. Eccles to the discussion at Dundee in 1912, which was reprinted in *The Electrician* for September 27, 1912, under the title, "On Certain Phenomena Accompanying the Propagation of Electric Waves over the Surface of the Globe," to which we are indebted for some of the information in this section.

To conclude, we may say that as regards the theory of long distance radiotelegraphy and its diurnal and secular variations none of the theories so far proposed seem to cover the facts so well as this theory of ionic refraction and reflection. The latest contribution made to the theory of diffraction of long electric waves round the earth by Dr. G. N. Watson (see *Proc. Roy. Soc. Lond.*, 1918, vol. 95, p. 83) shows that at the antipodes of the radiator there would be a point of zero magnetic force. Meanwhile, Marconi's Wireless Telegraph Company have announced that from their Carnarvon Station in North Wales messages have been transmitted directly to Australia, and that the signal strengths are very good. This almost amounts to a proof that diffraction alone will not account for such long distance radiotelegraphy,

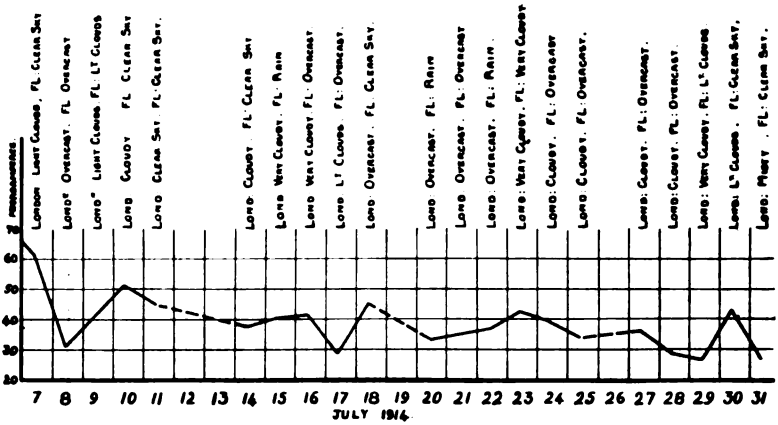


FIG. 18.—Curve representing the Variations in Signal Strength from day to day, observed at 11 A.M. for Signals from the Eiffel Tower Station, Paris, measured at University College, London. The weather at London and at Paris (FL) is indicated opposite each day, and the ordinates of the curve are proportional to the current in the receiving antenna at London reckoned in microamperes.

but that we must call to aid some hypotheses of atmospheric ionization to account both for normal transmission and for the occasional and diurnal variations.

Much, however, remains to be done before the whole of the phenomena are accounted for. This requires the assistance of various scientific workers. Hence, in opening a discussion on "The Scientific Theory and Outstanding Problems of Wireless Telegraphy," which took place at a joint meeting of sections A and G at the British Association meeting at Dundee in September 1912, the author concluded by suggesting the formation of a British Association Committee to guide and formulate research on some of these unsolved problems.

The suggestion was taken up, and a committee was formed containing the names of eminent physicists, mathematicians, radiotelegraphists and meteorologists, and plans were at once arranged for systematic observations on atmospheric stray waves. Much time, however, must elapse before the results of these observations can be reduced and extracted.

8. The Measurement of Signal Strength and Antenna Currents.—In order that quantitative measurements leading to real knowledge of the causes of

variation of signal strength may be made it is necessary to be able to measure accurately the feeble alternating electric currents set up in the antenna.

These currents vary from about 5 to 30 or 40 microamperes or more for good decipherable signals. In laboratories or stations where steady supports can be obtained, instruments are available such as the Einthoven galvanometer (see Chap. VII. § 22), which enable this to be accurately done. On board ship, where no such steadiness can be obtained, more imperfect methods have to be used, such as the shunted telephone where the observer judges the strength of the signals by gradually reducing the resistance of a shunt placed across the terminals of the telephone until the signal sounds just cease to be heard.

The theory of this method is as follows. Let T be the impedance of the telephone coils and S the resistance of the non-inductive shunt which just quenches the signal sounds. Let I_0 be the current through the telephone corresponding to this just audible sound. Let I be the current in the circuit before division between the telephone and shunt. Then we have

$$\frac{I-I_0}{I_0}=\frac{T}{S} \text{ or } I=I_0\frac{T+S}{S}$$

Since I_0 may be considered as constant for the same observer, we have as the relative measure of the signal strength the ratio $(T+S)/S$. The arrangement can be calibrated by placing the shunted telephone in series with a previously calibrated crystal and a very sensitive galvanometer. The shunted telephone method is at best a rather rough method of measurement.

In using an Einthoven or any other form of direct current galvanometer for measuring oscillations it has to be placed in series with a rectifying contact or crystal detector, or else an oscillation valve.

The problem which then presents itself is to calibrate the measuring instrument so as to interpret the deflections in terms of microamperes.

The following method suggested and used by the author is convenient and easily applied. It is necessary first to provide two coils which may be flat spirals mounted in a stand so that they can be placed at various known distances from each other with their axes in line with each other.

The first step is to measure their mutual inductance by the Carey Foster bridge at different distances and to delineate this by a curve.

If, then, a damped or undamped oscillation be sent through the primary coil the R.M.S. value of it can be measured by a hot-wire ammeter placed in that circuit. Also we can measure by a sensitive hot-wire ammeter the current in the secondary coil when the coils are placed close together. If they are moved far apart, then, although the secondary current is too weak to be directly measured on the hot-wire ammeter, we can calculate its value from the known primary current strength and the decrease in mutual inductance resulting from moving the coils a known distance apart.

If, then, we have the usual coupled receiving circuit and an Einthoven galvanometer in series with a crystal detector shunted across the variable condenser we can obtain deflections of this galvanometer when signal waves fall on the receiving aerial. We have then to provide a means of creating in the primary coil of the two mutual inductance coils trains of oscillations of the same oscillation and group frequency as those of the received radiotelegraphic waves to be measured. This is done by means of a buzzer, which is a form of rotating contact maker driven by an electric motor, the number of contacts per second being adjustable. A battery charges a condenser C_3 of variable capacity, and when the buzzer makes contact this condenser discharges with oscillations of known and adjustable frequency through the primary coil L_4 (see Fig. 19) and creates trains of oscillation.

These induce feebler secondary oscillations in the secondary coil L_3 which flow through the primary coil L_1 of the receiving jigger, and then again induce other oscillations in the receiving circuit L_2 which includes the Einthoven galvanometer. The measurement then consists in creating in the primary coil of the jigger L_1 oscillations of known R.M.S. value which create the same deflection of the Einthoven as do the wireless signals. We can thus deduce from observations

made with the galvanometer G (see Fig. 19), placed in the circuit which includes the coil L_4, the true value of the current in the coil L_1 which gives this equal deflection, and therefore it must be identical with the antenna current. The process is, therefore, a substitutory one, and if the buzzer and buzzer circuit is adjusted to equal oscillation and group frequency it gives the value of the antenna current. The scheme of the connections and apparatus will be understood from Fig. 19. It is necessary to know the spark or group frequency of the station sending the message signals and also the wave-length of the same.

Working in this manner an arrangement can be made for measuring the mean strength of the antenna current for certain signals sent out from any station. In experiments carried out by the author in July 1914, the antenna current of the University College receiving station was thus measured for signals sent out at 11 A.M. from the Eiffel Tower, Paris, and the ordinates of the irregular line in Fig. 19 show the relative strength.[44] It will be seen that they vary greatly from day to day. The same kind of variation in signal strength was found by Prof. E. W. Marchant in experiments the year previously.[45]

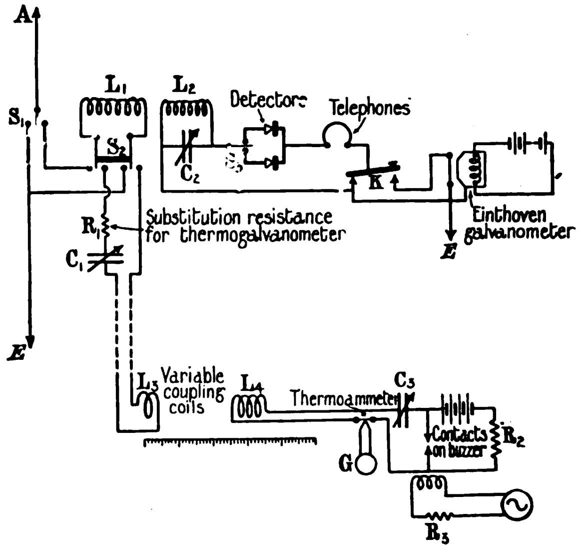


FIG. 19.—Arrangement of Circuits for measuring with an Einthoven Galvanometer the Receiving Antenna Current producing Signals of a certain strength (Fleming). A, antenna; G, thermal galvanometer.

With regard to the other methods of measuring the feeble receiving antenna currents it may be mentioned that M. Henri Abraham, in Paris, has constructed some very sensitive moving coil galvanometers with very narrow small coils of 400-500 turns of extremely fine wire; these coils being in a very strong magnetic field. With these he has been able to obtain quite large deflections with 1 micro-ampere, and has been able to measure at Arlington, near Washington, U.S.A., the receiving antenna current created by the Eiffel Tower, Paris, sending station.

In comparison with the above galvanometer methods the method with the shunted telephone is relatively very much less accurate. In fact the latter should be applied with considerable caution, and especially in comparisons made on different days or at different times during the same day. It depends upon the

[44] See remarks by the author in a discussion on the strength of wireless signals, *Journal of the Institution of Electrical Engineers*, London, vol. 53, p. 345, 1915.
[45] See E. W. Marchant, *Journ. Inst. Elec. Eng.*, vol. 53, p. 329, 1915.

reduction of a telephonic sound to just audible frequency, and is therefore affected by every change in the observer's hearing and by slight noises in the vicinity. For some additional information on the measurement of signal intensity the reader may refer to an article on this subject by Mr. J. L. Hogan, Jr., in *The Marconi Year Book of Wireless Telegraphy* for 1916, p. 662.

Some further references to measurements on the variation of signal strength are as follows:—

A. H. Taylor, "Radiotransmission and Weather," *The Electrician*, vol. 73, p. 45, June 19, 1914; also *Electrical World*, of New York, vol. 62, p. 425.

H. Mosler, *Elek. Zeitschrift*, August 1913, describes measurements made with signals from Norddeich and a station 420 kms. east of it.

The ratio of day to night strength was greatest in spring and autumn and least in June.

It has also been known for a long time that over large land areas the signal strength is greatest in winter and least in summer for signals sent with constant sending antenna current. Dr. L. W. Austin has given (see *Proc. Institute of Radio-Engineers*, U.S.A., vol. iii., June 1915) a curve showing the annual variation observed at the Bureau of Standards, Washington, for signals coming from radio stations at Philadelphia and Norfolk navy yards. The waves were 1000 metres in length and the spark frequency 1000, and sending antenna currents 10 amperes. The distances were 185 and 235 kms. respectively. All other conditions remaining the same, it was found that the receiving antenna currents were nearly twice as great in December as in July. The transmission was overland all the way.

CHAPTER X

RADIOTELEPHONY

1. The Problem of Radiotelephony.—Before the invention of the methods of radiotelephony described in this chapter, attempts had been made with some degree of success to transmit articulate speech over moderate distances without the aid of a connecting wire.

In addition to methods depending upon the induction of currents between distant circuits and their conduction through the earth, a method to which the attention of Sir William Preece and Sir John Gavey and others was at one time directed, another method was worked out based upon a peculiar property of selenium of varying its resistance under the action of light, and of the continuous-current electric arc of varying the intensity of its light when a periodic current is superimposed upon the continuous one operating the arc. This last method was the subject of much laborious work by E. Ruhmer, of Berlin.[1]

The above methods have, however, a very limited field of application. The inductive method labours under the disadvantage that the mutual induction between two circuits decreases very rapidly with the distances, varying almost inversely as the cube of the distances, and the method depending upon the use of an arc lamp is interfered with by daylight and by fog. We shall confine our attention, therefore, in this chapter to the details of the method employing electromagnetic waves which gives the greatest promise of ultimate utility, and is now generally called *radiotelephony*.

Radiotelephony consists, therefore, in the transmission to a distance of articulate speech through space without wires by means of electromagnetic waves, as distinguished from radiotelegraphy, which is the transmission of intelligence by means of arbitrary signs, whether audible or visible.

As soon as radiotelegraphy, as conducted by the methods already described in previous chapters, had made a certain progress, inventors naturally had their minds turned to the problem of the transmission of articulate speech by the same means. It soon became clear, however, that the attainment of any practical success was bound up with the invention of a transmitter for producing undamped electric radiation and upon a receiver which should be quantitative in action; that is to say, one not merely set in operation by oscillations like a coherer, but producing an effect proportional to the amplitude of the waves incident on the receiving antenna. The oscillation detector to be used in connection with radiotelephony must, therefore, be of such a character that it is capable of varying the current through a telephonic receiver in exact correspondence with the variations of air pressure due to the speaking voice taking place in proximity to the particular telephonic transmitter employed at the sending station.

In electric telephony conducted with wires, the apparatus usually employed consists of a transmitter of the microphone type, and a receiver of the Bell or magnetic type. For instance, in the simplest form of short distance transmitter and receiver, a microphone transmitter consists of a metal diaphragm which is set in vibration by the variations of the air pressure taking place in proximity to the mouth of the speaker uttering near it articulate words. Connected to the diaphragm is some mechanism by which an imperfect contact between carbon surfaces is altered by pressure. In the ordinary type of granular carbon micro-

[1] For a detailed account of this work the reader is referred to Herr Ruhmer's book, "Wireless Telephony." English translation by Dr. Erskine-Murray.

phone the movements of the diaphragm due to the vibrations of the air produced by the voice are made to press more or less together small fragments of graphitic carbon contained in a shallow chamber, and so alter the electric conductivity of the mass. This variable carbon resistance M (see Fig. 1) is placed in series with a few voltaic cells, B, and with the primary circuit of a small induction coil, T. One end of the secondary circuit of the induction coil is connected to one of the line wires, L, and the other to the earth or to a duplicate line wire if a complete metallic circuit, is employed. At the receiving end the current in the line passes through magnetizing coils which are placed on the polar extremities of a permanent magnet, and close to these poles is held a thin flexible sheet-iron diaphragm. When the variable current passes through the magnetizing coils, the diaphragm is more or less drawn in and its vibrations, therefore, reproduce, and are similar to, the variations of the current in the line wire. If, then, an articulate sound is created near the diaphragm of the transmitter, there will be variations of air pressure which may be represented by the ordinates of a periodic curve. In the case of a purely musical sound this curve approximates in form to a simple sine curve, but for any sound such as a vowel sound the form of the curve will be complicated and periodic if the vowel sound is continued. In Fig. 2 are shown curves taken with a Duddell oscillograph which represent the variation of current through a telephonic circuit when the various sounds are being made against the diaphragm of the transmitter. It will be seen that these curves are periodic and yet very irregular. By Fourier's theorem these complex curves can be resolved into the sum of a number of simple periodic curves or sine curves of different

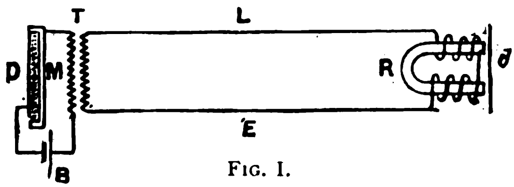
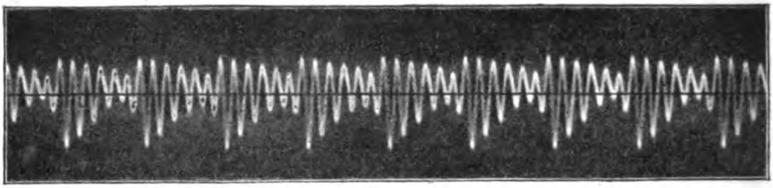


FIG. I.

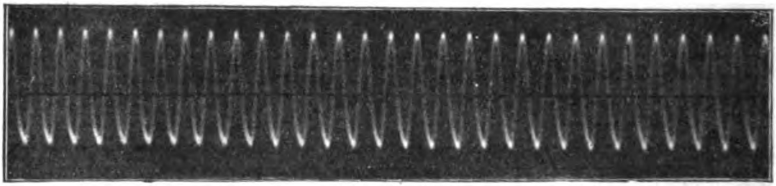
frequency and amplitude, these component sine curves differing from one another in phase. These curves, therefore, confirm von Helmholtz's celebrated synthesis of the vowel sounds. In the case of articulate sounds, variations in air pressure are non-repetitive, but they can, nevertheless, be represented by the ordinates of a single valued curve. Thus, for instance, in speaking to a phonograph the voice creates variations of air pressure in front of the speaking diaphragm and at the back of this diaphragm, or connected with it by a system of levers, is a delicate cutting tool which carves out upon the surface of the revolving plastic cylinder or disc which forms the receiving surface, a little channel or groove the bottom of which is irregular, the depth of this groove corresponding from instant to instant to the variations of air pressure produced against the diaphragm by the vocal organ. If, therefore, a section could be made of this groove and the outline at the bottom enlarged, it would present the appearance of a very irregular non-repetitive curve, each change in the ordinate of which, however, corresponds to a change in air pressure of the air in front of the diaphragm against which speech is being uttered, and which, therefore, has a vocal signification.

In the curves shown in Fig. 2, certain vowel sounds produced in conjunction with a consonantal sound are exhibited.

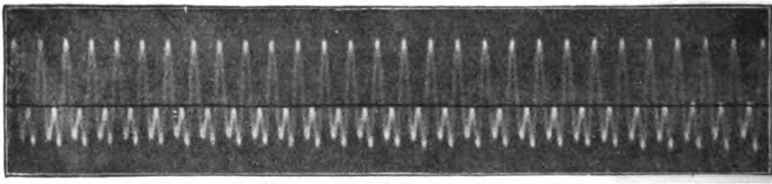
The problem of telephony is, therefore, to cause some other diaphragm at a distance to be moved from instant to instant in a similar manner to that of the diaphragm against which speech is being made. This receiving diaphragm will then reproduce at the distant end the same variations of air pressure as those which actuated the transmitting diaphragm, and the human ear placed in proximity to the receiver will, therefore, hear the speech being made at the distant place. In telephony with wires the movements of the transmitting diaphragm are made to translate themselves into corresponding variations in the strength of an electric



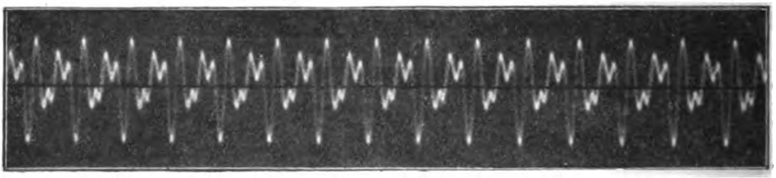
Vowel ā as in Ma.



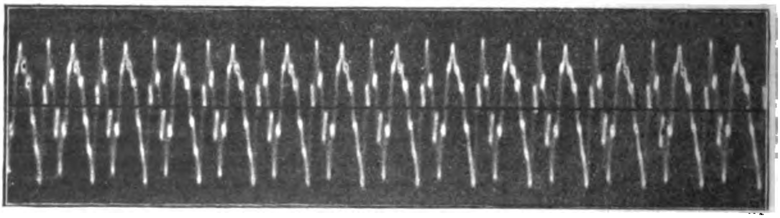
Simple Form of ōō Sound as in Coo.



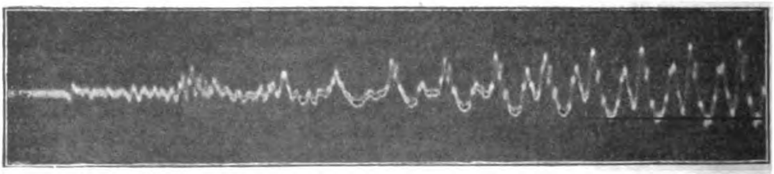
Complex Form of ōō Sound as in Coo.



Vowel ō as in Ho.



Vowel ē as in Me.



K and First Part of e as in Key

[*By permission from "The Proceedings of the Royal Institution of Great Britain,"* 1906.

FIG. 2.—Oscillograms or Wave Forms of Various Sounds taken with the Oscillograph by Mr. W. Duddell, F.R.S.

current in the connecting wire by means of the variation in resistance which takes place when carbon surfaces are more or less pressed together, and the re-translation of this variable electric current into the movement of a receiving diaphragm is made to take place by means of the variations in the polar strength of a magnet, which result when an electric current of varying strength circulates round these magnetic poles.

To achieve radiotelephony, we remove the interconnecting wire and substitute for it a train of electromagnetic waves passing through space. These waves must be either undamped trains or else trains of closely sequent waves with the train or group frequency greater than about 40,000 per second. We have then to modulate the amplitude or wave-length by the speaking voice at the transmitting end. At the receiving end we must cause these waves of variable amplitude to actuate a mechanism which shall cause them to set in vibration a receiving diaphragm, so that its displacements create aerial vibrations which reproduce in wave form those which are being made against the diaphragm of the transmitting instrument.

We have therefore to consider, first, the arrangements for generating the required electric waves in the transmitter; secondly, the means for modulating the amplitude of these waves in accordance with the wave form of articulate speech; and, thirdly, the arrangements for receiving these electric waves of variable amplitude, and causing them to affect a speaking telephone.

Sending and receiving antennæ at the two stations are requisite, as in connection with radiotelegraphy, to radiate and to absorb the electromagnetic waves. Owing to the divergence of this wave energy, and to the small fraction of the emitted power which is captured by the receiving antenna, it is necessary to modulate or control much larger currents in radiotelephony than it is in connection with telephony with wires. The difficulties still outstanding in radiotelephony are largely those of modulating large high frequency currents by means of some form of speaking microphone as described in the following sections.

2. Methods for Generating Undamped High Frequency Oscillations for Radiotelephony.—For the accomplishment of radiotelephony it is most desirable, though not absolutely necessary, that the generator should create steady undamped oscillations in the sending antenna, and the amplitude or wave-length of these waves is then varied in accordance with the wave form of the speech sound.

We have already described in Chap. I. various methods for the production of such waves by means of high frequency alternators or an electric arc. Hence this information need not be repeated.

An essential condition of success in the transmission of articulate speech by electromagnetic waves is that there should be no interruptions in the uniform flow of the oscillations, at least not below such a frequency as forms the upper limit for the creation of the sensation of sound. If regular vibrations are set up in the air, these are appreciated as sound by the normal ear, if they lie in frequency between about 40 and 20,000 per second. Human ears vary, however, a great deal in the value of the highest frequency which can be heard as sounds. As regards musical sounds, the highest note employed in music does not generally exceed in frequency 4000 or 5000. If, then, intermittent trains of damped waves were employed, even if the frequency of the trains were as much as 4000 or 5000, they would affect the oscillation detector at the receiving station and produce in any telephone connected with it a musical sound of high pitch which would drown out the variations of lesser frequency constituting articulate speech. Hence, if an alternator producing alternating current having a frequency less than 20,000 were connected to a radiating antenna, it is probable that most persons would hear a sound in a telephone connected to an electrolytic oscillation detector in a corresponding receiving antenna. If, however, the frequency were forced above 20,000 most persons would probably hear no sound. We may say, therefore, that to be of practical use in radio-telephony, a high frequency alternator should give a current having a frequency of not less than 30,000, and preferably of 40,000 or 50,000. We have already seen in Chap. I. that to obtain such frequencies by a simple single phase alternator means rather a high speed of revolution. Hence it is not generally possible to employ a wound armature or rotor. The machine must be an inductor alternator to secure good balancing in the rotating part.

There is then some difficulty in obtaining an alternating current of pure sine-wave form, and hence more difficulty in raising voltage by resonance. Although early experiments were made by R. A. Fessenden with small alternators of Mordey type driven at very high speeds by means of a De Laval steam turbine (see Fig. 8, Chap. I.), the mechanical difficulties involved were very great. These have been partly overcome in more modern high-speed alternators, such as that of Alexanderson, and the invention of the Goldschmidt frequency-raising alternator (see Chap. I.) provided a new means of obtaining the necessary high frequency currents. On the other hand, the invention of methods by which static transformers can be used to increase frequency, as explained in Chap. I., renders it possible to employ alternators of moderate frequency, say, 10,000, which are not difficult to construct, and then to raise this by static transformers by the method of Joly to 20,000 or 40,000. A quadrupling of the frequency by this means is in practical use, and has been found to be a convenient method for generating the required frequency.

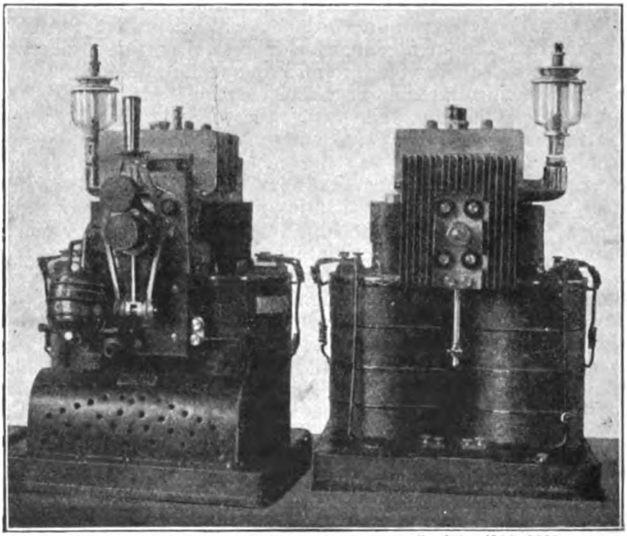


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FIG. 3.—Poulsen Arc Transmitter for Radiotelephony.

In pioneer experiments, owing to the less initial outlay, experimentalists have generally employed for radiotelegraphy some form of arc generator.

Thus V. Poulsen conducted radiotelegraphy with a carbon-copper electric arc, working in an atmosphere of hydrogen and in a strong magnetic field as the generator. His apparatus has been already described in § 14 of Chap. I. To avoid the necessity of water cooling and the provision of hydrogen or coal-gas for the arc chamber the apparatus was modified as follows: The metal box in which the arc burns was constructed with radiator flanges so as to employ air cooling. In place of gas, methylated spirit or some volatile hydrocarbon liquid was introduced drop by drop into the box through an arrangement like a sight-feed lubricator (see Fig. 3). By the employment of several arcs in series, it is possible to obtain high voltage and small currents.

Instead of placing the arc in a strong magnetic field and enclosing it in an atmosphere of hydrogen or hydrocarbon, it has been found that fair results can be obtained by the use of a number of arcs in series burning in air, or at any rate in an atmosphere deprived of oxygen. It has already been explained in Chap. I.

§ 14, that the characteristic curve of a continuous current arc is steeper for small currents than for large ones. If a number of electric arcs taking a small current are joined in series, and a condenser circuit possessing inductance shunted over the whole number, oscillations will be created in this circuit. A possible arrangement is as follows:—

A copper tube has a concave bottom fixed to it, and this tube is filled with water to keep it cool. The tube forms the positive terminal of the arc. The negative terminal is formed by a solid carbon rod, C, and the arc is struck in the cavity formed by the recessed end of the copper tube T (see Fig. 4). Six or twelve such tubes may be arranged in a row or two rows, and a number of carbon rods attached to a lever with an adjustment such that each carbon rod can be moved up or down independently at pleasure, or the whole number moved together slightly downwards by means of a touch on a single lever (see Fig. 5). By this mechanism all the carbons can be put in contact with the concave copper anodes, and then by one movement all the arcs are struck together. The carbon rods and copper cylinders are connected up in series, so that a current passing through them all forms an arc between the carbon tips and the concave copper roof above it, and these arcs are controlled simultaneously. A very similar arrangement of the author's, in which a series of arcs are formed in oil vapour, has been described in Chap. I. (see Fig. 81).

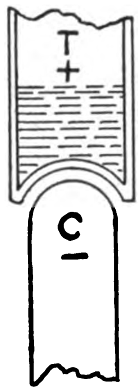

FIG. 4.

In place of a number of carbon-copper arcs in series, E. Ruhmer has found it possible to use a single arc produced by a high tension continuous current between aluminium wires. Two bobbins of square-sectioned aluminium wire are provided, and these wires are caused to travel slowly over two insulated pulleys in such fashion that the wires at one place are separated by a few millimetres. These wires are connected to a series of secondary cells giving a voltage of 2000 volts or upwards. Between these wires a high tension aluminium arc is formed. If this arc is shunted by a condenser and inductance, then in the latter circuit persistent or undamped electric oscillations are set up. These can be employed to create other oscillations by induction in a coupled antenna, and the antenna oscillation modulated by a microphone.

Several variations of the arc generator have been tried. Thus in France, Colin and Jeance conducted radiotelephony, using as arc electrodes thin carbon discs in an atmosphere of acetylene and hydrogen in certain proportions, which reduces the wear of the carbons to practically nothing. They also dispensed with the magnetic field used by Poulsen.

In the United States A. A. Jahnke has employed an electric arc with carbon and copper electrodes under liquid alcohol.

In Italy Vanni used a Moretti arc, in which an electric arc is formed between a copper electrode and a water surface (see Chap. I. § 14).

In Japan three inventors, Messrs. Torikata, Yokoyama, and Kitamura (T.Y.K. system), have devised a generator in which a continuous current of about 0·2 ampere is passed between electrodes of magnetite (oxide of iron) and brass. These electrodes are small flat surfaces of about 1 square cm. in area. The distance between them is regulated by an electromagnetic mechanism like an arc lamp. The voltage employed is 500. These electrodes are shunted by a circuit having a capacity of about 0·05 mfd. and also inductance in series. In this last circuit high frequency oscillations are created. The inventors consider that the discharge between the electrodes is more of the nature of a very rapid series of electric sparks than a true arc.

We can also employ as a generator of undamped electric oscillations the double anode thermionic amplifier or modified Fleming valve, which has been described as a receiver or detector in Chap. VI. § 15 (Fig. 45). In this appliance we have an exhausted glass bulb containing a metallic filament which can be rendered incandescent by a local battery. We have also two cold metal anodes, one a grid and the other a metal plate. These are carried on separate insulated terminals sealed through the glass. The arrangement and connections when used as a

telegraphic receiver have been shown in Fig. 45, Chap. VI. It has, however, been
419 found that this thermionic detector can be used as a receiver for radiotelephony also, as follows:—

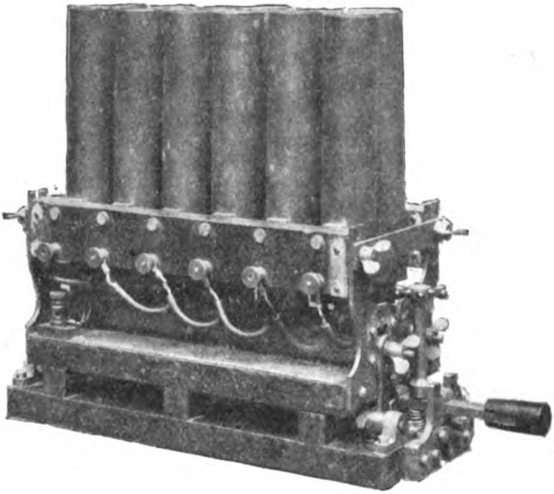

FIG. 5.—Multiple Arc Apparatus for the Production of Persistent Oscillations.

Let A (see Fig. 6) be an antenna having in series with it the primary coil P_1 of a transformer, and let the secondary coil S_1 have a condenser C in series with the

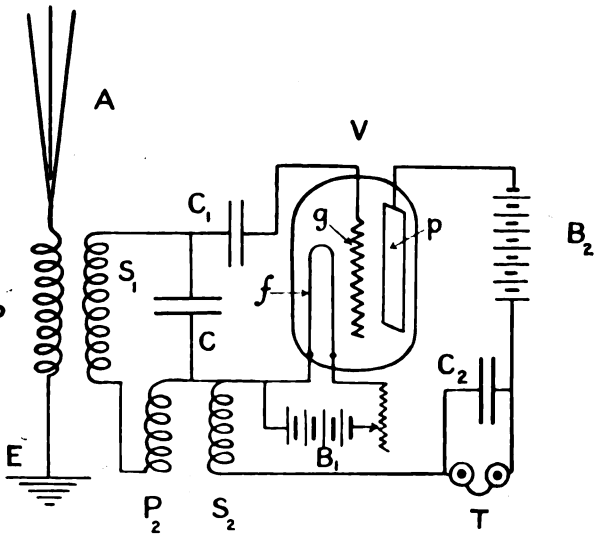

FIG. 6.—Thermionic Valve used as a receiver in Wireless Telephony.

primary coil P_2 of another transformer joined across the terminals. Let one terminal of C be connected through a condenser, C_1, called the grid condenser, with

the grid g of a double anode oscillation valve V. Again, let the plate p in the bulb be connected through a high potential battery, B_2, a condenser, C_2, and the secondary coil S_2 of the transformer P_2S_2 with the negative terminal of the

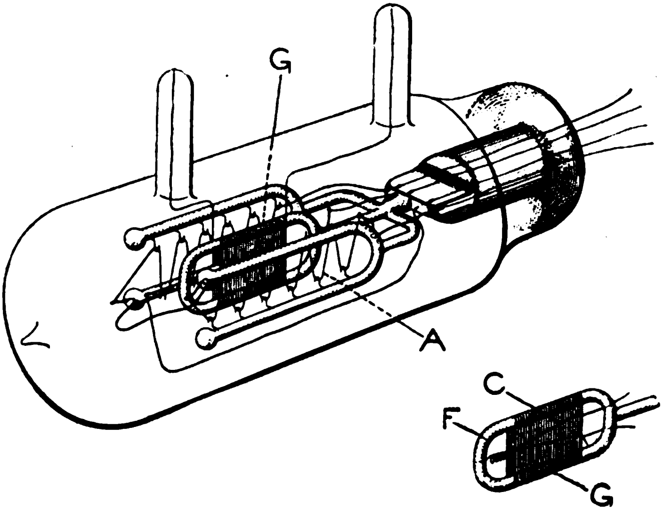

FIG. 7.—Small Pliotron or Double Anode Thermionic Amplifier, suitable for reception of signals.

incandescent filament. Suppose then that electric waves fall on the antenna and create oscillations in the primary coil P_1 and hence in the coupled secondary circuit S_1CP_2. Alternations of potential, therefore, occur on the filament and grid, and in consequence of the thermionic emission from the filament the condenser C_1

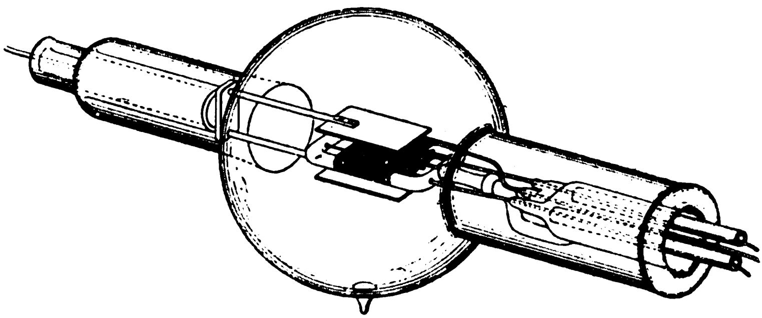

FIG. 8.—Large Pliotron or Double Anode Thermionic Generator or Controller of Oscillations.

becomes negatively charged on its plate next to the grid. The variation of negative potential of the grid then causes a corresponding variation in the stream of electrons from the filament which falls upon the plate, and therefore a variation in the current which the boosting battery B_2 is passing through the coil S_2 and this vacuous space. Hence alternating currents exist in the two circuits P_2 and S_2, which may be called the grid and plate current respectively. By proper tuning of

these circuits and coupling of the two coils P_2 and S_2, but not too closely, these currents may be made to exalt each other. Variations in the amplitude of the antenna current, therefore, cause variations of grid potential and these again amplified variations of plate current or telephone current in the telephone T.

In order that such a double anode thermionic detector may have a reasonable life it is essential that the bulb should be evacuated very completely and the metal filaments and plates entirely deprived of all occluded air. In the form of thermionic amplifier which Dr. Langmuir has called a Pliotron, the filament, grid, and anode are made of tungsten wire, so that a current can be passed through them during exhaustion of the bulb and assist to get rid of the occluded air. If this is not done the bombardment of the positive ions left in the bulb will destroy the filament. If the bulb is highly evacuated the phenomena are determined solely by the electronic emission from the incandescent filament. Figs. 7 and 8 show a

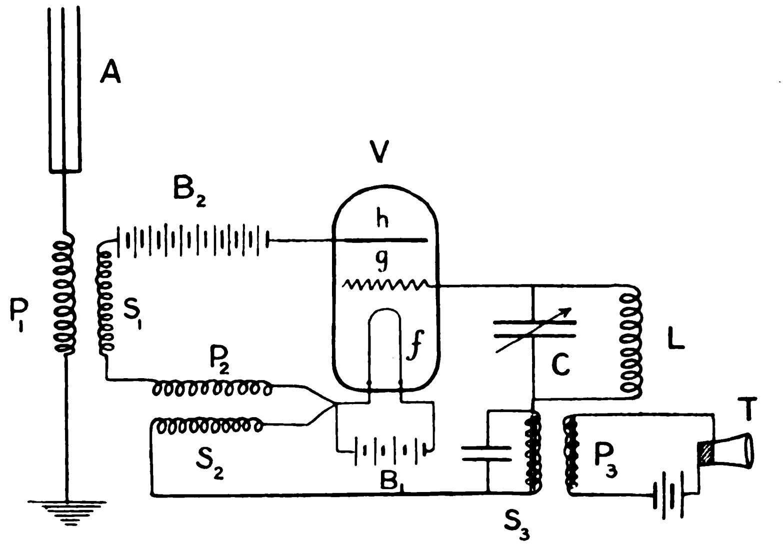

FIG. 9.—Connections of a Thermionic Oscillator creating undamped oscillations in an antenna which are modulated in amplitude by a Microphone Transmitter for the production of Wireless Telephony.

view of this Pliotron in two forms, small and large, suitable for the reception of signals and for the control of 1 kw. of power for radio-telephonic transmission.

An arrangement of circuits as shown in Fig. 6 was employed in 1914 in certain experiments on radiotelephony by Senatore Marconi and Mr. Round. The connections of the transmitter are as shown in Fig. 9. In this diagram A is the antenna and V is the double anode thermionic amplifier which may have the same construction as the large Pliotron shown in Fig. 8, or may resemble a large Fleming oscillation valve as shown in Fig. 10.

The filament *f* (see Fig. 9) is rendered incandescent by a battery, B_1, and the grid *g* is connected through an oscillation circuit L, C, and the transformer coils S_2, S_3 with the filament. The coil S_3 couples this circuit with a microphone, T, and the coil S_2 couples it to a circuit terminating on the anode plate *h*. This last circuit contains a battery, B_2, and is coupled to the antenna A by a transformer, P_1S_1.

The effect of this is to establish persistent oscillations both in the grid plate and antenna circuits which are modulated as to amplitude by speaking to the microphone T.

The reaction principle here employed to create oscillations may be illustrated by the behaviour of an ordinary telephone receiver and microphone transmitter when near each other. Suppose we pass through an ordinary carbon microphone-telephone transmitter the current from a small voltaic battery and let this current circulate also through the coils of a Bell magneto telephone receiver, then when the receiver diaphragm is placed near to the microphone diaphragm, the arrangement will usually emit a shrill continuous sound, or if it does not start at once a small tap on the microphone diaphragm will make it. The explanation is as follows: Feeble sound waves due to the voice or other causes fall on the transmitter diaphragm and cause vibrations in it, and hence cause also variations in the current passing through the receiver. The receiver, therefore, emits a sound, and these waves keep up and augment the vibrations of the transmitter diaphragm. Hence the vibrations of the two diaphragms sustain each other. A corresponding electrical oscillation system may be made to create sustained electrical oscillations by the use of a thermionic amplifier with grid and plate in the bulb, as above explained.

The arrangement devised by Mr. H. J. Round for the Marconi Company is as follows:—

The glow lamp with grid and plate has one oscillatory circuit, L, C, comprising an inductance coil and a condenser joined to the incandescent filament and to the inner grid (see Fig. 9), and a second oscillatory circuit, comprising an inductance coil and battery B_2 joined between the filament and the outer plate. The two inductance coils P_2, S_2 are so placed relatively to each other that they have mutual inductance. These two circuits are closely coupled to each other. Then it is clear that if the filament in the bulb is rendered incandescent, it emits torrents of negative ions, and any variation of the potential of the grid varies the ionic current and therefore the potential of the outer plate or cylinder. But these potentials, owing to the inductive connection of the two oscillatory circuits, act and react on each other just as is the case with the telephone receiver above mentioned. Accordingly we have set up in the two oscillatory circuits oscillations which sustain and augment each other, and can be made to induce other syntonic oscillations in a properly tuned antenna circuit coupled inductively to one of them. The external view of the complete apparatus is as shown in Fig. 10. Hence we have the following available methods for the production of the sustained oscillations required in radiotelegraphy.

(i) We may employ a high frequency alternator having a frequency of not less than 40,000, which is coupled inductively and loosely to the antenna, and creates in it the necessary oscillations; or, we may employ a Goldschmidt alternator, excited with moderate frequency, say 10,000, and raising frequency by three stages to 40,000; or, we may make use of an alternator of moderately high frequency, say 10,000, and raise this frequency by two tiers of Joly static transformers as explained in Chap. I. to a frequency of 40,000. This last method is quite practicable and is being employed at Nauen. It obviates to some extent the necessity for very high speeds in the rotating part of the alternator, and is on the whole less expensive than a Goldschmidt alternator with the necessary condensers. These methods are, however, only applicable in large power stations.

(ii) For smaller plants we can make use of a Poulsen electric arc or else a series of arcs burning in a hydrocarbon atmosphere, with or without magnetic fields across the arcs. This method is suitable for moderate distances and has been largely adopted for experimental work, but owing to the incessant adjustment of the arc required it is doubtful whether any such generator will prove itself permanently useful for practical radiotelephony.

(iii) The glow-lamp grid and plate generator arranged with two mutually sustaining and reacting oscillatory circuits. This last method is suitable for short or moderate distance radiotelephony, and especially so in the case of ship communications. It seems quite possible that some such method of oscillation generation will come into extensive commercial use as it is simple and easy to use, and requires only a supply of electricity at constant potential for the incandescent filament.

(iv) For still smaller apparatus and for ship purposes it is possible to substitute

for the train of perfectly undamped waves a very rapid series of quenched sparks, provided the spark frequency is sufficiently high to be above the limit of audition, that is not less than 20,000-30,000. Although this spark method has been success-

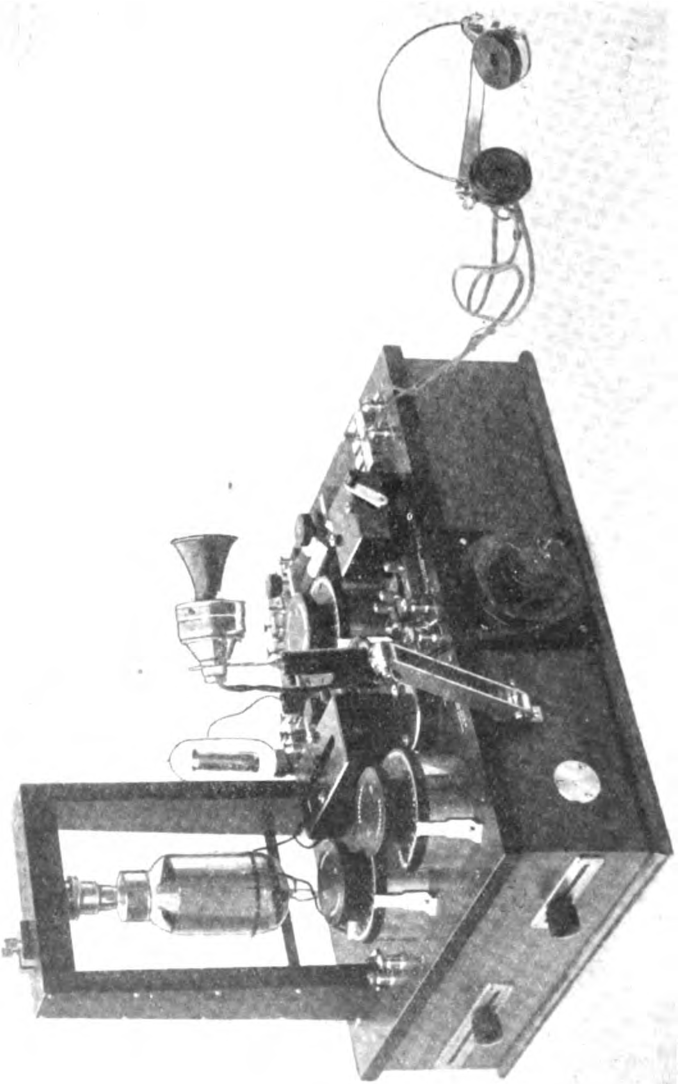

Fig. 10.—Transmitting and Receiving Apparatus for Wireless Telephony, based on the use of the Thermionic Amplifier (Marconi-Round System).

ful up to 50 miles or so, the speech quality or voice inflexion is very inferior to that obtained by using persistent oscillations.. The human ear has, however, a wonderful power of guessing the meaning of a sound from a mere suggestion of it. It is obvious that with the irregular trains of electric waves due to such a quenched spark it is impossible to modulate the amplitude so exactly to the wave form of

speech as in the case of perfectly undamped waves. Whichever method is adopted for creating these persistent oscillations in a circuit having capacity and inductance they have then to be transferred to an aerial wire or antenna tuned to the oscillating circuit so as to set up in this antenna persistent oscillations and radiate from it practically continuous electric waves. This is done by the usual inductive coupling, employing a two-coil or else single coil oscillation transformer.

To transmit speech we have then to modulate either the amplitude or else the wave-length of these continuous oscillations in accordance with the wave form of the spoken sound. As already explained, every vowel sound that can be uttered has a certain corresponding air wave or pressure-variation curve.

The consonantal sounds are represented by irregularities at the beginning or end of a series of more or less uniform waves of particular wave form.

3. Microphonic Control of Electric Oscillations.—The control or modulation of the amplitude or wave-length of the antenna oscillations is conducted by means of some form of microphone, generally of that type in which the speaking

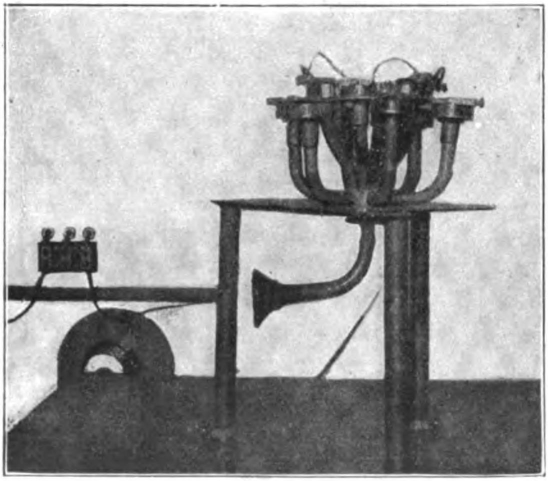

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FIG. 11.—Multiple Microphone.

voice makes a change in the resistance in some material. An ordinary telephone transmitter contains such a microphone, which consists of a shallow metal chamber closed by a flexible metal diaphragm which is insulated from the metal chamber, the space between the diaphragm and the solid back containing carbon granules which are more or less pressed together by the vibrations of the diaphragm. Hence when speech is made against the mouthpiece terminating on the diaphragm, the aerial vibrations set up similar vibrations in the diaphragm, and these movements, by pressing more or less the carbon granules, vary the resistance of the carbon included between the diaphragm and the solid back. It is found, however, that a single microphone transmitter cannot be operated satisfactorily with a current exceeding half an ampere, or at most one ampere, passing through it, nor if inserted in a circuit in which rupture of the circuit brings into operation large potential differences between the points at which rupture is made. Hence if the microphone is inserted in a condenser circuit, it must be put at a central or symmetrical point, that is, at a place which has a node of potential, or otherwise there is a tendency to produce sparking between the carbon contacts. The microphone must, therefore, be inserted in the earth wire of the antenna if it is used in the antenna circuit, or if it is used in any tertiary circuit it must be placed at a symmetrical point which has a node of potential.

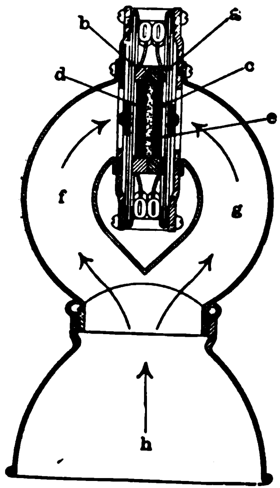

FIG. 12.—Collins Microphone for Radiotelephony.

FIG. 13.—Water-cooled Collins Microphone.

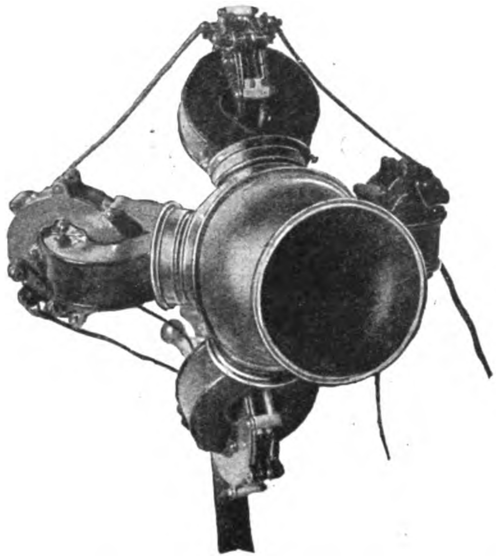

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FIG. 14.—Collins Multiplex Microphone for Radiotelegraphy.

If the current in the circuit is larger than about half an ampere, it is necessary to employ a number of microphones in parallel, and these may be arranged at the ends of tubes which are operated by a common mouthpiece (see Fig. 11) so that the diaphragms are all affected together, and the carbon resistances can then be connected in parallel, or otherwise they may be connected, if desired, in series.

In the arrangements adopted by Poulsen, this microphone transmitter, or variable resistance, is inserted either in the circuit of the antenna or in a circuit shunted across some of the turns in the coil in series with it. When speech is uttered against the microphones it varies the resistance of this microphone, and therefore alters the resistance of the antenna circuit slightly, and thus affects the current in the sending antenna. Words spoken to the mouthpiece, therefore, produce an effect upon the amplitude of the emitted electric waves, and these amplitudes are, so to speak, moulded into the form of speech ; that is to say, made to vary as the ordinates of a wave curve representing the changes of air pressure taking place near the mouthpiece of the transmitter. In some cases the microphone resistance may be inserted in the circuit of the electric arc itself and operate directly upon the continuous current affecting the arc. In this case the variation of the condenser current, and also of the amplitude of the waves radiated from the antenna, takes place in the same manner as the variations in the arc

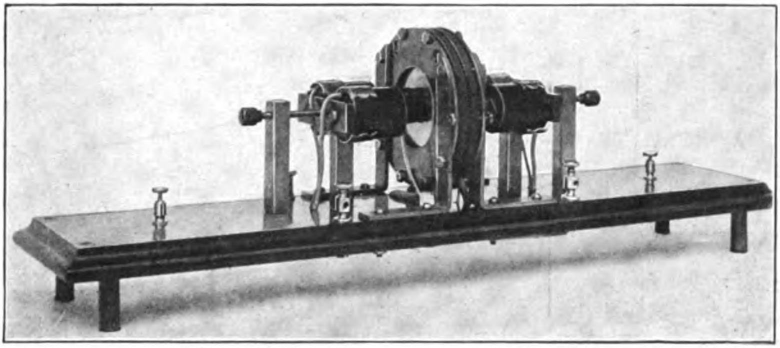

FIG. 15.—Double Diaphragm Magnet-actuated Carbon Microphone of Dubilier.

current produced by the changes in resistance of the microphone and of the action of the articulate sounds, or again, the microphone may be inserted as a shunt to the secondary circuit of the oscillation transformer connecting the antenna to the condenser circuit, so that the current into the antenna is more or less shunted to earth.

Finally, the microphone may be inserted in the earth connection of the antenna, so as to vary the current flowing into the antenna itself, and therefore the intensity of the radiated waves.

The moment we attempt to employ any of these methods to modulate any but small currents difficulties present themselves. If a number of microphones are placed in parallel then it is not easy to compel the current to divide itself equally between them. The one which takes most current gets hottest, and this again lowers its resistance and aggravates the trouble. Hence inventors have endeavoured to construct single microphones which shall carry current of 8 or 10 amperes which are water cooled.

One such microphone is shown in Figs. 12, 13, 14, invented by A. F. Collins (see *The Electrician*, vol. 65, p. 472, July 1, 1910). In this microphone the carbon granules are contained in a cell the opposite walls of which form the diaphragms set in vibration by the voice. The carbon granules are placed between two polished carbon surfaces attached to the backs of these diaphragms, and a mouthpiece leading into a bifurcated tube enables the variations of the air pressure to

be communicated simultaneously to the two diaphragms, which thus move in and out together. Fig. 12 shows a section, and Fig. 13 an outside view. A number of these microphones can be joined in parallel or in series, as in Fig. 14. The variable carbon resistance can also be water cooled.

A not very dissimilar double diaphragm microphone has been designed by W. Dubilier, which is shown in Fig. 15. In this case the double diaphragms are moved in and out together by two electromagnets which are energized by a small current controlled by an ordinary speaking carbon microphone. The main diaphragm can be water cooled. To overcome the heating of the carbon granules an ingenious plan has been devised by the Italian telephonists—Marzi, father and son, who construct a microphone in which the carbon particles are continually

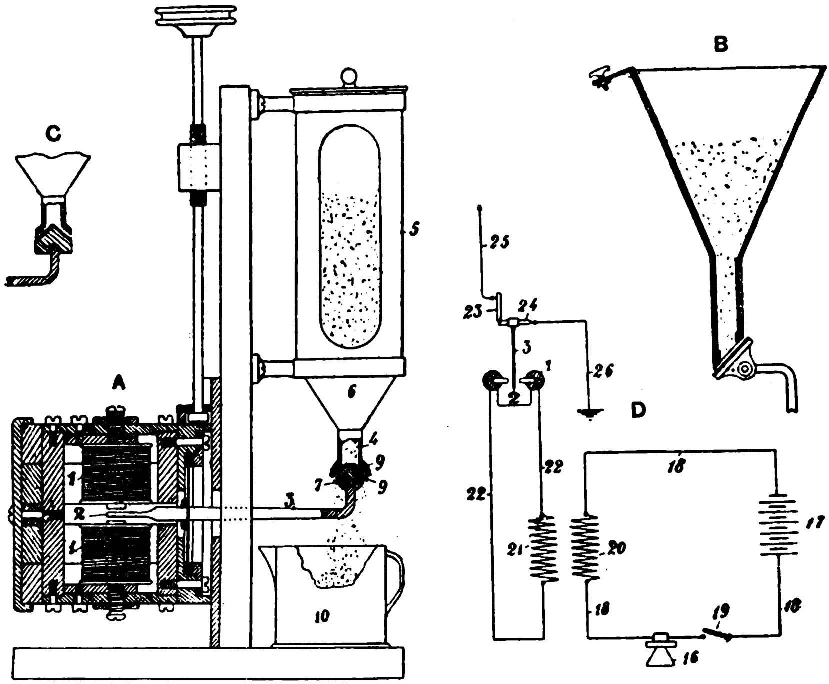

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FIG. 16.—Marzi Carbon Powder Microphone. The carbon powder is contained in a vessel, 5, and falls out of a spout, 9, which is partly closed by a ball moved by a lever, 3, which is actuated by an electromagnet, 1. This in turn is controlled by a current passing through an ordinary speaking microphone, the connections being as shown in the adjacent diagram.

renewed. In this manner the oxidation of the surfaces which deteriorates them is prevented.

In the Marzi microphone fine-carbon powder is contained in a vessel, and falls out of a spout at the bottom which is more or less closed by a ball or conical valve (see Fig. 16).

The carbon particles continually slip past the valve. This valve is moved to and fro so as to more or less compress the stream of carbon particles against the curved lip of the spout by a lever, which in turn is moved by the attraction of two electromagnets which are actuated by a current controlled by a small current which is modulated by a simple microphone.

The variable compression of the carbon particles falling out of the spout of the containing vessel and the vibrating valve causes a change of resistance in a circuit which includes that part of the carbon powder between the lip and the valve. The surface of the valve and the edge or flange of the spout of the carbon powder reservoir are platinized to prevent oxidation. The scheme of connections for the transmitter is shown in Fig. 17.

Other inventors, such as Majorana and Vanni, have constructed microphones by means of liquid jets, which are capable of modulating currents of considerable strength. Majorana's liquid microphone is made as follows :—

If a stream of water to which a little acid has been added flows from a suitably constructed jet under pressure it divides itself into drops which follow each other at practically constant intervals. This can be verified by throwing the shadow of the jet upon a screen and illuminating it by means of intermittent flashes of light. When these flashes of light come at intervals corresponding to the time required for one drop to move into the position of the drop next in front, the shadow of the jet will appear stationary upon the screen and be seen broken up into separate drops. If these drops fall on a level surface at right angles to their direction, a

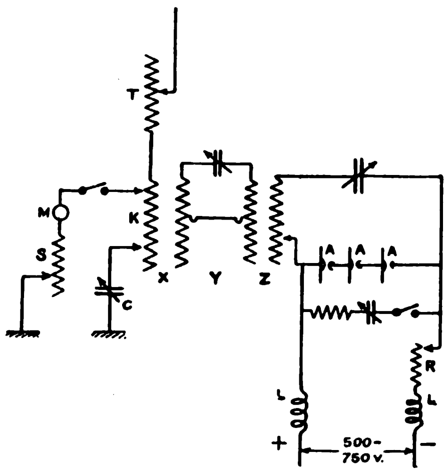

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FIG. 17.—Scheme of Connections showing the manner in which the Marzi Carbon Powder Microphone is connected to the radiotelephonic transmitter system. The arcs in series forming the generator are shown at A, A, A. The oscillations are created in the antenna circuit T, K, and the microphone M shunts a portion of this current to earth and so modulates the antenna current.

layer of liquid is formed upon that surface, the thickness depending upon the frequency with which the drops strike the surface. This fact can be applied to construct a microphone.

A mouthpiece of the usual shape and a diaphragm which can be set in vibration by the speaking voice are connected by means of a rigid attachment with a flexible tube out of which the liquid flows, so that vibrations imposed upon the diaphragm set up vibrations in the stream of liquid issuing from the jet (see Fig. 18). This liquid falls upon a collector, which consists of a circular pin or cylinder of platinum surrounded by a cylinder of insulating material, and this again by another outer cylinder of platinum. The jet, therefore, covers the end of this compound cylinder with a layer of conducting liquid which connects the two platinum electrodes, interposing a resistance depending upon its thickness (see Fig. 18). Hence if the liquid jet is made to vibrate, and the frequency of the drops into which it breaks altered, the thickness of the film of liquid is varied, and

its resistance is therefore changed in the same manner. A comparatively large current can be passed through this film of liquid from one platinum cylinder to the other owing to the fact that the liquid being continually removed cannot be overheated, and owing to the high specific resistance of the fluid, a considerable voltage can be applied to the film. The liquid film is then inserted between the antenna and the earth (see Fig. 18) in the arrangement already described, and the vibrations of the speaking voice operating on the jet of liquid are thus made to affect the thickness and therefore resistance of the liquid film, and thus to vary the intensity of the oscillations in the antenna and therefore the amplitude of the radiated waves.

Professor J. Vanni, of Rome, has also devised and used a very ingenious form of liquid microphone, constructed as follows :—

A jet of water made slightly conducting by the addition of salt or acid emerges from an ebonite mouthpiece and falls between two inclined metal plates A and B (see Fig. 19). One of these plates is fixed, and the other is attached to a diaphragm having a mouthpiece attached (see upper diagram in Fig. 19).

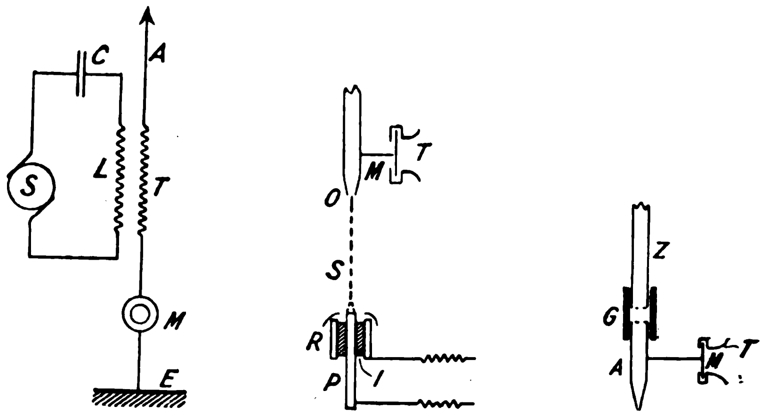

FIG. 18.—Details of Majorana's Liquid Microphone.

The liquid falls on one plate and bounces off on to the other, and connects the two conductively together. When vocal sounds are uttered to the mouthpiece the diaphragm vibrates and throws into vibration the liquid between the plates, and thus alters the length of the conducting liquid bridge between the plates. This liquid bridge is included in the circuit of the antenna near the earth. Hence any variation in the resistance of the liquid column or layer changes the resistance of the antenna, and hence the amplitude of the emitted persistent waves, without altering their wave-length. This variation in amplitude follows the wave form of the speech made, and therefore transmits it to the receiving antenna.

As this jet can convey a considerable current, such a liquid microphone is capable of dealing with large antenna currents. The liquid is circulated by means of a rotary pump, R. Instead of making the speaking diaphragm act directly on the plate, Prof. Vanni sometimes connects them electrically. Thus speech is made to an ordinary microphone transmitter, H (see Fig. 19, lower diagram), and the batter current transmitted through this acts on one coil of a transformer, P, the other coil, S, of which is in series with a magnetic telephone receiver. The diaphragm of the receiver is connected to the inclined plate A, on which the liquid jet falls. The other plate B is connected to the earth, whilst the plate A is connected to the transmitting antenna.

This arrangement is particularly effective, because we call into play by the voice the power of the local battery to move the plate A.

Wherever the microphone may be inserted, whether in the antenna circuit or in a shunt, or in an inductively coupled circuit, it has to carry a current of some magnitude, and the difficulties introduced by the heating of the variable resistance,

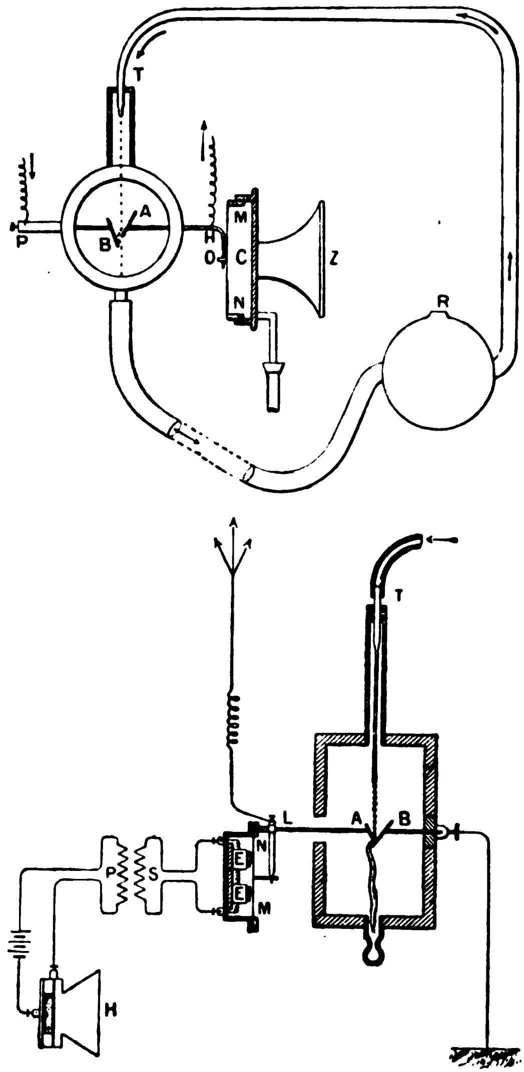

FIG. 19. – Arrangements of Liquid Microphone of Professor J. Vanni.

whether carbon or liquid, make it essential to keep the current which is varied as small as possible.

The difficulties of operating several microphone resistance changers in parallel successfully are considerable. Though an arrangement of microphones in parallel, as is shown in Fig. 8, may work just well enough for experimental tests in skilled hands, it is not satisfactory in ordinary practical use. One microphone

will fall to a lower resistance than the others, and will then take most of the current and begin to heat. It has, therefore, become evident that to conduct practical radiotelephony it is a necessary condition of success to use a single ordinary carbon or liquid microphone traversed by not more than, say, half an ampere, and to cause the changes in this small current created by the speaking voice acting on the diaphragm to modulate some other and much larger current. In other words, we require a telephonic relay or repeater. This has been quite recently achieved by the use of the thermionic amplifier, consisting of an incandescent filament in a vacuous bulb also containing two cold metal electrodes, one a grid or network placed in front of a plate or second member (see Chap. VI. Fig. 45). The thermionic current or stream of electrons or negative ions proceeding from the incandescent filament to the plate may be varied or changed in a very considerable degree by small variations in the potential of the grid or net placed in front of the plate.

By employing a number of thermionic amplifiers with their main or plate circuits in parallel it is possible by an ordinary single microphone coupled through an induction coil with their grids to modulate a current of sufficient strength to actuate some mechanism capable of varying in a similar manner the amplitude of the oscillations in the antenna. By a large group of such thermionic repeaters it is possible to modulate very considerable currents. Thus a microphone may act through an induction coil on the intermediate grid of the first amplifier. The thermionic current of this instrument may be made to vary the grid potentials of a number of other similar amplifiers placed with their main thermionic currents in parallel with each other.

By means of such a series or cascade arrangement of thermionic amplifiers, each stage consisting of an increasing number of such instruments worked in parallel, we can make a large percentage variation in some direct or continuous current which may be the magnetizing current of a high frequency alternator.

In a suitably designed high frequency alternator, the necessary exciting current can be reduced to a few amperes, and by thermionic variation of this current the voltage of the alternator can be varied, and therefore the radiating antenna current. We can use a single microphone in series with a chain of amplifiers, the last of which supplies the D.C. current required for excitation. For long-distance radiotelephony a possible method of producing the speech-modulations of the antenna current would seem to be to act in the above manner on the exciting current of the high frequency alternator supplying the energy.

Also in those cases in which frequency is raised by Joly transformers the microphone, in conjunction with an amplifier, may be used to vary the continuous current which passes through the third winding on the transformers, and this in turn varies the amplitude of the double frequency secondary oscillation.

We can then say that as regards the microphone, whilst the most obvious mode of using it is to insert either a carbon or a liquid microphone in the circuit of the antenna, and so vary the antenna resistance, this method is not very promising as a practical solution of the problem owing to the difficulties of obtaining a microphone which will carry continuously a sufficiently large current.

The present practice and the probable ultimate solution of the problem will involve only the use of a single ordinary carbon resistance microphone as used in ordinary wire telephony, but the small variations in current or potential which can be created by speech made to the diaphragm will be multiplied up by the use of thermionic valve amplifiers, and corresponding variations created in a magnified current which will be employed either in excitation of the H.F. alternator, or of the frequency changing transformers.

The control of the antenna oscillations can also be effected as follows: Suppose that a double anode thermionic valve (see Fig. 45, Chap. VI.) has its plate anode connected to some point on the antenna the potential variations of which are fairly large. Then when the filament is incandescent there will be a leak of positive electricity from the antenna. This can be prevented by imparting to the intermediate or grid anode a certain negative charge by a battery. If, then, a variation of this negative potential is created, either an increase or diminution,

there will be a corresponding variation in the amplitude of the antenna oscillations.

This can be done by connecting between the grid and the negative terminal of the filament the secondary circuit of an induction coil, in the primary circuit of which there is a local battery and microphone. The necessary negative potential can be applied to the grid by a battery of small cells inserted in the secondary circuit of the induction coil. In this manner Mr. Irving Langmuir has been able by the use of a single such thermionic valve to control the delivery of about 2 kw. of energy to an antenna.

4. Receiving Arrangements in Radiotelephony.—Assuming that a transmitting station is sending out undamped waves which are being moulded into speech form by means of a microphone controlled as already described, these waves may be absorbed by a properly syntonized receiving antenna tuned to the wave-length employed, and by suitable arrangements can be made to affect a telephone so as to translate back the oscillations of constant frequency but variable amplitude induced in the receiving antenna into articulate sounds. For this purpose it is necessary to employ in the receiving circuit an oscillation detector which is quantitative, that is, not merely affected by oscillations, but affected to some extent proportionately to their amplitude. Thus, for example, a coherer or oscillation detector of the imperfect-contact type will be of no use, because it is

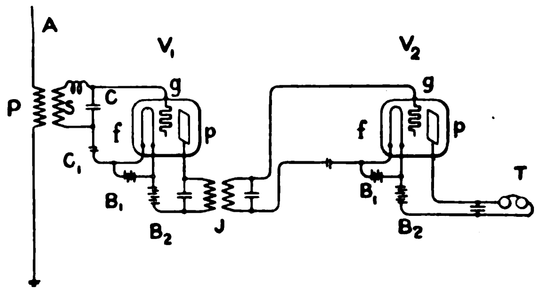

FIG. 20.—A Pair of Coupled Thermionic Amplifiers used in series as a Radiotelephonic Receiver.

only affected by a certain alternating voltage, and is at once affected almost to the full degree when that voltage reaches a certain limit. Several forms of oscillation detector, already described in Chap. VI., are very suitable for radiotelephonic reception, such as Fessenden's electrolytic detector, the author's glow-lamp or ionized gas or thermionic detectors, and the crystal detectors of Dunwoody and Pierce. Thus, for instance, if a receiving circuit is constructed by inductively coupling the receiving antenna to another oscillation circuit comprising a condenser and inductance properly syntonized to the antenna circuit, and if an electrolytic detector in series with a telephone and a shunted local cell is put across the terminals of the condenser, then oscillations passing through the electrolytic detector will not only alter its apparent electric resistance, but alter it in some sense proportionately to their intensity, and hence if undamped waves are falling upon the antenna of constant wave-length and varying amplitude, a corresponding variation in the apparent resistance of the electrolytic detector will take place, and therefore a corresponding variation in the currents through the telephone. The telephone diaphragm, therefore, emits a sound which corresponds with the fluctuation in the amplitude of the incident waves, and it therefore reproduces a speech made against the diaphragm of the transmitting microphone.

Since the energy absorbed by the receiving antenna is very small, at most a small fraction of a microwatt, it is evident that any receiving arrangement suitable for radiotelephony must act on the "trigger" principle, that is the energy absorbed from the wave is merely employed to release energy from a local battery, which last actuates the receiving telephone. Therefore any such receiver to be effective

must be of the nature of a relay or amplifier. At present the most effective means is the use of two or more double anode thermionic amplifiers in cascade, the arrangement being as shown in Fig. 20. In this diagram A is the aerial which is coupled through an oscillation transformer P, S with the usual oscillation circuit comprising the condenser C and coil S. V_1 and V_2 are two double anode thermionic valves, the filament in each being rendered incandescent by its own battery, B_1. The anode plates p are joined with the filament through a circuit which contains a local boosting battery B_2 so that a thermionic current tends to flow in each bulb from the filament to the plate.

In the first valve the grid g becomes negatively charged by the rectifying action, and any variation of the potential of g is reproduced on an increased scale in variation of the potential of the anode plate p. This last circuit is coupled inductively through the transformer J with the grid circuit of the second valve, and in the plate circuit of this last is placed a telephone, T. Hence any variation in the amplitude of the antenna oscillations is reproduced on a magnified scale in the current flowing through the telephone. In this manner a great amplification is secured of the slightest variation in the amplitude of the antenna oscillations, and speech made to the transmitter microphone is reproduced with great accuracy by the receiving telephone in the thermionic circuit of the second amplifier.

By a chain of such amplifiers it is possible to hear as speech modulations of antenna current which would not produce any audible sounds with a simple rectifying crystal contact in series with a telephone if applied as a shunt to the condenser terminals in the closed receiving circuit. For a fuller explanation of the actions taking place in such a thermionic amplifier the reader is referred to Chap. VI. § 15. We have in the thermionic amplifier probably the most sensitive of radiotelephonic receivers.

5. Achievements of Radiotelephony.—In this combined radiotelephonic transmitter and receiver we have then a wonderful transformation of energy. The variations of air pressure made by the speaking voice against the diaphragm of the transmitter microphone produce corresponding variations in its resistance. This, again, varies in the same manner the intensity of the electric oscillations in the oscillation circuit connected with the arc or alternator, and also the oscillations in the antenna. Electromagnetic waves are then emitted, the amplitude of which is changing in the same manner. A small fraction of the energy of these waves is captured by the receiving antenna, and oscillations set up, therefore, in the receiving condenser circuit, the amplitude of which varies in the same manner as that of the incident waves, and these acting on the particular detector coupled to the telephone reproduce movements in the receiving telephone diaphragm which imitate those made by the diaphragm of the transmitting microphone. Although this operation is complicated, nevertheless it has been so far perfected that articulate speech has been transmitted over three or four thousand miles or more by these methods.

The picture in Fig. 21 shows the complete arrangements for employing the Poulsen arc as a radiotelephonic transmitter, and the diagram in Fig. 22 shows the receiving arrangements made as described. By these methods Poulsen succeeded in transmitting articulate speech from Berlin to Copenhagen, a distance of 290 miles, and also from Lyngby to Esbjerg, a distance of 170 miles.

Fessenden states he has conducted radiotelephonic communication between Brant Rock and New York, a distance of 200 miles, using 1 kw. steam turbine driven alternator, giving alternating currents of a frequency from eighty to a hundred thousand at 150 volts, and a disc armature of a resistance of 6 ohms, and a field exciting current of 5 amperes. Using a transmitting antenna 200 feet high at New York, and the Atlantic Tower, 400 feet high, at Brant Rock, an expenditure of 200 watts in the antenna was required to cover 200 miles. He also made successful demonstrations in 1906 between Brant Rock and Plymouth, Mass., a distance of 11 miles, in which speed was transmitted satisfactorily to telephone experts who were present. For a detailed account of Fessenden's work in Radiotelegraphy up to the beginning of 1907, the reader may be referred to a series of articles by him in the *Electrical Review* for February 15, 22, March 1, 15, 1907.

In 1908, similar experiments were made by Professor Majorana in Italy, between

Monte Mario and Porto Danzig, a distance of 60 kilometres. In France, Lieutenants Colin and Jeance, and Chief Engineer Mercier, achieved the feat of transmitting speech radio-telephonically from the Eiffel Tower in Paris to Dieppe, and musical sounds from Paris to the coast of Finisterre, a distance of 310 miles.

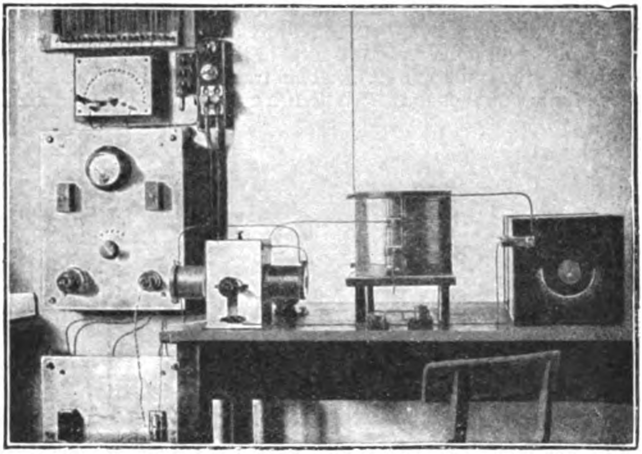

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FIG. 21.—General View of the Poulsen Transmitting Apparatus for Radiotelephony.

The apparatus used by Colin and Jeance was as follows [2]:—

The continuous waves used were produced by three electric arcs in series A (see Fig. 23), each having a carbon and copper electrode. The negative carbon

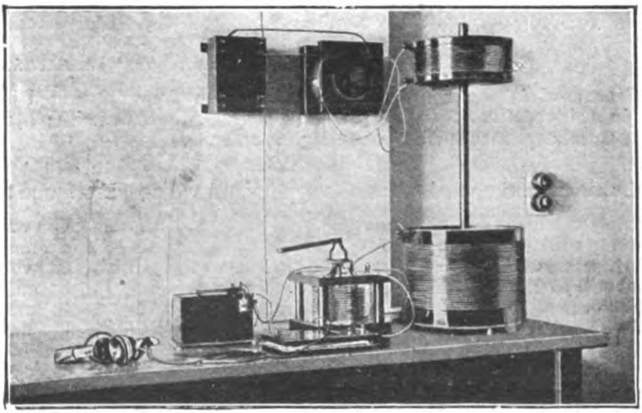

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FIG. 22.—Poulsen Receiving Apparatus for Radiotelephony.

electrode is a rod 1·5 mm. in diameter. The copper or positive electrode is in the form of a disc which forms the base of a cylinder cooled with water. The arcs are struck in an atmosphere of hydrocarbon produced by mixing acetylene and hydrogen in certain proportions. The carbons, therefore, do not wear away.

[2] See *The Electrician*, vol. 73, p. 655, July 24, 1914.

The supply voltage was from 500 to 750 volts, and the current used from 3·5 to 4·5 amperes. Choking coils were inserted and also a ballast resistance in the supply circuit. (See Fig. 23.) The arcs are shunted by a condenser in series with an inductance, and an intermediate circuit B was used between the arc shunt inductance and the antenna to purify the wave form. The microphone used consisted of nine carbon microphones in series, operated simultaneously by a megaphone, and was connected between the earth and the antenna end of the inductance coil in the antenna as shown at C in the diagram. (See Fig. 23.) This avoids sparking at the microphones. The transmitter is provided with a pair of such series of microphones for alternate use as they become heated.

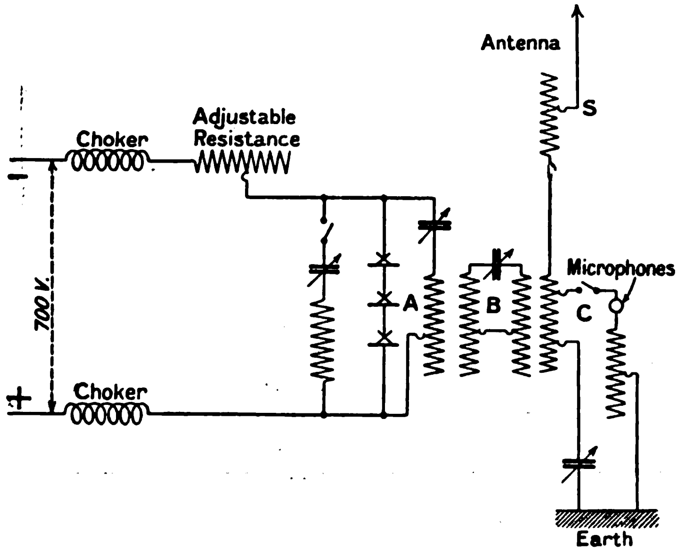

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FIG. 23.—Scheme of Arrangements of the Circuits of the Transmitter used by Colin and Jeance in their experiments on Wireless Telephony. A, arcs in series; B, intermediate circuit; C, microphones in circuit shunting the antenna.

The following are the particulars of currents and voltages from one test :—

Supply voltage of D.C. dynamo	650 volts.
Supply current of dynamo	4·2 amps.
P.D. across arcs in series	350 volts.
Antenna current with microphones in circuit	3·2 amps.
Current in microphones	0·5 amp.
Wave-length	985 metres.

With these data the French Naval authorities carried out successful speech tests between Paris and Mettraz, a distance of 200 km. or 125 miles.

Using his liquid microphone as above described and a Fleming glow-lamp oscillation valve as a receiver, Prof. Vanni achieved the feat of speaking clearly by radiotelephony from Rome to the island of Ponza (120 km.), to Maddelena (260 km.), to Palermo (420 km.), to Vittoria (600 km.), and finally to Tripoli (1000 km.). The timbre or quality of the voice is said to be reproduced with great accuracy.

J. B. Marzi and Son, using a series of four Moretti arcs in series as generator and their own microphone, viz. the Marzi carbon powder microphone above described, have radiotelephoned 520 miles in Italy from Spezia to Messina. Mr,

Marconi, using not very dissimilar methods, has also achieved considerable success in the same field between ships of the Italian Navy.

Mr. W. T. Ditcham has conducted experiments on wireless telephony using a form of quenched spark transmitter producing a series of nearly sequent feebly damped trains of oscillations in the antenna. These experiments were conducted between Letchworth and Northampton, and speech was received up to a distance of 155 km. or about 100 miles.

The spark discharger he used appears to have been not unlike the Chaffee generator described in Chap. I., in which the quenched spark is taken between copper and aluminium surfaces in an atmosphere of hydrogen across a gap between parallel plates a fraction of a millimetre in width, several such gaps being used in series.

The microphone control of the oscillations was by a pair of carbon Berliner pattern of microphones arranged in series. Several such pairs were arranged on a turn-table so as to be used for two minutes successively. The receiver consisted of a Pickard silicon-arsenic rectifier in series with a magneto telephone.

Using four of the above spark dischargers in series on 1000 volt D.C. circuit with a supply current of 1·5 amperes it was possible to obtain in the transmitting antenna a current of 8 amperes and a wave-length of 550 metres.

The aerial at each station was in the form of an inverted L and consisted of four wires suspended between masts with a vertical part 20 metres high and a nearly horizontal part 60 metres long.

With this arrangement good telephonic speech was obtained between Letchworth and Northampton, a distance of 35 miles. The total power used seems to have been about 2 kw.

All who have experimented with wireless telephony are agreed that the voice sounds are transmitted with remarkably good quality. This is evidence that there is very little distortion in the wave form. In telephony with wires the wire or cable exercises a great influence upon the wave form. The complicated and highly irregular wave form of the current impressed on the cable at the sending end may be resolved in virtue of Fourier's theorem into the sum of a number of simple harmonic oscillations differing in phase and amplitude. The higher harmonics travel faster but degrade and attenuate more rapidly than the fundamental tone and lower harmonics. Hence as the wave form travels along the cable its irregularities, due to the higher harmonics, are smoothed out and the sound quality changed. Hence it becomes less recognizable by the ear. This distortion can be to some extent removed by inserting inductance coils in the cable at intervals small compared with the wave-length. These are called *loading coils.*

In the case of waves travelling through the æther waves of all wave-length travel at the same speed. Hence there is no tendency for the wave form to become distorted with increase of distance, and the quality of the sound is unaltered although it is enfeebled.

It is therefore merely a question of strength of antenna current and sensitiveness in the detecting arrangement.

In the month of September 1915, important experiments on radiotelephony were made at Arlington (Virginia, U.S.A.), where there is a large radio station, owned by the United States Navy Department. The experiments were conducted by the American Telephone and Telegraph Company and Western Electric Company. In these experiments articulate speech was transmitted from Arlington to Mare Island in San Francisco Bay, a distance of 2500 miles, and on September 30, 1915, it was reported that radio speech had been received at night at Pearl Harbour, Honolulu, a distance of 4000 miles, and also received across the Atlantic at the Eiffel Tower Station in Paris.

The wave-length of the waves sent out from Arlington was 6000 metres, and the antenna current from 50 to 100 amperes.

Great secrecy was at the time observed about the actual methods employed for modulating the continuous oscillations and receiving the feeble speech waves, though it was surmised that it had been accomplished by a modification of the author's oscillation valve. Subsequently it was claimed to have been done by

generating the oscillations in the antenna by a 4-step chain of thermionic repeaters acting as a generator, and consisting of a series of one such valve, the grid circuit of which contained a speaking microphone, and the wing circuit influencing the grid circuit of a larger valve and this again of twelve such valves in parallel, and these acting on 300 to 550 similar thermionic valves in parallel, the main current of which was supplied by a 600-volt continuous current dynamo. This system was set in oscillation, and the oscillating energy transferred to the antenna and modulated into speech form by the single microphone transmitter.[3]

The reception was conducted by the thermionic amplifiers in series as already described (see Fig. 45, Chap. VI.). The result, though extremely interesting, can only be described as a gigantic experiment in radiotelephony. The successful commercial operation of wireless telephony for distances up to 200 or 300 miles or so, and on board ship will depend upon the production of some tolerably simple form of transmitter comprising a wave generator of undamped waves and a wave modulator operated by a single carbon microphone. This apparatus must be capable of being worked as easily as the present forms of wireless transmitter. Then for longer distances and for high power stations, some form of alternator with frequency raised by internal reactions or else externally by static transformers will probably hold the field.

The expense of working such an apparatus would most probably be prohibitive of great public use. It must be remembered that in normal times by far the greater part of long distance telegraphy is conducted in code, in which a great deal of information can be put into a single word.

The great field for radiotelephony conducted over a distance of a few hundred miles, if the apparatus can be simplified and rendered certain in action, would be for intercommunication between ships at sea. For perfected transoceanic and long distance working we shall have to wait for some simple and easily adjusted method of controlling the power output of large high frequency alternators by means of some current amplifier which in turn is actuated by a single carbon transmitter.

Reference has been made above to so-called *magnetic amplifiers* which work on a different principle to the thermionic amplifiers already explained. These are appliances by which the variation of a small direct current can be made to vary in a greater ratio the amplitude of the current from a high frequency alternator. One such device due to Messrs. Alexanderson and Nixdorff is as follows. A laminated iron core like a transformer core has on its circuits two coils. Through one of these the high frequency current from the alternator passes. This coil, therefore, acts as a choking coil and throttles the alternating current. If we pass a direct current through the other winding and arrange the ampere turns so as to saturate the core this will reduce the permeability and therefore the impedance of the alternating current winding. If, then, this direct current is varied in strength by a carbon microphone, corresponding variations will be created in the alternating current, and hence by proper proportioning of the circuits we can make the small variations of the direct current create similar variations in the alternating current.

If such a microphone-controlled choker is placed as a shunt across the terminals of a high frequency alternator, and if one of these terminals is connected to earth and the other to an antenna tuned to the alternator, then variations in the impedance of the choker produced by speaking to the microphone will vary proportionately the antenna current and enable radiotelephony to be conducted.

For details of the appliance the reader is referred to *The Proceedings of the Institute of Radio-Engineers*, New York, U.S.A., April 1916, vol. 4, p. 101 ; or to *The Electrician* for June 9th, 1916, vol. 77, p. 312. By such means large powers may be controlled by a single ordinary carbon microphone.

[3] See *The Electrician*, vol. 76, p. 466, December 31, 1915, letter by L. de Forest; also *Electrical World*, New York, vol. 66, p. 788, October 9, 1915.

INDEX

Cmjance 226

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